

## 1.5 行の交換、逆行列、まるめの誤差

交換行列

```
P = [0,1;1,0]
```

```
P =  
     0     1  
     1     0
```

```
A = [0,2;3,4]
```

```
A =  
     0     2  
     3     4
```

```
P*A
```

```
ans =  
     3     4  
     0     2
```

$P$ は交換行列

### 1.5.1

```
clear  
A = [1,4,2;-2,-8,3;0,1,1]
```

```
A =  
     1     4     2  
    -2    -8     3  
     0     1     1
```

```
b = [-2;32;1]
```

```
b =  
    -2  
    32  
     1
```

```
E21 = eye(3);  
E21(2,1) = 2
```

```
E21 =  
     1     0     0  
     2     1     0  
     0     0     1
```

```
U1 = E21 * A
```

```
U1 =  
     1     4     2
```

0	0	7
0	1	1

第2行と第3行を交換

```
P32 = [1,0,0;0,0,1;0,1,0];
U = P32 * U1
```

```
U =
     1         4         2
     0         1         1
     0         0         7
```

```
c1 = E21 * b
```

```
c1 =
    -2
    28
     1
```

```
c = P32*c1
```

```
c =
    -2
     1
    28
```

$$\begin{bmatrix} 1 & 4 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 7 \end{bmatrix} x = \begin{bmatrix} -2 \\ 1 \\ 28 \end{bmatrix}$$

```
% 要 Symbolic Math Toolbox
syms x y z;
eqn = U * [x;y;z] == c;
solx = solve(eqn, x,y,z);
Answer = [solx.x;solx.y;solx.z]
```

```
Answer =
```

$$\begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$$

## 1.5.2

```
clear
A = [1,1;3,3]
```

```
A =
     1         1
     3         3
```

```
E21 = [1,0;-3,1]
```

```
E21 =  
      1      0  
     -3      1
```

```
U = E21*A
```

```
U =  
      1      1  
      0      0
```

```
% 要 Symbolic Math Toolbox  
syms b1 b2  
b = [b1;b2]
```

```
b =  

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

```

```
c = E21 * b
```

```
c =  

$$\begin{pmatrix} b_1 \\ b_2 - 3 b_1 \end{pmatrix}$$

```

よって  $0 = b_2 - 3b_1$  なので  $b_2 = 3b_1$

交換行列は  $P_{kl} = P_{kl}^{-1}$  となる

### P32 例

```
clear  
A = [1,2,3;2,4,2;0,1,1]
```

```
A =  
      1      2      3  
      2      4      2  
      0      1      1
```

```
E21 = eye(3);  
E21(2,1) = -2
```

```
E21 =  
      1      0      0  
     -2      1      0  
      0      0      1
```

```
U1 = E21 * A
```

```
U1 =
    1      2      3
    0      0     -4
    0      1      1
```

```
P23 = [1,0,0;0,0,1;0,1,0]
```

```
P23 =
    1      0      0
    0      0      1
    0      1      0
```

```
U = P23*U1
```

```
U =
    1      2      3
    0      1      1
    0      0     -4
```

```
PA = P23*A
```

```
PA =
    1      2      3
    0      1      1
    2      4      2
```

```
E31 = eye(3);
E31(3,1) = -2
```

```
E31 =
    1      0      0
    0      1      0
   -2      0      1
```

```
U = E31 * PA
```

```
U =
    1      2      3
    0      1      1
    0      0     -4
```

```
L = E31^-1
```

```
L =
    1      0      0
    0      1      0
    2      0      1
```

### 1.5.3

```
clear
A = [0,1;2,3]
```

```
A =
```

$$\begin{array}{cc} 0 & 1 \\ 2 & 3 \end{array}$$

$$P21 = [0,1;1,0]$$

$$P21 = \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}$$

$$PA = P21 * A$$

$$PA = \begin{array}{cc} 2 & 3 \\ 0 & 1 \end{array}$$

$$L = [1,0;0,1]$$

$$L = \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}$$

$$D = [2,0;0,1]$$

$$D = \begin{array}{cc} 2 & 0 \\ 0 & 1 \end{array}$$

$$U = [1,3/2;0,1]$$

$$U = \begin{array}{cc} 1 & 3/2 \\ 0 & 1 \end{array}$$

#### 1.5.4

$$\begin{array}{l} \text{clear} \\ A = [1,1,1;1,1,2;1,2,5] \end{array}$$

$$A = \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 5 \end{array}$$

$$E21 = [1,0,0;-1,1,0;0,0,1]$$

$$E21 = \begin{array}{ccc} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

$$U1 = E21 * A$$

$$U1 = \begin{array}{ccc} 1 & 1 & 1 \\ 0 & 0 & 1 \end{array}$$

1	2	5
---	---	---

P32 = [1,0,0;0,0,1;0,1,0]

