

# Document Classification

Statistical Analysis and Document Mining

Spring 2019

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## 1 Introduction

### 1.1 Motivation

### 1.2 Outline

## 2 First Look at the Problem

### 2.1 Preprocessing

### 2.2 Bag-of-Words

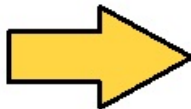
### 2.3 Naive Bayes Document Classification

## 3 Advanced Look at the Problem

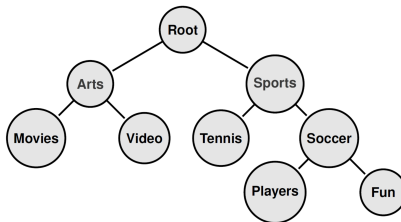
### 3.1 Sparse Files

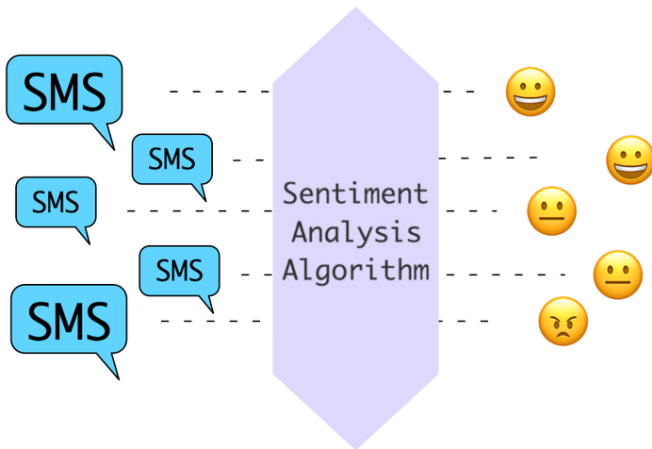
### 3.2 Two Laws of IR

### 3.3 tf-idf and Other Techniques

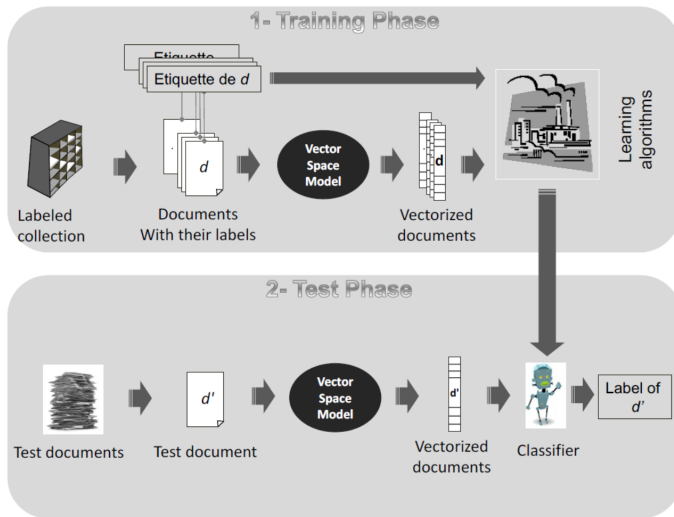




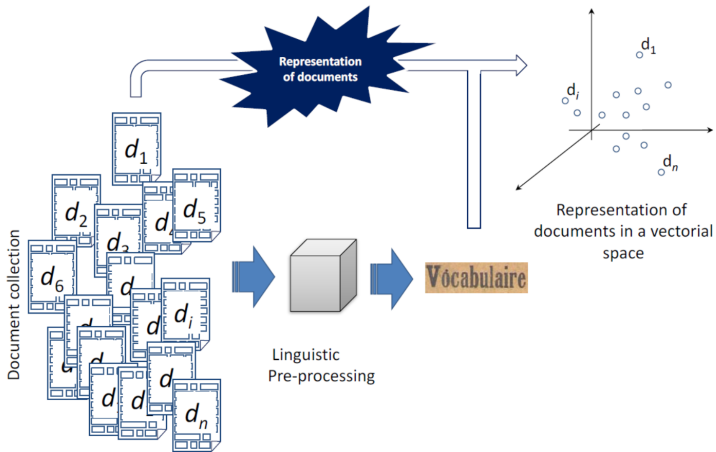












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- *Segmentation* (tokenization): separate a sequence of characters into semantic elements, or *words*.
- *Term* (type of words): class of all words having the same sequence of characters.
- *Example*:  
    *"The cat sat on the mat."*  
    *Words*: The, cat, sat, on, the, mat  
    *Terms*: the, cat, sat, on, mat
- *Difficulty*: Tokenization is language specific.

