# 1.5行の交換、逆行列、まるめの誤差

交換行列

P = [0,1;1,0]

P =

0 1 1

A = [0,2;3,4]

A =

0 3 2

P\*A

ans =

3 0 4

Pは交換行列

1.5.1

clear

A = [1,4,2;-2,-8,3;0,1,1]

A =

1 -2

- 8 1 2 3

b = [-2;32;1]

b =

-2 32 1

E21 = eye(3);E21(2,1) = 2

E21 =

1 2 0 0 1 0

U1 = E21 \* A

```
1 4 2
0 0 7
0 1 1
```

## 第2行と第3行を交換

P32 = [1,0,0;0,0,1;0,1,0]; U = P32 \* U1

U =

1 4 2 0 1 1 0 0 7

c1 = E21 \* b

c1 = -2 28 1

c = P32\*c1

c = -2 1 28

 $\begin{bmatrix} 1 & 4 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 7 \end{bmatrix} x = \begin{bmatrix} -2 \\ 1 \\ 28 \end{bmatrix}$ 

% 要 Symbolic Math Toolbox syms x y z; eqn = U \* [x;y;z] == c; solx = solve(eqn, x,y,z); Answer = [solx.x;solx.y;solx.z]

Answer =

 $\begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$ 

# 1.5.2

clear A = [1,1;3,3]

A =

1 1 3 3 E21 = [1,0;-3,1]

-3

E21 =

0 1

U = E21\*A

U =

1

1

% 要 Symbolic Math Toolbox

syms b1 b2

b = [b1; b2]

b =

c = E21 \* b

c =

 $\begin{pmatrix} b_1 \\ b_2 - 3 b_1 \end{pmatrix}$ 

よって $0 = b_2 - 3b_1$ なので $b_2 = 3b_1$ 

交換行列は $P_{kl} = P_{kl}^{-1}$ となる

**P32** 例

clear

A = [1,2,3;2,4,2;0,1,1]

A =

1 2

3 2

E21 = eye(3);

E21(2,1) = -2

E21 =

1 -2 0 1 0 0

0

1

U1 = E21 \* A

# P23 = [1,0,0;0,0,1;0,1,0]

## U = P23\*U1

## PA = P23\*A

## E31 = eye(3);E31(3,1) = -2

## U = E31 \* PA

## $L = E31^-1$

# 1.5.3

```
0
2
                        1
3
  P21 = [0,1;1,0]
  P21 =
                        1
         1
  PA = P21 * A
  PA =
                        3
  L = [1,0;0,1]
  L =
         1
                        0
         0
                        1
  D = [2,0;0,1]
  D =
                        0
         2
  U = [1,3/2;0,1]
  U =
         1
                        3/2
         0
                        1
1.5.4
  clear
 A = [1,1,1;1,1,2;1,2,5]
  A =
         1
                        1
                                       1
2
5
         1
                        1
         1
                        2
  E21 = [1,0,0;-1,1,0;0,0,1]
  E21 =
         1
                        0
                                       0
        - 1
                        1
                                       0
  U1 = E21 * A
```

U1 =

 

```
1 2 5
```

```
P32 = [1,0,0;0,0,1;0,1,0]
```

$$U3 = E21 * U2$$

#### ======

$$E31 = eye(3);$$
  
 $E31(3,1) = -1$ 

## その1

clear A = [0,1,-1;1,-1,0;1,0,-1]

A =

0 1 1

1 -1 0 -1 0 -1

b = [2;2;2]

b =

2 2 2

P21 = [0,1,0;1,0,0;0,0,1]

P21 =

0 1 0

1 0 0 0 0 1

PA = P21 \* A

PA =

1 0 1 -1 1 0 0 -1 -1

E31 = eye(3);

E31(3,1) = -1

E31 =

1 0 -1 0 1 0 0 0 1

U = E31 \* PA

U =

1 0 0 -1 1 1

0 -1 -1

c = E31 \* b

c =

2 2 0

2行目と3行目が同じなのにbは異なるので特異&解なし

その2

clear A = [0,1,-1;1,-1,0;1,0,-1]

A =

0 1

1 - 1

-1 0 -1

b = [0;0;0]

b =

0 0 0

P21 = [0,1,0;1,0,0;0,0,1]

