

## PRIVACY:

2.0

(APP2):

Idea: Operations on encrypted data making the decrypted result as the wanted aggregate.

- Each voter can check his vote is unmodified. **VERIFIABILITY**
- Each voter can compute the results (tally). **AGGREGATE VISIBILITY**
- Each voter can be the only one who knows for who he voted. **PRIVACY**

~~We need to break~~ ~~break~~ VERIFIABILITY / PRIVACY

Trust independence towards voters and institutions.

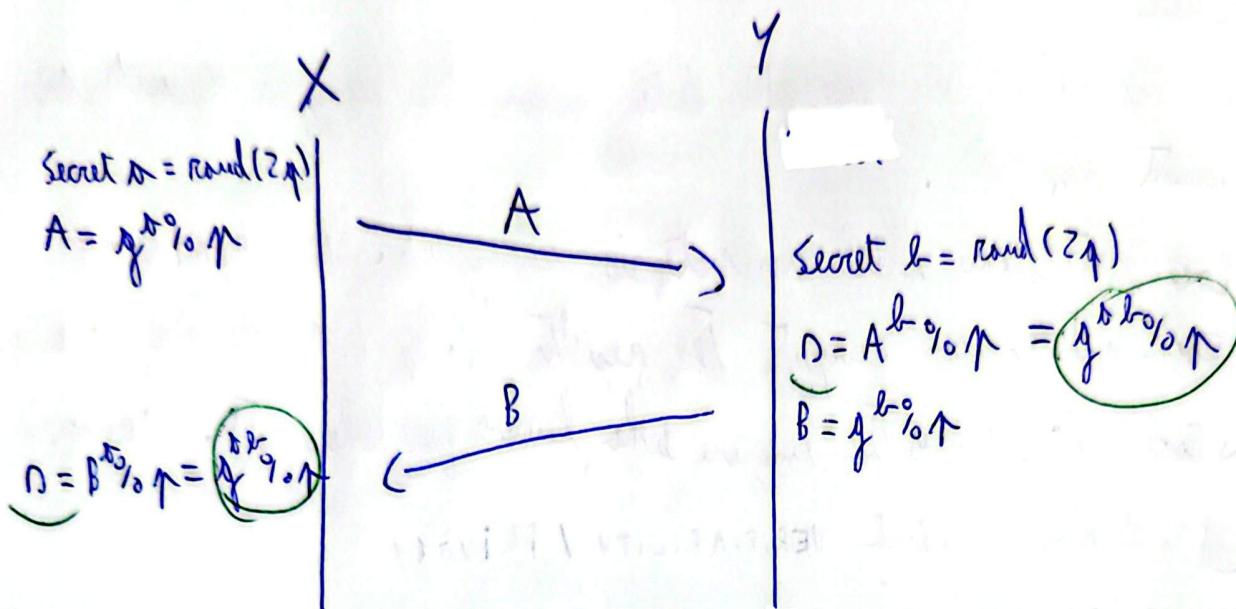
Distributed Homomorphic Public Key Encryption:

(ELGAMAL, RSA)

- Public Key Encryption: Anyone can encrypt a message with the public key but only the holder of the private key can decrypt it.
- Homomorphic Encryption: Allows computations on encrypted data without decrypting it.  $\text{Enc}(m_1) \times \text{Enc}(m_2) = \text{Enc}(m_1 + m_2)$
- Distributed Key Management: Private key split across different parties. Parties must collaborate to be able to decrypt.

