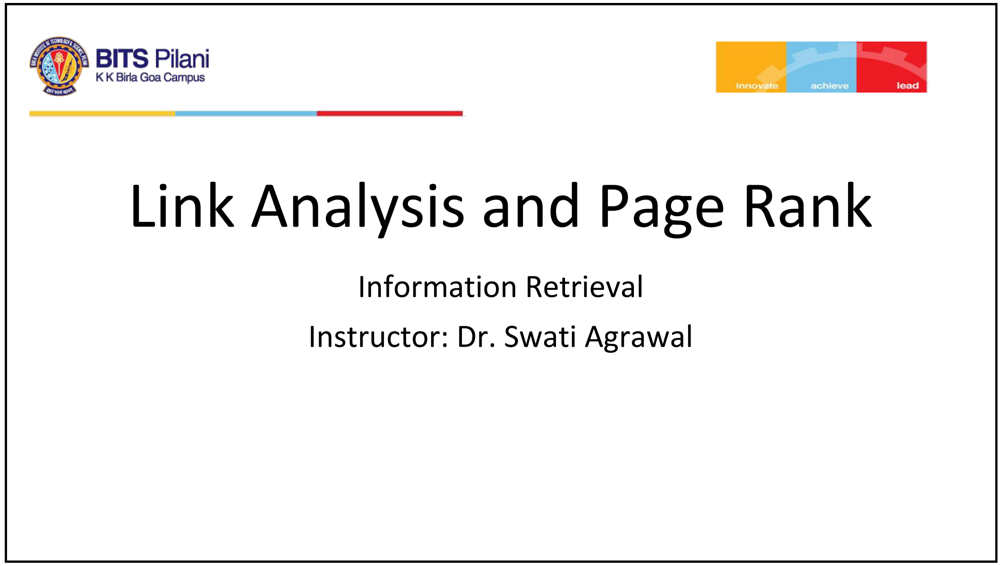

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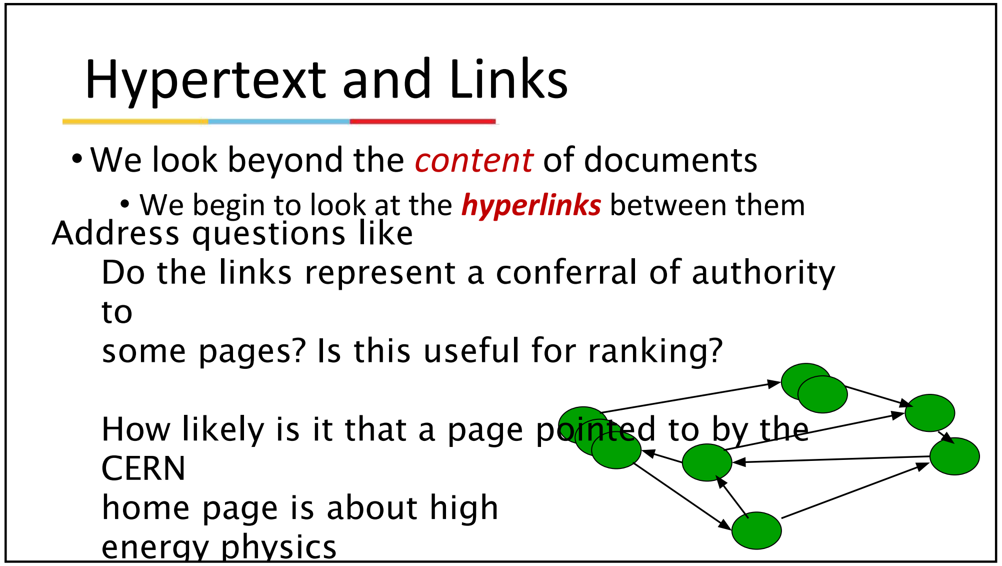
Link Analysis and Page Rank

Information Retrieval

Instructor: Dr. Swati Agrawal



Hypertext and Links

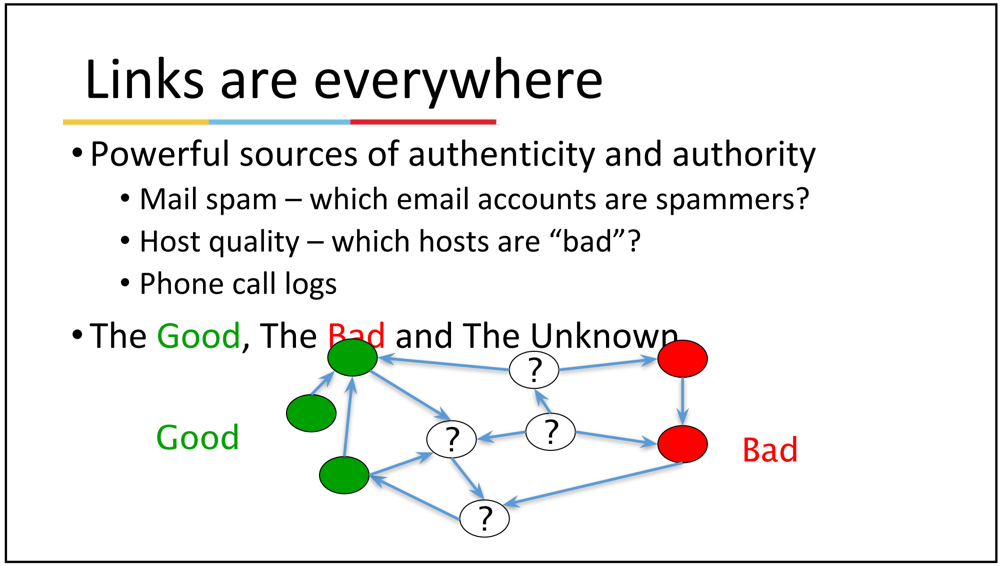
- We look beyond the *content* of documents
  - We begin to look at the ***hyperlinks*** between them

Address questions like

Do the links represent a conferral of authority to
some pages? Is this useful for ranking?

