

HW # 7

17% OS,

$$T_s \text{ comp} = T_s / 1.5$$

$$1) \#s = \frac{K(s+0.1)}{s^2(s+3)(s+8)}$$

$$z = \frac{-\ln(.17)}{\sqrt{\pi^2 + \ln^2(.17)}} = \frac{1.772}{3.607} = .491$$

a) plot

RL

$$SR \quad T_s = 10.93s$$

$$b) K = 42.1$$

$$T_s \text{ comp} = 7.287s$$

$$= \frac{4}{\sigma_d \text{ comp}}$$

$$c) \sigma_d \text{ comp} = .5489$$

$$w_d \text{ comp} = w_d \frac{\sigma_d \text{ comp}}{\sigma_d} = 1.85 \frac{.5489}{1.04} = .9765$$

$$\text{new pole} : -\sigma_d \pm jw_d = -.5489 \pm j.9765$$

d) compensating zero

$$\theta = \left[\tan^{-1} \left(\frac{.9765}{-.5489 + j} \right) + 180 \right] - 2 \left[\tan^{-1} \left(\frac{.9765}{-.5489} \right) + 180 \right] - \left[\tan^{-1} \left(\frac{.9765}{3 - .5489} \right) \right] - \left[\tan^{-1} \left(\frac{.9765}{8 - .5489} \right) \right]$$

$$\theta_c - 153.181 = (K+1) 180^\circ$$

$$\theta_c = 180 - \theta = 180 + 153.181 = 333.1814$$

$$\tan \theta_c = \frac{w_d}{z_0 - \sigma_d}$$

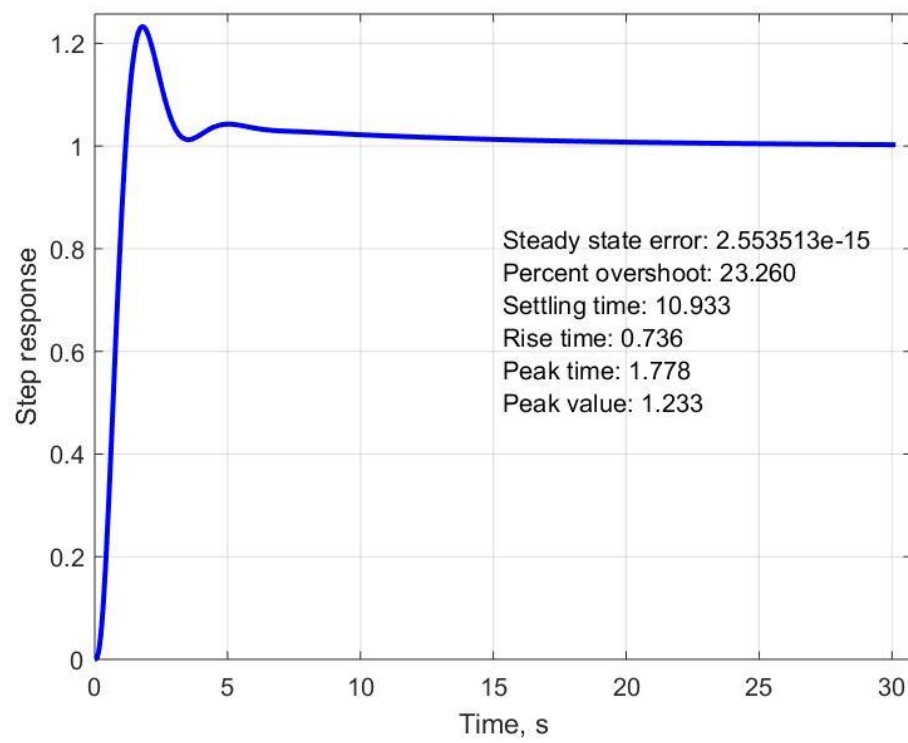
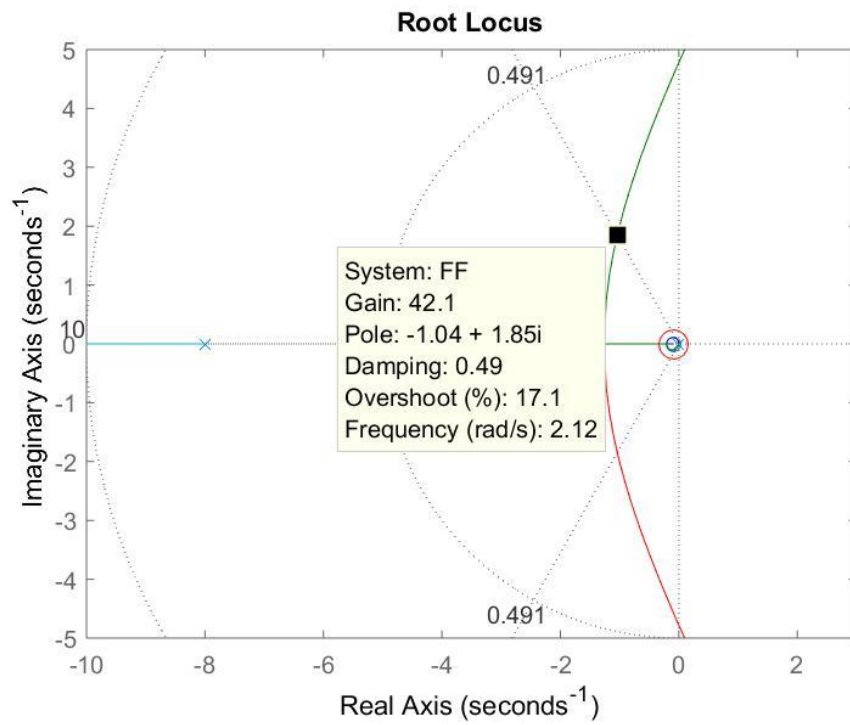
$$z_0 = \frac{w_d}{\tan \theta_c} + \sigma_d = \frac{.9765}{\tan(333.1814)} + (-.5489) = -1.382$$