P32 =

1	0	0
0	0	1
0	1	0

U2 = P32 \* U1

U2 =

1	1	1
1	2	5
0	0	1

U3 = E21 \* U2

U3 =

1	1	1
0	1	4
0	0	1

=====

PA = P32 \* A

PA =

1	1	1
1	2	5
1	1	2

PU1 = E21 \* PA

PU1 =

1	1	1
0	1	4
1	1	2

E31 = eye(3);  
E31(3,1) = -1

E31 =

1	0	0
0	1	0
-1	0	1

U = E31 \* PU1

U =

1	1	1
0	1	4
0	0	1

## 1.5.5

その1

```
clear  
A = [0,1,-1;1,-1,0;1,0,-1]
```

```
A =  
      0      1     -1  
      1     -1      0  
      1      0     -1
```

```
b = [2;2;2]
```

```
b =  
      2  
      2  
      2
```

```
P21 = [0,1,0;1,0,0;0,0,1]
```

```
P21 =  
      0      1      0  
      1      0      0  
      0      0      1
```

```
PA = P21 * A
```

```
PA =  
      1     -1      0  
      0      1     -1  
      1      0     -1
```

```
E31 = eye(3);  
E31(3,1) = -1
```

```
E31 =  
      1      0      0  
      0      1      0  
     -1      0      1
```

```
U = E31 * PA
```

```
U =  
      1     -1      0  
      0      1     -1  
      0      1     -1
```

```
c = E31 * b
```

```
c =  
      2  
      2  
      0
```