In French, the following issues may arise during the segmentation process:

- Lexical components with hyphens:  
*chassé-croisé, peut-être, rendez-vous*
- Lexical components with an apostrophe:  
*jusqu'où, aujourd'hui, prud'homme*
- Idiomatic expressions:  
*au fait, poser un lapin, tomber dans les pommes*
- Contracted forms:  
*j', M'sieur, Gad'zarts (les gars des Arts et Métiers)*
- Acronyms:  
*K7, A.R., CV, càd, P.-V.*

- 1 *Textual normalization*: consists in reducing the words of a same family to their canonical forms.
  - Punctuation: suppression of points and hyphens;
  - Lower-upper case: transform all upper cases to lower cases;
  - Accents: suppression of accents.
  
- 2 *Linguistic normalization* consists in
  - Rooting: replace each word by its root;
  - Stemming: replace each word by its canonical form.

## Non-spam message before preprocessing

Subject: Re: 5.1344 Native speaker intuitions The discussion on native speaker intuitions has been extremely interesting, but I worry that my brief intervention may have muddied the waters. I take it that there are a number of separable issues. The first is the extent to which a native speaker is likely to judge a lexical string as grammatical or ungrammatical per se. The second is concerned with the relationships between syntax and interpretation (although even here the distinction may not be entirely clear cut).

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## Spam message after preprocessing

financial freedom follow financial freedom work ethic extraordinary desire earn  
least per month work home special skills experience required train personal  
support need ensure success legitimate homebased income opportunity put  
back control finance life ve try opportunity past fail live promise



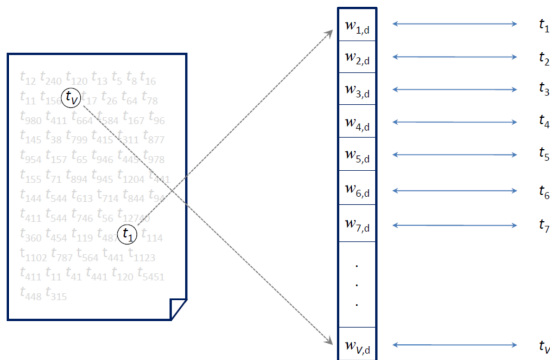
# The Bag of Words Representation

I love this movie! It's sweet, but with satirical humor. The dialogue is great and the adventure scenes are fun... It manages to be whimsical and romantic while laughing at the conventions of the fairy tale genre. I would recommend it to just about anyone. I've seen it several times, and I'm always happy to see it again whenever I have a friend who hasn't seen it yet!



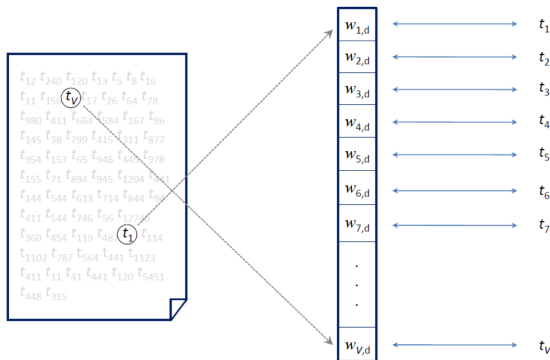
it	6
I	5
the	4
to	3
and	3
seen	2
yet	1
would	1
whimsical	1
times	1
sweet	1
satirical	1
adventure	1
genre	1
fairy	1
humor	1
have	1
great	1
...	...

