1D Deforestation Model with Agribusiness Dynamics

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1 Initialization

Let $N \in \mathbb{N}$. This code takes some parameters and is supposed to generate spatial-temporal series of a 1 dimensional, discrete grid with N entries[1] for these variables: Deforestation $(D_{nt})_{t=0}^{\infty}$, emissions $(O_{nt})_{t=0}^{\infty}$, Rents $(R_{nt})_{t=0}^{\infty}$ and Agribusinesses locations $(S_{nt})_{t=0}^{\infty}$.

The most important decisions to initialize the model are

- Where the Agribusiness begins. This should be a $1 \times N$ matrix with entries 1 meaning that there is an agribusiness in that plot and 0 if there is not;
- Which plots are protected/unprotected. Also a $1 \times N$ matrix with 1 meaning protected and 0 unprotected;
- The initial deforestation. This is at least equal to the initial Agribusiness.
- Productivity is also a $1 \times N$ matrix. **Its entries need to be bigger than** 1.
- $\rho, \sigma > 1$

The following is an example on how to set up these fundamentals. After the initialization, you will have to call the function `SolveDeforestation1D` and feed it the fundamentals you determined. The rest of this note looks into what this function does.

```
Agribusiness = zeros(1, N, Tmax); Agribusiness(1,end,1) = 1;
D = zeros(1, N, Tmax); D(1,end,1) = 1; deforested.
O = zeros(1, N, Tmax); O_aux = rand(1,N); O(:,:,1) =
    D(1,1,1).*O_aux;
R = zeros(1, N, Tmax);
A = 2.*ones(1,N);
F = 2*ones(1,N);
L = zeros(1,N); L(1,1:N/2) = 1;
w = wage * ones(1,N);

SolveDeforestation1D(N,Tmax,A,F,O,D,L,Agribusiness,O_aux)
```

[1] You can think of it as a $1 \times N$ matrix.

2 Simulation Loop

The model iterates over time steps to update land use, emissions, and agribusiness expansion. Before going into the loop, it is important to calculate the neighbors vector, as it will be important in the computation of the profits in the future. Neighbors are defined as plots share a border with an Agribusiness. In one spatial dimension, Agribusinesses can have 1 (if they are in a border) or 2 neighbors.

The following code returns the positions of the neighbors of every Agribusiness. The first step is to feed the `clusters` vector with the Agribusiness data. This vector will compute the position of each Agribusiness no matter its size. For example, the data

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

shows 3 Agribusiness, represented as clusters of ones. Once you separate this clusters, you can calculate each cluster's neighbors (and check if they are or not valid).

```
clusters = {};
    isOne = (Agribusiness(1,:,t) == 1);
    d = diff([0 isOne 0]);
    startIdx = find(d == 1);
    endIdx   = find(d == -1) - 1;

    for k = 1:length(startIdx)
        clusters{k} = startIdx(k):endIdx(k);
    end

    left  = cellfun(@(c) min(c(:)), clusters);
    right = cellfun(@(c) max(c(:)), clusters);

    cand  = [left-1; right+1];
    valid = cand >= 1 & cand <= N;              % in-bounds mask

    neighbors = arrayfun(@(i) cand(valid(:,i), i).', ...
                         1:numel(left), 'UniformOutput', false);
```

We must update our 4 variables of interest. The first one that concerns us is R_{nt}, as it is required to determine both deforestation and the Agribusiness dynamics. Remember that the expression that rules over rents in plot n at period t is

$$\Psi\{S,n\} - \Psi\{S\} \geq R_{nt} \geq \max\left\{\varrho_1 F + \varrho_2 w_n - \varrho_3 A_{nt} - \frac{w_n}{1-\beta}, \frac{A_{nt} - w_n}{1-\beta}\right\} \tag{1}$$

