Bridge Paper 6: Admissible Geometry and Reconstruction Dynamics

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Abstract

Bridge Paper 5 (BP5) established that apparent motion of localised field configurations can be reinterpreted as sequential reconstruction: a time-indexed family of stabilised geometries related by reconstruction maps, rather than literal transport of objects through space. The present paper elevates this reconstruction picture from an interpretive framework to a *selection principle*. We define the configuration space $\mathcal{G}$ of stabilised geometries and formulate admissibility constraints—stability, bounded mismatch, and internal mode regularity—that filter which reconstruction sequences are physically allowed. Dynamics thus emerges not as forces acting on objects, but as constrained selection within $\mathcal{G}$: admissible histories are selected by reconstruction constraints, not generated by equations of motion. In the deep-stabilisation and slow-evolution regime, we show that admissible paths approximate geodesics on a metric inherited from the mismatch functional, recovering the effective mass and kinetic structure of BP4. Multi-configuration systems introduce constraint coupling, which yields effective interaction potentials in the infrared limit. Conservation laws arise from symmetries of the admissibility conditions. We explicitly delineate the regime of validity and enumerate claims that this framework does not make.

Status: First public draft. Comments welcome.

Contents

1 Introduction: From Interpretation to Selection

1.1 Recap of BP4 and BP5

Bridge Paper 4 (BP4) [3] demonstrated that a localised, stabilised field configuration Φ_0 possesses an effective mass M_{eff} derived from the moduli space metric of the underlying field theory. Slow collective motion along translation modes yields an effective kinetic energy

$$T_{\text{eff}} = \frac{1}{2} M_{\text{eff}} \dot{X}^2, \tag{1}$$

where $X(t)$ is the collective coordinate labelling the configuration's spatial localisation. Interactions between well-separated configurations produce effective potentials, and in the weak-field limit, the Newtonian structure is recovered.

Bridge Paper 5 (BP5) [4] reinterpreted $X(t)$ not as the literal position of a transported object, but as a *reconstruction label*: at each instant, the field is independently stabilised as a geometry $\Phi_0(X(t))$, and sequential geometries are related by a reconstruction map $\mathcal{R}_\Delta$ with mismatch functional $\mathcal{M}$. Motion is thus reconstruction; apparent transport is the time-ordered correspondence between stable configurations.

1.2 The Selection Problem

BP5 provides an interpretation of what motion *is*, but does not specify *which* reconstruction sequences are allowed. The reconstruction map $\mathcal{R}_\Delta$ relates consecutive geometries, and the mismatch $\mathcal{M}$ quantifies their difference, yet infinitely many sequences $\{\gamma_k\}$ could in principle be constructed. The **selection problem** is: what determines the admissible set of reconstruction histories?

This paper answers that question by formulating *admissibility constraints* on histories in the configuration space of stabilised geometries. Dynamics is thereby recast as selection: the physically realised history is one (or one among few) that satisfies all constraints.

1.3 Goal of BP6

The goal of this paper is to:

(i) Define the configuration space $\mathcal{G}$ of stabilised geometries and the space of histories over $\mathcal{G}$.

(ii) Formulate admissibility constraints—stability, bounded mismatch, internal mode regularity—as precise mathematical conditions.

(iii) Show that admissible evolution under these constraints yields, in the appropriate regime, the geodesic/inertial structure recovered in BP4/BP5.

(iv) Demonstrate that multi-configuration constraint coupling produces effective interactions.

BP5 resolved the *interpretation* of motion; BP6 specifies *admissibility and selection*.

1.4 Conceptual Dictionary

To orient the reader, Table 1 contrasts standard dynamical language with the admissibility-based vocabulary employed here.

Table 1: Conceptual dictionary: transport/force language versus admissibility/constraint language.

