Resource Depletion in a Spatial Economy

Problem Set 1 - ECON 33550

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Contents

Abstract

This PSet develops a quantitative spatial equilibrium model in which economic activity can endogenously erode local amenities through environmental damage, weakening the standard agglomeration forces emphasized in quantitative spatial models. Motivated by resource-intensive economies, where extraction regions are often productive yet remain distant from major urban centers, I introduce a mechanism in which deforestation generated by production erodes amenities in the location of residence and acts as a congestion force. I quantify the model using new data from the Brazilian Amazon timber market and conduct counterfactual simulations to evaluate how changes in the perception of the intensity of environmental damage can reshape the spatial distribution of production, population, trade, and deforestation.

1 Introduction

In many examples of Quantitative Spatial Models, productivity and output are strong agglomeration forces. Individuals want to be positioned in a place where they can access varieties via trade, while still producing some output and observing their idiosyncratic amenities. This model seems to be able to explain many spatial disparities observed in the developed economies (such as the US), but it misses a key aspect that prevents its application in countries in which trade is intensive in natural resources: **what if the extraction of those goods harms the environment, damaging amenities in locations and acting as an additional congestion force?**

A model with this feature would possibly help explain the spatial disparity of economic, and even environmental variables in countries where logging, agriculture or mining are the main activities. In particular, it would capture the stylized fact that extraction locations, despite being relatively productive, are often far from major agglomerations of workers.[1].

This PSet develops a model in which amenities are harmed by economic output, and quantifies it using new data on the timber market in the Brazilian Amazon. I collect bilateral trade shares between municipalities in the region, as well as wage and population data that comes from the census.

Since the objective is to relate economic output to the extraction of natural resources, I will restrict the dataset to raw timber products, so that trade flows can be directly connected to deforestation. Figure 1 displays data in the Legal Amazon geography.