The upper bound of this inequality is Agribusinesses profits and the lower bound is the farmer selling price. The idea is to set values of R_{nt} as an weighted average of the bounds. The weights will determine the bargaining power of each side. This next code calculates the upper bound, which accounts for the farmer's bargaining power (the maximum value they can sell the land for the Agribusiness). As of now, this code makes an extra assumption on Ψ, which is that **the only plots whose rent is affected by the presence of the agribusiness are neighbors**. Another way to see this is

$$\Psi\{S,n\}-\Psi\{S\} = \begin{cases} \Gamma \sum_{i\in\{S,n\}} \delta_i \left(\frac{\tau_i w_i}{B(A_n,S+1)}\right)^{1-\sigma} - \Gamma \sum_{i\in\{S\}} \delta_i \left(\frac{\tau_i w_i}{B(A_n,S)}\right)^{1-\sigma}, n \text{ is a neighbor} \\ 0, n \text{ is a neighbor} \end{cases}$$

```
for i =1:numel(neighbors)
        cluster = clusters{i};
        ContAgri = numel(cluster); % Counting the S
        term1 = sum(profit(A,ContAgri+1,cluster));
        term2 = sum(profit(A,ContAgri,cluster));
        for k = 1:numel(neighbors{i})
              if Agribusiness(1,neighbors{i}(k),t) == 1
                     continue;
              end
              DiffProfit(1,neighbors{i}(k)) = term1 - term2 +
                 profit(A,ContAgri+1,neighbors{i}(k));
        end
    end
```

Note that rents will not be calculated in sites that are already Agribusinesses as this computation would be useless. For each $n = 1, \cdots, N$, the loop begins by checking if the site is an Agribusiness and skipping to the next site if it is the case. After that, we store the number of contiguous sites that are Agribusinesses in the ContAgri object and for each of them (whose positions are stored in neighbors(n)), we compute that **difference of profits**. In the end of the loop, we add the profit of plot n. This process should make the rents higher when distance to the closest Agribusiness decreases. Forests contiguous to an Agribusiness and located in unprotected sites should be the most valuable, ceteris paribus.

In the previous code, we have used the functions `profit`, `b`, `Sigma` which are in the same code of this document. In particular, the function `Sigma` controls the increasing or diminishing scale effects of incorporating a new plot to the Agribusiness. The lower bound is a trivial matrix calculation.

```
FarmersProfit = max(q1*F+q2*wage-q3*A-wage/(1-beta),
   (A-wage)/(1-beta));

R(1,:,t) = 1.*DiffProfit + 0*FarmersProfit; % Full bargaining
   power to the Agribusiness
```

Next, we turn to deforestation. Farmers make deforestation choices based on discounted earnings of each plot. Legal status will play an important role in these decisions. The value functions for each legal status are coded in different files and appear here as functions. The main difference between then is that the value of unprotected land depends on rents, because there is a possibility of selling it.

A plot will be deforested if the following happens: (i) it has not been deforested and (ii) is value is bigger than zero.

```
Vu = value_u(A,R(1,:,t),F);
Vp = value_p(A,F);

Du = L == 0 & D(1,:,t) ~= 1 & Agribusiness(1,:,t) ~= 1 & Vu > 0;
Dp = L == 1 & D(1,:,t) ~= 1 & Agribusiness(1,:,t) ~= 1 & Vp > 0;
```

Finally, we update deforestation, emissions and the Agribusiness. The latter is the most complex. Firstly, every deforested plot that is not an Agribusiness has the chance of (i) obtaining property rights and (ii) being sold to the Agribusiness. The first thing happens with probability π, and the latter happens if the difference in profits (values of Ψ) is superior to R_{nt}. If a land does not get the property rights in one period, it will have another shot in the next. Every deforested plot is eventually sold to an Agribusiness with probability 1.

