*The purpose of this assignment is to give you practice writing programs with recursion.*

1. **Trinomial coefficients (brute force).** Write a program `TrinomialBrute.java` that takes two integer command-line arguments *n* and *k* and computes the corresponding *trinomial coefficient*. The trinomial coefficient $T(n, k)$ is the coefficient of $x^{n+k}$ in the expansion of $(1 + x + x^2)^n$. For example,

   $$(1 + x + x^2)^3 = 1 + 3x + 6x^2 + 7x^3 + 6x^4 + 3x^5 + x^6$$

   Thus, $T(3, 3) = 1$, $T(3, 2) = 3$, $T(3, 1) = 6$, and $T(3, 0) = 7$.

   Implement a recursive function `trinomial()` to compute $T(n, k)$ by using the following recurrence relation:

   $$T(n, k) = \begin{cases} 1 & \text{if } n = 0 \text{ and } k = 0 \\ 0 & \text{if } k < -n \text{ or } k > n \\ T(n-1, k-1) + T(n-1, k) + T(n-1, k+1) & \text{otherwise} \end{cases}$$

   To do so, organize your program according to the following public API:

   ```
   public class TrinomialBrute {

       // Returns the trinomial coefficient T(n, k).
       public static long trinomial(int n, int k)

       // Takes two integer command-line arguments n and k and prints T(n, k).
       public static void main(String[] args)
   }
   ```

   As you will see, this approach is hopeless slow for moderately large values of *n* and *k*.

   ```
   ~/Desktop/recursion> java-introcs TrinomialBrute 3 3
   1

   ~/Desktop/recursion> java-introcs TrinomialBrute 3 2
   3

   ~/Desktop/recursion> java-introcs TrinomialBrute 3 1
   6

   ~/Desktop/recursion> java-introcs TrinomialBrute 3 0
   7

   ~/Desktop/recursion> java-introcs TrinomialBrute 24 12
   287134346

   ~/Desktop/recursion> java-introcs TrinomialBrute 30 0
   [takes too long]
   ```

   *Note*: you may assume that *n* is a non-negative integer.

   *Trinomial coefficients arise in combinatorics. For example, $T(n, k)$ is the number of permutations of n symbols, each of which is $-1$, 0, or $+1$, which sum to exactly k and $T(n, k - n)$ is the number of different ways of randomly drawing k cards from two identical decks of n playing cards.*

2. **Trinomial coefficients (dynamic programming).** Write a program `TrinomialDP.java` that takes two integer command-line arguments *n* and *k* and computes the trinomial coefficient $T(n, k)$ using *dynamic programming*. To do so, organize your program according to the following public API:

   ```
   public class TrinomialDP {

       // Returns the trinomial coefficient T(n, k).
       public static long trinomial(int n, int k)

       // Takes two integer command-line arguments n and k and prints T(n, k).
       public static void main(String[] args)
   }
   ```

This version should be fast enough to handle larger values of *n* and *k*.

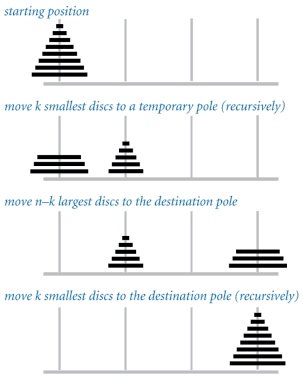
```
~/Desktop/recursion> java-introcs TrinomialDP 3 0
7

~/Desktop/recursion> java-introcs TrinomialDP 24 12
287134346

~/Desktop/recursion> java-introcs TrinomialDP 30 0
18252025766941

~/Desktop/recursion> java-introcs TrinomialDP 40 0
934837217271732457
```

3. **Reve's puzzle.** *Reve's puzzle* is identical to the *towers of Hanoi* problem, except that there are 4 poles (instead of 3). The task is to move *n* discs of different sizes from the starting pole to the destination pole, while obeying the following rules:

    - Move only one disc at a time.
    - Never place a larger disc on a smaller one.

    The following remarkable algorithm, discovered by Frame and Stewart in 1941, transfers $n$ discs from the starting pole to the destination pole using the fewest moves (although this fact was not proven until 2014).

    - Let $k$ denote the integer nearest to $n + 1 - \sqrt{2n + 1}$.
    - Transfer (recursively) the $k$ smallest discs to a single pole other than the start or destination poles.
    - Transfer the remaining $n - k$ disks to the destination pole (without using the pole that now contains the smallest $k$ discs). To do so, use the algorithm for the 3-pole towers of Hanoi problem.
    - Transfer (recursively) the $k$ smallest discs to the destination pole.

