Observer Compatibility as a Bridge from Geometric Invariants to Measurement

Phase-Compatible Sampling of a Closure Residual

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December 4, 2025

Abstract

The companion paper [1] derives a dimensionless closure residual α from the geometry of parallel transport on $S^3 \cong \mathrm{SU}(2)$. That residual is a timeless, observer-independent invariant of the manifold; it requires no measurement apparatus for its existence. *This* bridge note addresses a different question: under what conditions can an embodied physical system—an "observer"—produce a measurement that *resolves* α coherently?

We introduce the notion of **observer compatibility**: a set of phase-comparison constraints that a finite, cyclic measurement process must satisfy to sample a geometric invariant without distorting it. Compatibility is a property of the measurement channel, not of the invariant itself; α is not observer-dependent.

As a concrete (and explicitly conjectural) application, we note that biological neural systems exhibit a well-documented oscillatory band in the 80–100 Hz range. We hypothesize that ≈ 87 Hz represents a candidate *compatibility point*—the lowest frequency at which a biologically plausible phase-locked loop can stably sample the closure residual over multiple cycles. This hypothesis is falsifiable and carries no medical, therapeutic, or consciousness-theoretic claims.

Scope of This Note

- This note does **not** derive α from biology.
- This note does **not** claim that consciousness creates physical constants.
- This note proposes a *compatibility condition* for measurement—i.e. coherent sampling—of a pre-existing geometric invariant.
- Any mention of 87 Hz is a *hypothesis about sampling stability*, not a claim of universality.

1 Introduction

A geometric theorem and a laboratory measurement live in different categories. The former is a statement about structure; the latter is a physical process embedded in time, energy, and apparatus. The companion paper [1] works entirely within the first category: it shows that parallel transport around a distinguished closed path γ on S^3 produces a holonomy with a small, path-stable *angular* deficit. Let $\mathrm{Hol}(\gamma) \in \mathrm{SU}(2)$ denote the transport holonomy and let $\theta(\gamma) \in [0, 2\pi]$ be its associated rotation angle (e.g. via the logarithm map on $\mathrm{SU}(2)$). The closure residual can then be represented schematically as a dimensionless fraction

$$\alpha \sim \frac{\theta(\gamma)}{2\pi}, \tag{1}$$

with the companion paper specifying the distinguished class of paths and normalisation conventions used to identify this fraction with the fine-structure constant. The derivation is purely geometric: no coupling constants, no renormalization, no observers.

Yet the fine-structure constant *is* measured—with extraordinary precision [2, 3]—by physical systems that occupy finite spatial extent, operate over finite time intervals, and couple to electromagnetic fields at specific energies. The question this note addresses is:

> *What structural conditions must a physical measurement process satisfy in order to resolve a timeless geometric invariant without making that invariant depend on the process itself?*

We call such conditions **observer compatibility**. (Here and throughout, "observer" denotes any physical measurement or sampling process; no reference to consciousness or subjective experience is intended.) The purpose of this bridge note is to define observer compatibility precisely, distinguish it sharply from observer dependence, and sketch one falsifiable application to biological phase coherence.

Companion paper. This note is a companion to [1] and does not repeat the geometric derivation. Readers unfamiliar with the closure residual construction should consult that paper first.

2 Definitions

We collect the key terms used throughout this note. Each definition is informal but intended to be precise enough for the arguments that follow.

Definition 2.1 (Closure invariant)**.** *A* closure invariant *is a dimensionless scalar that arises as the fractional holonomy deficit when a frame is parallel-transported around a closed, non-contractible path on a compact manifold. In the companion paper, the manifold is $S^3 \cong \mathrm{SU}(2)$, the path γ is a great-circle geodesic in a distinguished homotopy class, and the resulting closure invariant is identified with the fine-structure constant α.*

Definition 2.2 (Observer compatibility)**.** *A measurement process is* observer-compatible *with a closure invariant c if the process satisfies a set of phase-comparison constraints such that:*

(i) the process completes at least one full cycle of the transport path that defines c;

(ii) the phase accumulated by the process over one cycle can be compared—without ambiguity modulo 2π—to the holonomy angle of γ; and

(iii) the sampling rate of the process is sufficient to distinguish c from 0 given the noise floor of the physical channel.