Standard Dynamics	**Admissibility Framework (BP6)**
Object/particle	Stabilised geometry $\gamma \in \mathcal{G}$
Position $X(t)$	Collective coordinate / reconstruction label
Trajectory	Admissible history in $\mathcal{G}$
Force	Constraint coupling gradient
Acceleration	Deviation of admissible path from free geodesic
Inertia	Mismatch cost for configuration change
Potential energy	Constraint-induced effective energy
Equation of motion	Admissibility selection rule

2 Configuration Space of Stabilised Geometry

2.1 Field Configurations and Stabilised Minima

We work with a real scalar field $\Phi : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$, though the framework extends to more general field content. The energy functional is

$$E[\Phi] = \int_{\mathbb{R}^3} \mathrm{d}^3x \left[\frac{1}{2c^2}(\partial_t \Phi)^2 + \frac{1}{2}(\nabla\Phi)^2 + V(\Phi) \right], \tag{2}$$

where $V(\Phi)$ is a potential admitting localised, stable minima. The gradient energy $E_{\text{geom}}[\Phi]$ is defined by

$$E_{\text{geom}}[\Phi] \;=\; \int_{\mathbb{R}^3} \mathrm{d}^3x \left[\frac{1}{2}(\nabla\Phi)^2 + V(\Phi) \right]. \tag{3}$$

A **stabilised configuration** is a static or quasi-static field profile Φ_0 satisfying

$$\left.\frac{\delta E_{\text{geom}}}{\delta \Phi}\right|_{\Phi_0} = 0, \qquad \text{with positive-definite second variation (modulo zero modes).} \tag{4}$$

Definition 2.1 (Stabilised Geometry)**.** A *stabilised geometry* γ is an equivalence class $[\Phi_0]$ of stabilised configurations related by spatial translations and any internal symmetry transformations of the field theory.

The equivalence removes redundancy: two configurations differing only by a global translation represent the same intrinsic geometry.

2.2 The Configuration Space $\mathcal{G}$

Definition 2.2 (Configuration Space)**.** The *configuration space of stabilised geometries* is

$$\mathcal{G} \;=\; \big\{\gamma = [\Phi_0] : \Phi_0 \text{ satisfies } (4)\big\} / \sim, \tag{5}$$

where $\sim$ denotes the equivalence under spatial translations and internal symmetries.

Elements of $\mathcal{G}$ are not "particles at positions" but intrinsic stable field geometries. A representative $\Phi_0(x; q)$, parameterised by collective coordinates $q = (X, \xi)$—where $X \in \mathbb{R}^3$ labels translation and ξ labels internal moduli—can be chosen for computational purposes, but the physical object is the equivalence class.

Figure 1 illustrates the structure of $\mathcal{G}$.

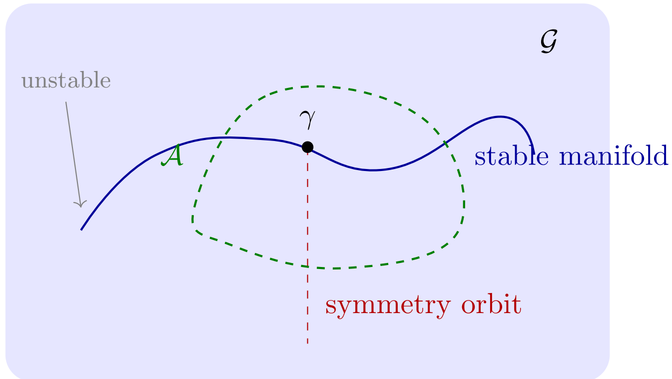

Figure 1: Schematic of the configuration space $\mathcal{G}$ of stabilised geometries. The stable manifold (solid curve) contains configurations satisfying (4). Symmetry orbits (dashed vertical lines) are quotiented out. The admissible region $\mathcal{A}$ (dashed boundary) is defined by constraints in Section 3.

2.3 Histories in $\mathcal{G}$

Definition 2.3 (Discrete History)**.** A *discrete history* is a sequence $\{\gamma_k\}_{k=0}^N$ with $\gamma_k \in \mathcal{G}$ for each k, indexed by discrete parameter k corresponding to reconstruction steps of duration Δ.

Definition 2.4 (Continuous History)**.** A *continuous history* is a smooth curve $\gamma : [0, T] \to \mathcal{G}$, $t \mapsto \gamma(t)$.

The discrete formulation is primary; the continuous limit is derived in Section 5 and Appendix A.

2.4 Observables on $\mathcal{G}$

Physical observables are functionals $\mathcal{O} : \mathcal{G} \to \mathbb{R}$ invariant under the equivalence relation. Examples include:

- Geometric energy: $E_{\text{geom}}(\gamma)$.
- Internal mode amplitudes: projections onto non-translational eigenmodes of the stability operator.

