Feedback — Problem Set #1

Help Center

You submitted this quiz on **Mon 6 Apr 2015 11:45 AM IST**. You got a score of **5.00** out of **5.00**.

Question 1

We are given as input a set of n requests (e.g., for the use of an auditorium), with a known start time s_i and finish time t_i for each request i. Assume that all start and finish times are distinct. Two requests *conflict* if they overlap in time --- if one of them starts between the start and finish times of the other. Our goal is to select a maximum-cardinality subset of the given requests that contains no conflicts. (For example, given three requests consuming the intervals $[0, 3]$, $[2, 5]$, and $[4, 7]$, we want to return the first and third requests.) We aim to design a greedy algorithm for this problem with the following form: At each iteration we select a new request i, including it in the solution-so-far and deleting from future consideration all requests that conflict with i. Which of the following greedy rules is guaranteed to always compute an optimal solution?

Your Answer		Score	Explanation
At each iteration, pick the remaining request with the earliest finish time.	✔	1.00	Let R_j denote the requests with the j earliest finish times. Prove by induction on j that this greedy algorithm selects the maximum-number of non-conflicting requests from S_j.
At each iteration, pick the remaining request with the earliest start time.			
At each iteration, pick the remaining request which requires the least time (i.e., has the smallest value of $t_i - s_i$) (breaking ties arbitrarily).			
At each iteration, pick			

the remaining request with the fewest number of conflicts with other remaining requests (breaking ties arbitrarily).

Total	1.00 / 1.00

Question 2

We are given as input a set of n jobs, where job j has a processing time p_j and a deadline d_j. Recall the definition of *completion times* C_j from the video lectures. Given a schedule (i.e., an ordering of the jobs), we define the *lateness* l_j of job j as the amount of time $C_j - d_j$ after its deadline that the job completes, or as 0 if $C_j \leq d_j$. Our goal is to minimize the maximum lateness, $\max_j l_j$. Which of the following greedy rules produces an ordering that minimizes the maximum lateness? You can assume that all processing times and deadlines are distinct.

Your Answer		Score	Explanation
Schedule the requests in increasing order of deadline d_j	✔	1.00	Proof by an exchange argument, analogous to minimizing the weighted sum of completion times.
Schedule the requests in increasing order of processing time p_j			
None of the other answers are correct.			
Schedule the requests in increasing order of the product $d_j \cdot p_j$			
Total		1.00 / 1.00	

Question 3

Consider an undirected graph $G = (V, E)$ where every edge $e \in E$ has a given cost c_e. Assume that all edge costs are positive and distinct. Let T be a minimum spanning tree of G and P a shortest path from the vertex s to the vertex t. Now suppose that the cost of every edge e of G is increased by 1 and becomes $c_e + 1$. Call this new graph G'. Which of the following is true about G'?

Your Answer		Score	Explanation
T is always a minimum spanning tree and P is always a shortest s-t path.			
T may not be a minimum spanning tree but P is always a shortest s-t path.			
T may not be a minimum spanning tree and P may not be a shortest s-t path.			
T must be a minimum spanning tree but P may not be a shortest s-t path.	✔	1.00	The positive statement has many proofs (e.g., via the Cut Property). For the negative statement, think about two different paths from s to t that contain a different number of edges.
Total		1.00 / 1.00	

Question 4

Suppose T is a minimum spanning tree of the connected graph G. Let H be a connected induced subgraph of G. (I.e., H is obtained from G by taking some subset $S \subseteq V$ of vertices, and taking all edges of E that have both endpoints in S. Also, assume H is connected.) Which of the following is true about the edges of T that lie in H? You can assume that edge costs are distinct, if you wish. [Choose the strongest true statement.]

Your Answer		Score	Explanation
For every G and H, these edges form a spanning tree (but not necessary minimum-cost) of H			
For every G and H, these edges are contained in some minimum spanning tree of H	✔	1.00	Proof via the Cut Property (cuts in G correspond to cuts in H with only fewer crossing edges).
For every G and H, these edges form a minimum spanning tree of H			
For every G and H and spanning tree T_H of H, at least one of these edges is missing from T_H			
Total		1.00 / 1.00	

Question 5

Consider an undirected graph $G = (V, E)$ where edge $e \in E$ has cost c_e. A *minimum bottleneck spanning tree* T is a spanning tree that minimizes the maximum edge cost $\max_{e \in T} c_e$. Which of the following statements is true? Assume that the edge costs are distinct.

Your Answer	Score	Explanation
A minimum bottleneck spanning tree is not always a minimum spanning tree and a minimum spanning tree is not		

always a minimum bottleneck spanning tree.			
A minimum bottleneck spanning tree is always a minimum spanning tree and a minimum spanning tree is always a minimum bottleneck spanning tree.			
A minimum bottleneck spanning tree is always a minimum spanning tree but a minimum spanning tree is not always a minimum bottleneck spanning tree.			
A minimum bottleneck spanning tree is not always a minimum spanning tree, but a minimum spanning tree is always a minimum bottleneck spanning tree.	✔	1.00	For the positive statement, recall the following (from correctness of Prim's algorithm): for every edge e of the MST, there is a cut (A, B) for which e is the cheapest one crossing it. This implies that every other spanning tree has maximum edge cost at least as large. For the negative statement, use a triangle with one extra high-cost edge attached.
Total		1.00 / 1.00	