7. Little Oh and Omega

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Agenda

1. Little Oh o
2. Little Omega ω

Little Oh o

The function $f(n) = o(g(n))$
For all positive constants $c > 0$ there exists a constant n_0 such that $f(n) < cg(n)$, $\forall n \geq n_0$ where $n_0 \geq 1$

Examples:

1. $f(n) = 3n + 2$ $\quad f(n) = o(n^2)$
2. $f(n) = 4n^3 + 5$ $\quad f(n) = o(n^4)$

Little Oh o

Intuitively, in o-notation, the function $f(n)$ becomes insignificant relative to $g(n)$ as n approaches infinity

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$$

An upper bound that is not asymptotically tight.

Little Omega ω

The function $f(n) = \omega(g(n))$
For all positive constants $c>0$ there exists a constant n_0 such that $f(n)>cg(n), \forall n \geq n_0$ where $n_0 \geq 1$

Examples:

1. $f(n) = 3n + 2$ $\quad f(n) = \omega(1)$
2. $f(n) = 4n^3 + 5$ $\quad f(n) = \omega(n^2)$

Little Omega ω

Intuitively, in ω-notation, the function $f(n)$ becomes arbitrarily large relative to $g(n)$ as n approaches infinity

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty$$

An lower bound that is not asymptotically tight.

Little o and ω

- $o(n) = \mathcal{O}(n) - \Theta(n)$ (can't touch the upper bound)
- $\omega(n) = \Omega(n) - \Theta(n)$ (can't touch the lower bound)
- Intersection of Little o and ω is an empty set that's why Little θ does not exist.

Interpretation

- $f(n) = \mathcal{O}(g(n))$ $f(n)$ grows no faster than $g(n)$
- $f(n) = \Omega(g(n))$ $f(n)$ grows no slower than $g(n)$
- $f(n) = \Theta(g(n))$ $f(n)$ grows at the same rate as $g(n)$
- $f(n) = o(g(n))$ $f(n)$ grows slower than $g(n)$
- $f(n) = \omega(g(n))$ $f(n)$ grows faster than $g(n)$