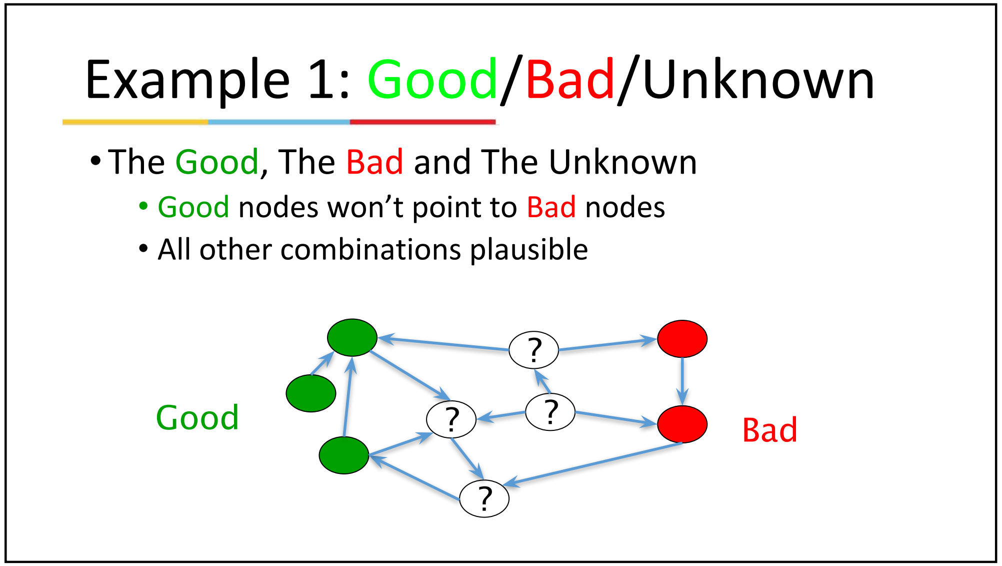
How likely is it that a page pointed to by the CERN
home page is about high
energy physics

A good link will point to a good link always.
Bad links can point to both.



Links are everywhere

- Powerful sources of authenticity and authority
  - Mail spam – which email accounts are spammers?
  - Host quality – which hosts are "bad"?
  - Phone call logs
- The Good, The Bad and The Unknown

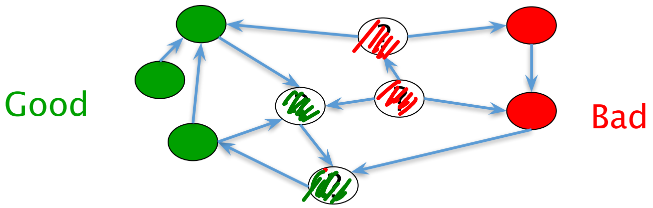


Example 1: Good/Bad/Unknown

- The Good, The Bad and The Unknown
  - Good nodes won't point to Bad nodes
  - All other combinations plausible

Simple iterative logic

- Good nodes won't point to Bad nodes
  - If you point to a Bad node, you're Bad
  - If a Good node points to you, you're Good



Simple iterative logic

- Good nodes won't point to Bad nodes
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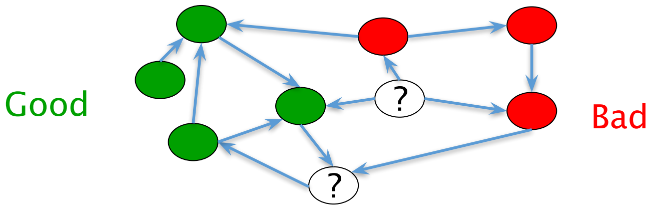
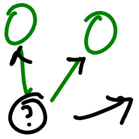


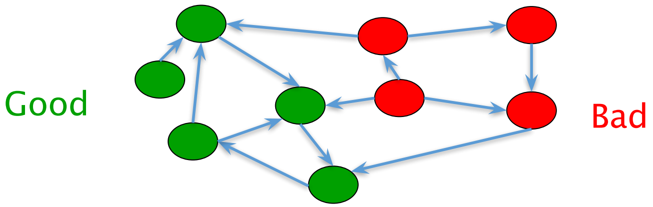
Unknowns are possible:



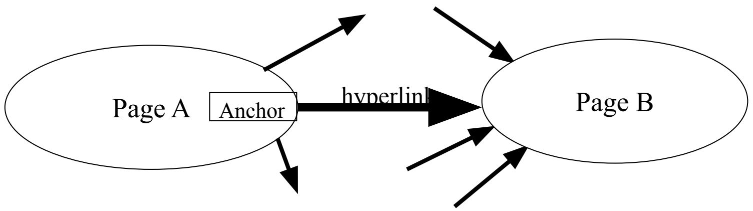
→ can be good or bad

Simple iterative logic

- Good nodes won't point to Bad nodes
  - If you point to a Bad node, you're Bad
  - If a Good node points to you, you're Good



The Web as a Directed Graph

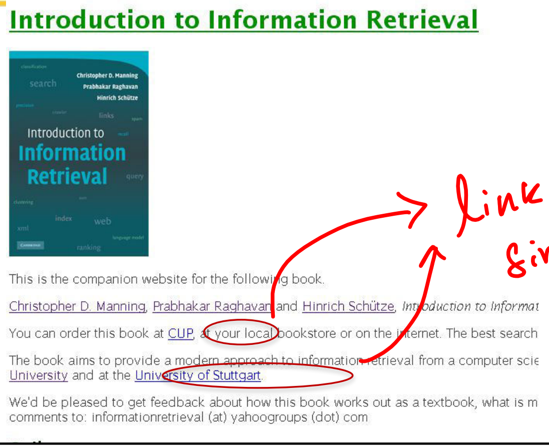


✓ **Hypothesis 1:** A hyperlink between pages denotes a conferral of authority (quality signal)

✓ **Hypothesis 2:** The text in the anchor of the hyperlink on page A describes the target page B

↳ may or may not be true .