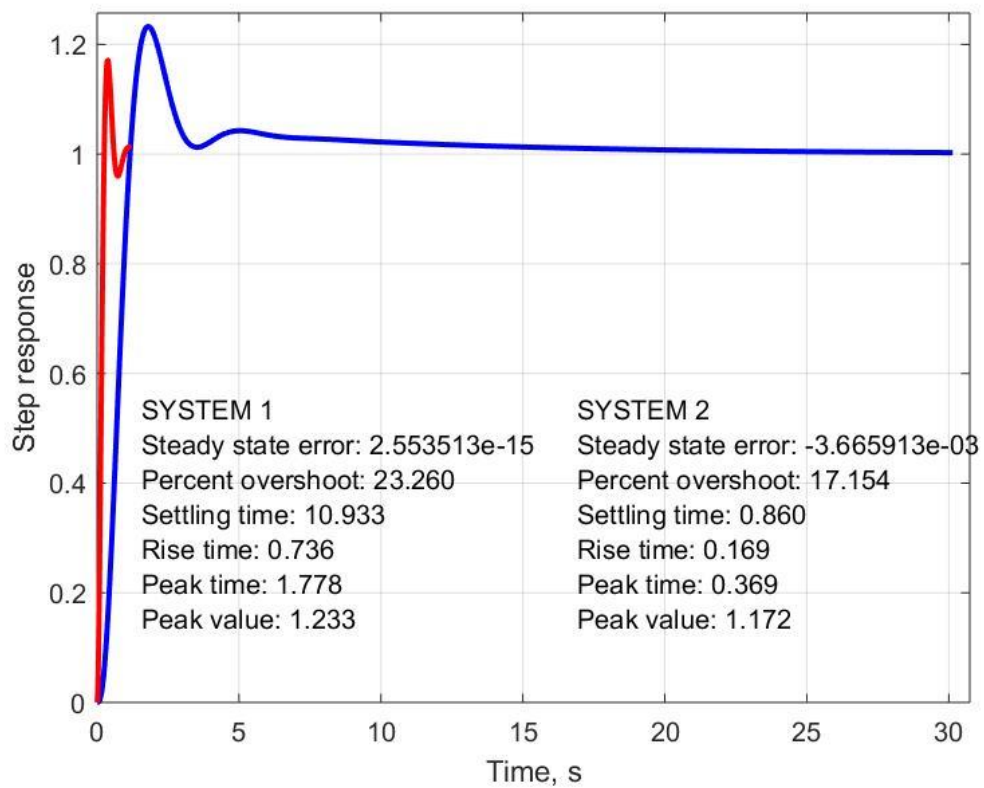
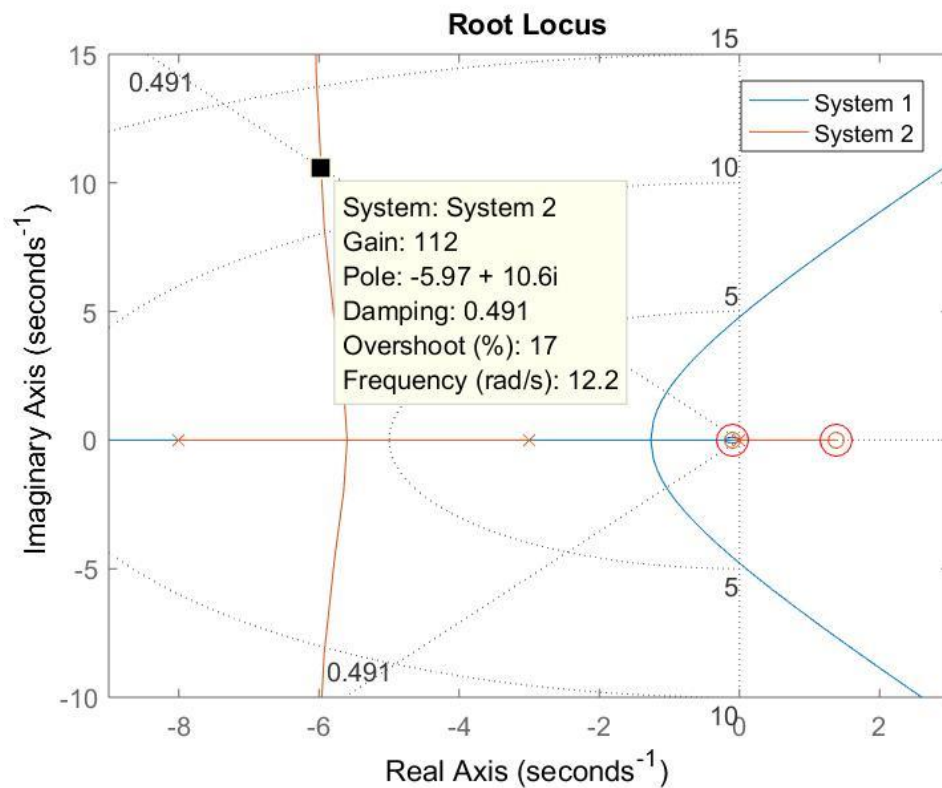
$$K = \frac{|s+3||s+8||s|}{|s+0.1||s-1.382|} \text{ for } -.5489 \pm j.9765 = \frac{(2.638)(7.515)(1.12)^2}{(1.075)(2.164)} = 10.7$$