Deforestation is simply updated adding the plots for which `Vu` or `Vp` is positive and emissions use the identity

$$O(n,t+1) = O(n,t) + \underbrace{D(n,t)}_{\text{Deforestation in period t}} \cdot \underbrace{E(n)}_{\text{Constant Emissions matrix}} \tag{2}$$

```
D(1,:,t+1) = D(1,:,t) + Du+Dp;
O(1,:,t+1) = O(1,:,t) +  D(1,:,t) .* O_aux;
Candidates = D(1,:,t) - Agribusiness(1,:,t);
PP = Candidates ==1 & rand(1,N) < pi & L==0;
NewAgri = PP==1 & DiffProfit >= R(1,:,t);
Agribusiness(1,:,t+1) = Agribusiness(1,:,t) + NewAgri;
```

3 Some exercises

This section will show some parameters, results and thoughts on 1D applications of the model. For each exercise, we present two plots: (i) initial/final deforestation (white = deforested, black = not deforested) and (ii) deforestation (in number of plots) and emissions over time (stock).

3.1 Everything protected and A decreasing (static model).

This exercise will present no dynamics, as $\pi = 0$. Indeed, without the Agribusiness purchasing new territories and influencing the values of the contiguous plots via R_{nt}, all deforestation will be made in $t = 1$. We set all territories to be unprotected and productivity to be decreasing in n. We initiate the agribusiness in the left vertex of the grid, i.e, in the one with the highest productivity.

```
A = linspace(3.1,1.1, N);
F = 1.8*ones(1,N);
L = zeros(1,N); L(1,1:N/2) = 1;
w = wage * ones(1,N);
```

Table 1: Agribusiness in one vertex

Variable	Value	Description
N	15	Number of spatial plots
T_{max}	25	Number of time periods
β	0.10	Discount factor)
α	0.75	Land productivity parameter
π	0	Probability of acquiring property rights
T_{lag}	2	Time producing at low productivity
δ	1	Demand shifter
τ	1.2	Transport cost
w_n	1	Wage level
σ	1.5	Elasticity parameter
$\bar{S}$	6	Σ parameter
p_I	1	Investment price parameter
ρ	1.5	Interest rate

```
D = zeros(1, N, Tmax); D(1,1,1) = 1;
O = zeros(1, N, Tmax); O_aux = rand(1,N); O(:,:,1) =
    D(1,1,1).*O_aux;
Agribusiness = zeros(1, N, Tmax); Agribusiness(1,1,1) = 1;
```

Figure 1 and Figure 2 shows deforestation before and after these simulations. Note that all deforestation is done in the first period, as we predicted given the lack of the Agribusiness. The Agribusiness gives life to this model. In a world in which the amount of deforested plots is constant, emissions are linear.