Assumption 1: reputed sites



Introduction to Information Retrieval

This is the companion website for the following book.

Christopher D. Manning, Prabhakar Raghavan and Hinrich Schütze, *Introduction to Informat*

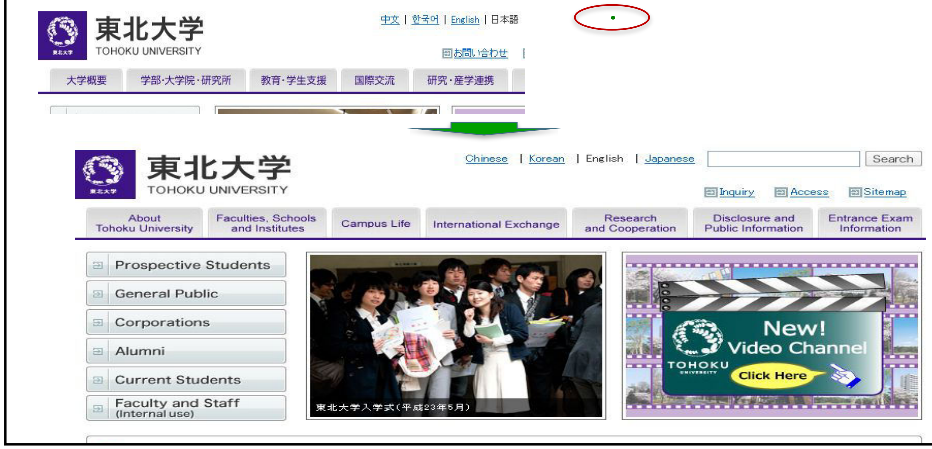
You can order this book at CUP, at your local bookstore or on the internet. The best search

The book aims to provide a modern approach to information retrieval from a computer scie
University and at the University of Stuttgart.

We'd be pleased to get feedback about how this book works out as a textbook, what is m
comments to: informationretrieval (at) yahoogroups (dot) com

links are authentic since web page is authentic

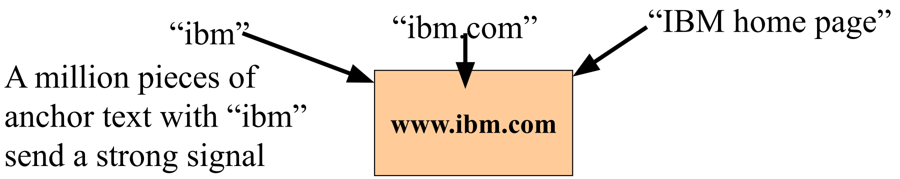
Assumption 2: annotation of target



Anchor texts directly link to pages. So if document frequency for such anchor texts pointing to same webpage is high, then assign high weights to that word for that document. Term Freq doesn't matter much as we are considering the document that is being pointed and not the curr page.

Anchor Text *WWW Worm* - McBryan [Mcbr94]

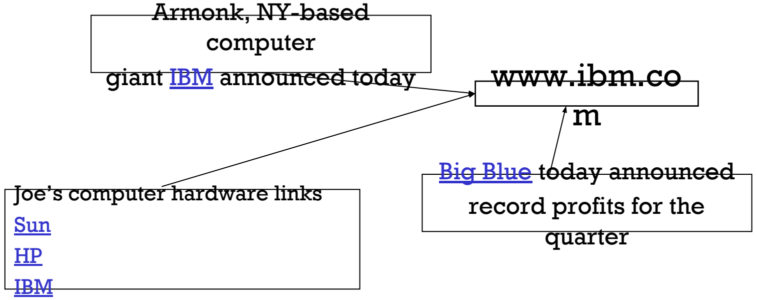
- For ***ibm*** how to distinguish between:
  - IBM's home page (mostly graphical)
  - IBM's copyright page (high term freq. for 'ibm')
  - Rival's spam page (arbitrarily high term freq.)



A million pieces of anchor text with "ibm" send a strong signal

Indexing anchor text

- When indexing a document *D*, include (with some weight) anchor text from links pointing to *D*.



Armonk, NY-based computer giant IBM announced today

www.ibm.com

Joe's computer hardware links
Sun
HP
IBM

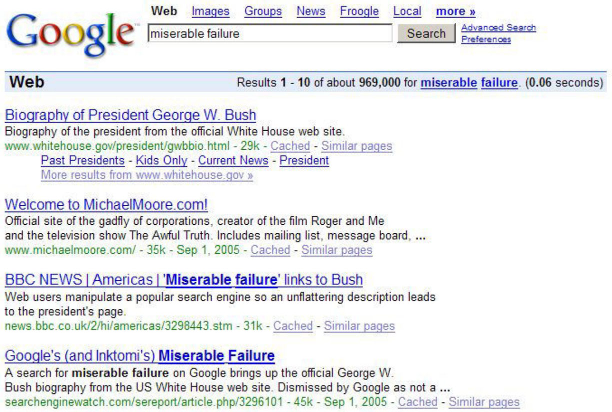
Big Blue today announced record profits for the quarter

