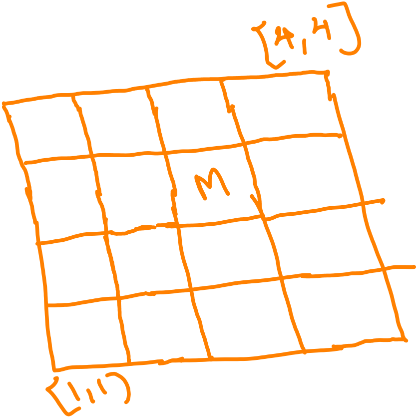
Questions regarding assignment2



Curiosity example

1. Query: Who killed the cat?
   KB $\models$ $Kills(x, Tuna)$?

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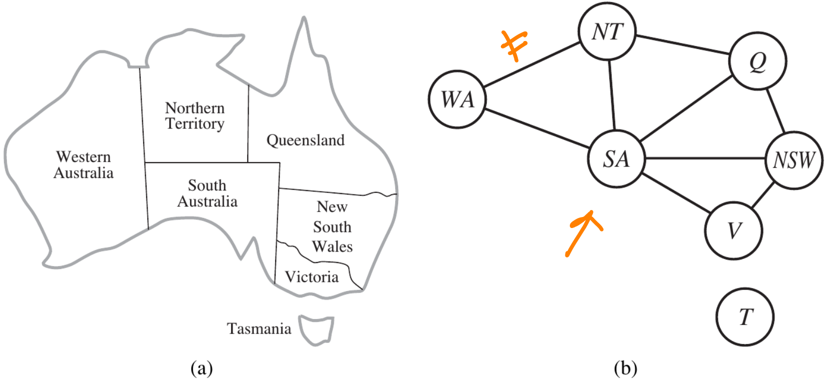

Figure 6.1 (a) The principal states and territories of Australia. Coloring this map can be viewed as a constraint satisfaction problem (CSP). The goal is to assign colors to each region so that no neighboring regions have the same color. (b) The map-coloring problem represented as a constraint graph.

Constraint Satisfaction Problem

- Australian map coloring problem

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- Problem definition : X, D and C

$\{Red, G, B\}$

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  1. $X = \{X_1, X_2, \ldots, X_n\}$
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  3. C is a set of constraints.

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- Consistent assignment
- Complete assignment
- Solution : consistent and complete

Constraint Satisfaction Problem (CSP)

- constraint graph vs. state space graph

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- $X = \{WA, NT, Q, NSW, V, SA, T\}$
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- $C = \{WA \neq NT, WA \neq SA, \ldots, V \neq NSW\}$

Constraint Satisfaction Problem (CSP)

- constraint graph vs. state space graph
- $X = \{WA, NT, Q, NSW, V, SA, T\}$
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- Multiple solutions are possible.

Constraint Satisfaction Problem (CSP)

- constraint graph vs. state space graph
- $X = \{WA, NT, Q, NSW, V, SA, T\}$
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- Multiple solutions are possible.
- Goal: understand general approaches for solving CSPs.

Car assembly scheduling

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  - $Axle_F + 10 \leq Wheel_{RF}$; $Axle_B + 10 \leq Wheel_{RB}$

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  - $(Axle_F + 10 \leq Axle_B)$ or $(Axle_B + 10 \leq Axle_F)$

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  - $Cap_{RF} + 1 \leq Inspect$

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  - $(Axle_F + 10 \leq Axle_B)$ or $(Axle_B + 10 \leq Axle_F)$
  - $Cap_{RF} + 1 \leq Inspect$
- $D_i = \{1, 2, 3, \ldots, 27\}$

Types of constraints

- Unary constraint

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Types of constraints

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- Binary constraint
  $WA \neq NT$
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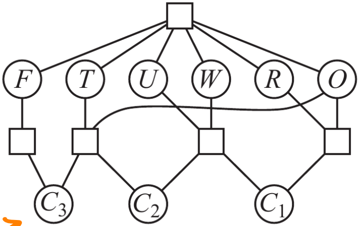
  $Alldiff(A, B, C, D)$

Cryptoarithmetic Problem

$C_3 \; C_2 \; C_1$

$$\begin{array}{cccc} & T & W & O \\ + & T & W & O \\ \hline F & O & U & R \end{array}$$



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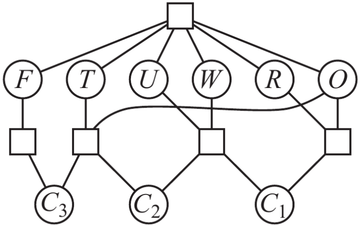

- *Alldiff* (F, T, U, W, R, O)
  It is given that all letters are distinct.