P21 =

0 1 0 1 0 0

0 0 1

PA = P21 \* A

PA =

1 0 1 -1 1 0

0 -1 - 1

E31 = eye(3);E31(3,1) = -1

E31 =

0 1 0 0 1

U = E31 \* PA

U =

1 0 0

0 -1 -1

c = E31 \* b

c =

0 0 0

特異&解はu = v = w

逆行列

```
BA = IかつAC = IならばB = C
つまり AA<sup>-1</sup> = A<sup>-1</sup>A = I
                                                  \left(\mathbf{A}\mathbf{B}\right)^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}
P34 例
  clear
  A = [2,1,1,1,0,0;4,1,0,0,1,0;-2,2,1,0,0,1]
   A =
            2
                               1
                                                  1
                                                                     1
           4
                               1
                                                  0
                                                                     0
                                                                                        1
                                                                                                           0
           - 2
  E21 = eye(3);
  E21(2,1) = -2
   E21 =
            1
                               0
                                                  0
           - 2
                               1
                                                  0
            0
                                                  1
  U1 = E21*A
   U1 =
                                                 1
-2
1
                                                                    1
-2
            2
                                                                                                           0
            0
                              -1
                                                                                        1
                                                                                                           0
           -2
  E31 = eye(3);
  E31(3,1) = 1
   E31 =
            1
                                                  0
            0
                                                  0
                               1
            1
  U2 = E31 * U1
   U2 =
                                                 1
-2
2
                                                                    1
-2
1
            2
                                                                                                           0
                               1
                              - 1
                                                                                        1
            0
                                                                                                           0
  E32 = eye(3);
  E32(3,2) = 3
   E32 =
            1
                               0
1
3
                                                  0
            0
                                                  0
            0
```

U = E32 \* U2

最初の3列がいつもの上3角行列U、後ろの3列が $L^{-1}$ 

つまり
$$\boldsymbol{L}^{-1} = \boldsymbol{E}_{32}\boldsymbol{E}_{31}\boldsymbol{E}_{21}$$

## Gauss-Jordan法

clear

$$AI = [A, I]$$

$$AI = [2,1,1,1,0,0;0,-1,-2,-2,1,0;0,0,-4,-5,3,1]$$

第2列のピボットより上を0にする

$$U1 = E12*AI$$

第3列のピボットより上を0にする

E13 = 
$$eye(3)$$
;  
E13(1,3) =  $-1/4$ 

$$U2 = E13 * U1$$

$$U2 = 2$$
 0 0 1/4 1/4 -1/4

$$E23 = eye(3);$$
  
 $E23(2,3) = -1/2$ 

## 各ピボットで割り算を行う

$$pdiv = [1/2,0,0;0,-1,0;0,0,-1/4]$$

$$U = pdiv * U3$$

$$\boldsymbol{U} = \left[ \boldsymbol{I}, \boldsymbol{A}^{-1} \right]$$

#### 1.5.6

**AB**<sup>-1</sup>**C** の逆行列

$$\left(\mathbf{A}\mathbf{B}^{-1}\boldsymbol{C}\right)^{-1} = \boldsymbol{C}^{-1}\mathbf{B}\mathbf{A}^{-1}$$

## 1.5.7

$$A = [0,1;1,0]$$

#### A^-1

#### A\*A

$$\mathbf{B} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\mathbf{B}^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

# 1.5.8

# B = eye(2)

## AB = [A,B]

$$E21 = eye(2);$$
  
 $E21(2,1) = -3$ 

## U1 = E21\*AB

```
A^-1
```

警告: 行列は、特異行列に近いか、正しくスケーリングされていません。結果は不正確になる可能性があります。RCOND = 6.938894e-18。

ans =

1.5.9

$$\boldsymbol{E} = \begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Eは3行目の8倍を1行目に加える行列

clear E = [1,0,8;0,1,0;0,0,1]

E =

E^-1

ans =

A = [E, eye(3)]

A =

E13 = eye(3); E13(1,3) = -8

E13 =

U = E13 \* A

U =

1 1 0 0 0 -8 0 1 0 1 0 0 0 1 0 0 1

U(:,4:6)

## 1.5.10

clear
syms a b c d;
A = [a,b;c,d]

 $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ 

A = [A, eye(2)]

 $\begin{array}{cccc} \mathsf{A} & = & \\ \begin{pmatrix} a & b & 1 & 0 \\ c & d & 0 & 1 \end{pmatrix} \end{array}$ 

E21 = [1,0;-c/a,1]

 $E21 = \begin{pmatrix} 1 & 0 \\ -\frac{c}{a} & 1 \end{pmatrix}$ 

U1 = E21\*A

 $\begin{pmatrix}
a & b & 1 & 0 \\
0 & d - \frac{bc}{a} & -\frac{c}{a} & 1
\end{pmatrix}$ 