Another thing that matters would be the reputation of the page that has the link embedded. If that page has good rank, then the pointed page would also have a good rank "idiot": Donald Trump

Google bombs

- Indexing anchor text can have unexpected side effects: Google bombs.
  - whatelse does not have side effects?
- A Google bomb is a search with “bad” results due to maliciously manipulated anchor text
  - Miserable failure-> George Bush Wikipedia page
  - Bad writers, Dumb and Dumber-> Game of Thrones Season 08
- Google introduced a new weighting function in January 2007 that fixed many Google bombs

Google bomb example



Web Results **1** - **10** of about **969,000** for **miserable failure**. (**0.06** seconds)

Biography of President George W. Bush
Biography of the president from the official White House web site.
www.whitehouse.gov/president/gwbbio.html - 29k - Cached - Similar pages
Past Presidents - Kids Only - Current News - President
More results from www.whitehouse.gov »

Welcome to MichaelMoore.com!
Official site of the gadfly of corporations, creator of the film Roger and Me and the television show The Awful Truth. Includes mailing list, message board, ...
www.michaelmoore.com/ - 35k - Sep 1, 2005 - Cached - Similar pages

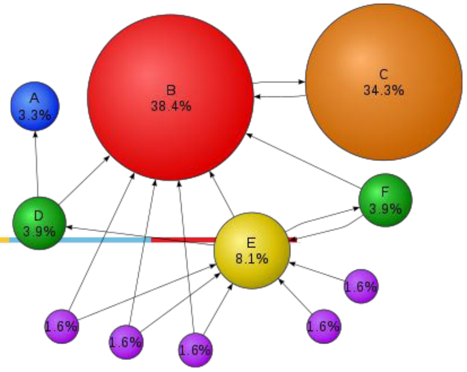
BBC NEWS | Americas | '**Miserable failure**' links to Bush
Web users manipulate a popular search engine so an unflattering description leads to the president's page.
news.bbc.co.uk/2/hi/americas/3298443.stm - 31k - Cached - Similar pages

Google's (and Inktomi's) **Miserable Failure**
A search for **miserable failure** on Google brings up the official George W. Bush biography from the US White House web site. Dismissed by Google as not a ...
searchenginewatch.com/sereport/article.php/3296101 - 45k - Sep 1, 2005 - Cached - Similar pages

Indexing anchor text

- Can score anchor text with weight depending on the authority of the anchor page’s website
  - E.g., if we were to assume that content from cnn.com or yahoo.com is authoritative, then trust (more) the anchor text from them
  - Increase the weight of off-site anchors (non-nepotistic scoring)

Link Analysis: PageRank



Google Pagerank System

Google was developed by Sergey Brin and Larry Page

This is the method that Larry Page developed to rank and order the pages.

Hence, the Pagerank.

PageRank is a trademark of Google. The PageRank process has been patented.

Citation Analysis

- Citation frequency
- Bibliographic coupling frequency
  - Articles that co-cite the same articles are related
- Citation indexing
  - Who is this author cited by? (Garfield 1972)
- Pagerank preview: Pinsker and Narin ’60s
  - Asked: which journals are authoritative?

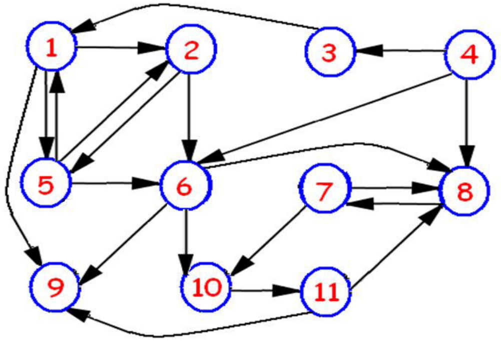
The web isn’t scholarly citation

- Millions of participants, each with self interests
- Spamming is widespread
- Once search engines began to use links for ranking (roughly 1998), link spam grew
  - You can join a *link farm* – a group of websites that heavily link to one another

Multiple spammers join their networks and point to each others' pages to increase probability.

How would you order these site?

- Suppose each of the nodes at right have the links shown in the directed graph. Which node is most important and should appear first?



If a high reputed page points to many other pages, the reputation passed on is divided among all.

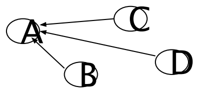
The Basic Idea

1. How important a page is on the web
2. The more incoming edges to a page, the more important the page must be
3. How important the connecting page is

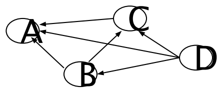
source: http://www.webworkshop.net/pagerank.html

Simplified PageRank algorithm

- Assume four web pages: **A**, **B**,**C** and **D**. Let each page would begin with an estimated PageRank of 0.25.



$$PR(A) = PR(B) + PR(C) + PR(D).$$



$$PR(A) = \frac{PR(B)}{2} + \frac{PR(C)}{1} + \frac{PR(D)}{3}.$$

- L(A) is defined as the number of links going out of page A. The PageRank of a page A is given as follows:

$$PR(A) = \frac{PR(B)}{L(B)} + \frac{PR(C)}{L(C)} + \frac{PR(D)}{L(D)}.$$

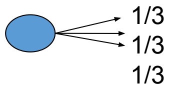
PageRank algorithm including damping factor

Assume page A has pages B, C, D ..., which point to it. The parameter d is a damping factor which can be set between 0 and 1. Usually set d to 0.85. The PageRank of a page A is given as follows:

$$PR(A) = 1 - d + d\left(\frac{PR(B)}{L(B)} + \frac{PR(C)}{L(C)} + \frac{PR(D)}{L(D)} + \cdots\right)$$

Pagerank scoring

- Imagine a user doing a random walk on web pages:
  - Start at a random page
  - At each step, go out of the current page along one of the links on that page, equiprobably



- “In the long run” each page has a long-term visit rate - use this as the page’s score.