2行目と3行目が同じなのに**b**は異なるので特異&解なし

その2

```
clear
A = [0,1,-1;1,-1,0;1,0,-1]
```

```
A =
```

0	1	-1
1	-1	0
1	0	-1

```
b = [0;0;0]
```

```
b =
```

0
0
0

```
P21 = [0,1,0;1,0,0;0,0,1]
```

```
P21 =
```

0	1	0
1	0	0
0	0	1

```
PA = P21 * A
```

```
PA =
```

1	-1	0
0	1	-1
1	0	-1

```
E31 = eye(3);
E31(3,1) = -1
```

```
E31 =
```

1	0	0
0	1	0
-1	0	1

```
U = E31 * PA
```

```
U =
```

1	-1	0
0	1	-1
0	1	-1

```
c = E31 * b
```

```
c =
```

0
0
0

特異&解は  $u = v = w$

逆行列



$BA = I$ かつ $AC = I$ ならば $B = C$

つまり  $AA^{-1} = A^{-1}A = I$

$$(AB)^{-1} = B^{-1}A^{-1}$$

### P34 例

```
clear
A = [2,1,1,1,0,0;4,1,0,0,1,0;-2,2,1,0,0,1]
```

A =

2	1	1	1	0	0
4	1	0	0	1	0
-2	2	1	0	0	1

```
E21 = eye(3);
E21(2,1) = -2
```

E21 =

1	0	0
-2	1	0
0	0	1

```
U1 = E21*A
```

U1 =

2	1	1	1	0	0
0	-1	-2	-2	1	0
-2	2	1	0	0	1

```
E31 = eye(3);
E31(3,1) = 1
```

E31 =

1	0	0
0	1	0
1	0	1

```
U2 = E31 * U1
```

U2 =

2	1	1	1	0	0
0	-1	-2	-2	1	0
0	3	2	1	0	1

```
E32 = eye(3);
E32(3,2) = 3
```

E32 =

1	0	0
0	1	0
0	3	1

```
U = E32 * U2
```

U =

2	1	1	1	0	0
0	-1	-2	-2	1	0
0	0	-4	-5	3	1

最初の3列がいつもの上3角行列  $U$ 、後ろの3列が  $L^{-1}$

つまり  $L^{-1} = E_{32}E_{31}E_{21}$

## Gauss-Jordan法

```
clear
```

$$AI = [A, I]$$

```
AI = [2,1,1,1,0,0;0,-1,-2,-2,1,0;0,0,-4,-5,3,1]
```

A =

2	1	1	1	0	0
0	-1	-2	-2	1	0
0	0	-4	-5	3	1

第2列のピボットより上を0にする

```
E12 = eye(3);  
E12(1,2)=1
```

E12 =

1	1	0
0	1	0
0	0	1

```
U1 = E12*AI
```

U1 =

2	0	-1	-1	1	0
0	-1	-2	-2	1	0
0	0	-4	-5	3	1

第3列のピボットより上を0にする

```
E13 = eye(3);  
E13(1,3) = -1/4
```

E13 =

1	0	-1/4
0	1	0
0	0	1

```
U2 = E13 * U1
```

U2 =

2	0	0	1/4	1/4	-1/4
---	---	---	-----	-----	------

0	-1	-2	-2	1	0
0	0	-4	-5	3	1

```
E23 = eye(3);
E23(2,3) = -1/2
```

E23 =

1	0	0
0	1	-1/2
0	0	1

```
U3 = E23 * U2
```

U3 =

2	0	0	1/4	1/4	-1/4
0	-1	0	1/2	-1/2	-1/2
0	0	-4	-5	3	1

各ピボットで割り算を行う

```
pdiv = [1/2,0,0;0,-1,0;0,0,-1/4]
```

pdiv =

1/2	0	0
0	-1	0
0	0	-1/4

```
U = pdiv * U3
```

U =

1	0	0	1/8	1/8	-1/8
0	1	0	-1/2	1/2	1/2
0	0	1	5/4	-3/4	-1/4

$$U = [I, A^{-1}]$$

## 1.5.6

$AB^{-1}C$  の逆行列

$$(AB^{-1}C)^{-1} = C^{-1}BA^{-1}$$

## 1.5.7

```
A = [0,1;1,0]
```

A =

0	1
1	0

```
A^-1
```

```
ans =  
      0      1  
      1      0
```

```
A*A
```

```
ans =  
      1      0  
      0      1
```

$$\mathbf{B} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\mathbf{B}^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

### 1.5.8

```
clear  
A = [1,1;3,3]
```

```
A =  
      1      1  
      3      3
```

```
B = eye(2)
```

```
B =  
      1      0  
      0      1
```

```
AB = [A,B]
```

```
AB =  
      1      1      1      0  
      3      3      0      1
```

```
E21 = eye(2);  
E21(2,1) = -3
```

```
E21 =  
      1      0  
     -3      1
```

```
U1 = E21*AB
```

```
U1 =  
      1      1      1      0  
      0      0     -3      1
```

特異になってしまうため逆行列が作れない

A<sup>-1</sup>

警告：行列は、特異行列に近いが、正しくスケーリングされていません。結果は不正確になる可能性があります。RCOND = 6.938894e-18。

ans =

*	*
*	*

## 1.5.9

$$E = \begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$E$  は3行目の8倍を1行目に加える行列

```
clear
```

```
E = [1,0,8;0,1,0;0,0,1]
```

E =

1	0	8
0	1	0
0	0	1

E<sup>-1</sup>

ans =

1	0	-8
0	1	0
0	0	1

```
A = [E,eye(3)]
```

A =

1	0	8	1	0	0
0	1	0	0	1	0
0	0	1	0	0	1

```
E13 = eye(3);
```

```
E13(1,3) = -8
```

E13 =

1	0	-8
0	1	0
0	0	1

```
U = E13 * A
```

U =

1	0	0	1	0	-8
0	1	0	0	1	0
0	0	1	0	0	1

U(:,4:6)

```
ans =
```

1	0	-8
0	1	0
0	0	1

### 1.5.10

```
clear
syms a b c d;
A = [a,b;c,d]
```

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

```
A = [A,eye(2)]
```

$$A = \begin{pmatrix} a & b & 1 & 0 \\ c & d & 0 & 1 \end{pmatrix}$$

```
E21 = [1,0;-c/a,1]
```

$$E21 = \begin{pmatrix} 1 & 0 \\ -\frac{c}{a} & 1 \end{pmatrix}$$

```
U1 = E21*A
```

$$U1 = \begin{pmatrix} a & b & 1 & 0 \\ 0 & d - \frac{bc}{a} & -\frac{c}{a} & 1 \end{pmatrix}$$

```
U1(2,2) = (a*d-b*c)/a
```

$$U1 = \begin{pmatrix} a & b & 1 & 0 \\ 0 & \frac{ad-bc}{a} & -\frac{c}{a} & 1 \end{pmatrix}$$

```
E12 = [1,-(a*b)/(a*d-b*c);0,1]
```

$$E12 = \begin{pmatrix} 1 & -\frac{ab}{ad-bc} \\ 0 & 1 \end{pmatrix}$$

$$U2 = E12 * U1$$

$$U2 =$$

$$\begin{pmatrix} a & 0 & \frac{bc}{ad-bc} + 1 & -\frac{ab}{ad-bc} \\ 0 & \frac{ad-bc}{a} & -\frac{c}{a} & 1 \end{pmatrix}$$