U1(2,2) = (a\*d-b\*c)/a

 $\begin{pmatrix}
a & b & 1 & 0 \\
0 & \frac{ad - bc}{a} & -\frac{c}{a} & 1
\end{pmatrix}$ 

E12 = [1,-(a\*b)/(a\*d-b\*c);0,1]

E12 =  $\begin{pmatrix} 1 & -\frac{ab}{ad - bc} \\ 0 & 1 \end{pmatrix}$ 

$$U2 = E12 * U1$$

$$\begin{pmatrix} a & 0 & \frac{bc}{ad - bc} + 1 & -\frac{ab}{ad - bc} \\ 0 & \frac{ad - bc}{a} & -\frac{c}{a} & 1 \end{pmatrix}$$

U2(1,3) = (a\*d)/(a\*d-b\*c)

$$\begin{pmatrix}
a & 0 & \frac{ad}{ad - bc} & -\frac{ab}{ad - bc} \\
0 & \frac{ad - bc}{a} & -\frac{c}{a} & 1
\end{pmatrix}$$

E11 = [1/a, 0; 0, 1]

$$\begin{pmatrix} \frac{1}{a} & 0 \\ 0 & 1 \end{pmatrix}$$

U3 = E11 \* U2

$$\begin{pmatrix}
1 & 0 & \frac{d}{ad-bc} & -\frac{b}{ad-bc} \\
0 & \frac{ad-bc}{a} & -\frac{c}{a} & 1
\end{pmatrix}$$

E22 = [1,0;0,a/(a\*d - b\*c)]

$$\begin{pmatrix} 1 & 0 \\ 0 & \frac{a}{ad - bc} \end{pmatrix}$$

U4 = E22 \* U3

$$\begin{pmatrix}
1 & 0 & \frac{d}{ad-bc} & -\frac{b}{ad-bc} \\
0 & 1 & -\frac{c}{ad-bc} & \frac{a}{ad-bc}
\end{pmatrix}$$

Ain = U4(:,3:4)

Ain =

$$\left(\begin{array}{ccc}
\frac{d}{ad-bc} & -\frac{b}{ad-bc} \\
-\frac{c}{ad-bc} & \frac{a}{ad-bc}
\end{array}\right)$$

$$A^{-1} = \frac{1}{\text{ad - bc}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

clear A = [2,-1,0;-1,2,-1;0,-1,2]

A = [A, eye(3)]

E21 = eye(3);E21(2,1) = 1/2

U1 = E21 \* A

E32 = eye(3);E32(3,2) = 2/3

E32 =

1 0 0
0 1 0
0 2/3 1

U2 = E32 \* U1

E12 = eye(3); E12(1,2) = 2/3

```
E12 =
       1
                      2/3
                                     0
       0
                      1
                                     0
       0
                      0
                                     1
U3 = E12 * U2
U3 =
                                    -2/3
       2
                                                   4/3
                                                                  2/3
                                                                                 0
       0
                      3/2
                                    - 1
                                                   1/2
                                                                  1
                                                                                 0
       0
                      0
                                    4/3
                                                   1/3
                                                                  2/3
                                                                                 1
E13 = eye(3);
E13(1,3) = 1/2
E13 =
       1
                      0
                                     1/2
       0
                      1
                                     0
       0
                                     1
                      0
U4 = E13 * U3
U4 =
                                     *
       2
                                                   3/2
                                                                  1
                                                                                 1/2
       0
                      3/2
                                    - 1
                                                   1/2
                                                                  1
                                                                                 0
       0
                                     4/3
                                                   1/3
                                                                  2/3
                                                                                 1
E23 = eye(3);
E23(2,3) = 3/4
E23 =
       1
                      0
                                     0
       0
                      1
                                     3/4
       0
U5 = E23 * U4
U5 =
       2
                                                   3/2
                                                                  1
                                                                                 1/2
       0
                      3/2
                                                   3/4
                                                                  3/2
                                                                                 3/4
       0
                                     4/3
                                                   1/3
                                                                  2/3
                                                                                 1
E11 = eye(3);
E11(1,1) = 1/2
E11 =
       1/2
                      0
                                     0
       0
                      1
                                     0
       0
U6 = E11 * U5
U6 =
       1
                                                   3/4
                                                                  1/2
                                                                                 1/4
```