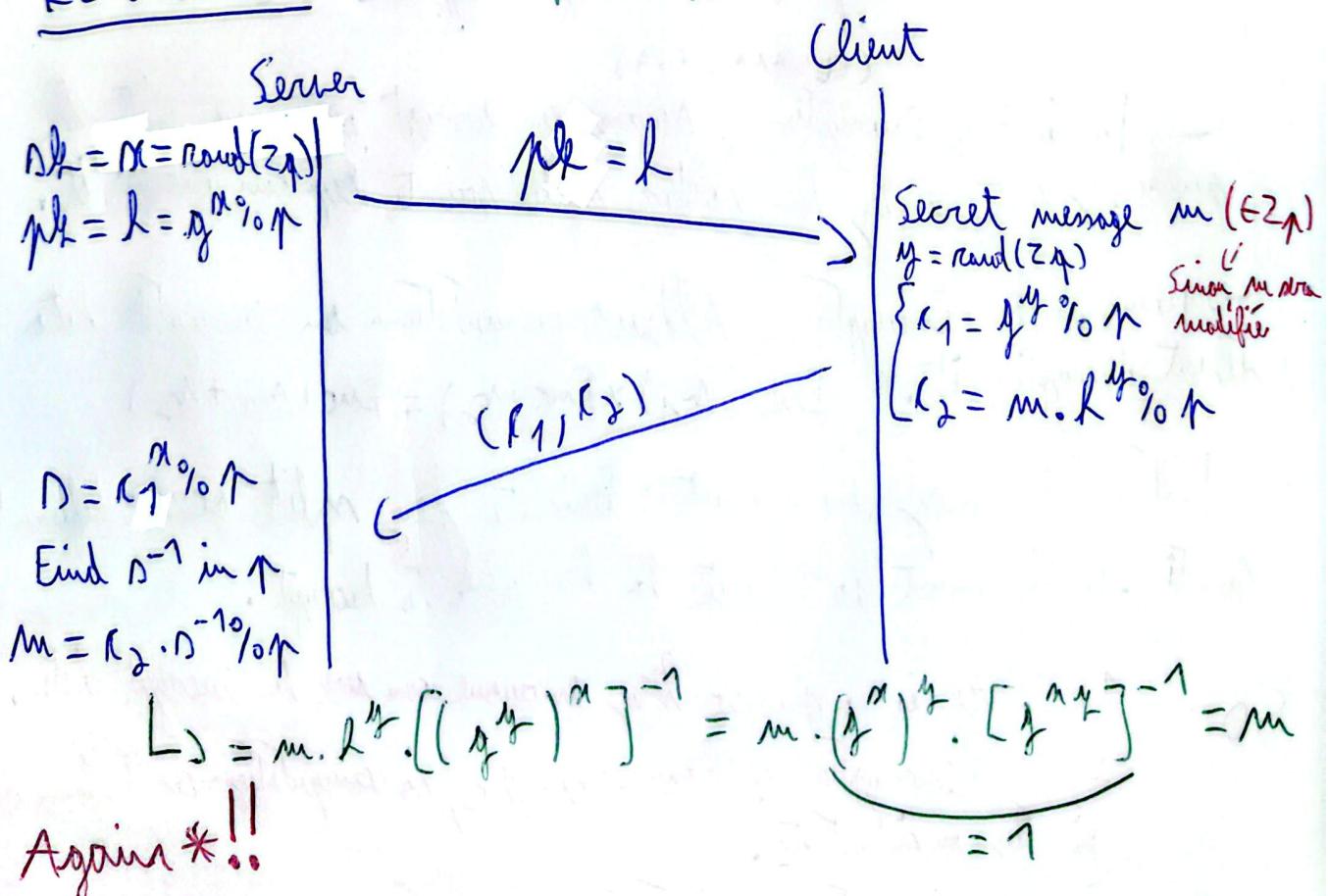
- So:
- 1) There is a public key everyone can use to encrypt data.
  - 2) The encryption is homomorphic, so computations can be done directly on ciphertexts.
  - 3) The decryption is distributed among multiple parties, so only a collaboration of them can decrypt the result.

## Diffie - Hellman: Agreement on a shared secret key. 2. 7



\* Only a discrete log can break the algo but it's NP-HARD to compute.

