

## Drawing Circumcircles and Incircles

Given the coordinates of three non-collinear points  $A, B$ , and  $C$ , how can the circumscribed and inscribed circles be drawn?

Let  $a, b$ , and  $c$  be the lengths of the sides of the triangle, calculated from the coordinates using the distance formula. Then the radii of the incircle ( $r$ ) and circumcircle ( $R$ ) can be calculated using the semiperimeter ( $s$ ):

$$\begin{aligned}s &= \frac{1}{2}(a + b + c) \\ r &= \sqrt{\frac{(s - a)(s - b)(s - c)}{s}} \\ R &= \frac{abc}{4rs}.\end{aligned}$$

How can we use these to detect when the points are collinear?

We also need the coordinates of the centers of the incircle and circumcircle. To calculate the coordinates of the circumcenter, find the intersection of the perpendicular bisectors of any two of the triangle's sides. To calculate the coordinates of the incenter, find the equation of the line perpendicular to  $\overline{AB}$  that passes through the point on side  $\overline{AB}$  that lies a distance  $s - a$  from vertex  $A$ . Then find the equation of the line perpendicular to  $\overline{AC}$  that passes through the point on  $\overline{AC}$  that is a distance  $s - a$  from  $A$ . The intersection of these two lines is the incenter.