3/4

1/3

4/3

3/2

2/3

3/4

1

0

0

3/2

$$E22 = eye(3);$$
  
 $E22(2,2) = 2/3$ 

E33 = 
$$eye(3)$$
;  
E33(3,3) = 3/4

$$Ain = U8(:,4:6)$$

$$\mathbf{A}^{-1} = 4 \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

# まるめの誤差

部分選択

同じ列の中の選び得るピボットを比較、最大値を選び、それがピボットになるように行交換する

$$\mathbf{A'} = \begin{bmatrix} .0001 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\mathbf{A''} = \mathbf{PA'} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} .0001 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ .0001 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ .0001 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & .9999 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

#### 1.5.11

```
clear
%format rat
%format short
A = [1,1/2,1/3;1/2,1/3,1/4;1/3,1/4,1/5]
```

```
A = 

1.0000     0.5000     0.3333     0.2500     0.3333     0.2500     0.2000
```

## A = [A, eye(3)]

E21 = eye(3);E21(2,1) = -1/2

$$U1 = E21 * A$$

$$E31 = eye(3);$$
  
 $E31(3,1) = -1/3$ 

$$U2 = E31 * U1$$

```
U2 =
                                1.0000 0 0
-0.5000 1.0000 0
-0.3333 0 1.0000
    1.0000 0.5000
0 0.0833
                       0.3333
                       0.0833
        0.0833
                       0.0889
E32 = eye(3);
E32(3,2) = -1
E32 =

\begin{array}{ccc}
1 & 0 \\
0 & 1
\end{array}

                 0
         1
                 0
    0 -1
               1
U3 = E32 * U2
U3 =
   1.0000 0.5000
0 0.0833
                                                    0
0
                       0.3333
                                1.0000
                                           0
                                1.0000 0
-0.5000 1.0000
                       0.0833
        0.0000
                       0.0056
                               0.1667
                                        -1.0000
                                                     1.0000
E12 = eye(3);
E12(1,2) = -6
E12 =
    1 -6 0
0 1 0
0 0 1
U4 = E12 * U3
U4 =
                                        -6.0000
                                                    9
9
    1.0000 0.0000
                      -0.1667
                                4.0000
     0 0.0833
                     0.0833
                                -0.5000 1.0000
         0.0000
                       0.0056
                                0.1667
                                          -1.0000
                                                     1.0000
E13 = eye(3);
E13(1,3) = 30
E13 =
    1 0 30
0 1 0
0 0 1
U5 = E13 * U4
U5 =
                                                    30.0000
    1.0000 0.0000
                       0.0000
                                9.0000 -36.0000
```

0.0833 -0.5000 1.0000 0.0056 0.1667 -1.0000

0 0.0833

0.0000

0

0

1.0000

```
E23 = eye(3);
E23(2,3) = -15
E23 =
        0 0
    1
    0 1 -15
        0
    0
             1
U6 = E23 * U5
U6 =
   1.0000 0.0000
                   0.0000
                           9.0000 -36.0000
                                           30.0000
                          -3.0000
      0 0.0833
                   -0.0000
                                  16.0000 -15.0000
         0.0000
                   0.0056
                           0.1667
                                  -1.0000
                                          1.0000
E22 = eye(3);
E22(2,2) = 12
E22 =
    1
        0
    0
        12
              0
       0
    0
U7 = E22 * U6
U7 =
         0.0000
                          9.0000 -36.0000
   1.0000
                   0.0000
                                           30.0000
                   -0.0000 -36.0000 192.0000 -180.0000
           1.0000
      0
           0.0000
       0
                   0.0056
                           0.1667
                                  -1.0000
                                          1.0000
E33 = eve(3);
E33(3,3) = 180
E33 =
       0
           0
0
    1
    0
         1
    0
       0 180
U8 = E33 * U7
U8 =
                   1.0000
           0.0000
```

30.0000 -180.0000 180.0000

0

0

1.0000

0.0000

1.0000

```
clear
A = [0.001,0;1,1000]
```

A =

1.0e+03 \*

0.000001000000000

0

0.0010000000000000

1.00000000000000000

$$E21 = eye(2);$$
  
 $E21(2,1) = -1000$ 

E21 =

1 0 -1000 1

#### U1 = E21 \* A

U1 =

1.0e+03 \*

0.000001000000000

0

1.0000000000000000

$$Ad = [0,1;1,0]*A$$

Ad =

1.0e+03 \*

0.001000000000000

1.00000000000000000

0.000001000000000

0

$$E21 = eye(2);$$
  
 $E21(2,1) = -0.001$ 

E21 =

1.0000000000000000

0

-0.0010000000000000

1.0000000000000000

#### U1 = E21\*Ad

U1 =

1 1000 0 -1

一番目の計算だとピボットが0.001と1000になる

二番目の計算だとピボットが1と-1になる