Page Rank Algo:

1. Initialize the score of each node in the network = 1
2. Now, simultaneously update each of the nodes' value using the page rank equation
3. Keep iterating 2 until an equilibrium value is reached

$$PR(A) = 1 - d + d\left(\frac{PR(B)}{L(B)} + \frac{PR(C)}{L(C)} + \frac{PR(D)}{L(D)} + \cdots\right)$$

↳ The PR(A) depends on the incoming edges from the other pages.

Note: The value 'd' is the damping factor. It represents the probability of walking randomly to another hyperlink in the current page.
1-d represents the probability of teleporting. i.e. the user changes the url on his own and doesn't traverse using a link of the page itself.

Let the page rank vector be denoted by $\vec{F}$

$$\vec{F}_{i+1} = (1-d).\overset{\begin{bmatrix}1\\1\\\vdots\end{bmatrix}}{\vec{1}} + d.X_{n\times n}\vec{F}_i$$

where X is :

$$\begin{array}{c|cccc}
\text{To}\backslash\text{From} & A & B & C & D\cdots \\
\hline
A & P_{AA} & P_{BA} & P_{CA} & P_{DA}\cdots \\
B & & & & \\
C & & & & \\
D & & & & \\
\vdots & & & &
\end{array}$$

where P_{MN} is probability of reaching N from page M.

$$P_{MN} = \frac{1}{\text{outdeg}(M)} \quad \text{if } M \to N \text{ exists}$$

else $\quad 0$.

Note that each column will have eigenvalue = 1

Simplifying:

$$\vec{F}_{i+1} = \underbrace{\left[\frac{(1-d)}{n} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots \\ 1 & \ddots & & & \\ 1 & & \ddots & & \\ \vdots & & & & \ddots \end{bmatrix}_{n\times n} + d\, X_{n\times n} \right]}_{A} \cdot \vec{F}_i$$

$$\Rightarrow \quad \vec{F}_{i+1} = A\vec{F}_i$$

$$\boxed{\vec{F}_{i+1} = A\vec{F}_i} \longrightarrow \text{Page Rank matrix}$$

Our final aim is to find $\vec{F}_n$ where $\vec{F}_n = \vec{F}_{n+1}$

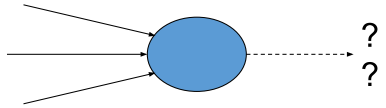
So, $A\vec{F}_i = \vec{F}_i$ has to be solved.

Two methods

- Since A is a stochastic matrix, i.e. an eigen value = 1. Use this to get eigenvector for A with $\lambda = 1$. (costly for large data)
- Just iterate and try to get close. ✓

Not quite enough

- The web is full of dead-ends.
  - Random walk can get stuck in dead-ends.
  - Makes no sense to talk about long-term visit rates.



Teleporting

- At a dead end, jump to a random web page.
- Probability to jump = 1/N
- Can jump to current position as well
- Two ways to teleport
  - When at a node with no out-links, surfer invokes the teleport operation
  - When at a node with out-links, teleport operation with probability 0<α<1
- At any non-dead end, with probability 10%, jump to a random web page.
  - With remaining probability (90%), go out on a random link.
  - 10% - α parameter.

Result of teleporting

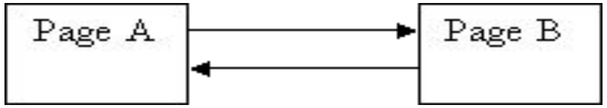
- Now cannot get stuck locally.
- There is a long-term rate at which any page is visited
- How do we compute this visit rate?

Intuitive Justification

- A "random surfer" who is given a web page at random and keeps clicking on links, never hitting "back“, but eventually gets bored and starts on another random page.
  - The probability that the random surfer visits a page is its PageRank.
  - The d damping factor is the probability at each page the "random surfer" will get bored and request another random page.
- A page can have a high PageRank
  - If there are many pages that point to it
  - Or if there are some pages that point to it, and have a high PageRank.

Compute the Page Rank of A and B

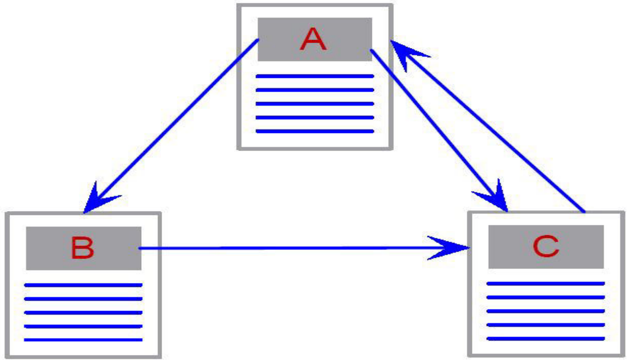
d=0.
5



- Each page has one outgoing link. So that means L(A) = 1 and L(B) = 1.
- PR(A) = PR(B) =1

What is the PageRank of A, B, C

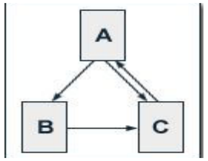
d=0.
5



The Characteristics of PageRank™

D=0.5

- PR(A) = 0.5 + 0.5 PR(C)
- PR(B) = 0.5 + 0.5 (PR(A) / 2)
- PR(C) = 0.5 + 0.5 (PR(A) / 2 + PR(B))