We do not develop the theory of observables further here; it suffices that well-defined functionals on $\mathcal{G}$ exist.

3 Reconstruction Admissibility

3.1 The Reconstruction Map

Following BP5 [4], we define a reconstruction map that relates consecutive geometries in a discrete history.

Definition 3.1 (Reconstruction Map)**.** The *reconstruction map* $\mathcal{R}_\Delta : \mathcal{G} \to \mathcal{G}$ associates to each stabilised geometry γ_k a candidate successor geometry $\mathcal{R}_\Delta(\gamma_k)$, representing the "natural continuation" under reconstruction at time-step Δ.

The precise form of $\mathcal{R}_\Delta$ depends on the dynamics of the underlying field theory. For present purposes, it suffices that $\mathcal{R}_\Delta$ encodes the tendency of a configuration to persist or evolve according to intrinsic field dynamics over one reconstruction step.

3.2 The Mismatch Functional

Definition 3.2 (Mismatch Functional)**.** The *mismatch functional* $\mathcal{M} : \mathcal{G} \times \mathcal{G} \to \mathbb{R}_{\geq 0}$ quantifies the discrepancy between two geometries:

$$\mathcal{M}(\gamma', \gamma; \mathcal{R}) \; = \; \inf_{\substack{\Phi_0 \in \gamma \\ \Phi_0' \in \gamma'}} \int_{\mathbb{R}^3} \mathrm{d}^3x \left| \Phi_0'(x) - \mathcal{R}_\Delta[\Phi_0](x) \right|^2 w(x), \tag{6}$$

where $w(x) \geq 0$ is a weight function (e.g., $w = 1$ or localised to the configuration's support), and the infimum is over representatives in each equivalence class.

The mismatch measures how well γ' matches the natural successor $\mathcal{R}_\Delta(\gamma)$. A small mismatch indicates that the transition $\gamma \to \gamma'$ is consistent with the reconstruction dynamics; a large mismatch indicates a "jump" that reconstruction cannot naturally produce.

3.3 Admissibility Constraints

We now define the constraints that filter admissible histories.

Hypothesis 3.1 (Stability Constraint)**.** Each geometry in an admissible history must lie on the stable manifold:

$$\gamma_k \in \mathcal{G}_{\text{stable}} \; : \; E_{\text{geom}}(\gamma_k) \leq E^* \quad \text{and} \quad \delta^2 E_{\text{geom}}|_{\gamma_k} > 0 \text{ (mod zero modes)}. \tag{7}$$

Hypothesis 3.2 (Bounded Mismatch)**.** The mismatch between consecutive geometries is bounded:

$$\mathcal{M}(\gamma_{k+1}, \gamma_k; \mathcal{R}) \; \leq \; \mathcal{M}^*, \tag{8}$$

where $\mathcal{M}^* > 0$ is a threshold set by the reconstruction fidelity.

Hypothesis 3.3 (Internal Mode Regularity)**.** Excitation of internal (non-collective) modes is bounded:

$$\left\| P_{\text{int}} \delta\gamma_k \right\| \; \leq \; \epsilon_{\text{int}}, \tag{9}$$

where P_{int} projects onto internal mode directions and $\delta\gamma_k = \gamma_{k+1} - \gamma_k$ in a suitable linearisation.

Definition 3.3 (Admissible History)**.** A discrete history $\{\gamma_k\}_{k=0}^N$ is *admissible* if it satisfies Hypotheses 3.1–3.3 at every step k. We write $\{\gamma_k\} \in \mathcal{A}$.

A continuous history $\gamma(t)$ is admissible if it is the $\Delta \to 0$ limit of admissible discrete histories (see Appendix A).

Figure 2 illustrates the discrete admissible update.

4 Constraints as Selection Rules

4.1 Core Postulate: Admissible Evolution via Constrained Selection

We now state the central postulate of this paper.

Hypothesis 4.1 (Selection Principle)**.** Physical evolution corresponds to admissible histories. At each step, the successor geometry is determined by constrained selection within $\mathcal{A}$.

We provide two equivalent formulations of this selection.