K =

plot, K = 88.1

$$f) G_c(s) = s - 1.383$$





HW#7

#2) $G(s) = \frac{K}{(s^2 + 4s + 8)(s+10)}$ w 30% OS + $T_s = 1s$

$$\zeta = \frac{-\ln(0.3)}{\sqrt{\pi^2 + \ln^2(0.3)}} = \frac{1.204}{3.364} = 0.358$$

a) plot

b) $K = 83.8$

$$\tau_s = 2.679$$

$$\tau_{s \text{ comp}} = 1s = \frac{4}{\sigma_d \text{ comp}} = 4$$

$$\sigma_d \text{ comp} = 4$$

$$W_d \text{ comp} = W_d \frac{\sigma_d \text{ comp}}{\sigma_d} = \frac{3.8 (4)}{1.52} = 10$$

$$\text{new pole: } -\sigma_d \pm j W_d = -4 \pm j(10)$$

c) compensating zero PD

$$s^2 + 4s + 8, \quad \lambda = \frac{-4 \pm \sqrt{16 - 4(8)}}{2} = \frac{-4 \pm \sqrt{-16}}{2} = \frac{-4 \pm j4}{2} = -2 \pm j2$$

$$G(s) = \frac{K}{(s+2-j2)(s+2+j2)(s+10)}$$

$$\theta_c = -\left[\tan^{-1}\left(\frac{10-2}{-4+2}\right) + 180^\circ\right] - \left[\tan^{-1}\left(\frac{10+2}{-4+2}\right) + 180^\circ\right] - \left[\tan^{-1}\left(\frac{10}{-4+10}\right)\right]$$

$$\theta_c = 262.535 = (2k+1)180^\circ$$

$$\theta_c = -180 + 262.535 = 82.535$$

$$Z_D = \frac{W_d}{\tan \theta_c} + \sigma_d = \frac{10}{\tan(82.535)} + 4 = 5.310$$