## EL GAMAL: Public Key Encryption



Euler's Little: Efficiently find the inverse of a number in a modular group. 2.2

$$((\mathbb{Z}_n)^\times \setminus \{1\}) \cong \mathbb{Z}_{n-1}$$

constant

$$\mathbb{Z}_{n-1} = \{1, 2, \dots, (n-1)\} \pmod{n}$$

$$\text{Remember: } a, b \in \mathbb{Z}_n \Rightarrow (a \cdot b) \pmod{n} = r \in \mathbb{Z}$$

$$\text{So: } (\prod_{i=1}^{n-1} \mathbb{Z}_n^{\setminus \{i\}})^{\frac{1}{n-1}} = (\prod_{i=1}^{n-1} (\mathbb{Z}_n^{\setminus \{i\}})^{\frac{1}{n-1}})^{\frac{1}{n-1}} \stackrel{\substack{\text{As } (\cdot) \text{ has an inverse in } \mathbb{Z}_n \\ (n-1) \text{ is prime}}}{} \stackrel{\substack{\text{This multiplication} \\ \text{is just a} \\ \text{permutation.}}}{=} ((1 \cdot 1) \times (2 \cdot 1) \times \dots \times ((n-1) \cdot 1))^{\frac{1}{n-1}} \pmod{n}$$

$$= (n-1)! \pmod{n}$$

$$= \frac{(n-1)!}{(n-1)} \pmod{n}$$

$$\Rightarrow (n-1)! \pmod{n} = (n-1) \cdot (n-2) \cdot \dots \cdot 1 \pmod{n}$$

$$\Rightarrow 1 \pmod{n} = (n-1) \pmod{n}$$

$$\Rightarrow 1 \cdot (-1) \pmod{n} = (n-1) \cdot (-1) \pmod{n}$$

$$\boxed{\Rightarrow (-1) \pmod{n} = (n-2) \pmod{n}}$$

## Expo ElGamal: for message $m \in \{0, 1\}$

### Simple ElGamal w/o

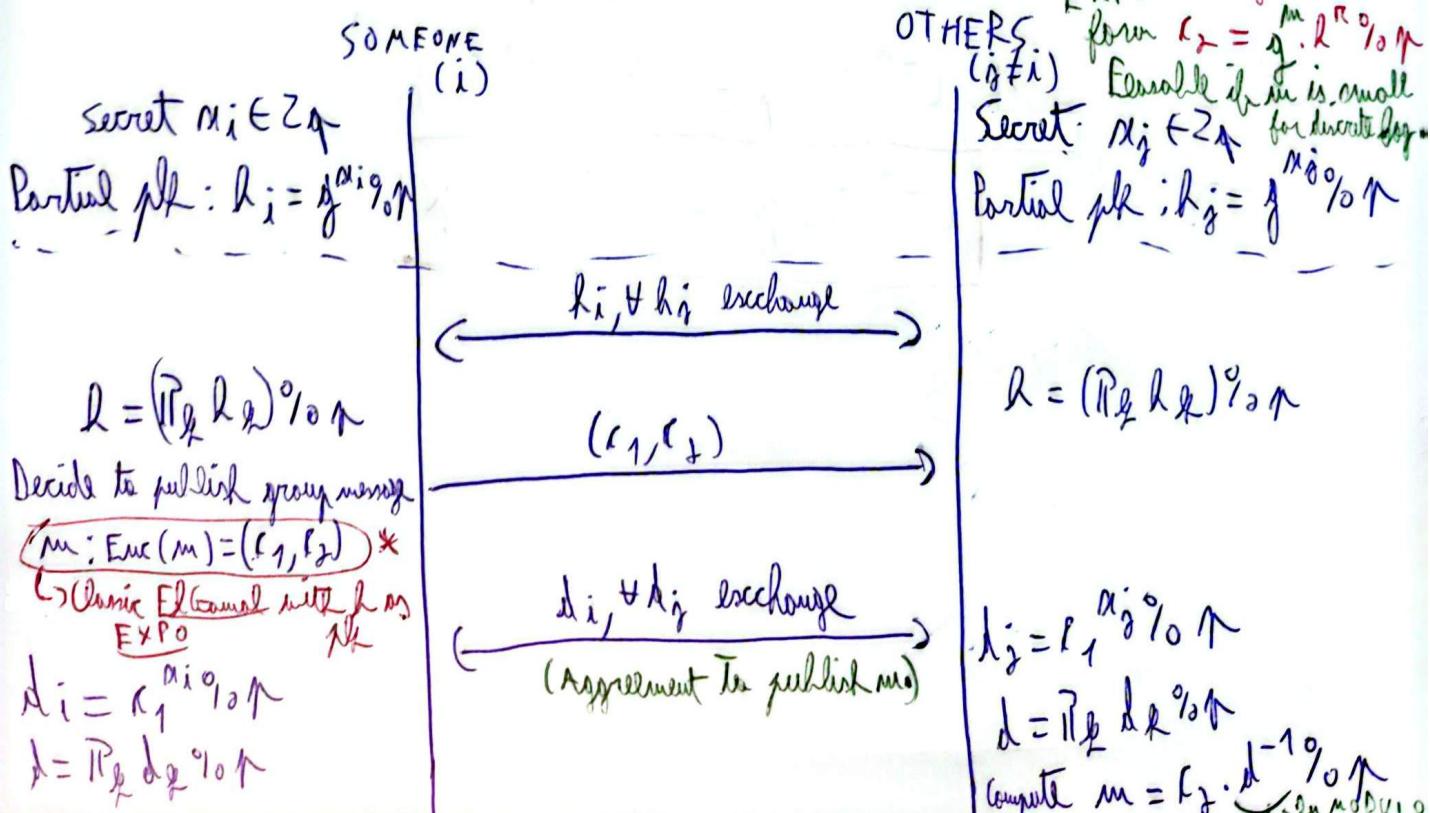
$$\text{Classic ElGamal} + m' = g^{m \% p} / m \in \{0, 1\}$$