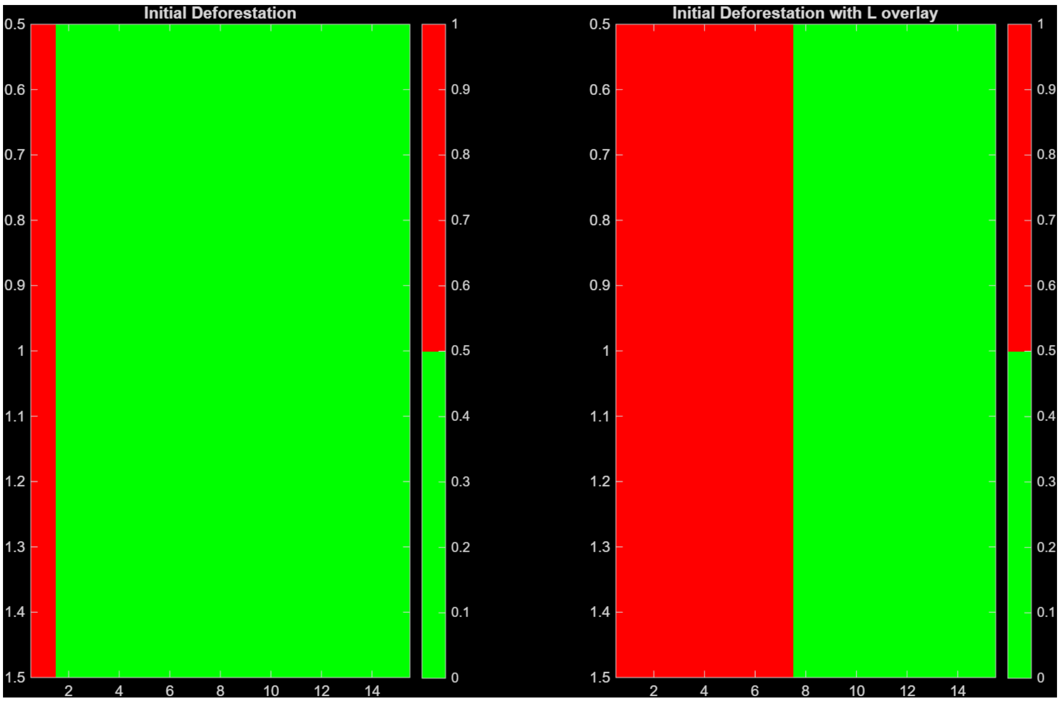


Figure 1: $\pi = 0$ and A decreasing: initial and final deforestation. 1 = deforested, 0 = not deforested

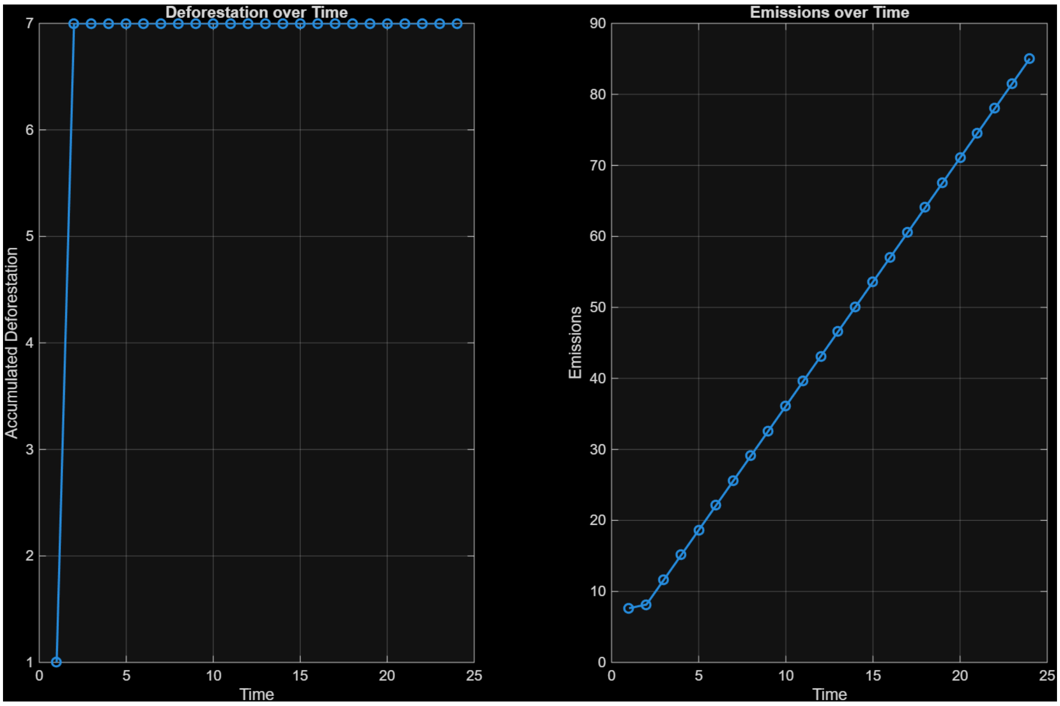


Figure 2: $\pi = 0$ and A decreasing: accumulated deforestation and emissions

3.2 Agribusiness in one vertex

Here we will see some dynamics. We begin by creating the a deforestation point in the last position of the deforestation matrix in the first period. Include the same position in the Agribusiness map. Consider a geography in each the first half positions are protected, and the last half, where the agribusiness stars, is unprotected. Table 4 shows the rest of the parameters.