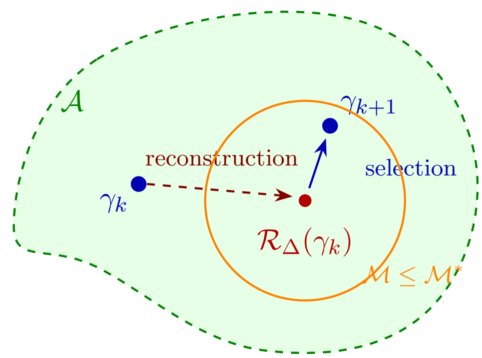

Figure 2: Discrete admissible update. Starting from γ_k, reconstruction suggests $\mathcal{R}_\Delta(\gamma_k)$. The admissible successor γ_{k+1} is selected within the admissible region $\mathcal{A}$ and the mismatch bound (orange circle).

Formulation (A): Constrained Minimisation. The successor geometry minimises the mismatch subject to admissibility:

$$\gamma_{k+1} = \operatorname*{argmin}_{\gamma' \in \mathcal{A}} \mathcal{M}(\gamma', \gamma_k; \mathcal{R}). \tag{10}$$

Formulation (B): Feasibility Projection. The successor geometry is the admissible point closest to the unconstrained reconstruction:

$$\gamma_{k+1} = \Pi_{\mathcal{A}}\big(\mathcal{R}_\Delta(\gamma_k)\big), \tag{11}$$

where $\Pi_{\mathcal{A}}$ denotes projection onto the admissible set with respect to the metric induced by $\mathcal{M}$.

Under regularity conditions (convexity of $\mathcal{A}$ in local coordinates, smoothness of $\mathcal{M}$), these formulations are equivalent. Both are computational implementations of a selection rule, not fundamental equations of motion; neither minimisation nor projection is ontologically primitive.

4.2 Selection on $\mathcal{G}$, Not Forces on Objects

The selection rule (10) or (11) operates on the configuration space $\mathcal{G}$, not on "objects in space." There is no force pushing a particle; rather, the admissibility constraints filter which reconstruction sequences are realised. The language of "forces" and "acceleration" is recovered only in the effective description (Section 5).

Remark 4.1. This is analogous to the principle of least action: the trajectory is not "pushed" by forces instant-by-instant, but selected as the extremum of an action functional over the entire path space. Here, selection is by admissibility rather than action extremisation.

4.3 Well-Posedness of the Selection Rule

Proposition 4.1 (Existence and Uniqueness)**.** *Suppose:*

(a) $\mathcal{A} \subset \mathcal{G}$ is non-empty, closed, and convex in local coordinates.

(b) $\mathcal{M}(\cdot, \gamma_k; \mathcal{R})$ is continuous and coercive on $\mathcal{G}$.

(c) $\mathcal{R}_\Delta(\gamma_k) \in \mathcal{G}$ for all $\gamma_k \in \mathcal{A}$.

Then the selection rule (10) admits a unique solution $\gamma_{k+1} \in \mathcal{A}$, and the map $\gamma_k \mapsto \gamma_{k+1}$ is continuous.

Proof. Continuity and coercivity of $\mathcal{M}$ ensure that the infimum is attained on the closed set $\mathcal{A}$. Strict convexity of $\mathcal{M}$ (inherited from the L^2-type structure in (6)) guarantees uniqueness. Continuity of the update map follows from the maximum theorem. □

5 Emergent Inertial Structure as Extremal Admissibility

5.1 Continuous-Time Limit of the Mismatch

We derive the continuous-time limit of the mismatch functional. Let $q = (q^1, \ldots, q^n)$ be local coordinates on $\mathcal{G}$ (collective coordinates for the configuration). Over a time step Δ, write

$$q_{k+1} = q_k + \dot{q}_k \Delta + O(\Delta^2). \tag{12}$$

Proposition 5.1 (Mismatch Expansion)**.** *Under the assumption that $\mathcal{R}_\Delta$ implements free propagation to leading order, the mismatch functional admits the expansion*

$$\mathcal{M}(\gamma_{k+1}, \gamma_k; \mathcal{R}) \;=\; \frac{\Delta}{2} g_{ij}(q)\dot{q}^i \dot{q}^j + O(\Delta^2), \tag{13}$$

where $g_{ij}(q)$ is a positive-definite metric on $\mathcal{G}$ given by

$$g_{ij}(q) \;=\; \int_{\mathbb{R}^3} \mathrm{d}^3x \, \frac{\partial \Phi_0}{\partial q^i} \frac{\partial \Phi_0}{\partial q^j} \, w(x). \tag{14}$$

Proof. See Appendix A. □

The metric (14) is precisely the moduli space metric familiar from soliton dynamics [1, 2]. In the translation sector, $q = X$, and

$$g_{XX} = \int_{\mathbb{R}^3} \mathrm{d}^3x \, |\nabla \Phi_0|^2 = M_{\text{eff}} c^2, \tag{15}$$

recovering the effective mass of BP4 [3].