$$G_c^{PD}(s) = s + 5.31$$

$$K = \frac{|s+2-j2||s+2+j2||s+10|}{|s+5.31|} \text{ for } -4+j10 = \frac{|8.246||12.166||11.64|}{|110.091|} = 116$$

d) plot

$$G(s) = \frac{K}{(s^2 + 4s + 8)(s + 10)}$$

e) PI

$$G_c^{PI}(s) = \frac{s + z_1}{s}$$

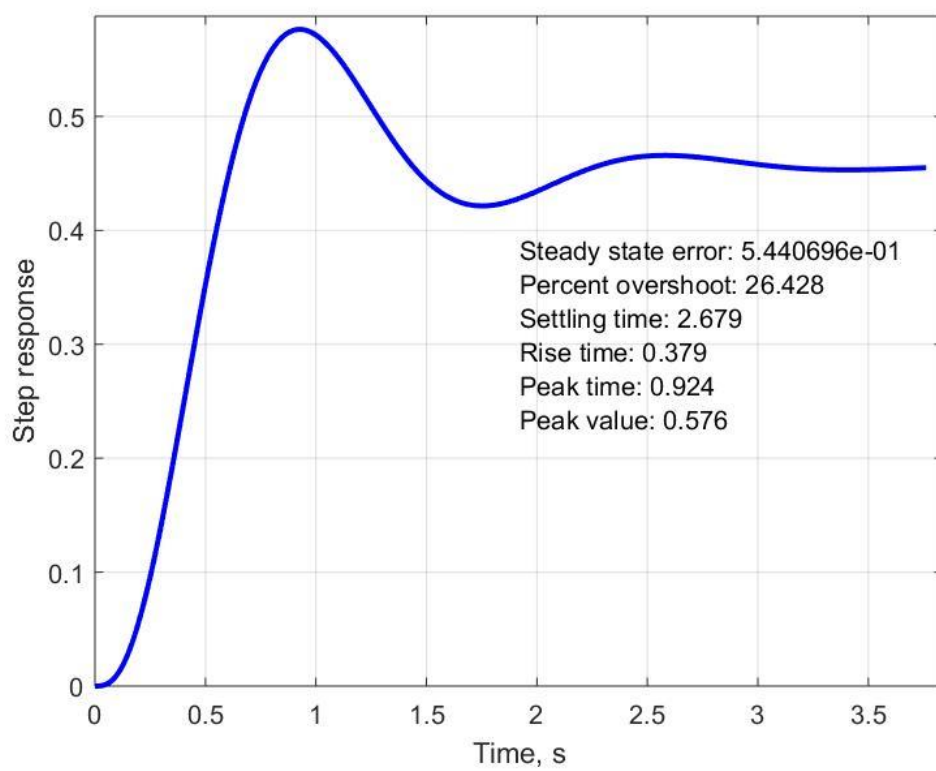
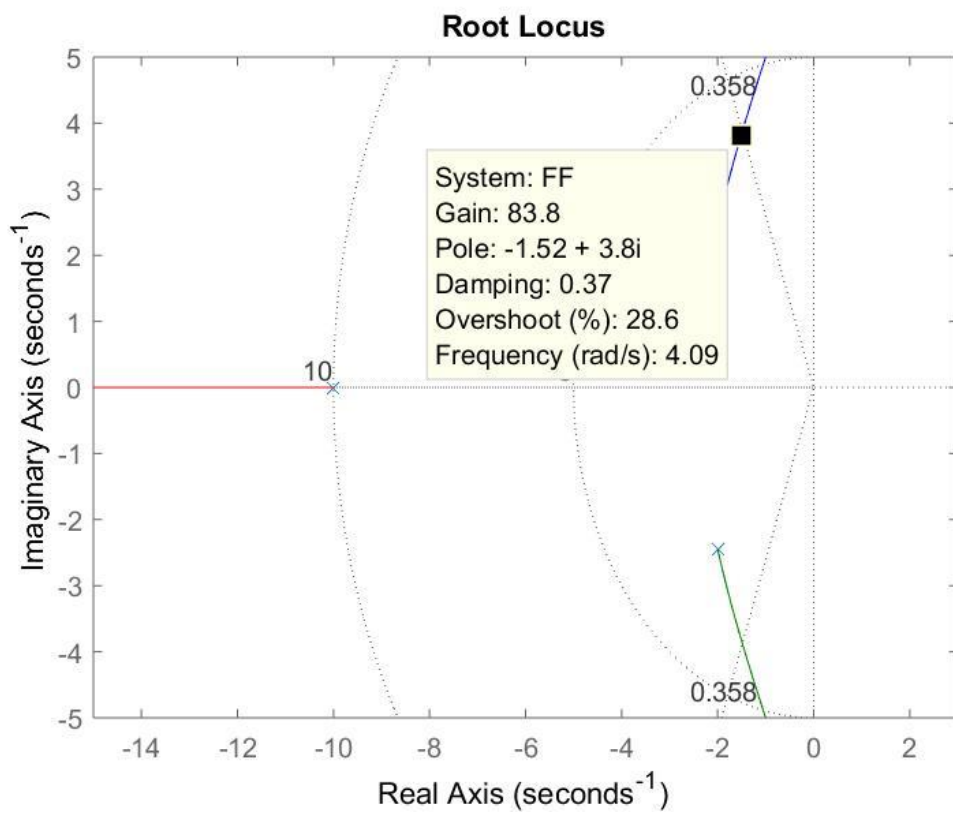
$$\text{Type 0, } e(\infty) = \frac{1}{1 + K_p}$$

