

## 2.2 未知数 $n$ の方程式 $m$ の解

長方形の行列に対する消去法

例

$$A = \begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 5 \\ -1 & -3 & 3 & 0 \end{bmatrix}$$

```
clear  
A = [1,3,3,2; 2,6,9,5; -1,-3,3,0]
```

```
A =  
     1     3     3     2  
     2     6     9     5  
    -1    -3     3     0
```

```
E21 = eye(3);  
E21(2,1) = -2
```

```
E21 =  
     1     0     0  
    -2     1     0  
     0     0     1
```

•

```
U1 = E21*A
```

```
U1 =  
     1     3     3     2  
     0     0     3     1  
    -1    -3     3     0
```

•

```
E31 = eye(3);  
E31(3,1) = 1
```

```
E31 =  
     1     0     0  
     0     1     0  
     1     0     1
```

```
U2 = E31 * U1
```

```
U2 =  
     1     3     3     2  
     0     0     3     1  
     0     0     6     2
```

```
E32 = eye(3);  
E32(3,2) = -2
```

```
E32 =
    1     0     0
    0     1     0
    0    -2     1
•
```

```
U3 = E32 * U2
```

```
U3 =
    1     3     3     2
    0     0     3     1
    0     0     0     0
•
```

ここで、U3は上台形行列となる

```
E32*E31*E21
```

```
ans =
    1     0     0
   -2     1     0
    5    -2     1
```

```
L = ans^-1
```

```
L =
    1.0000     0     0
    2.0000    1.0000     0
   -1.0000    2.0000    1.0000
```

```
L*U3
```

```
ans =
    1.0000    3.0000    3.0000    2.0000
    2.0000    6.0000    9.0000    5.0000
   -1.0000   -3.0000    3.0000    0.0000
•
```

$A = LU$ の関係は今まで通り成り立つ

行の交換を行った場合、交換行列を $P$ とすると $PA = LU$ の関係になる

$U$ は上台形行列

```
syms u v w y
x = [u;v;w;y]
```

```
x =
```

$$\begin{pmatrix} u \\ v \\ w \\ y \end{pmatrix}$$

```
exp = U3*x==[0;0;0]
```

exp =

$$\begin{pmatrix} u + 3v + 3w + 2y = 0 \\ 3w + y = 0 \\ 0 = 0 \end{pmatrix}$$

この時、 $u$ と $w$ は0でないピボットをもつ列に対応する未知数（基底変数）である

また、 $v$ と $y$ はピボットのない列に対応する未知数（自由変数）である

この方程式で一般な解を得るためには自由変数に任意の値を与えればよい

```
sol = solve(exp,u,w)
```

```
sol = フィールドをもつ struct:  
  u: [1x1 sym]  
  w: [1x1 sym]
```

```
x = [sol.u;v;sol.w;y]
```

x =

$$\begin{pmatrix} -3v - y \\ v \\ -\frac{y}{3} \\ y \end{pmatrix}$$

自由変数を含む解はこうなる

また、

$$x = \begin{bmatrix} -3v - y \\ v \\ -\frac{1}{3}y \\ y \end{bmatrix} = v \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} -1 \\ 0 \\ -\frac{1}{3} \\ 1 \end{bmatrix}$$

となり、すべての解はこの2つのベクトルの線形結合となる

## 2.2.1

$$\begin{bmatrix} a & b & c & d & e & f & g & h & i \\ 0 & j & k & l & m & n & o & p & q \\ 0 & 0 & r & s & t & u & v & w & x \\ 0 & 0 & 0 & y & z & aa & bb & cc & dd \\ 0 & 0 & 0 & 0 & ee & ff & gg & hh & ii \end{bmatrix}$$

```
clear
A = [1,1;0,0]
```

```
A =
     1     1
     0     0
```

•

```
B = [-1,0;0,1]
```

```
B =
    -1     0
     0     1
```

•

```
A + B
```

```
ans =
     0     1
     0     1
```

## P62

```
clear
syms u v w y b1 b2 b3
x = [u;v;w;y]
```

```
x =
      (
      u
      v
      w
      y
      )
```

```
b = [b1;b2;b3]
```

```
b =
      (
      b1
      b2
      b3
      )
```

```
A = [1,3,3,2; 2,6,9,5; -1,-3,3,0]
```

```
A =
```

$$\begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 5 \\ -1 & -3 & 3 & 0 \end{bmatrix}$$

•

```
E21 = eye(3);
E21(2,1) = -2
```

```
E21 =
     1     0     0
    -2     1     0
     0     0     1
```

•

```
U1 = E21*A
```

```
U1 =
     1     3     3     2
     0     0     3     1
    -1    -3     3     0
```

•

```
E31 = eye(3);
E31(3,1) = 1
```

```
E31 =
     1     0     0
     0     1     0
     1     0     1
```