We get the following PageRank™ values for the single pages:

- PR(A) = 14/13 = 1.07692308
- PR(B) = 10/13 = 0.76923077
- PR(C) = 15/13 = 1.15384615

Topic Specific PageRank

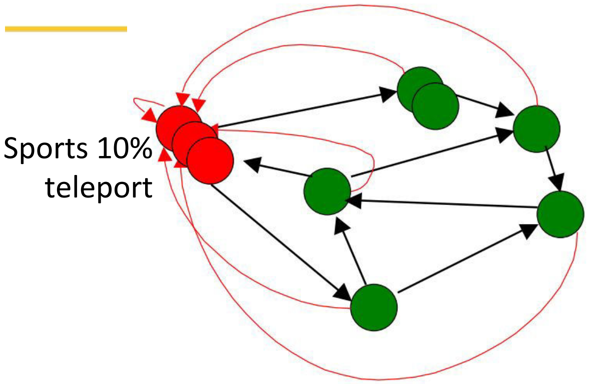
- aka personalized pagerank
- Previous pagerank computation: teleporting to a random web page chosen uniformly at random
- Now, teleporting to a web page chosen non-uniformly
- Computing the pagerank values tailored to particular interests

Example

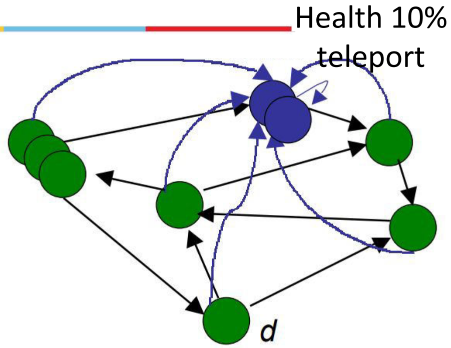
- A person having interest in sports might wish that pages on sports be ranked higher than non-sports page
- For a random surfer, in case of teleport operation as before, teleports to *a random web page on sports topic* instead of teleporting to a uniformly chosen random web page
- Requirement: a non-zero subset *s* of pages on sports topic to make the teleport operation feasible
- Topic specific pagerank

Topic Specific PageRank

- The distribution over teleporting s need not to be uniformly chosen but could be arbitrary
- Similar topic specific pagerank distribution can be done for other domains as well
- Advantage: gives the potential of considering settings in which the search engine knows what topic a user is interested in
- Challenge: how to know users’ interests
  - users explicitly register their interests
  - the system learns by observing each user's behavior over time



Non- dead nodes:
10% teleport, 90% random walk



Non- dead nodes:
10% teleport, 90% random walk

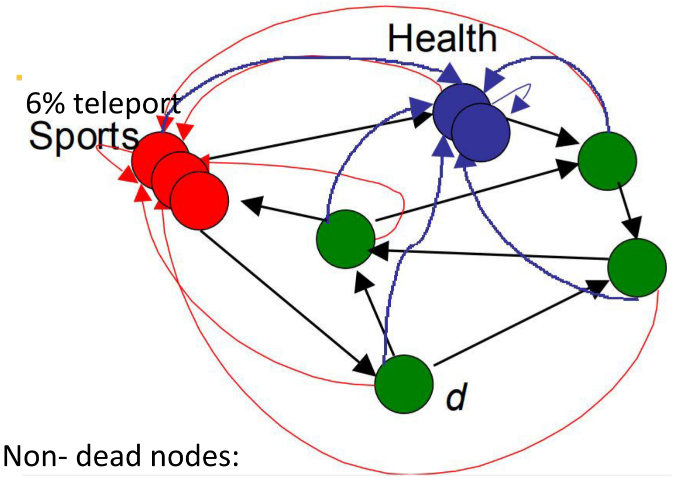
Multiple Topics

- what if a user is known to have a mixture of interests from multiple topics?
- 60% sports and 40% health
- can we compute a *personalized PageRank* for this user and How?
- Assumption:
  - an individual's interests can be well-approximated as a linear combination of a small number of topic page distributions

Multiple Topics

- A user with this mixture of interests could teleport as follows:
  - determine first whether to teleport to the set *s* of known sports pages, or to the set of known health pages.
  - This choice is made at random, choosing sports pages 60% of the time and health pages 40% of the time
  - Once we choose that a particular teleport step is to (say) a random sports page, we choose a web page in s uniformly at random to teleport to.

4% teleport



Non- dead nodes:
10% teleport, 90% random walk

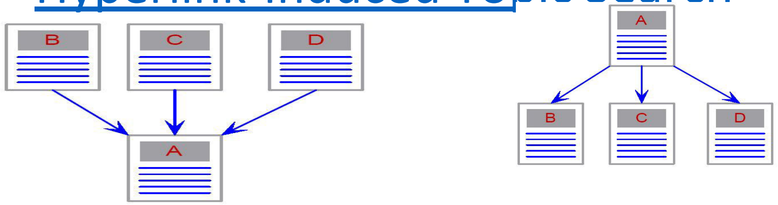
Multiple Topics

- Idea is intuitive but difficult to implement or bring in practice
- Why?
  - for each user, we compute a transition probability matrix and compute its steady-state distribution
  - Pagerank computation for every distinct combination of user interests over topics

Hubs and Authorities

- Given a query, every web page is assigned two scores
  - Hub score
  - Authority score
- Two ranked lists of results are computed
- Useful for broad search topics
- Query: I wish to learn “PageRank”
- Results:
  - authoritative source of information on pagerank
  - Hand-compiled lists (not authoritative in themselves) of links to authoritative web pages on specific topic
- Good hub page: pointing to many good authorities
- Good authority page: pointed by many good hub pages

HITS – Hubs and Authorities - Hyperlink-Induced Topic Search



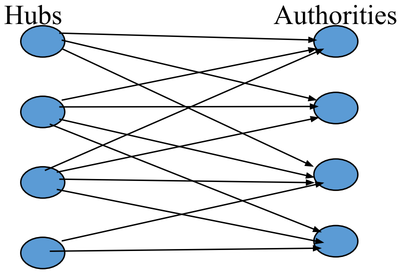
- **A** on the left is an **authority**
- **A** on the right is a **hub**

Hubs and Authorities

- Thus, a good hub page for a topic *points* to many authoritative pages for that topic.
- A good authority page for a topic is *pointed* to by many good hubs for that topic.
- Circular definition - will turn this into an iterative computation.