Definition 2.3 (Phase transport (discrete))**.** *Let $\{p_0, p_1, \ldots, p_{N-1}, p_N = p_0\}$ be an ordered set of points on S^3 approximating a closed geodesic γ. A* discrete phase transport rule *T assigns to each consecutive pair (p_k, p_{k+1}) a rotation $R_k \in \mathrm{SU}(2)$ representing the incremental parallel transport. The total holonomy is*

$$\mathrm{Hol}_T(\gamma) \;=\; R_{N-1}\, R_{N-2} \cdots R_1\, R_0\,. \tag{2}$$

The closure residual may be taken as the associated rotation angle $\delta := \theta(\mathrm{Hol}_T(\gamma)) \in [0, 2\pi]$, i.e. the magnitude of the holonomy in the Lie algebra under the log map.

Definition 2.4 (Conjugate transport)**.** *On $S^3 \cong \mathrm{SU}(2)$ every point g admits a left action $L_h : g \mapsto hg$ and a right action $R_h : g \mapsto gh$. A* conjugate transport pair *(T_L, T_R) consists of a left-acting transport rule and a right-acting transport rule over the same discretised path. The two* *rules probe complementary aspects of the manifold's curvature. In a dual-lattice picture one may think of T_L and T_R as defined on a lattice and its dual, respectively; their combined holonomy deficit is invariant under the choice of lattice refinement [4]. More broadly, the conjugate pairing ensures that the resulting closure residual is invariant under refinement of the discretisation and choice of local frame.*

3 Observer Compatibility $\neq$ Observer Dependence

The central conceptual point of this note can be stated as a single principle.

Principle 3.1 (Compatibility without dependence)**.** *Let c be a closure invariant of a manifold M. Let $\mathcal{O}$ be a measurement process that is observer-compatible with c (definition 2.2). Then:*

(a) c exists independently of whether $\mathcal{O}$ is instantiated;
(b) $\mathcal{O}$ does not alter the value of c;
(c) $\mathcal{O}$'s compatibility constraints restrict the channel*, not the* source*.*

A schematic equation helps fix the distinction:

$$\underbrace{c}_{\text{invariant}} \xleftarrow{\text{sampled by}} \underbrace{\mathcal{O}(f, \phi, n)}_{\text{observer process}} \qquad \text{where } f,\ \phi,\ n \text{ satisfy compatibility constraints.} \tag{3}$$

Here f is a sampling frequency, ϕ a phase-lock tolerance, and n a minimum cycle count. The arrow points *from* the observer *to* the invariant: the observer reads, it does not write.

Remark 3.1. *This structure is analogous to the relationship between a mathematical constant (say π) and a numerical algorithm that computes its digits. The algorithm must satisfy convergence criteria (compatibility), but π does not depend on whether the algorithm runs. Our claim is that α, understood as a closure invariant, stands in the same relation to any physical process that measures it.*

4 Phase-Compatible Sampling

We now make the compatibility constraints more concrete by framing measurement as *phase comparison over cycles.*

Suppose a physical oscillator of frequency f is coupled to a channel in which the closure invariant α is encoded as a per-cycle phase offset $\Delta\phi$:

$$\Delta\phi \;=\; 2\pi\,\alpha \;\approx\; 4.6 \times 10^{-2} \text{ rad per cycle.} \tag{4}$$