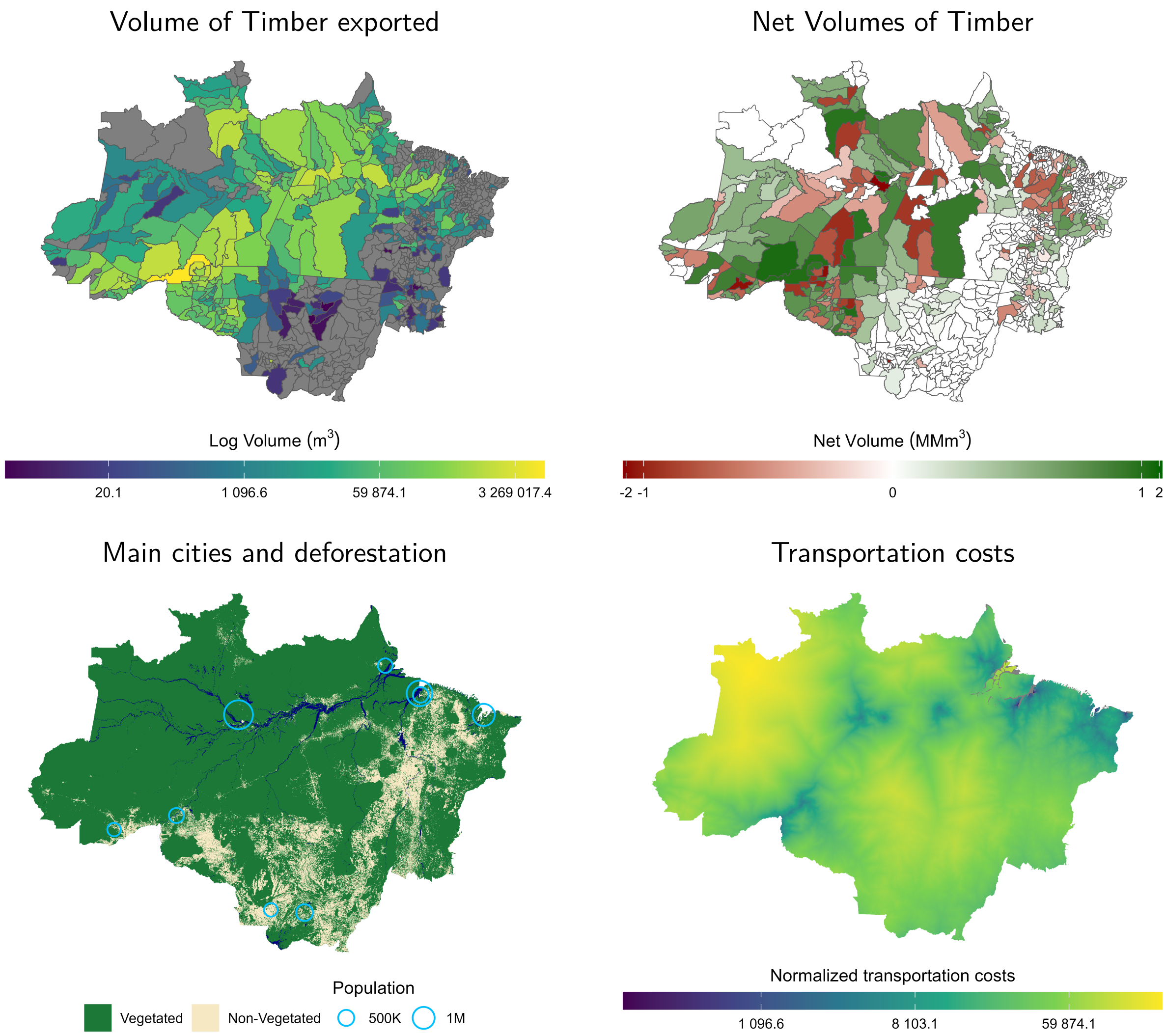

Figure 1: Descriptive Statistics

[1]Saudi Arabia (Oil and Gas), Nigeria (Oil), Indonesia (Timber and Agriculture), Congo (Mining), Peru (Mining), Chile (Mining), Brazil (Timber, Mining and Agriculture) are examples of countries in which extraction of resources happen far away from the main urban centers, but are the main sources of economic output.

The first panel in Figure 1 shows the volume of timber exported by each municipality, while the second one shows the net volume (exports - imports). Note that some municipalities have net volume 0: they consume all of the logs they extract. Panel 3 summarizes the key point: non-vegetated areas tend to be far from major agglomerations. Finally, Panel 4 shows a unit-free transportation costs measure estimated by Araújo et al. (2023) that will be used in the model.

In Panel 1, it's clear that that the highest exporter municipality (bright yellow, on the left near the border with Bolivia) is located close to the borders of the states of *Rondônia* and *Acre*. By this municipality, connecting these two states, passes the road BR-340. Since this road connects Acre to the rest of the country, decreasing transportation costs, many environmentalists argue that it is one of the causes of deforestation in the region. A specific question I would like to answer is what is the impact of this road on overall deforestation and trade in the region.

The remaining of the text is organized as the following way. In section 2 we develop a quantitative spatial model with trade, migration, commuting and a mechanism through which output can erode amenities. Section 3 describes the data and the main calibration decisions. Section 4 performs the counterfactual exercise of shocking commuting costs that reflect the removal of the infrastructure discussed in the previous paragraph. Finally, section 5 outlines possible extensions of this work.

2 Model

The model follows Monte et al. (2018) in preferences and trade and introduces more structure on commuting through the bilateral amenities term B_{ni}. Amenities will be equal to a scale parameter $\overline{B_{ni}}$ reduced by the level of deforestation in the location of residence D_n weighted by an elasticity ϕ, which determines how much deforestation damages amenities. This specification imposes some changes in the commuting structure. Later on, we add one equation to the equilibrium that serves as an environmental restriction, connecting deforestation and production.

2.1 Preferences

Let Ω be a finite index of locations. Preferences are such that a representative worker who lives and works in locations $n, i \in \Omega$ chooses consumption, C_n, and housing H_n taking into account amenities tied to the residence-workplace pair, b_{ni}, and commuting costs $\kappa_{ni} \in [1, +\infty)$ using the following Cobb-Douglas utility function

$$U_{ni} = \frac{b_{ni}}{\kappa_{ni}} \left(\frac{C_n}{\alpha}\right)^{\alpha} \left(\frac{H_n}{1-\alpha}\right)^{1-\alpha}. \tag{1}$$