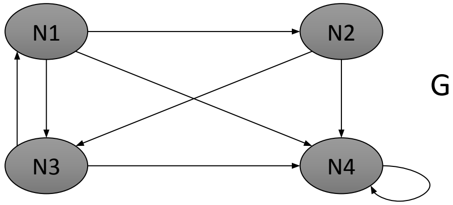
Hubs and Authorities

- Together they tend to form a bipartite graph:



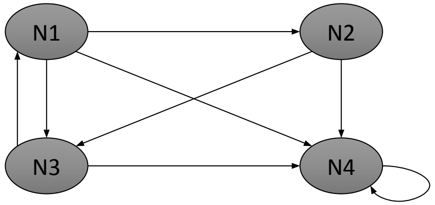
Example

$$\begin{array}{cccc} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{array} \Rightarrow \begin{array}{c|cccc} & N1 & N2 & N3 & N4 \\ \hline N1 & 0 & 1 & 1 & 1 \\ N2 & 0 & 0 & 1 & 1 \\ N3 & 1 & 0 & 0 & 1 \\ N4 & 0 & 0 & 0 & 1 \end{array}$$



Graph with the nodes

Example



Nodes	Hub (out-degree)	Authority (in-degree)
N1	3	1
N2	2	1
N3	2	2
N4	1	4

Ranks using out-degree and in-degree

Hub: N1, N2, N3 (tie), N4
Authority: N4, N3, N2, N1 (tie)

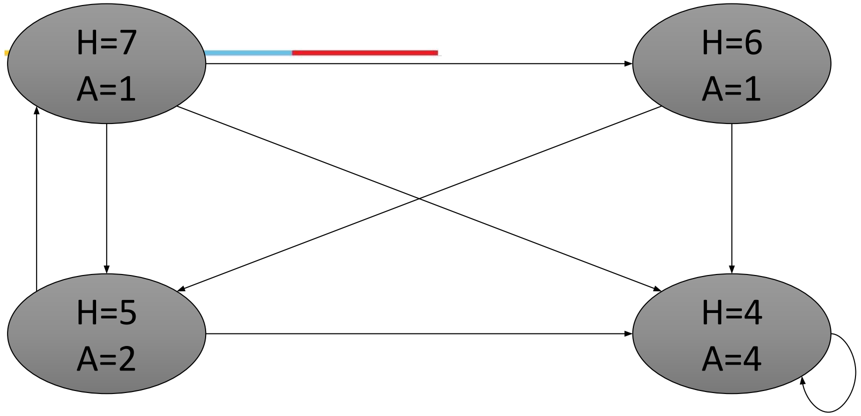
HITS algorithm

- **Input**: an adjacency matrix A, initial hub weight vector u
- Authority weight vector, v= $A^T * u$
- Updated hub weight vector, u= A * v
- Recreate the node graph with new hub and authority values (k=1)
- Recreate the rank table using new hub and authority scores

$$\begin{array}{c|cccc} & N1 & N2 & N3 & N4 \\ \hline N1 & 0 & 1 & 1 & 1 \\ N2 & 0 & 0 & 1 & 1 \\ N3 & 1 & 0 & 0 & 1 \\ N4 & 0 & 0 & 0 & 1 \end{array} \xrightarrow{\text{transpose}} \underset{A^T}{\begin{array}{c|cccc} & N1 & N2 & N3 & N4 \\ \hline N1 & 0 & 0 & 1 & 0 \\ N2 & 1 & 0 & 0 & 0 \\ N3 & 1 & 1 & 0 & 0 \\ N4 & 1 & 1 & 1 & 1 \end{array}} * \underset{u}{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}} = \underset{v}{\begin{bmatrix} 1 \\ 1 \\ 2 \\ 4 \end{bmatrix}}$$

$$\underset{A}{\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}} * \underset{v}{\begin{bmatrix} 1 \\ 1 \\ 2 \\ 4 \end{bmatrix}} = \underset{\text{hub u}}{\begin{bmatrix} 7 \\ 6 \\ 5 \\ 4 \end{bmatrix}}$$

K=1



Ranks using new hub and authority values (k=1)

Nodes	Hub (u)	Authority (v)
N1	7	1
N2	6	1
N3	5	2
N4	4	4

K=2

Authority $v_i{'} = \frac{v_i}{\sqrt{\sum_{i=1}^{n} {v_i}^2}}$

Hub $u_i{'} = \frac{u_i}{\sqrt{\sum_{i=1}^{n} {u_i}^2}}$

Hub: N1, N2, N3, N4

Authority: N4, N3, N2, N1 (tie)