5.2 Geodesic Structure in the Deep-Stabilisation Regime

Definition 5.1 (Deep-Stabilisation Regime)**.** The *deep-stabilisation regime* is the parameter domain where:

Internal modes are suppressed: $\|P_{\text{int}} \delta\gamma\| \ll 1$.

Evolution is slow: $|\dot{q}|\Delta \ll \ell$, where ℓ is the characteristic scale.

Mismatch is small: $\mathcal{M} \ll \mathcal{M}^*$.

In this regime, the admissibility constraints are not saturated, and the evolution is determined by minimising the mismatch.

Proposition 5.2 (Geodesic Approximation)**.** *In the deep-stabilisation regime, admissible paths $\gamma(t)$ satisfy*

$$\frac{\mathrm{d}^2 q^i}{\mathrm{d}t^2} + \Gamma^i_{jk}(q)\dot{q}^j \dot{q}^k = 0, \tag{16}$$

where Γ^i_{jk} are the Christoffel symbols of the metric g_{ij}.

Proof. Summing the mismatch (13) over N steps with $\Delta = T/N$, the total mismatch becomes

$$\sum_{k=0}^{N-1} \mathcal{M}(\gamma_{k+1}, \gamma_k) \;\to\; \frac{1}{2}\int_0^T \mathrm{d}t\, g_{ij}(q)\dot{q}^i\dot{q}^j \tag{17}$$

as $N \to \infty$. Minimising this integral yields the geodesic equation (16). □

5.3 Recovery of the BP4/BP5 Effective Kinetic Term

Restricting to the translation sector $q = X \in \mathbb{R}^3$, the metric (15) is flat (for homogeneous underlying space), and the geodesic equation (16) reduces to

$$M_{\text{eff}}\ddot{X} = 0. \tag{18}$$

This is precisely the inertial motion of BP4, now derived from the admissibility selection principle.

Remark 5.1. The effective mass M_{eff} is not an input parameter but emerges from the moduli space metric (14), which in turn derives from the mismatch functional $\mathcal{M}$.

5.4 Domain of Validity

Hypothesis 5.1 (Regime Assumptions)**.** The geodesic approximation (Proposition 5.2) is valid when:

(a) The configuration remains in the stable manifold throughout.

(b) $|\dot{q}| \ll c$ (non-relativistic collective motion).

(c) Internal mode excitation remains bounded: radiation losses negligible.

(d) Multi-configuration interactions are weak (treated perturbatively in Section 6).

Outside this regime, corrections to the geodesic structure appear (Section 8).

6 Interactions as Constraint Coupling

6.1 Two-Configuration Systems

Consider two stabilised geometries $\gamma^A, \gamma^B \in \mathcal{G}$. The composite system is described by a point in the product space $\mathcal{G} \times \mathcal{G}$, with collective coordinates (q^A, q^B).

Definition 6.1 (Composite Configuration Space)**.** For n configurations, the composite configuration space is

$$\mathcal{G}^{(n)} = \mathcal{G} \times \cdots \times \mathcal{G} \quad (n \text{ factors}), \tag{19}$$

with points $\boldsymbol{\gamma} = (\gamma^1, \ldots, \gamma^n)$.

6.2 Coupled Mismatch Functional

When configurations have overlapping support or interact via the field, the total mismatch is not simply additive.

Definition 6.2 (Coupled Mismatch)**.** The *coupled mismatch functional* for a two-configuration system is

$$\mathcal{M}_{\text{total}}(\boldsymbol{\gamma}', \boldsymbol{\gamma}; \mathcal{R}) \;=\; \mathcal{M}_A(\gamma'^A, \gamma^A) + \mathcal{M}_B(\gamma'^B, \gamma^B) + \varepsilon\, \mathcal{M}_{\text{int}}(\boldsymbol{\gamma}', \boldsymbol{\gamma}), \tag{20}$$

where ε parameterises the interaction strength and $\mathcal{M}_{\text{int}}$ captures cross-terms.