### Distributed Key Generation ElGamal:

In DKG for ElGamal, a group of people jointly computes together a shared public key, and the private key is split into shares.

Only a threshold of participants working together can decrypt a ciphertext. An individual participant cannot decrypt alone with their own share.

Once all trustees publish their partial decryption, any outsider can recover the message. Only security here is joint consent but there is not security after consent.

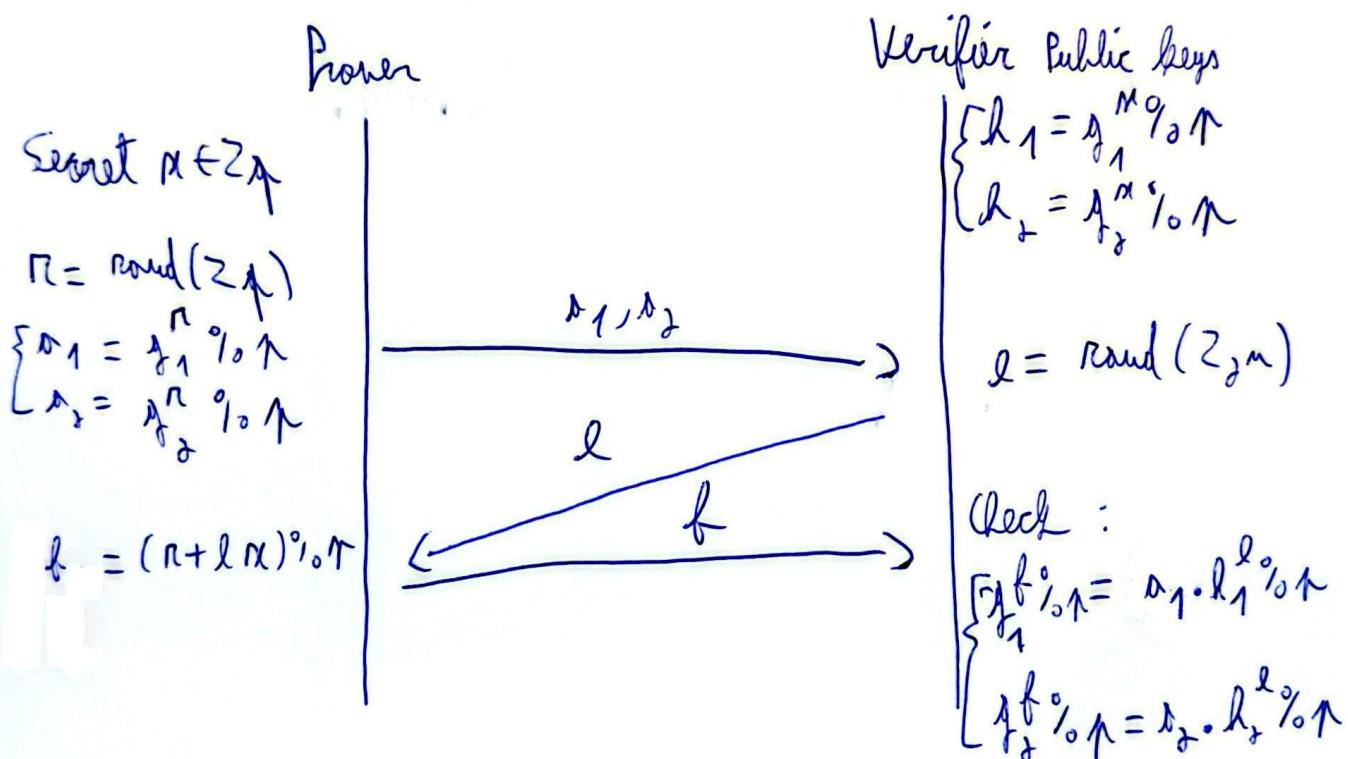


Chow - Pederson:

Schnorr proves knowledge of a discrete log.  
 Chow - Pederson proves equality of discrete logarithms. → Chow - Pederson is a stronger proof.  