Individuals might have idiosyncratic reasons for choosing a particular pair (n, i), so we take b_{ni} as draws of a Fréchet distribution such as in Eaton and Kortum (2002). The cumulative distribution function of amenities is

$$G(b) = \exp(-B_{ni} b^{-\varepsilon}), \qquad \varepsilon > 0,$$

where B_{ni} is the mean of amenities in each pair (n, i). I introduce more structure on this parameter, by considering a decrease in the level of amenities $\overline{B_{ni}}$ given by the amount of deforestation in the location of residence D_n using the following functional form

$$B_{ni} \doteq \overline{B_{ni}} \cdot \exp(-\phi D_n), \qquad \overline{B_{ni}} > 0 \tag{2}$$

in which $\phi > 0$ is a parameter that governs how much deforestation damages amenities. Equation 2 can be seen as a finite resource assumption, since increasing D_n for fixed ϕ and $\overline{B_{ni}}$ will take amenities arbitrarily close to zero. In this PSet, D_n will be quantified as deforestation, but it can be interpreted as any environmental harm caused by production, such as mining

Consumption is characterized by the standard CES consumption and price indexes following the original model.

$$C_n = \left\{ \sum_{i \in \Omega} \int_0^{M_i} c_i(j)^\rho \mathrm{d}j \right\}^{\frac{1}{\rho}}, \qquad P_n = \left\{ \sum_{i \in \Omega} \int_0^{M_i} p_i(j)^{1-\sigma} \mathrm{d}j \right\}^{\frac{1}{1-\sigma}}$$

where $\sigma \doteq 1/(1-\rho) > 1$. I also make the same assumption as in the original paper that land is owned by immobile landlords that only consume locally. This assumption, alongside with land market yields the following expression for land prices Q_n,

$$Q_n = (1-\alpha) \frac{\overline{v_n} R_n}{H_n} \tag{3}$$

where R_n is the number of residents, $\overline{v_n}$ is the income and H_n is the land supply in location n.

2.2 Production and Trade

The production and trade structure is identical to the original model. For each variety j, $x(j)$ units take $l_i(j) = F + x_i(j)/A_i$ of labour output to be produced. Maximizing profits under monopolistic competition, we conclude that the $p_{ni}(j) = \mu \frac{d_{ni} w_i}{A_i}$ where d_{ni} are the shipment costs, w_i is the wage and $\mu = \sigma/(\sigma - 1)$ is a mark-up. Combining the first order conditions of the firm's problem with the zero profits assumption, we conclude that the equilibrium output of each variety is $x(j) = x_i = A_i F \cdot (\sigma - 1)$, which does not depend on j. Then, the number of varieties produced in i, M_i, can be obtained by substituting the total output $Y_i \doteq M_i x_i$ in the labor market clearing equation

$$L_i = F + \frac{M_i A_i F \cdot (\sigma - 1)}{A_i} \Rightarrow M_i = \frac{L_i}{\sigma F} \tag{4}$$

Using 4 and the equilibrium prices, we can obtain bilateral trade shares and the price index.

$$\pi_{ni} = \frac{M_i p_{ni}^{1-\sigma}}{\sum_{k \in \Omega} M_k p_{nk}^{1-\sigma}} = \frac{L_i \left(\frac{d_{ni} w_i}{A_i}\right)^{1-\sigma}}{\sum_{k \in \Omega} L_k \left(\frac{d_{nk} w_k}{A_k}\right)^{1-\sigma}} \tag{5}$$

$$P_n = \mu \left(\frac{L_n}{\sigma F \pi_{nn}} \right)^{\frac{1}{1-\sigma}} \cdot \left(\frac{d_{ni} w_i}{A_i} \right) \tag{6}$$

2.3 Deforestation

I model deforestation as being proportional to the total output in i, Y_i. The interpretation is that output imposes some sort of congestion force, a congestion force that may reflect deforestation, industrial residues or perhaps worse natural conditions (extreme heat or frequent floods for instance) or disasters. The constant that governs how much the output feeds this forces is $\delta > 0$ such that for each $i \in \Omega$

$$D_i = \delta Y_i = \frac{\delta}{\mu} A_i L_i. \tag{7}$$

Equation 7 can be seen as an environmental equilibrium condition, as it states that the observed levels of deforestation have to match output in each location.