The interaction term $\mathcal{M}_{\text{int}}$ arises when the reconstruction of γ^A depends on the presence of γ^B (and vice versa), typically through field overlap or boundary conditions.

Figure 3 illustrates the coupled system.

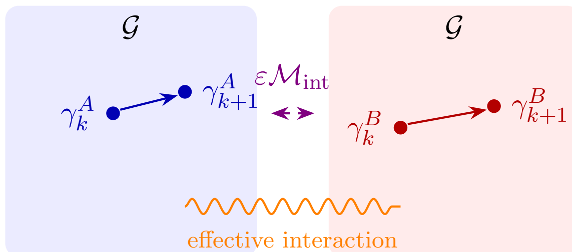

Figure 3: Two-configuration system $(\gamma^A, \gamma^B) \in \mathcal{G} \times \mathcal{G}$. Constraint coupling via $\mathcal{M}_{\text{int}}$ (dashed arrow) modifies admissible evolution and produces an effective interaction (wavy line) in the IR.

6.3 Effective Interaction Potential

In the continuous-time limit, the coupled mismatch (20) yields an effective Lagrangian:

$$L_{\text{eff}} = \frac{1}{2} g^A_{ij} \dot{q}^{A,i} \dot{q}^{A,j} + \frac{1}{2} g^B_{ij} \dot{q}^{B,i} \dot{q}^{B,j} - V_{\text{int}}(q^A, q^B), \tag{21}$$

where the effective interaction potential is

$$V_{\text{int}}(q^A, q^B) = \varepsilon \lim_{\Delta \to 0} \frac{1}{\Delta} \mathcal{M}_{\text{int}}. \tag{22}$$

Proposition 6.1 (Emergence of Effective Forces)**.** *The equations of motion derived from (21) are*

$$M^A_{\text{eff}} \ddot{X}^A = -\nabla_{X^A} V_{\text{int}}, \qquad M^B_{\text{eff}} \ddot{X}^B = -\nabla_{X^B} V_{\text{int}}, \tag{23}$$

in the translation sector, recovering the structure of interacting effective particles.

6.4 Reduction to the BP4/BP5 Two-Body Structure

In the weak-coupling and large-separation limit:

- $|X^A - X^B| \gg \ell^A + \ell^B$ (configurations well-separated).
- $\varepsilon \ll 1$ (perturbative coupling).

The interaction potential V_{int} reduces to the form derived in BP4 [3]:

$$V_{\text{int}}(X^A, X^B) \approx \kappa \frac{f(|X^A - X^B|)}{|X^A - X^B|^p}, \tag{24}$$

where κ is a coupling constant and f, p depend on the field theory details. In the gravitational analogy sector, $p = 1$ and the Newtonian limit is recovered.

7 Conservation Laws from Symmetries of Admissibility

7.1 Symmetry Assumptions

Hypothesis 7.1 (Translation Invariance)**.** The admissibility constraints are invariant under spatial translations: if $\{\gamma_k\} \in \mathcal{A}$, then $\{T_a \gamma_k\} \in \mathcal{A}$ for any $a \in \mathbb{R}^3$, where T_a is the translation action.

Hypothesis 7.2 (Time-Homogeneity)**.** The constraints depend only on the relation between consecutive geometries, not on the absolute step index k.

7.2 Emergent Conserved Quantities

Proposition 7.1 (Momentum Conservation)**.** *Under Hypothesis 7.1, the total momentum*

$$P^i_{\text{total}} = \sum_\alpha g^\alpha_{ij}(q^\alpha)\dot{q}^{\alpha,j} \tag{25}$$

is conserved along admissible histories, up to corrections of order $O(\varepsilon^2)$ from constraint coupling.