•

```
U2 = E31 * U1
```

```
U2 =
     1     3     3     2
     0     0     3     1
     0     0     6     2
```

•

```
E32 = eye(3);
E32(3,2) = -2
```

```
E32 =
     1     0     0
     0     1     0
     0    -2     1
```

```
U3 = E32 * U2
```

```
U3 =
     1     3     3     2
     0     0     3     1
     0     0     0     0
```

•

```
E32*E31*E21
```

```
ans =
    1     0     0
   -2     1     0
    5    -2     1
```

```
L = ans^-1
```

```
L =
    1.0000         0         0
    2.0000    1.0000         0
   -1.0000    2.0000    1.0000
```

```
c = L^-1 * b
```

```
c =
    (
      b_1
      b_2 - 2 b_1
      5 b_1 - 2 b_2 + b_3
    )
```

$$\begin{bmatrix} 1 & 3 & 3 & 2 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} u \\ v \\ w \\ y \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 - 2 b_1 \\ 5 b_1 - 2 b_2 + b_3 \end{pmatrix}$$

## 2.2.2

$$\begin{aligned} x + y + z &= 1 \\ x + y + z &= 0 \end{aligned}$$

## 2.2.3

```
clear
A = [1,2,0,1;0,1,1,0;1,2,0,1]
```

```
A =
    1     2     0     1
    0     1     1     0
    1     2     0     1
```

```
E31 = eye(3);
E31(3,1) = -1
```

```
E31 =
    1     0     0
    0     1     0
   -1     0     1
```

•

$$U = E31 * A$$

$$U =$$

$$\begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

•

$$L = E31^{-1}$$

$$L =$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

•

```
syms u v w y
x = [u;v;w;y]
```

$$x =$$

$$\begin{pmatrix} u \\ v \\ w \\ y \end{pmatrix}$$

$$\text{eqn} = U * x == 0$$

$$\text{eqn} =$$

$$\begin{pmatrix} u + 2v + y = 0 \\ v + w = 0 \\ 0 = 0 \end{pmatrix}$$

$$\text{sol} = \text{solve}(\text{eqn}, u, v)$$

```
sol = フィールドをもつ struct:
u: [1x1 sym]
v: [1x1 sym]
```

$$[\text{sol}.u; \text{sol}.v; w; y]$$

$$\text{ans} =$$

$$\begin{pmatrix} 2w - y \\ -w \\ w \\ y \end{pmatrix}$$

$$x = \begin{bmatrix} 2w - y \\ -w \\ w \\ y \end{bmatrix} = w \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix} + y \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

## 2.2.4

```
clear
A = [0,1,4,0;0,2,8,0]
```

```
A =
     0     1     4     0
     0     2     8     0
```

```
E21 = eye(2);
E21(2,1) = -2
```

```
E21 =
     1     0
    -2     1
```

•

```
U = E21 * A
```

```
U =
     0     1     4     0
     0     0     0     0
```

•

```
L = E21^-1
```

```
L =
     1     0
     2     1
```

```
L*U
```

```
ans =
     0     1     4     0
     0     2     8     0
```

```
syms u v w y
x = [u;v;w;y]
```

```
x =
```

$$\begin{pmatrix} u \\ v \\ w \\ y \end{pmatrix}$$



```
eqn = U*x == 0
```

eqn =

$$\begin{pmatrix} v + 4w = 0 \\ 0 = 0 \end{pmatrix}$$

$$x = \begin{bmatrix} u \\ -4w \\ w \\ y \end{bmatrix}$$

```
syms b1 b2  
b = [b1;b2]
```

b =

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

```
L^-1*b
```

ans =

$$\begin{pmatrix} b_1 \\ b_2 - 2b_1 \end{pmatrix}$$

```
eqnv = v + 4*w == b1
```

eqnv =  $v + 4w = b_1$

```
solve(eqnv,v)
```

ans =  $b_1 - 4w$

$$x = \begin{bmatrix} u \\ b_1 - 4w \\ w \\ y \end{bmatrix} = u \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + w \begin{bmatrix} 0 \\ -4 \\ 1 \\ 0 \end{bmatrix} + y \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ b_1 \\ 0 \\ 0 \end{bmatrix}$$

## 2.2.5

```
clear  
A = [0,0;1,2;4,8;0,0]
```

A =

$$\begin{bmatrix} 0 & 0 \\ 1 & 2 \\ 4 & 8 \\ 0 & 0 \end{bmatrix}$$

0 0

```
P12 = [0,1,0,0;1,0,0,0;0,0,1,0;0,0,0,1]
```

P12 =

0	1	0	0
1	0	0	0
0	0	1	0
0	0	0	1

```
PA = P12*A
```

PA =

1	2
0	0
4	8
0	0

```
P23 = [1,0,0,0;0,0,1,0;0,1,0,0;0,0,0,1]
```

P23 =

1	0	0	0
0	0	1	0
0	1	0	0
0	0	0	1

```
PA = P23 * PA
```

PA =

1	2
4	8
0	0
0	0

```
E21 = eye(4);  
E21(2,1) = -4
```

E21 =

1	0	0	0
-4	1	0	0
0	0	1	0
0	0	0	1

```
U = E21 * PA
```

U =

1	2
0	0
0	0
0	0

```
syms u v  
eqn = U * [u;v]
```

eqn =

$$\begin{pmatrix} u + 2 v \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$u == -2*v$$