Table 2: Agribusiness in one vertex

Variable	Value	Description
N	15	Number of spatial plots
$T_{\max}$	25	Number of time periods
β	0.10	Discount factor)
α	0.75	Land productivity parameter
π	0.5	Probability of acquiring property rights
T_{lag}	2	Time producing at low productivity
δ	1	Demand shifter
τ	1.2	Transport cost
w_n	1	Wage level
σ	1.5	Elasticity parameter
$\bar{S}$	6	Σ parameter
p_I	1	Investment price parameter
ρ	1.5	Interest rate

```
A = 2.*ones(1,N);            % Productivity > 1
```

```
F = 1.8*ones(1,N);          % Homogeneous deforestation cost
L = zeros(1,N); L(1,1:N/2) = 1;% Legal status
w = wage * ones(1,N);         % Constant wage
D = zeros(1, N, Tmax); D(1,end,1) = 1; % Agrinusiness starts in
    one of the vertices
O = zeros(1, N, Tmax); O_aux = rand(1,N); O(:,:,1) =
    D(1,1,1).*O_aux;
R = zeros(1, N, Tmax);
Agribusiness = zeros(1, N, Tmax); Agribusiness(1,end,1) = 1; %
    Agrinusiness starts in one of the vertices
```

Figure 3 and Figure 4 shows deforestation before and after these simulations. Note that the Agribusiness enters the grid over time, following the Agribusiness dynamic. Protected land is kept intact: its value is too low and cannot be influenced by the presence of the Agribusiness. Emissions have superlinear behavior.

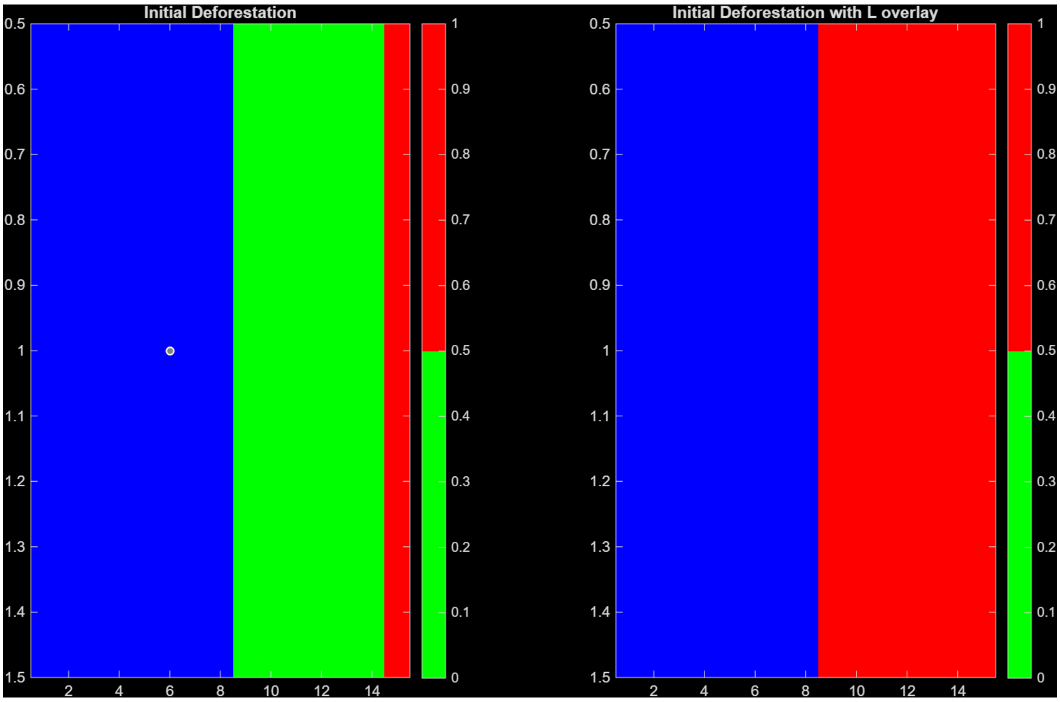


Figure 3: Agribusiness on the vertex: initial and final deforestation. 1 = deforested, 0 = not deforested