 Both without revealing  $\alpha$ .

Completeness: If a user is legitimate, he'll always succeed maths.

Soundness: If I can prove 5.32

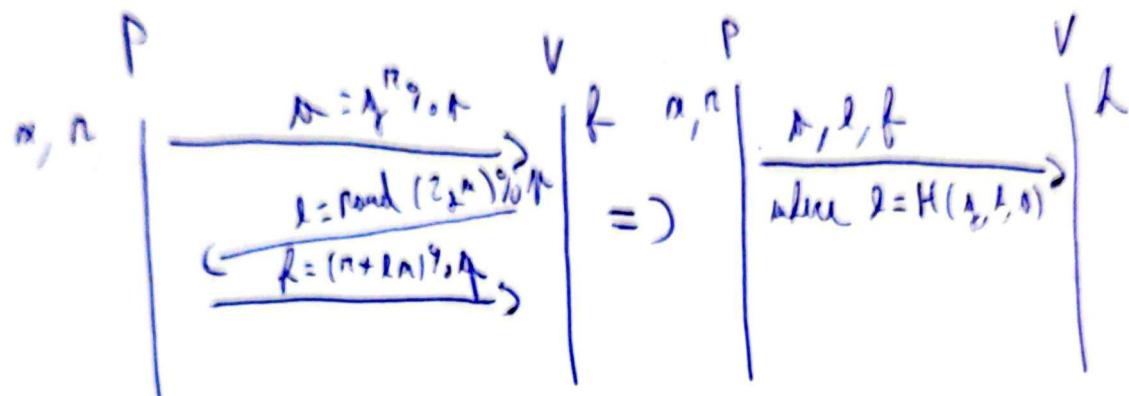


Two groups  $\mathbb{Z}_{p_1}$  and  $\mathbb{Z}_{p_2}$  have different  $g_i$  but common  $p$  and  $q$ .

From  $G = (g, p, q)$  you can find  $G' = (g', p, q)$  as:  
 $r = \text{rand}(\mathbb{Z}_{p_1}^*)$  ( $0 < r < p$ )  $\wedge \text{gcd}(r, p) = 1$   
 $g' = g^r \% p$

## Non-Interactive ZK - Knowledge Proof:

Idea: The prover computes a predictable challenge.



Challenge  $\beta$  becomes an hash function value.