Conjunctive Normal Form

- Clause

$(a \vee b \vee c)$ $(\neg a \vee b)$

$\underbrace{a \wedge b}$

Conjunctive Normal Form

- Clause
- Conjuctive Normal Form (CNF) : Conjunction of Clauses

$$\underbrace{(a \vee b) \wedge (\neg b \vee c)}$$

Conjunctive Normal Form

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- Can every sentence α be written in a logically equivalent CNF?

Conjunctive Normal Form

- Clause
- Conjuctive Normal Form (CNF) : Conjunction of Clauses
- Can every sentence α be written in a logically equivalent CNF?
- What is the CNF of $B_{1,1} \Leftrightarrow P_{2,1} \vee P_{1,2}$?

$$(B_{1,1} \Rightarrow P_{2,1} \vee P_{1,2}) \wedge ((P_{2,1} \vee P_{1,2}) \Rightarrow B_{1,1})$$

$$(\neg B_{1,1} \vee P_{2,1} \vee P_{1,2}) \wedge ((\neg P_{2,1} \wedge \neg P_{1,2}) \vee B_{1,1})$$

$$\ldots\ldots \wedge (\neg P_{2,1} \vee B_{1,1}) \wedge (\neg P_{1,2} \vee B_{1,1})$$

Resolution Algorithm

- Deduction theorem :

  $\beta \models \alpha$ if and only if $\beta \Rightarrow \alpha$ is valid.

Resolution Algorithm

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  $\beta \models \alpha$ if and only if $\beta \Rightarrow \alpha$ is valid.

  $\beta \models \alpha$ if and only if $\underbrace{\neg\beta \vee \alpha}$ is valid.

$\neg(\neg\beta \vee \alpha)$

Resolution Algorithm

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  $\beta \models \alpha$ if and only if $\beta \wedge \neg\alpha$ is a contradiction.

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- Is this sentence in CNF?

  $(a \vee \neg b) \wedge (\neg a \vee \neg b) \wedge (b)$

Resolution Algorithm

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- Is this sentence in CNF? Is it a contradiction?

  $(a \vee \neg b) \wedge (\neg a \vee \neg b) \wedge (b)$

$\neg a$

$\neg b$

$\boxed{}$

Resolution Algorithm

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  $\beta \models \alpha$ if and only if $\beta \Rightarrow \alpha$ is valid.

  $\beta \models \alpha$ if and only if $\neg\beta \vee \alpha$ is valid.

  $\beta \models \alpha$ if and only if $\beta \wedge \neg\alpha$ is a contradiction.
- Is this sentence in CNF? Is it a contradiction?

  $(a \vee \neg b) \wedge (\neg a \vee \neg b) \wedge (b)$
- Ground resolution theorem

Resolution Algorithm

- How can we use the Resolution Algorithm to check whether $KB \models \alpha$?

Resolution Algorithm

- How can we use the Resolution Algorithm to check whether $KB \models \alpha$?
- $KB \models \alpha$ if and only if $KB \wedge \neg\alpha$ is a contradiction.

$R_1 : \underbrace{C_1}_{} \wedge \underbrace{C_2}_{}$

$R_2 : \underbrace{C_3}_{} \wedge \underbrace{C_4}_{}$

$\vdots$

$R_5 : \underbrace{C_6}_{} \wedge \underbrace{C_7}_{} \wedge \underbrace{C_9}_{}$

$\underbrace{\underbrace{KB}_{} \wedge \underbrace{\neg\alpha}_{}}_{}$

$\neg\alpha \equiv \underbrace{C_{11} \wedge C_{12} \wedge \cdots}_{}$

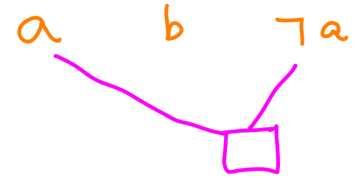
Resolution Algorithm

KB : $a \wedge b$

$KB \models a$?

$\neg\alpha \equiv \neg a$

$KB \wedge \neg\alpha \equiv a \wedge b \wedge \neg a$

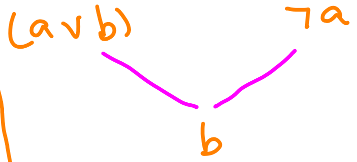


KB : $a \vee b$

$KB \models a$?

$\neg\alpha \equiv \neg a$

$KB \wedge \neg\alpha \equiv (a \vee b) \wedge \neg a$



Resolution Algorithm Inference

$KB \models \underbrace{\neg P_{1,2}}$

$KB \wedge P_{1,2}$

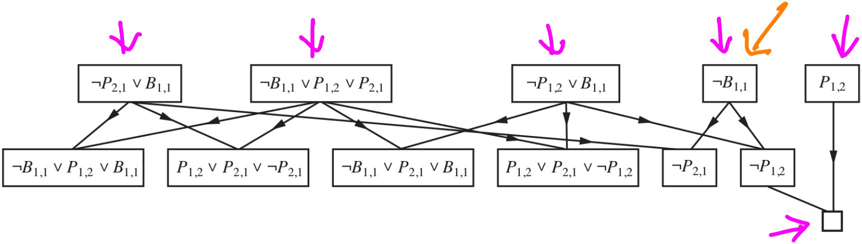


Figure 7.13 Partial application of PL-RESOLUTION to a simple inference in the wumpus world. $\neg P_{1,2}$ is shown to follow from the first four clauses in the top row.

Resolution Algorithm

$a \vee \neg b, \; b \vee c \vee \neg a$

function PL-RESOLUTION(KB, α) **returns** *true* or *false*
  inputs: KB, the knowledge base, a sentence in propositional logic
    α, the query, a sentence in propositional logic

  $clauses \leftarrow$ the set of clauses in the CNF representation of $KB \wedge \neg\alpha$
  $new \leftarrow \{\,\}$
  loop do
    for each pair of clauses C_i, C_j **in** $clauses$ **do**
      $resolvents \leftarrow$ PL-RESOLVE(C_i, C_j)
      if $resolvents$ contains the empty clause **then return** *true*
      $new \leftarrow new \cup resolvents$
    if $new \subseteq clauses$ **then return** *false*
    $clauses \leftarrow clauses \cup new$

Soundness and Completeness of Resolution

- Is resolution algorithm sound?

Soundness and Completeness of Resolution

- Is resolution algorithm sound? Deduction theorem

Soundness and Completeness of Resolution

- Is resolution algorithm sound? Deduction theorem
- Complete?

Soundness and Completeness of Resolution

- Is resolution algorithm sound? Deduction theorem
- Complete? Ground resolution theorem

A more efficient algorithm

- SAT is NP-complete.

A more efficient algorithm

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- Can we come up with a more efficient algorithm by making some assumptions?

A more efficient algorithm

- SAT is NP-complete.
- Can we come up with a more efficient algorithm by making some assumptions?
- Definite clause

$\underbrace{a \vee \neg b}$ $\quad a \vee b \quad$ $\underbrace{\neg a \vee b \vee \neg c}$

A more efficient algorithm

- SAT is NP-complete.
- Can we come up with a more efficient algorithm by making some assumptions?
- Definite clause
- Horn clause

$\neg a \vee \neg b$

A more efficient algorithm

- SAT is NP-complete.
- Can we come up with a more efficient algorithm by making some assumptions?
- Definite clause
- Horn clause
- Will the resolvent of two definite clauses be a definite clause?

$\neg a \vee b$ $\qquad\qquad$ $\neg c \vee \neg b$

$\neg a \vee \neg c$