2.4 Commuting

While, commuting is also the same from the standard literature, the additional structure imposed in amenities will cause changes in the equilibrium commuting shares. In this section, I will redo the important computation and discuss the relevant differences from the original model. Indirect utility of workers can be expressed as monotonic transformation of amenities for each pair (n, i)

$$U_{ni} = \frac{b_{ni} w_i}{\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}} = \left(\frac{w_i}{\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}} \right) \cdot b_{ni}$$

Therefore, utility is also distributed Fréchet such that its cumulative probability distribution function is $G_{ni}(U) = \exp(-\Psi_{ni} U^{-\varepsilon})$ where

$$\Psi_{ni} = \overline{B_{ni}} \cdot \exp(-\phi D_n) \cdot \left(\frac{w_i}{\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}} \right)^{\epsilon}$$

Defining $\Psi \doteq \sum_{(n,i)\in\Omega^2} \Psi_{ni}$, we can express the probability of a worker choosing the pair (n, i) as $\lambda_{ni} = \Psi_{ni}/\Psi$. Summing all residences for a fixed workplace location (conversely), we obtain the probability that an individual chooses to work (live) on a particular location, such that

$$\lambda_n^R = \frac{R_n}{\overline{L}} = \sum_{r\in\Omega} \frac{\Psi_{nr}}{\Psi}, \qquad \lambda_i^L = \frac{L_i}{\overline{L}} = \sum_{s\in\Omega} \frac{\Psi_{si}}{\Psi} \tag{8}$$

In order to obtain the equilibrium conditions on L_i, R_i and $\overline{v_i}, w_i$ we need to compute the probability of workers commute to i conditional on living on n. That is

$$\lambda_{ni|n}^R \doteq \frac{\lambda_{ni}}{\lambda_n^R} = \frac{\Psi_{ni}/\Psi}{\sum_{r\in\Omega} \Psi_{nr}/\Psi} = \frac{\overline{B_{ni}} \cdot \exp(-\phi D_n) \cdot \left(\frac{w_i}{\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}} \right)^{\epsilon}}{\sum_{r\in\Omega} \overline{B_{nr}} \cdot \exp(-\phi D_n) \cdot \left(\frac{w_r}{\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}} \right)^{\epsilon}}$$

In te expression above, the terms that depend on the residence location n simplify, including the deforestation term $\exp(-\phi D_n)$. Therefore, the workplace decision is invariant to deforestation once the residence decision has been made, as expected. However, the damage deforestation does to amenities is still important to determine the network of residence-workplace decisions through 8. The conditional probability simplifies to

$$\lambda_{ni|n}^R = \frac{\overline{B_{ni}} \cdot \left(\frac{w_i}{\kappa_{ni}} \right)^{\epsilon}}{\sum_{r\in\Omega} \overline{B_{nr}} \cdot \left(\frac{w_r}{\kappa_{nr}} \right)^{\epsilon}}.$$

Finally, population mobility imposes that expected utility is the same for every pair (n, i). Using the expectation from the Fréchet distribution yields

$$\overline{U} = \mathbb{E}[U_{ni}] = \Gamma\left(\frac{\varepsilon-1}{\varepsilon}\right)\left\{\sum_{(s,r)\in\Omega^2} \overline{B_{sr}} \cdot \exp(-\phi D_s) \cdot \left(\frac{w_r}{\kappa_{si} P_n^{\alpha} Q_s^{1-\alpha}}\right)^{\epsilon}\right\}^{\frac{1}{\varepsilon}} \tag{9}$$

2.5 General Equilibrium

The general equilibrium variables of the model are defined by the following vector

$$\Sigma \doteq \{w_n, \overline{v_n}, Q_n, L_n, R_n, P_n, D_n, \overline{U}\}.$$

and the equilibrium vector is characterized by 7 equations. The first 6 of them are the ones that come from the original model and the extra one is the Resource Depletion assumption (7).

3 Data and Estimation

In this section I will discuss the data I will use to quantify the model. I begin with a brief description of the sources of the Trade and Deforestation, then I show a calibration table summarizing all the variables that I need to solve the model and finish with a discussion of some of my calibration choices.

The bilateral trade flows come from a database of timber trade in Brazil: DOF (*Documento de Origem Florestal*[2]). This database operates as a monitoring mechanism for native forestry products, requiring users to register each transaction, including details such as species, volume, price, and transportation mode (mostly trucks and boats). This registration process generates a document, which must be printed and transported in the same vehicle as the forestry product. Failure to present this document or inconsistencies in the reported data can lead to legal penalties. Therefore, this database contains every *legal* transaction in the market since 2007.

