Multiple Imputation:

Would this be a proper formulation from the maximum correntropy point of view?

Maximum correntropy criterion in regression:

$$y = f_w(x) + \varepsilon$$

where $f_w(x)$ is a function with parameters w to be found.

Optimization criterion is defined by:

$$J(w) = \sum_{i=1}^{n} K_\sigma\big(y_i - f_w(x_i)\big)$$

n data points

$$K_\sigma(y_i, f_w(x_i)) = \exp\left[-\left\|\frac{y_i - f_w(x_i)}{\sigma}\right\|^2\right]$$

$$\max_w J(w) \equiv \min_w -J(w)$$

As K_σ is the gaussian kernel, we can make use of the following fact:

There exists a convex conjugate function φ of $K_\sigma(z) = \exp\left(-\frac{|z|^2}{\sigma}\right)$ such that:

$$-K_\sigma(z) = \min_\alpha \left[\alpha \frac{|z|^2}{\sigma^2} + \varphi(-\alpha) \right] \quad (1)$$

and for a fixed z, the minimum is reached at $\alpha = K_\sigma(z)$

Let us substitute (1) into $J(w)$. We get

$$-J(w) = \sum_{i=1}^{n} -\min_{\alpha_i} \left[\alpha_i \frac{|y_i - f_w(x_i)|^2}{\sigma^2} + \varphi(-\alpha_i) \right]$$

and so $\min_w -J(w)$ becomes

$$\min_{w,\alpha} \sum_{i=1}^{n} \alpha_i \frac{|y_i - f_w(x_i)|^2}{\sigma^2} + \varphi(-\alpha_i)$$

(A) For a fixed z, as stated before, the minimum is found at

$$\hat{\alpha}_i = K_\sigma(y_i - f_w(x_i))$$

(B) For a fixed α, the optimization criterion becomes

$$\min_w \sum_{i=1}^{n} \alpha_i \frac{|y_i - f_w(x_i)|^2}{\sigma^2}$$

$$\equiv \min_w \sum_{i=1}^{n} \frac{|y_i - f_w(x_i)|^2}{\sigma^2} K_\sigma(y_i - f_w(x_i))$$

which corresponds to the form of the NW estimator, especially if $f_w(x_i) = b$. Furthermore, if we make $y_i = x_i$, meaning n complete points and b = missing point, we should get the method that is being proposed.

Therefore, by maximizing the correntropy with a gaussian kernel and taking $f_w(x)$ as a local point b, we get the proposed method.

Does this make sense?