Cryptoarithmetic Problem

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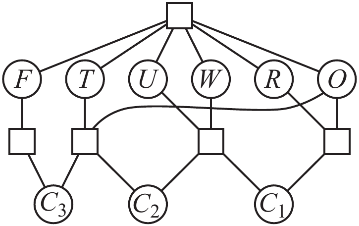

- *Alldiff*(F, T, U, W, R, O)
  It is given that all letters are distinct.
- $O + O = 10 \times C_1 + R$

Cryptoarithmetic Problem

$$
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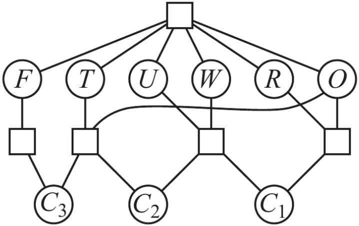
 & T & W & O \\
+ & T & W & O \\
\hline
F & O & U & R
\end{array}
$$



- $Alldiff(F, T, U, W, R, O)$
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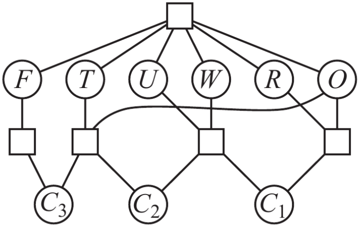

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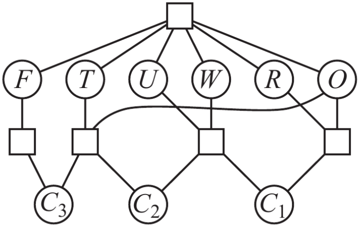

- $Alldiff(F, T, U, W, R, O)$
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- $O + O = 10 \times C_1 + R$
- $C_1 + W + W = 10 \times C_2 + U$
- $C_2 + T + T = 10 \times C_3 + O$
- $C_3 = F$

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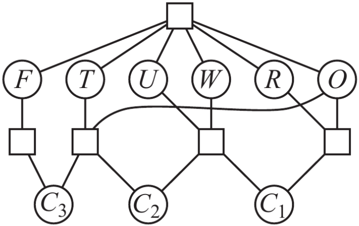

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- $C_2 + T + T = 10 \times C_3 + O$
- $C_3 = F$
- $F \neq 0$

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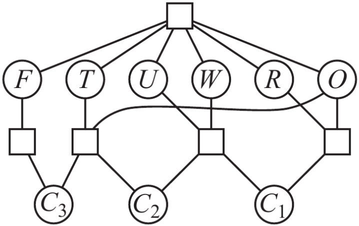

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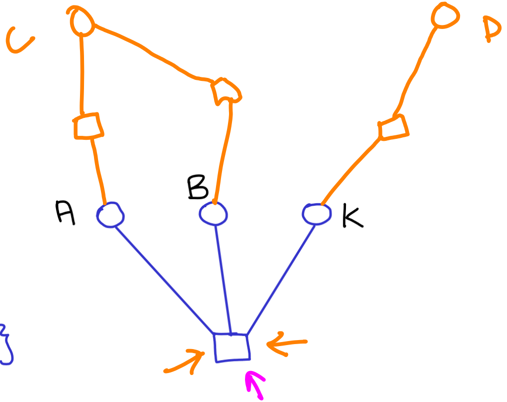
- $C_2 + T + T = 10 \times C_3 + O$
- $C_3 = F$
- $F \neq 0$
- $D_i = \{0, \ldots, 9\}$

Constraint satisfaction problem (CSP)