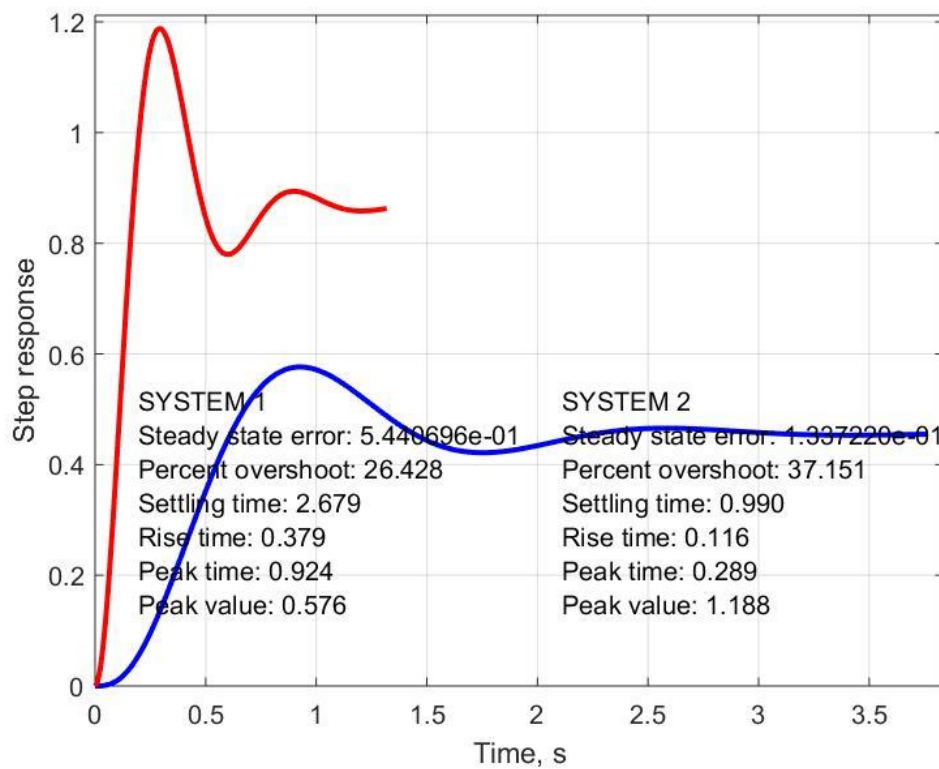
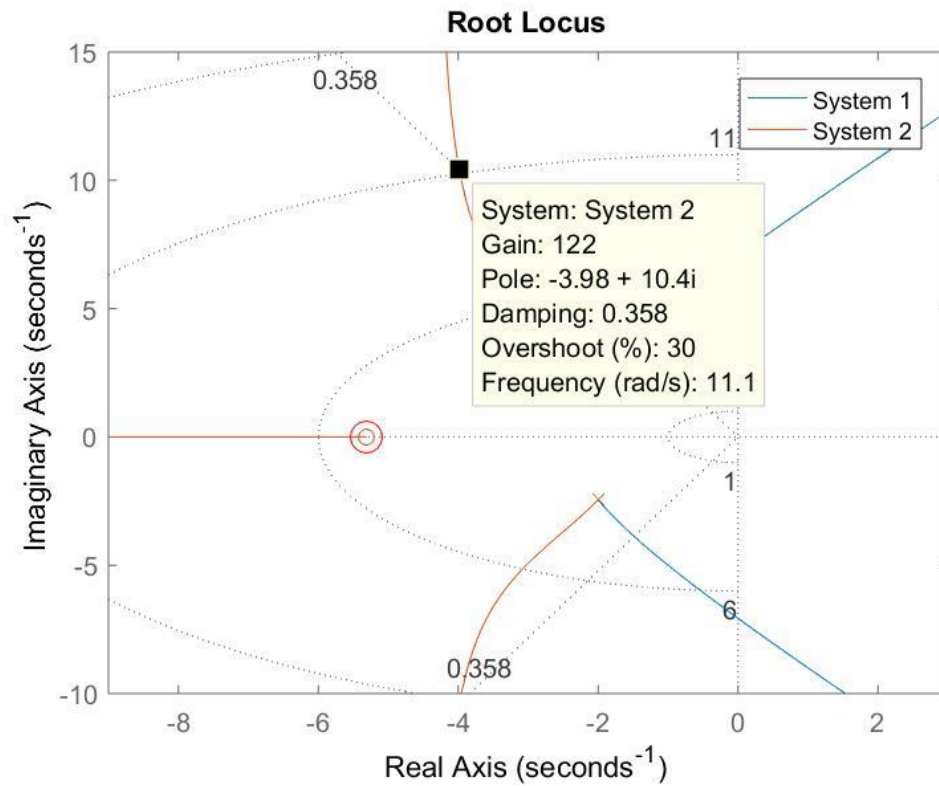
$$K_p = \frac{K}{G(0)} =$$