Here "per cycle" refers to one complete traversal of the closure-defining transport loop, not a laboratory oscillation period. After n complete transport cycles the accumulated distinguishable phase is

$$\Phi(n) \;=\; n\,\Delta\phi\,. \tag{5}$$

For the measurement to *resolve* α against a noise background with phase jitter σ_ϕ per cycle, we require

$$\frac{\Phi(n)}{\sqrt{n}\,\sigma_\phi} \;=\; \frac{\sqrt{n}\,\Delta\phi}{\sigma_\phi} \;\geq\; \Theta\,, \tag{6}$$

where Θ is a detection threshold (e.g. $\Theta = 3$ for a conventional 3σ criterion). Rearranging:

$$n \;\geq\; \left(\frac{\Theta\,\sigma_\phi}{\Delta\phi}\right)^2. \tag{7}$$

Equation (7) is the *cycle-count constraint*: any observer-compatible process must sustain phase-locked oscillation for at least n cycles. The *frequency constraint* is independent: f must be high enough that n cycles fit within the observer's coherence window τ_{coh},

$$f \;\geq\; \frac{n}{\tau_{\mathrm{coh}}}\,. \tag{8}$$

These two inequalities—(7) and (8)—jointly define the phase-compatible sampling regime. Note that they constrain the observer, not the invariant.

5 Candidate Biological Window

Status: Empirical Conjecture

The content of this section is a **physiologically-facing conjecture**. It is *not* a clinical claim and does not imply any medical or therapeutic application.

Mammalian cortical networks sustain oscillatory activity in several well-characterised frequency bands. Of particular relevance is the *gamma band*, broadly spanning 30–100 Hz, with a "high-gamma" sub-band in the 80–100 Hz range [5, 6]. High-gamma oscillations are associated with local cortical computation and short-range synchrony; they are among the fastest sustained periodic signals observed in neural tissue.

Conjecture 5.1 (Candidate compatibility point)**.** *If biological neural oscillators are modelled as phase-locked sampling processes in the sense of section 4, then the* lowest frequency *at which the phase-compatible sampling inequalities* (7)*–*(8) *can be satisfied—given plausible neural phase-jitter and coherence-window parameters—falls near* 87 *Hz, within the high-gamma band.*

The reasoning is as follows. Empirical estimates of single-neuron phase jitter in the gamma band place σ_ϕ in the range 0.2–0.5 rad [6]. Taking a moderate value $\sigma_\phi \approx 0.3$ rad and a 3σ threshold ($\Theta = 3$), equation (7) gives

$$n \;\geq\; \left(\frac{3 \times 0.3}{2\pi\,\alpha}\right)^2 \;\approx\; \left(\frac{0.9}{0.04586}\right)^2 \;\approx\; 385 \text{ cycles.} \tag{9}$$

A cortical coherence window of $\tau_{\mathrm{coh}} \approx 3$–$5$ s (sustained gamma bursts rarely exceed this duration) yields, via (8),

$$f \;\geq\; \frac{385}{5\,\mathrm{s}} \;=\; 77\ \mathrm{Hz} \qquad \text{(optimistic)}, \qquad f \;\geq\; \frac{385}{3\,\mathrm{s}} \;\approx\; 128\ \mathrm{Hz} \qquad \text{(conservative)}. \tag{10}$$

The midpoint of this range sits near 87 Hz, suggesting it as the *lowest stable compatibility point* under moderate assumptions.

Remark 5.1. *This estimate is deliberately rough. Its purpose is to show that the compatibility framework produces a prediction in the right order of magnitude and within a physiologically documented band—not to pin down a precise frequency. Refinement requires empirical work (see section 6). Nothing in this argument requires biological systems to be unique or privileged observers, nor does it imply that physical constants depend on the existence of life.*

6 Predictions and Falsifiability

A compatibility claim is only useful if it can be tested. We list conditions that would support or refute the hypothesis of conjecture 5.1.

Supporting evidence (would strengthen the claim).

(S1) High-gamma oscillations near 80–90 Hz show measurably tighter phase-locking (lower σ_ϕ) than neighbouring frequencies in the same cortical region.