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In the bag-of-words the word order in a sentence is ignored. Given a document, to each term  $t_j$  a specific value  $w_{j,d}$  is assigned.

$\Rightarrow$  A document can be represented as a vector:  $\mathbf{d} = (w_{j,d})_{j=1}^V$ .



- Observation is a document  $\mathbf{d} = (w_{j,d})_{i=1}^V$ , where
  - $V$  is a vocabulary size;
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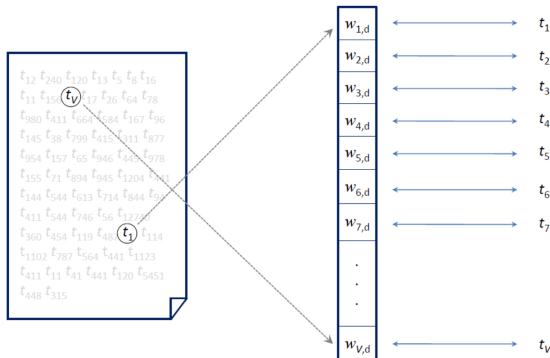
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- Training set:  $S = \{\mathbf{d}_i, y_i\}_{i=1}^n$ , where
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- *Target*: minimise the misclassification error:

$$P(h(\mathbf{D}) \neq Y) = \sum_{c \in \{1, \dots, K\}} P(Y = c) P(h(\mathbf{D}) \neq c | Y = c).$$

A document is represented as a vector:  $\mathbf{d} = (w_{j,d})_{j=1}^V$ .

*How we define importance  $w_{j,d}$ ,  $j \in \{1, \dots, V\}$ ?*



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- Suppose conditional independence of terms given a class.
- *Question:* How then the term  $t_j$  contributes into classification?
  - For each class, we can compute the probability that the term is present in a document from this class.  
⇒ Bernoulli distribution.

- Let  $W_{j,d}|Y = c$  be distributed acc. to *Bernoulli*. Then, probability that  $t_j$  is present in a document  $\mathbf{d}$  is:

$$P(W_{j,d}|Y = c) = p_{t_j|c}.$$

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$$P(\mathbf{D} = \mathbf{d}|Y = c) = \prod_{j=1}^V P(W_{j,d} = w_{j,d}|Y = c).$$

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- Bernoulli naive Bayes decision rule for the document classification task:

$$\begin{aligned} h_{NB}(\mathbf{d}) &:= \operatorname{argmax}_{c \in \mathcal{Y}} P(Y = c) \prod_{j=1}^V P(W_{j,d} = w_{j,d}|Y = c) \\ &= \operatorname{argmax}_{c \in \mathcal{Y}} P(Y = c) \prod_{j=1}^V p_{t_j|c}^{w_{j,d}} (1 - p_{t_j|c})^{1-w_{j,d}}. \end{aligned}$$

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$$\hat{p}_{t_j|c} = \frac{\sum_{i=1}^n \mathbb{I}(w_{j,d_i} = 1 \wedge y_i = c)}{\sum_{i=1}^n \mathbb{I}(y_i = c)} =: \frac{\text{df}_{t_j}(c)}{n_c}.$$

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- Considering the log scale, we classify new  $\mathbf{d}$  as follows:

$$h_{NB}(\mathbf{d}) := \underset{c \in \mathcal{Y}}{\operatorname{argmax}} \ln \frac{n_c}{n} + \sum_{\substack{j=\{1,\dots,V\} \\ t_j \in d}} \ln \hat{p}_{t_j|c} + \sum_{\substack{j=\{1,\dots,V\} \\ t_j \notin d}} \ln(1 - \hat{p}_{t_j|c}).$$

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Then,  $\sum_{j=1}^V \text{tf}_{t_j,d} = N_d$ .
- *Naive* assumption: all  $N_d$  words are *i.i.d.* by the following law:

$$P(\text{word} = t_j) = p_{t_j|c}, \quad j = \{1, \dots, V\}, \quad \sum_{j=1}^V p_{t_j|c} = 1.$$

$\Rightarrow$  Then, document  $d$  is distributed acc. to multinomial distribution with parameters  $(p_{t_j|c})_{j=1}^V$ .