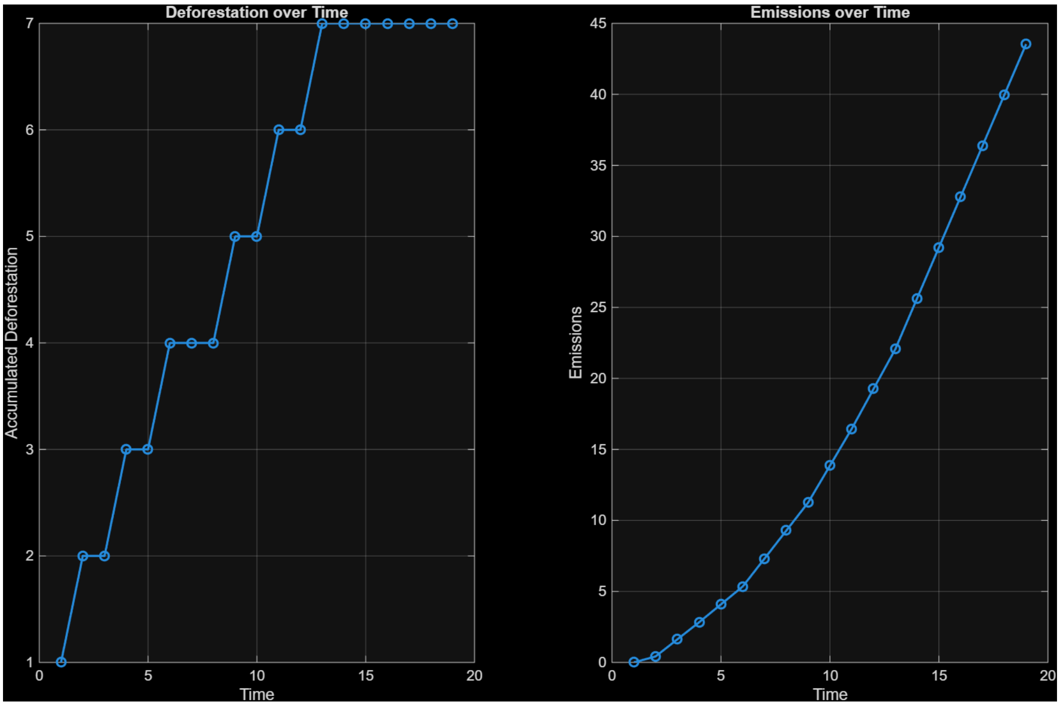
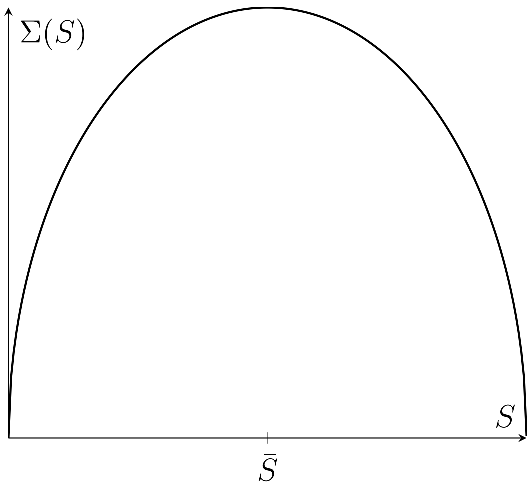


Figure 4: Agribusiness on the vertex: accumulated deforestation and emissions

3.3 Σ restraining Agribusiness expansion

If we change $\bar{S} = 5.5$ in the last computation, we obtain a slight change (Figure 5 shows the output): the deforestation forces do not get to the protected line. Instead, they stop early, due to the concavity Σ. Be aware that Σ can limit the Agribusiness. Maybe a standard like $\bar{S} = 0.2 \cdot N^2$ is useful. The following is a depiction of Σ.



Note that returns of scale are increasing if $\Sigma'(S) > 0$, that is, if $S < \bar{S}$. Shifting $\bar{S}$ to the right will make the Agribusiness more willing to incorporate a grater ammount of plots.

3.4 Agribusiness in the middle

This time, initiate the Agribusiness in position 8 ($N = 15$) and make only two protected sites, 4 and 11. This will stop the expansion of the Agribusiness.