Proof. Translation invariance implies that $\mathcal{M}_{\text{total}}$ depends only on relative coordinates. The standard Noether argument applied to the effective Lagrangian (21) yields momentum conservation. Higher-order coupling corrections may introduce small non-conservation. □

Corollary 7.1 (Energy Conservation)**.** *Under Hypothesis 7.2, the effective energy*

$$E_{\text{total}} = \sum_\alpha \frac{1}{2} g^\alpha_{ij}\dot{q}^{\alpha,i}\dot{q}^{\alpha,j} + V_{\text{int}} \tag{26}$$

is conserved along admissible histories in the continuous-time limit.

8 Breakdown Regimes and Failure of Admissibility

8.1 When Constraints Cannot Be Satisfied

The admissibility framework breaks down when the constraints (Hypotheses 3.1–3.3) cannot be simultaneously satisfied. This occurs in several scenarios:

Stability Loss. If external perturbations or rapid evolution drive the configuration toward the boundary of the stable manifold:

$$E_{\text{geom}}(\gamma_k) \to E^*, \qquad \delta^2 E_{\text{geom}}|_{\gamma_k} \to 0, \tag{27}$$

then admissible successors may not exist, signalling a phase transition or decay of the configuration.

Internal Mode Excitation. If the evolution excites internal modes beyond the regularity bound:

$$\|P_{\text{int}}\delta\gamma_k\| > \epsilon_{\text{int}}, \tag{28}$$

the collective-coordinate description fails, and energy leaks into radiation modes.

Reconstruction Lag. If the required rate of change exceeds what reconstruction can accommodate:

$$\mathcal{M}(\gamma_{k+1}, \gamma_k; \mathcal{R}) > \mathcal{M}^*, \tag{29}$$

no admissible successor exists, indicating that the evolution is too "fast" for the underlying field dynamics.

8.2 Effective Description of Breakdown

Near but not at breakdown, corrections to the geodesic equations appear:

Proposition 8.1 (Dissipative Corrections)**.** *When internal modes are weakly excited, the effective equations of motion acquire dissipative terms:*

$$M_{\text{eff}}\ddot{X} + \eta\dot{X} = -\nabla V_{\text{int}}, \tag{30}$$

where $\eta > 0$ is an effective damping coefficient arising from energy transfer to internal modes.

This corresponds to radiation damping in the underlying field theory and is consistent with the prediction structure of BP4 [3].

9 Scope and Non-Claims

To maintain the conservative posture of bridge papers, we explicitly state what this framework does *not* claim:

- **Not a derivation of special or general relativity.** The framework is compatible with Lorentz invariance in the IR, but does not derive it from first principles.
- **Not a quantum theory.** All constructions are classical. Quantisation of the configuration space $\mathcal{G}$ is not addressed.
- **No claims about quantum measurement or collapse.**
- **Not a fundamental ontology.** This is an effective bridge description, not a claim about ultimate reality.
- **No consciousness or observer primacy.** "Observer" in this paper means only frame choice, coarse-graining, or boundary conditions—not conscious agents.
- **No preferred reference frame.**
- **No derivation of the Standard Model.** The field content is taken as given; we do not derive particle spectra.

10 Conclusion

This paper has recast dynamics as a selection problem on the configuration space of stabilised geometries. Building on BP5's reconstruction interpretation of motion, we defined admissibility constraints—stability, bounded mismatch, internal mode regularity—that filter which histories are physically realised. In the deep-stabilisation regime, admissible paths approximate geodesics on a metric derived from the mismatch functional, recovering the effective mass and inertial structure of BP4. Multi-configuration systems exhibit constraint coupling, which yields effective interaction potentials in the infrared limit. Conservation laws emerge from symmetries of the admissibility conditions.

A natural next step is the formulation of a path-integral or other quantisation procedure on $\mathcal{G}$, which we defer to future work.

A Discrete-to-Continuous Limit Derivation

We derive the continuous-time limit of the mismatch functional and the emergence of the moduli space metric.

A.1 Setup

Let $\gamma(t) \in \mathcal{G}$ be a continuous history, discretised as $\gamma_k = \gamma(k\Delta)$ for step size Δ. A representative field configuration is $\Phi_0(x; q(t))$, where $q(t)$ are collective coordinates.

A.2 Mismatch Expansion

The reconstruction map $\mathcal{R}_\Delta$ acts on field configurations. For small Δ, we assume $\mathcal{R}_\Delta[\Phi_0(q_k)] \approx \Phi_0(q_k) + O(\Delta)$, encoding that reconstruction attempts to preserve the configuration.