A plot

6) PID open loop

$$[G_c^{PD}(s)][G_c^{PI}(s)][K][G(s)] =$$





20% OS, $\tau_p = .8$, $e_{ss} = .01$

#3) $G(s) = \frac{K}{s(s+4)(s+8)}$

$\zeta = \frac{\tau_d(2)}{\sqrt{\tau_d^2 + 4n^2(2)}} = \frac{1.609}{3.53} = .456$

a) plot $\tau_p = \frac{\pi}{\omega_d} = \frac{\pi}{2.49} = 1.261$

b) $K = 74$

$\tau_{p \text{ comp}} = .8$

$\omega_d \text{ comp} = \frac{\pi}{\tau_{p \text{ comp}}} = \frac{\pi}{.8} = 3.927$, $\theta_d^{\text{comp}} = \theta_d \left(\frac{\omega_d^{\text{comp}}}{\omega_d} \right) = 1.28 \left(\frac{3.927}{2.49} \right) = 2.019$

new pole

$-\sigma_d \pm j\omega_d = -2.019 \pm j3.927$

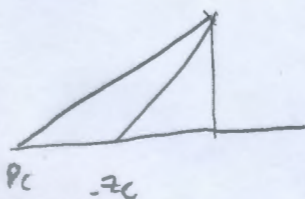
zero of PD controller

$\theta_c = \left[\tan^{-1} \left(\frac{3.927}{-2.019} \right) + 180 \right] - \left[\tan^{-1} \left(\frac{3.927}{4-2.019} \right) \right] - \left[\tan^{-1} \left(\frac{3.927}{8-2.019} \right) \right]$

$\theta_c - 213.728 = (2K+1)180^\circ \Rightarrow \theta_c = -180^\circ + 213.728 = 33.73^\circ$

$z_0 = \frac{\omega_d}{\tan \theta_c} + \sigma_d = \frac{3.927}{\tan(33.728)} + 2.019 = 7.901$, $G_c^{\text{PD}}(s) = s + 7.901$

c) $z_c^{\text{lead}} = 40$, $\theta_2 = \tan^{-1} \frac{\omega_d}{z_c^{\text{lead}} - \sigma_d} = \tan^{-1} \left(\frac{3.927}{40-2.019} \right) = -5.339^\circ$



$\theta_1 = \theta_2 - \theta_c = -5.339^\circ - 33.73^\circ = -39.069^\circ$

$p_c^{\text{lead}} = \sigma_d + \frac{\omega_d}{\tan \theta_1} = 2.019 + \frac{3.927}{\tan(-39.069^\circ)} = -4.837$

$G_c^{\text{lead}} = \frac{s - 40}{s - 4.837}$

loop gain

$K = \frac{|s - 4.837||s||s+4||s+8|}{|s-40|} = 26.025$