$$U2(1,3) = (a*d)/(a*d-b*c)$$

$$U2 =$$

$$\begin{pmatrix} a & 0 & \frac{ad}{ad-bc} - \frac{ab}{ad-bc} \\ 0 & \frac{ad-bc}{a} & -\frac{c}{a} & 1 \end{pmatrix}$$

$$E11 = [1/a, 0; 0, 1]$$

$$E11 =$$

$$\begin{pmatrix} \frac{1}{a} & 0 \\ 0 & 1 \end{pmatrix}$$

$$U3 = E11 * U2$$

$$U3 =$$

$$\begin{pmatrix} 1 & 0 & \frac{d}{ad-bc} - \frac{b}{ad-bc} \\ 0 & \frac{ad-bc}{a} & -\frac{c}{a} & 1 \end{pmatrix}$$

$$E22 = [1, 0; 0, a/(a*d - b*c)]$$

$$E22 =$$

$$\begin{pmatrix} 1 & 0 \\ 0 & \frac{a}{ad-bc} \end{pmatrix}$$

$$U4 = E22 * U3$$

$$U4 =$$

$$\begin{pmatrix} 1 & 0 & \frac{d}{ad-bc} - \frac{b}{ad-bc} \\ 0 & 1 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{pmatrix}$$

$$Ain = U4(:, 3:4)$$

$$Ain =$$

$$\begin{pmatrix} \frac{d}{ad-bc} & -\frac{b}{ad-bc} \\ -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{pmatrix}$$

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

```
clear
A = [2,-1,0;-1,2,-1;0,-1,2]
```

```
A =
     2     -1     0
    -1     2    -1
     0    -1     2
```

```
A = [A,eye(3)]
```

```
A =
     2     -1     0     1     0     0
    -1     2    -1     0     1     0
     0    -1     2     0     0     1
```

```
E21 = eye(3);
E21(2,1) = 1/2
```

```
E21 =
     1     0     0
    1/2     1     0
     0     0     1
```

```
U1 = E21 * A
```

```
U1 =
     2     -1     0     1     0     0
     0    3/2    -1    1/2     1     0
     0    -1     2     0     0     1
```

```
E32 = eye(3);
E32(3,2) = 2/3
```

```
E32 =
     1     0     0
     0     1     0
     0    2/3     1
```

```
U2 = E32 * U1
```

```
U2 =
     2     -1     0     1     0     0
     0    3/2    -1    1/2     1     0
     0     0    4/3    1/3    2/3     1
```

```
E12 = eye(3);
E12(1,2) = 2/3
```



E12 =

1	2/3	0
0	1	0
0	0	1

U3 = E12 \* U2

U3 =

2	*	-2/3	4/3	2/3	0
0	3/2	-1	1/2	1	0
0	0	4/3	1/3	2/3	1

E13 = eye(3);  
E13(1,3) = 1/2

E13 =

1	0	1/2
0	1	0
0	0	1

U4 = E13 \* U3

U4 =

2	*	*	3/2	1	1/2
0	3/2	-1	1/2	1	0
0	0	4/3	1/3	2/3	1

E23 = eye(3);  
E23(2,3) = 3/4

E23 =

1	0	0
0	1	3/4
0	0	1

U5 = E23 \* U4

U5 =

2	*	*	3/2	1	1/2
0	3/2	*	3/4	3/2	3/4
0	0	4/3	1/3	2/3	1

E11 = eye(3);  
E11(1,1) = 1/2

E11 =

1/2	0	0
0	1	0
0	0	1

U6 = E11 \* U5

U6 =

1	*	*	3/4	1/2	1/4
0	3/2	*	3/4	3/2	3/4
0	0	4/3	1/3	2/3	1

```
E22 = eye(3);  
E22(2,2) = 2/3
```

```
E22 =  
      1      0      0  
      0      2/3    0  
      0      0      1
```

```
U7 = E22 * U6
```

```
U7 =  
      1      *      *      3/4      1/2      1/4  
      0      1      *      1/2      1      1/2  
      0      0      4/3     1/3     2/3      1
```

```
E33 = eye(3);  
E33(3,3) = 3/4
```

```
E33 =  
      1      0      0  
      0      1      0  
      0      0     3/4
```

```
U8 = E33 * U7
```

```
U8 =  
      1      *      *      3/4      1/2      1/4  
      0      1      *      1/2      1      1/2  
      0      0      1     1/4     1/2     3/4
```

```
Ain = U8(:,4:6)
```

```
Ain =  
      3/4      1/2      1/4  
      1/2      1      1/2  
      1/4      1/2     3/4
```

```
Ain * 4
```

```
ans =  
      3      2      1  
      2      4      2  
      1      2      3
```

$$\mathbf{A}^{-1} = 4 \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$