    
    
    *Recursive plan for Reve's Puzzle*

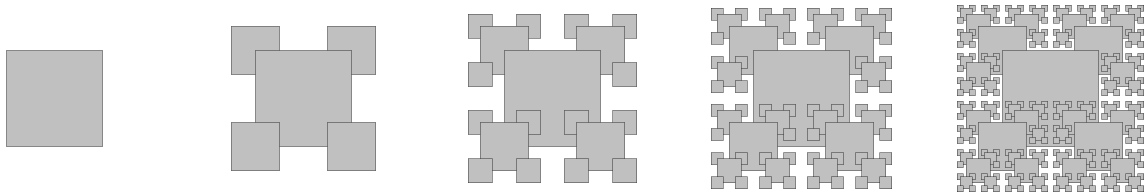
    Write a program `RevesPuzzle.java` that takes an integer command-line argument *n* and prints a solution to Reve's puzzle. Assume that the the discs are labeled in increasing order of size from 1 to *n* and that the poles are labeled *A*, *B*, *C*, and *D*, with *A* representing the starting pole and *D* representing the destination pole. Here are a few sample executions:

```
~/Desktop/recursion> java-introcs RevesPuzzle 3
Move disc 1 from A to B
Move disc 2 from A to C
Move disc 3 from A to D
Move disc 2 from C to D
Move disc 1 from B to D

~/Desktop/recursion> java-introcs RevesPuzzle 4
Move disc 1 from A to D
Move disc 2 from A to B
Move disc 1 from D to B
Move disc 3 from A to C
Move disc 4 from A to D
Move disc 3 from C to D
Move disc 1 from B to A
Move disc 2 from B to D
Move disc 1 from A to D

~/Desktop/recursion> java-introcs RevesPuzzle 5
Move disc 1 from A to D
Move disc 2 from A to C
Move disc 3 from A to B
Move disc 2 from C to B
Move disc 1 from D to B
Move disc 4 from A to C
Move disc 5 from A to D
Move disc 4 from C to D
Move disc 1 from B to A
Move disc 2 from B to C
Move disc 3 from B to D
Move disc 2 from C to D
Move disc 1 from A to D
```

*Note*: you may assume that *n* is a positive integer.

*Recall that for the towers of Hanoi problem, the minimum number of moves for a 64-disc problem is* $2^{64} - 1$. *With the addition of a fourth pole, the minimum number of moves for a 64-disc problem is reduced to* $18{,}433$.

4. **Recursive squares.** Write a program `RecursiveSquares.java` that takes an integer command-line argument *n* and plots a recursive square pattern of order *n*.

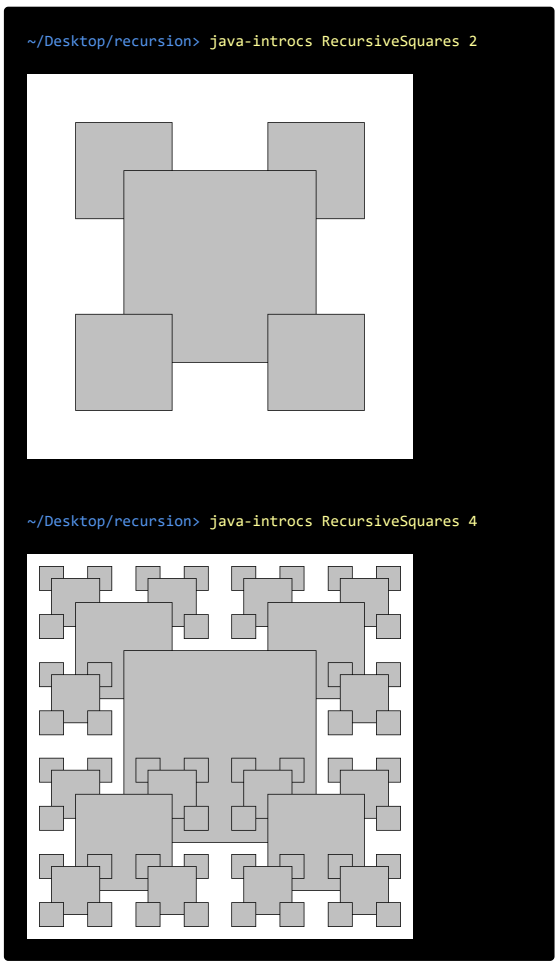


*order 1* *order 2* *order 3* *order 4* *order 5*

To do so, organize your program according to the following public API:

```
public class RecursiveSquares {

    // Draws a square centered on (x, y) of the given side length
    // with a light gray background and a black border.
    public static void drawSquare(double x, double y, double length)

    // Draws a recursive square pattern of order n, centered on (x, y)
    // of the given side length.
    public static void draw(int n, double x, double y, double length)

    // Takes an integer command-line argument n and draws a recursive
    // square pattern of order n, centered on (0.5, 0.5) with side length 0.5.
    public static void main(String[] args)
}
```

The largest square is centered on the canvas and has side length 0.5. The side length of each square is one-half that of the next largest square.




*Note*: you may assume that *n* is a non-negative integer.

**Submission.** Submit a `.zip` file containing `TrinomialBrute.java`, `TrinomialDP.java`, `RevesPuzzle.java`, and `RecursiveSquares.java`. You may not call library functions except those in the `java.lang` (such as `Integer.parseInt()` and `Math.sqrt()`). Use only Java features that have already been introduced in the course (e.g., functions and recursion, but not objects).