$$\text{ans} = u = -2 v$$

$$x = \begin{bmatrix} -2v \\ v \end{bmatrix} = v \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$\text{syms } b_1 \text{ } b_2 \text{ } b_3 \text{ } b_4$$

$$b = [b_1;b_2;b_3;b_4]$$

$$b =$$

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

$$c = P23 * (P12*b)$$

$$c =$$

$$\begin{pmatrix} b_2 \\ b_3 \\ b_1 \\ b_4 \end{pmatrix}$$

$$c = E21 * c$$

$$c =$$

$$\begin{pmatrix} b_2 \\ b_3 - 4 b_2 \\ b_1 \\ b_4 \end{pmatrix}$$

$$\text{eqnv} = \text{eqn} == c$$

$$\text{eqnv} =$$

$$\begin{pmatrix} u + 2 v = b_2 \\ 0 = b_3 - 4 b_2 \\ 0 = b_1 \\ 0 = b_4 \end{pmatrix}$$

$$x = \begin{bmatrix} -2v + b_2 \\ v \end{bmatrix} = v \begin{bmatrix} -2 \\ 1 \end{bmatrix} + \begin{bmatrix} b_2 \\ 0 \end{bmatrix}$$

### 2.2.6

```
clear
A = [1,2,2;2,4,5]
```

```
A =
     1     2     2
     2     4     5
```

```
E21 = eye(2);
E21(2,1) = -2
```

```
E21 =
     1     0
    -2     1
```

```
U = E21 * A
```

```
U =
     1     2     2
     0     0     1
```

```
syms u v w
x = [u;v;w]
```

```
x =
      (
      u
      v
      w
      )
```

```
b = [1;4]
```

```
b =
     1
     4
```

•

```
c = E21 * b
```

```
c =
     1
     2
```

•

```
eqn = U*x == c
```

```
eqn =
```

$$\begin{pmatrix} u + 2v + 2w = 1 \\ w = 2 \end{pmatrix}$$

```
solu = solve(eqn(1),u)
```

```
solu = 1 - 2w - 2v
```

```
u == 1- 2*2-2*v
```

```
ans = u = -2v - 3
```

$$x = v \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}$$

## 2.2.7

```
clear
A = [1,0;0,1;2,3]
```

```
A =
     1     0
     0     1
     2     3
```

```
E31 = eye(3);
E31(3,1) = -2
```

```
E31 =
     1     0     0
     0     1     0
    -2     0     1
```

```
U = E31 * A
```

```
U =
     1     0
     0     1
     0     3
```

```
E32 = eye(3)
```

```
E32 =
     1     0     0
     0     1     0
     0     0     1
```

```
E32(3,2) = -3
```

```
E32 =
     1     0     0
     0     1     0
```

$$\begin{array}{ccc} 0 & -3 & 1 \end{array}$$

$$U = E32 * U$$

$$U = \begin{array}{cc} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{array}$$

```
syms b1 b2 b3
b = [b1;b2;b3]
```

$$b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$c = E32 * (E31 * b)$$

$$c = \begin{pmatrix} b_1 \\ b_2 \\ b_3 - 3b_2 - 2b_1 \end{pmatrix}$$

$$\text{eqn} = c(3) == 0$$

$$\text{eqn} = b_3 - 3b_2 - 2b_1 = 0$$

## 2.2.8

```
clear
A = [1,1,2;2,3,-1;3,4,1]
```

$$A = \begin{array}{ccc} 1 & 1 & 2 \\ 2 & 3 & -1 \\ 3 & 4 & 1 \end{array}$$

```
E21 = eye(3);
E21(2,1) = -2
```

$$E21 = \begin{array}{ccc} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

$$U = E21 * A$$

$$U = \begin{array}{ccc} 1 & 1 & 2 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{array}$$

3      4      1

```
E31 = eye(3);  
E31(3,1) = -3
```

```
E31 =  
     1     0     0  
     0     1     0  
    -3     0     1
```

```
U = E31 * U
```

```
U =  
     1     1     2  
     0     1    -5  
     0     1    -5
```

```
E32 = eye(3);  
E32(3,2) = -1
```

```
E32 =  
     1     0     0  
     0     1     0  
     0    -1     1
```

```
U = E32 * U
```

```
U =  
     1     1     2  
     0     1    -5  
     0     0     0
```

•

```
L = (E32*E31*E21)^-1
```

```
L =  
     1     0     0  
     2     1     0  
     3     1     1
```

•

```
syms c  
b = [2;5;c]
```

```
b =  

$$\begin{pmatrix} 2 \\ 5 \\ c \end{pmatrix}$$

```

```
cc = L^-1 * b
```

```
cc =
```

$$\begin{pmatrix} 2 \\ 1 \\ c-7 \end{pmatrix}$$

```
solve(cc(3)==0,c)
```

```
ans = 7
```