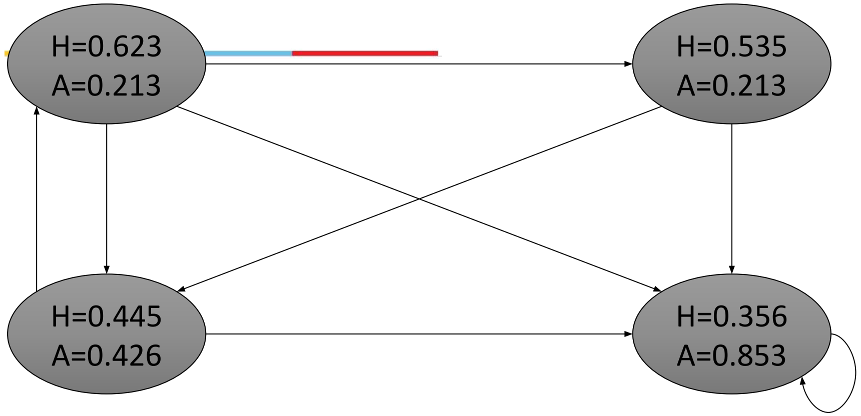
$V' = [\frac{1}{\sqrt{1^2+1^2+1^2+4^2}}, \frac{1}{\sqrt{1^2+1^2+1^2+4^2}}, \frac{2}{\sqrt{1^2+1^2+1^2+4^2}}, \frac{4}{\sqrt{1^2+1^2+1^2+4^2}}]$

$V' = [\frac{1}{\sqrt{22}}, \frac{1}{\sqrt{22}}, \frac{2}{\sqrt{22}}, \frac{4}{\sqrt{22}}] = [0.213, 0.213, 0.426, 0.853]$

Similarly,

$U' = [\frac{7}{\sqrt{126}}, \frac{6}{\sqrt{126}}, \frac{5}{\sqrt{126}}, \frac{4}{\sqrt{126}}] = [0.623, 0.535, 0.445, 0.356]$

K=2



Ranks using new hub and authority values (k=2)

Nodes	Hub (u')	Authority (v')
N1	0.623	0.213
N2	0.535	0.213
N3	0.445	0.426
N4	0.356	0.853

Hub: N1, N2, N3, N4

Authority: N4, N3, N2, N1 (tie)

For k=3

For v, $0.213^2+0.213^2+0.426^2+0.853^2=0.999$

For u, $0.623^2+0.535^2+0.445^2+0.356^2=0.999$

Hyperlink-Induced Topic Search (HITS)

- In response to a query, instead of an ordered list of pages each meeting the query, find <u>two</u> sets of inter-related pages:
  - *Hub pages* are good lists of links to pages answering the information need.
    - e.g., “Bob’s list of cancer-related links.”
  - *Authority pages* are direct answers to the information need
    - occur recurrently on good hubs for the subject.
- Most approaches to search do not make the distinction between the two sets

Recap

- Anchor text and hyperlinks
- Web as a graph
- Pagerank basic algorithm
- Random Surfing
- Teleporting
- Topic specific pagerank
  - Single topic
  - Multiple topics
- HITS algorithm (hubs and authorities)

Google PageRank

- The original formula

$$\mathbf{PR(A) = (1\text{-}d) + d\ (\frac{PR(T_1)}{C(T_1)} + \frac{PR(T_2)}{C(T_2)} + \ldots + \frac{PR(T_n)}{C(T_n)})}$$

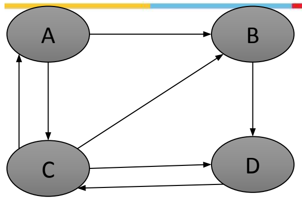
- a given **A** website page rank depends on other websites’ page rank that are linking to **A**.
- **C(T)** parameter is the number of outgoing links from website **T**
- **d** is the damping factor: a value in the range **0** and **1**

Google Pagerank

- We have to initialize the page ranks at the beginning
- The usual approach is to initialize every page rank to be **1/n**
- We make several iterations until convergence

$$PR_{t+1}(P_i) = \sum_{P_i} \frac{PR_t(P_j)}{C(P_j)}$$

Example

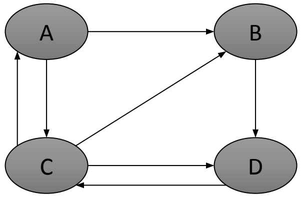


	I0	I1	I2	PR
A	1/4	1/12	1.5/12	1
B	1/4	2.5/12	2/12	2
C	1/4	4.5/12	4.5/12	4
D	1/4	4/12	4/12	3

$$PR_{t+1}(P_i) = \sum_{P_j} \frac{PR_t(P_j)}{C(P_j)}$$

Matrix Representation

- In iterative approach, we update the values one-by-one
- We can use matrix multiplication to do multiple calculations at a time



$$\begin{bmatrix} 0 & 0 & 1/3 & 0 \\ 1/2 & 0 & 1/3 & 0 \\ 1/2 & 0 & 0 & 1 \\ 0 & 1 & 1/3 & 0 \end{bmatrix}$$

Column stochastic matrix

Matrix Representation

$$\begin{bmatrix} 0 & 0 & 1/3 & 0 \\ 1/2 & 0 & 1/3 & 0 \\ 1/2 & 0 & 0 & 1 \\ 0 & 1 & 1/3 & 0 \end{bmatrix}$$

Power method

$$PR_{t+1} = \mathrm{H} * PR_t$$

	I0	I1	I2	PR
A	1/4	1/12	1.5/12	1
B	1/4	2.5/12	2/12	2
C	1/4	4.5/12	4.5/12	4
D	1/4	4/12	4/12	3

$$\mathrm{V} = \begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix}$$

$$v_2 = H\,v$$

$$v_3 = H\,v_2 = H\,(Hv) = H^2 v$$

$$v_4 = H\,v_3 = H^3 v$$