

# Heuristic Functions for 8-puzzle problem


7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

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Start State

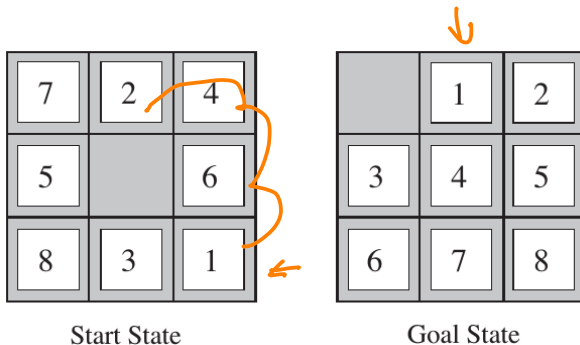
	1	2
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Goal State

1. Number of misplaced tiles ( $h_1$ )

8

# Heuristic Functions for 8-puzzle problem



1. Number of misplaced tiles ( $h_1$ )

8

2. Manhattan distance ( $h_2$ )

$$3 + \dots + = 18$$

# Effect of Heuristic on performance

- ▶ For now, let us assume that  $h_1$  and  $h_2$  are consistent.

$$(b^*)^d$$

- →  
→

→  
→  
→



# Effective Branching Factor

$$N + 1 = 1 + b^* + (b^*)^2 + \dots + (b^*)^d$$

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	Search Cost (nodes generated)			Effective Branching Factor		
$d$	IDS	$A^*(h_1)$	$A^*(h_2)$	IDS	$A^*(h_1)$	$A^*(h_2)$
2	10	6	6	→ 2.45	→ 1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	3644035	227	73	<u>2.78</u>	→ 1.42	→ 1.24
14	—	539	113	—	1.44	1.23
16	—	1301	211	—	1.45	1.25
18	—	3056	363	—	1.46	1.26
20	—	7276	676	—	1.47	1.27
22	—	18094	1219	—	1.48 ↵	1.28
24	—	39135	1641	—	1.48	1.26

**Figure 3.29** Comparison of the search costs and effective branching factors for the ITERATIVE-DEEPENING-SEARCH and  $A^*$  algorithms with  $h_1$ ,  $h_2$ . Data are averaged over 100 instances of the 8-puzzle for each of various solution lengths  $d$ .

# Heuristic domination

- Which heuristic is better —  $h_1$  or  $h_2$ ? Why?

$$f(n) < f(\text{Goal}) = g(\text{Goal}) + h(\text{Goal})$$
$$= C^*$$

$$\rightarrow g(n) < C^*$$

$$f(n) < C^*$$

$$g(n) + h(n) < C^*$$

$$g(n) < C^* - h(n)$$



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- ▶ How to find a consistent heuristic?

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- ▶ What will be the cost of the shortest path in this relaxed state space?  
Manhattan distance ( $h_2$ )



## $h_2$ is consistent in the relaxed state space for 8-puzzle

To prove :  $h_2(n) \leq c(n, a, n') + h_2(n')$

Proof:

$$h_2(n) > c(n, a, n') + h_2(n')$$