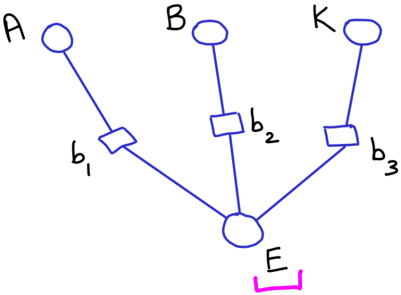
$$X = \{A, B, K\}$$

$$D_i = \{1, 2, 3\}$$

$$C = \{A + B = K\}$$



n-ary constraint to binary constraint



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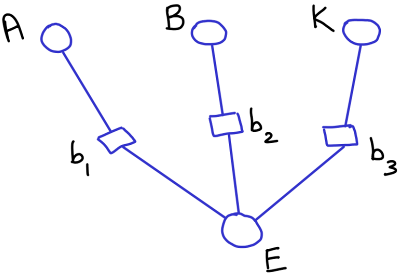

3-ary

- $\underbrace{D_E}_{} = \{< 1, 2, 3 >, < 2, 1, 3 >, < 1, 1, 2 >\}$

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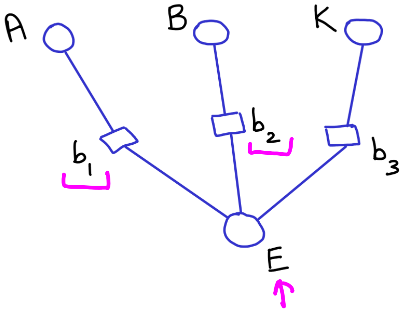
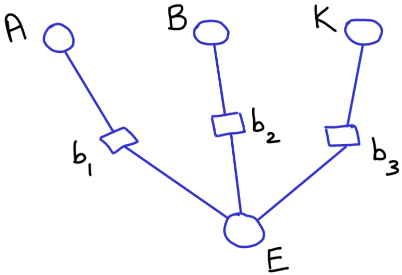


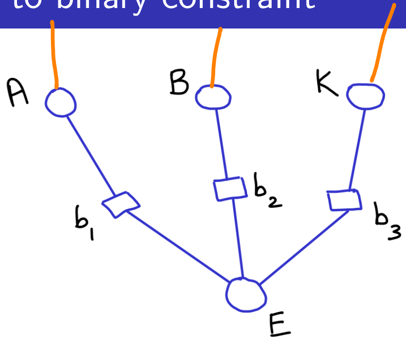
- $D_E = \{< 1,2,3 >, < 2,1,3 >, < 1,1,2 >\}$
- Constraint b_1 checks whether A equals first element of the tuple assigned to E.

n-ary constraint to binary constraint



- $D_E = \{< 1, 2, 3 >, < 2, 1, 3 >, < 1, 1, 2 >\}$
- Constraint b_1 checks whether A equals first element of the tuple assigned to E.
- Constraint b_2 checks whether B equals second element of the tuple assigned to E.

n-ary constraint to binary constraint



- $D_E = \{<1,2,3>, <2,1,3>, <1,1,2>\}$
- Constraint b_1 checks whether A equals first element of the tuple assigned to E.
- Constraint b_2 checks whether B equals second element of the tuple assigned to E.
- Constraint b_3 checks whether K equals third element of the tuple assigned to E.

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- Add a new variable corresponding to each n-ary constraint, where $n > 2$.

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- The domain of the new variable consists of n-tuples whose elements satisfy the corresponding n-ary constraint.
- Add a binary constraint between the new variable and each of the original n variables (that were participating in the original n-ary constraint).

Finding a solution for a CSP

- The problem : $O(d^n)$ possible assignments.

$d \times d \times \cdots d$

d

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- Achieve $\underbrace{\text{node consistency}}$

  $\underbrace{WA \neq Green}$

$D_{WA} = \{r, \cancel{g}, b\}$

Finding a solution for a CSP

- The problem : $O(d^n)$ possible assignments.
- Reduce the number of legal values for different variables.
- Achieve node consistency

  $WA \neq Green$
- Achieve arc consistency