- Let  $\mathbf{D}|Y = c$  be distributed acc. to the *multinomial law*.  
Then, the likelihood of  $\mathbf{D}$  to be from the class  $c$  is:

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  - Around 16 GB!



Variables	Values
<b># of documents in the collection</b>	<b>1,349,539</b>
Total # of occurrences of words	696,668,157
Average # of words per document	416
Size of the pre-processed collection on the disk	4.6 GB
Total # of types of words	757,476
Total # of types of words after rooting	604,244
<b>Size of the vocabulary</b>	<b>604,244</b>
<b>Average # of terms per document</b>	<b>225</b>
Size of the collection after removing a stop-list	2.8 GB

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- No need to store entries with zero values.
- *LibSVM format*: the observation is written in the following format:

y	index-value		index-value
2	5:0.356	...	9:1000
3	2:10.2	...	15:0.01

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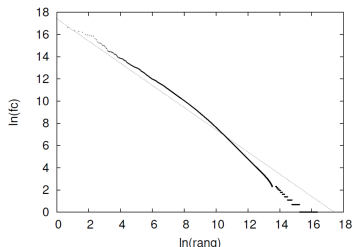
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- When the collection was filtered by removing a stop-list ("a", "the", "of", etc.) of size 200 words, the average number of terms was reduced in documents from 416 to 225 (around 45% reduction). Why?  
In addition, their filtering reduces the space on the disk of about 39% (from 4.6 GB to 2.8 GB).

*The number of occurrences  $fc(m)$  of a word  $m$  in a document collection is inversely proportional to its rank:*

$$\forall m : fc(m) \approx \frac{\lambda}{rang(m)}.$$

⇒ The k-th most frequent word is approximately k times less present than the most frequent one.



Rank	Word	Freq.	%
1	the	22,038,615	4.9%
2	be	12,545,825	2.79%
3	and	10,741,073	2.39%
4	of	10,343,885	2.3%
5	a	10,144,200	2.25%
6	in	6,996,437	1.56%
7	to (i.m.)	6,332,195	1.41%
8	have	4,303,955	0.96%
9	to (p.)	3,856,916	0.86%
10	it	3,872,477	0.86%

Top 10 frequent word from the 450 million word corpus  
 (<https://www.wordfrequency.info>).



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    *"The cat sat on the mat."*  
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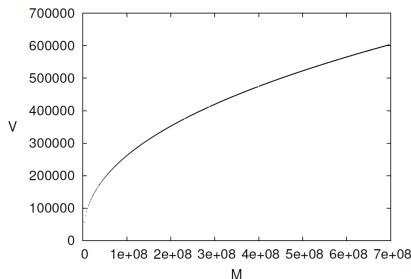
- We suppress very frequent words that are present in all of the documents and that do not bring any information.
- *Example:*  
    *"The cat sat on the mat."*  
    *Before filtering:* the, cat, sit, on, mat  
    *After filtering:* cat, sit, mat
- How many frequent words we should suppress?

*The size of the vocabulary  $V$  increases sub-linearly with respect to the number of words present in a collection  $M$ :*

$$V = k \cdot M^{\beta},$$

where  $k$  and  $\beta$  are parameters that are dependent on the collection. Typically, in English text corpora  $k \in [10, 100]$ , and  $\beta \in [0.4, 0.6]$ .

⇒ Larger the collection size, larger the vocabulary size.



You are willing to analyse a document containing 1,000,000 words.

- Let  $k = 10$ ,  $\beta = 0.5$ . Following the Heaps' law, What is the number of distinct words in the document?

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- Let  $k = 10$ ,  $\beta = 0.5$ . Following the Heaps' law, What is the number of distinct words in the document?
- It was found that the 7% of all words are "the" article. Following the Zipf's law, estimate the value of  $\lambda$ . What is the frequency of the second most frequent word? The third most frequent one?

- With filtering of stopwords the term frequency weighting may work better.