The mismatch (6) becomes:

$$\begin{aligned}
\mathcal{M}(\gamma_{k+1}, \gamma_k) &= \inf_{\text{reps}} \int \mathrm{d}^3x \left|\Phi_0(x; q_{k+1}) - \mathcal{R}_\Delta[\Phi_0(x; q_k)]\right|^2 w(x) && (31)\\
&\approx \int \mathrm{d}^3x \left|\Phi_0(x; q_k + \dot{q}_k\Delta) - \Phi_0(x; q_k)\right|^2 w(x) && (32)\\
&= \int \mathrm{d}^3x \left|\frac{\partial \Phi_0}{\partial q^i}\dot{q}^i \Delta\right|^2 w(x) + O(\Delta^3) && (33)\\
&= \Delta^2\, g_{ij}(q)\dot{q}^i\dot{q}^j + O(\Delta^3), && (34)
\end{aligned}$$

where

$$g_{ij}(q) = \int \mathrm{d}^3x\, \frac{\partial \Phi_0}{\partial q^i}\frac{\partial \Phi_0}{\partial q^j} w(x). \tag{35}$$

A.3 Action Integral

Summing over $N = T/\Delta$ steps:

$$\begin{aligned}
\sum_{k=0}^{N-1} \mathcal{M}(\gamma_{k+1}, \gamma_k) &= \sum_{k=0}^{N-1} \Delta^2\, g_{ij}\dot{q}^i\dot{q}^j + O(\Delta^2) && (36)\\
&= \Delta \sum_{k=0}^{N-1} \Delta\, g_{ij}\dot{q}^i\dot{q}^j + O(\Delta^2) && (37)\\
&\to \Delta \int_0^T \mathrm{d}t\, g_{ij}(q)\dot{q}^i\dot{q}^j \quad \text{as } \Delta \to 0. && (38)
\end{aligned}$$

Rescaling, the total mismatch cost is proportional to the kinetic action:

$$S_{\text{kin}} = \frac{1}{2}\int_0^T \mathrm{d}t\, g_{ij}(q)\dot{q}^i\dot{q}^j. \tag{39}$$

Minimising this action yields the geodesic equation (16).

B Notation Summary

Table 2 summarises the notation used in this paper and its relation to BP4/BP5.

Table 2: Notation summary.

Symbol	**Meaning**	**BP4/BP5 relation**
Φ	Field configuration	Same as BP4/BP5
Φ_0	Stabilised field configuration	Same as BP4/BP5
E_{geom}	Geometric (gradient + potential) energy	Same as BP4
E_0	Ground state energy	Same as BP4
M_{eff}	Effective mass	Same as BP4
κ	Coupling constant	Same as BP4
$X(t)$	Collective coordinate (translation)	Same as BP4/BP5
ℓ	Characteristic length scale	Same as BP4/BP5
ξ	Internal moduli	Same as BP5
$\mathcal{G}$	Configuration space of stabilised geometries	New in BP6
γ	Element of $\mathcal{G}$	New in BP6
$\mathcal{R}_\Delta$	Reconstruction map	From BP5
$\mathcal{M}$	Mismatch functional	From BP5
$\mathcal{A}$	Admissible region/set	New in BP6
g_{ij}	Moduli space metric	Implicit in BP4
$\mathcal{M}^*$	Mismatch threshold	New in BP6
E^*	Energy threshold for stability	New in BP6
ε	Interaction coupling strength	New in BP6
c	Speed of light	Same as BP4/BP5

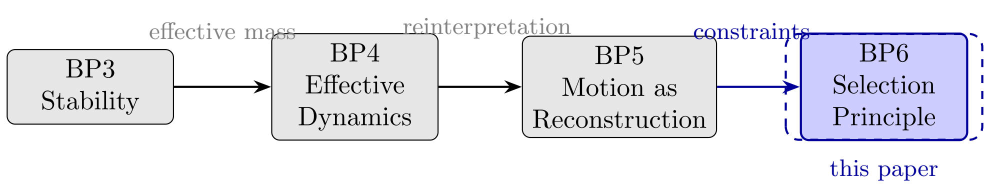

Figure 4: Bridge paper arc from BP3 to BP6. BP6 elevates the reconstruction interpretation (BP5) to a selection principle via admissibility constraints.

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