DOF was not adopted by all Brazilian states at the same time, so the data has some inconsistencies. I also observe the same bilateral trade flows from another source for robustness. These trades come from an NGO called Timberflow, that sent me this data in 2024. They have a database that complements DOF with older systems. While this database is more complete for most states in older years, my baseline estimations will use DOF as it is the official timber trade database in Brazil in the sense that Brazilian authorities can enforce compliance with DOF reporting.

Deforestation data comes from the Mapbiomes database. This data is computed using satellite imagery. Each pixel in the pictures is transformed into a category according to its land use: forest, pasture, agriculture, mining, urban and more.

Finally, the geography in which we will run this model will be the municipalities of the Brazilian Legal Amazon that have nonzero trade flows in the whole period of the DOF database. Figure 1 shows a depiction of this geography. The sample includes 364 municipalities.

Table 1 summarizes all the calibration decision of the model, as well as values and sources when appropriate.

I will invert A_n from the observed D_n and L_n^R. Nevertheless, I was not able to invert $\overline{B_{ni}}$ and $\overline{H_{ni}}$ from the data I have, so I will just suppose they are constant through all locations. II tried several simplifying assumptions, but none yielded a workable inversion. Obviously this

[2]This translates to Forestry Origin Document.

Table 1: Calibration Table

Variable	Value	Notes / Source
Exogenous fundamentals / data (inputs)		
A_i		Inversion
d_{ni}		Araújo et al. (2023)
κ_{ni}		$=(\text{distance}_{ni})^{-1.29}$
$\overline{B}_{ni}$	1	
$\overline{H}_n$	1	
$\overline{L}, L_n^R$	30,240,233	(2010 Brazilian Census)
Structural parameters		
σ	4.107	Dominguez-Iino (2026)
α	0.6	
ε	3.3	Monte et al. (2018)
F	1	
ϕ	$\{0, 1, 5\}$	Sensitivity Analysis
δ	0.1254	Estimated
Endogenous equilibrium objects (outputs)		
w_i		(2010 Brazilian Census)
D_n		Mapbiomes

is unfortunate, but since my main argument is about the effects of δ, ϕ and D_n, perhaps this is not a huge loss.

Deforestation in each location is computed as the share of Non-Vegetated area in each municipality, then dividing each share by their minimum and taking logs. The commuting costs are obtained elevating the geographical distance between municipalities centroids to a parameter estimated by the gravity equation in the original paper. Trade costs d_{ni}

For the structural parameters, we borrow α, F and ε from the original model, since none of these parameters are related to the main mechanism of this model, I will not try to estimate them using the available data. σ is taken from the across commodities price elasticity estimate from Dominguez-Iino (2026). This paper uses data from South America including that overlaps with our geography, what makes this estimate suitable for this exercise.

Unfortunately I do not have data to estimate ϕ directly, nor I found appropriate estimates in the literature to account for this parameter. The model is silent about how deforestation affects amenities. It could be due to health related reasons, such as mining pollution, or perhaps more violence since markets for illegal goods are connected to an increase in violent crimes (see section 5 for a further discussion).

In order to estimate the output-deforestation elasticity, I regress the accumulated deforestation on accumulated timber volume exported in the municipality level, controlling by their areas and state fixed effects. The exact equation used is

$$\log(\Delta\text{Deforestation}_i) = \delta \log(\Delta\text{TimberVolume}_i) + \gamma^T X + \delta_n + \eta_i. \tag{10}$$

Thus, the coefficient from that regression tells how much deforestation we should expect in a municipality with 1 extra volume of timber exported, controlling by the size of each unit.

The results for that estimation using data from DOF and Timberflow can be found in Table 2. While both estimates seem to be in the same ballpark, $\delta = 0.1254$ is my preferred estimate because it reaches a bigger R^2 and uses data from DOF, which is the official government instrument to enforce environmental law.

Table 2: Regression Results for δ calibration

	DOF		Timberflow	
Model:	(1)	(2)	(3)	(4)
Variables				
$\log(\Delta\text{TimberVolume})$	0.2343***	0.1254***	0.2333***	0.1679***
	(0.0176)	(0.0236)	(0.0154)	(0.0390)
Fixed effects				
State		✓		✓
Other Controls				
Constant	✓		✓	
log(Area)	✓	✓	✓	✓
Other statistics				
Observations	667	667	808	808
R^2	0.44	0.55	0.37	0.41
R^2 Within		0.16		0.24

*Significance: ***: 0.01, **: 0.05, *: 0.1*

4 Counterfactual Exercise

For the counterfactual exercise I will study how changes in ϕ affect deforestation and the residence decision of workers. Remember that ϕ is the parameter that controls the intensity of the resource depletion mechanism. I solve the model for $\phi\{0, 1, 5\}$ and compute the normalized D_n, the resident population R_n and also the statistic $\sum_{n\in\Omega} D_n$. Figure 2 shows the equilibrium outcomes for these three quantities for each value of ϕ.