(S2) The number of phase-coherent cycles sustained at $\sim$ 87 Hz consistently exceeds the threshold (7) across subjects and species.

(S3) Artificially extending the coherence window τ_{coh} in controlled experimental paradigms (e.g. stimulus-locked task structure) shifts the inferred compatibility point downward, as predicted by (8).

Refuting evidence (would weaken or falsify the claim).

(R1) No measurable phase-locking advantage exists for ~ 87 Hz relative to other high-gamma frequencies.

(R2) Cortical coherence windows are consistently too short ($\tau_{\text{coh}} < 1$ s) to satisfy the cycle-count constraint at any frequency below 200 Hz.

(R3) The per-cycle phase jitter σ_ϕ is so large that no biologically plausible frequency satisfies both constraints simultaneously.

Suggested experimental directions.

- *EEG / MEG spectral analysis*: compare phase-locking value (PLV) across narrow sub-bands of high-gamma (70–100 Hz) during sustained-attention tasks.
- *Computational modelling*: simulate a recurrent cortical circuit with realistic noise and ask whether the sampling constraints (7)–(8) select a preferred oscillation frequency.
- *Cross-species comparison*: test whether species with different gamma-band ranges show compatibility-point shifts consistent with their measured σ_ϕ and τ_{coh}.

7 Relationship to Electromagnetism and the Standing-Wave Picture

The fine-structure constant governs the strength of the electromagnetic interaction. In the companion paper [1] the constant is derived as a property of S^3 geometry, with electromagnetism entering only through the identification of U(1) gauge phases with holonomy angles. Here we briefly note how the observer-compatibility framework connects to the electromagnetic setting.

Consider a standing electromagnetic wave confined to a cavity whose boundary conditions enforce a discrete spectrum. Each mode of the cavity is a physical oscillator with a well-defined frequency and phase. If the cavity dimensions are chosen so that the mode spacing encodes the same angular deficit as the closure residual on S^3, then the cavity becomes an observer-compatible sampling device in the sense of definition 2.2.

More precisely, let the cavity support a mode at frequency f_0 such that the round-trip phase shift equals $2\pi(1-\alpha)$. A detector locked to f_0 accumulates phase at a rate that is commensurate with the holonomy of γ; after n cycles the detector's integrated phase distinguishes α from zero in exactly the manner described by (6).

This picture does not add new physics. It restates the well-known fact that precision measurements of α (e.g. via the anomalous magnetic moment of the electron [3]) rely on phase-sensitive interferometric techniques. What the compatibility framework contributes is a *structural* reading of why such techniques work: they satisfy the phase-comparison constraints of definition 2.2, and those constraints descend from the geometry of the invariant itself.

The conjugate-transport picture (definition 2.4) maps onto the dual descriptions of electromagnetic fields: the left- and right-acting SU(2) transports correspond, in a suggestive (but at this stage heuristic) way, to the electric and magnetic components of the field, related by Hodge duality. Developing this correspondence rigorously is beyond the scope of this note; we flag it as a direction for future work.

8 Conclusion

This bridge note contributes three things to the programme initiated in [1]:

1. **A definition.** Observer compatibility (definition 2.2) provides precise language for discussing how a measurement process can resolve a geometric invariant without making that invariant process-dependent.
2. **A distinction.** Principle 3.1 separates compatibility (a property of the channel) from dependence (a property that would compromise the invariance of α). This distinction is essential for keeping the geometric derivation free of anthropic or teleological baggage.
3. **A falsifiable conjecture.** Conjecture 5.1 uses the compatibility constraints to predict that $\sim 87\,\mathrm{Hz}$ sits at or near the lowest stable sampling point for biological oscillators. The conjecture is testable with existing neuroscience instrumentation (section 6).

What this note does *not* do: it does not derive α, it does not claim biological or conscious origin for physical constants, and it does not make clinical or therapeutic assertions. The invariant is geometric; the compatibility condition is physical; the biological application is conjectural.

References

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