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*Example:* Medical Prescription vs Recipe.

take	water	glass	eat	wait	...	paracetamol	sugar	stomach
7	6	4	4	4	...	0	2	0
6	7	4	3	5	...	1	0	1



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- However, in many applications less frequent words also play a crucial role.

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- We want to diminish the weight of terms that occur frequently in general and increase the weight of terms that occur rarely in average.

tf-idf weighting is a trade-off between term frequency and document frequency:

- Normalised term frequency (tf part):

$$\frac{\text{tf}_{t_j,d}}{\sum_{j=1}^V \text{tf}_{t_j,d}} = \frac{\text{tf}_{t_j,d}}{N_d}.$$

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- Then, the tf-idf weight is defined as:

$$w_{t_j,d} = \frac{\text{tf}_{t_j,d}}{N_d} \ln \frac{n}{\text{df}_{t_j}}.$$

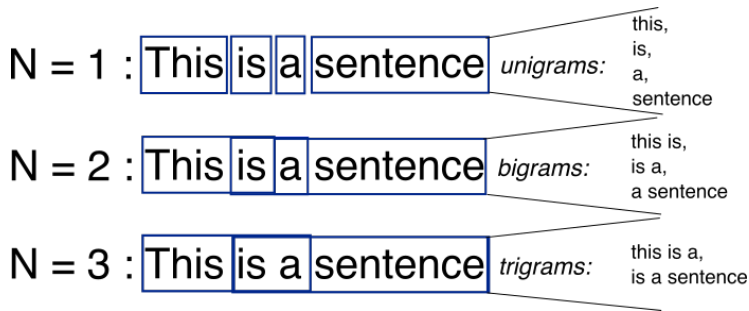
- We have the following training documents:

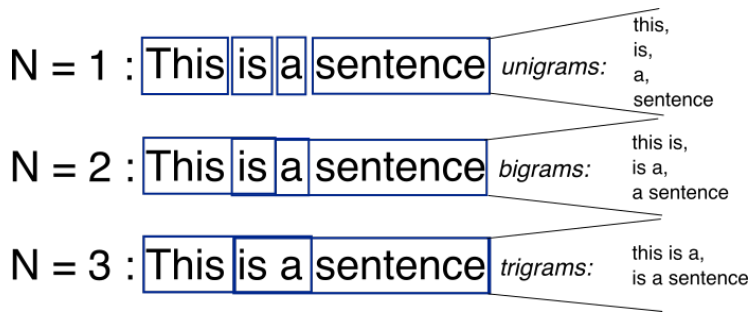
$d_1 : \{ \text{"cat", "sit", "mat", "cat", "jump", "bed", "cat", "good", "sit"} \}$

$d_2 : \{ \text{"cat", "dog", "jump", "sit", "dog", "cat", "animal"} \}$

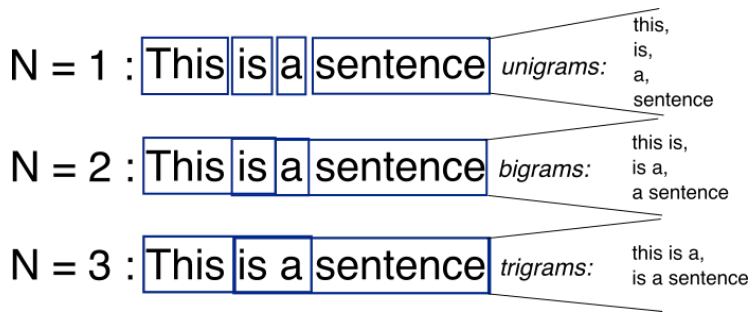
$d_3 : \{ \text{"table", "sit", "write", "think", "good", "book", "dog", "sit"} \}$

- What is the tf-idf representation for this dataset?





- Several words could be more important when they appear together.



- Several words could be more important when they appear together.
- The approach is very costly when  $V$  is large.



...ATTACACGGTGACCAACCTATT...

Gram	Frequency
ATTA	4
GACC	3
CGGT	5
...	...