Firstly, note that the total deforestation is decreasing in ϕ, as expected. In particular, when the resource depletion mechanism is completely shut down ($\phi = 0$), we observe the biggest total normalized deforestation. This number is approximately 6.7 times smaller when ϕ increases to 5. Since the deforestation values are normalized, we cannot interpret them directly, so this ratio gives an idea of the intensity of the change.

The spatial disparities when the mechanism is shut down also call attention. In this case, deforestation is concentrated in a few locations due to their high productivity, not always far from the decisions of residence. For instance, check out the state of *Rondônia*, in the west region of the maps in Panel A). Since there is no mechanism to back off deforestation and baseline amenities ($\overline{B_{ni}}$) are constant, the optimal for workers is to concentrate production in those few, high productivity places.

Panel B shows that when the mechanism is turned on, deforestation spreads everywhere, since concentrating everything in one place would deplete entirely the resources in that location. As a consequence, R_n also starts to spread out, but is still concentrated near the capital of the state of *Amazonas*, perhaps due to low trade/commuting costs.

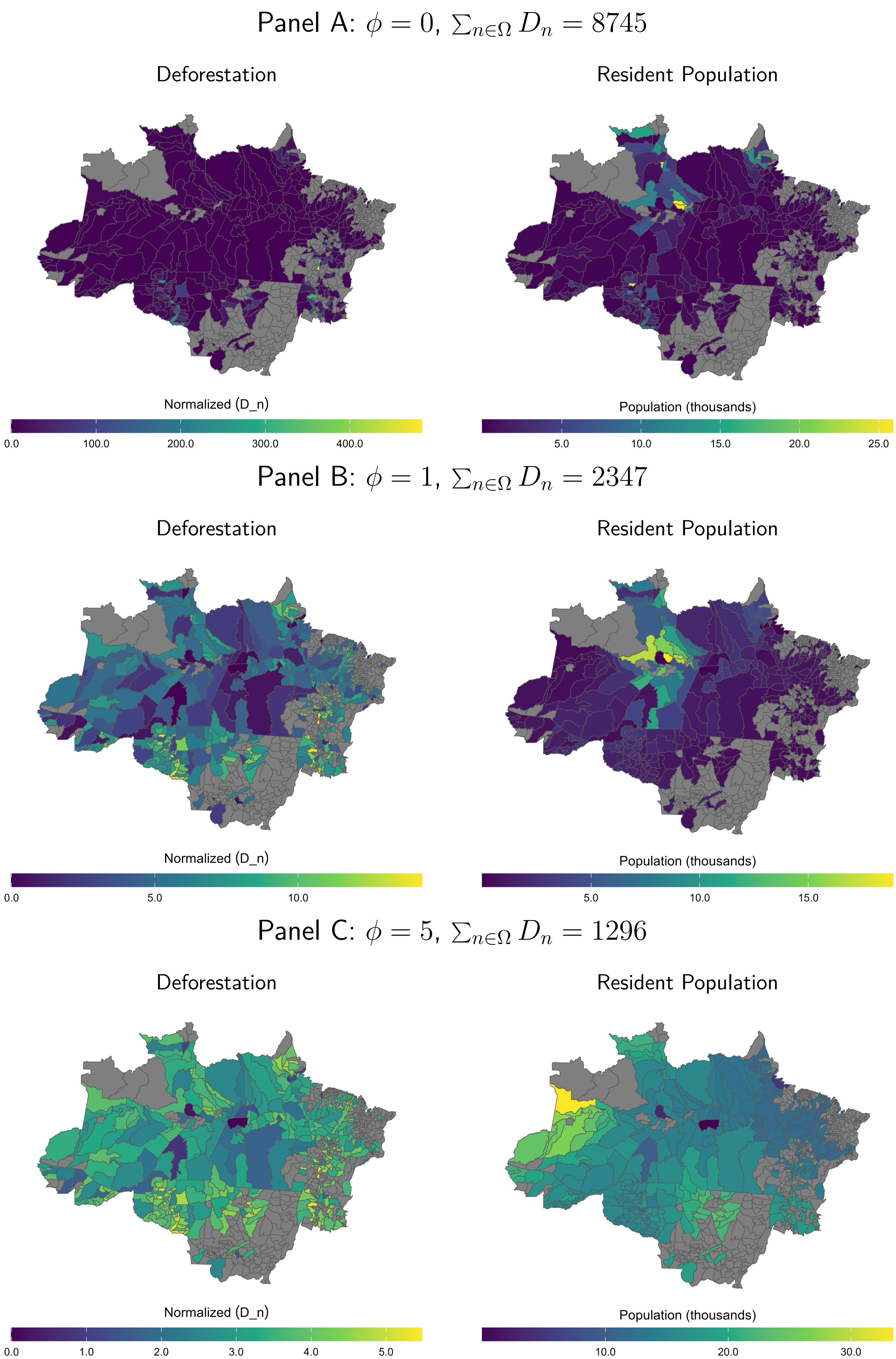

Figure 2: Counterfactual exercise.

Finally, if the mechanism is even more intensified, we can see that residence decisions tend to move far from most deforested places. Again, the example of *Rondônia*, in the west of the map is instructive.

It is hard to imagine how the government could impact ϕ by a policy. However, one can interpret this parameter as a *perceived* notion of how much economic exploration of natural resources hurt amenities. In this sense, its possible to imagine how some events can impact ϕ. Before the nuclear disaster in Chernobyl, for instance, workers might have been indifferent between living close or far from a nuclear plant, but this perception definitely changed after the accident. In Brazil, the recent Dam accidents of Mariana and Brumadinho also serve as examples.

5 Next Steps: adding an illegal market of natural goods

This PSet enhanced the quantitative spatial model on Monte et al. (2018) by adding an extra congestion force, that is based on the depreciation of amenities by economic output. The idea of this formulation was to provide a more suitable framework to study developing economies based on extraction of natural resources (Oil, Agriculture, Minerals and, in this case, Timber).

Nevertheless, the model is silent on an important feature of the production side of such economies and that is immensely correlated to the harmful exploration of natural resources: illegal markets and violence. In the case of Brazil, many papers recently reported the existence of illegal markets of Gold Pereira and Pucci (2021) and Timber Chimeli and Soares (2017). This literature seems to have reached a conclusion that the illegal markets of natural resources tend to be positively correlated to legal markets, as if firms acting in illegality benefited from the infrastructure of the legal market, rather than being harmed by it.

Data from DOF seems to yield arguments consistent with that conclusion. e-estimating equation 10 using accumulated deforestation within (i) Public Protected Areas, where deforestation is illegal and enforcement of property rights high and (ii) Public Not Protected areas, where deforestation is also illegal, but enforcement is low, shows dramatically different estimates as shown in Table 3. In particular, the estimate of the model (6) in Table 3 generates consistent evidence with the argument since the outcome is illegal deforestation and the regressor is the legal market of timber.

Table 3: Regression Results for δ calibration with deforestation breakdown

	Total		Public Protected		Public Not Protected		Private	
Model:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Variables								
$\log(\Delta \text{TimberVolume})$	0.2333***	0.1679***	0.0318***	0.0224	0.2153***	0.1340***	0.2348***	0.1755***
	(0.0154)	(0.0390)	(0.0113)	(0.0294)	(0.0149)	(0.0130)	(0.0151)	(0.0404)
Fixed effects								
State		✓		✓		✓		✓
Other Controls								
Constant	✓		✓		✓		✓	
log(Area)	✓	✓	✓	✓	✓	✓	✓	✓
Other statistics								
Observations	808	808	808	808	808	808	808	808
R^2	0.43	0.51	0.37	0.41	0.46	0.61	0.41	0.48
R^2 Within		0.21		0.24		0.24		0.21

*Significance: ***: 0.01, **: 0.05, *: 0.1*

This evidence points to the fact that we should develop a theory in which the production side is given the option to act in the legal or illegal market. Firms across different regions would get different incentives since enforcement varies in space (perhaps it is a function of the trade costs). The key assumption is that illegal firms would cause more damage to amenities than legal firms, because they are more harmful for the environment and use violence more frequently.

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