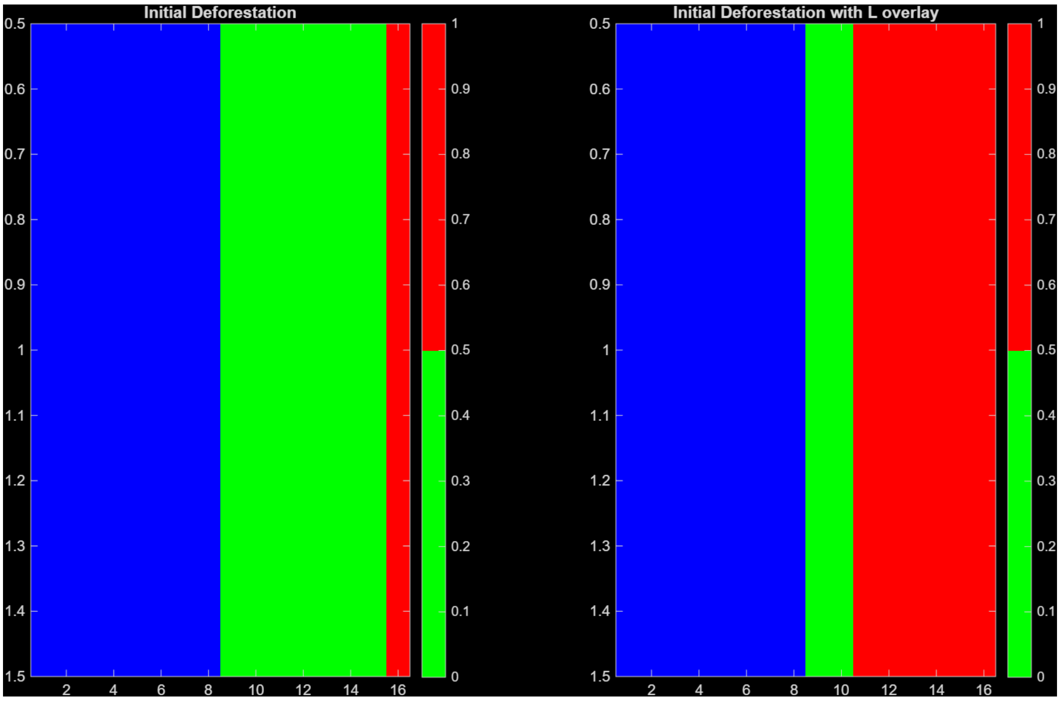


Figure 5: Agribusiness on the vertex: initial and final deforestation. 1 = deforested, 0 = not deforested

Table 3: Agribusiness in one vertex

Variable	Value	Description
N	15	Number of spatial plots
$T_{\max}$	25	Number of time periods
β	0.10	Discount factor)
α	0.75	Land productivity parameter
π	0.5	Probability of acquiring property rights
T_{lag}	2	Time producing at low productivity
δ	1	Demand shifter
τ	1.2	Transport cost
w_n	1	Wage level
σ	1.5	Elasticity parameter
$\bar{S}$	6	Σ parameter
p_I	1	Investment price parameter
ρ	1.5	Interest rate

```
A = 2.1.*ones(1,N);
F = 1.65*ones(1,N);
L = zeros(1,N); L(1,4) = 1; L(1,11) = 1;
w = wage * ones(1,N);

D = zeros(1, N, Tmax); D(1,8,1) = 1;
O = zeros(1, N, Tmax); O_aux = rand(1,N); O(:,:,1) =
    D(1,1,1).*O_aux;
R = zeros(1, N, Tmax);
```

```
Agribusiness = zeros(1, N, Tmax); Agribusiness(1,8,1) = 1;
```

Figure 3 and Figure 4 shows deforestation before and after these simulations. Note that the Agribusiness enters the grid over time, following the Agribusiness dynamic. Not all land is protected, but the 2 protected plot manage to stop the Agribusiness to deforest everything, even some unprotected areas.

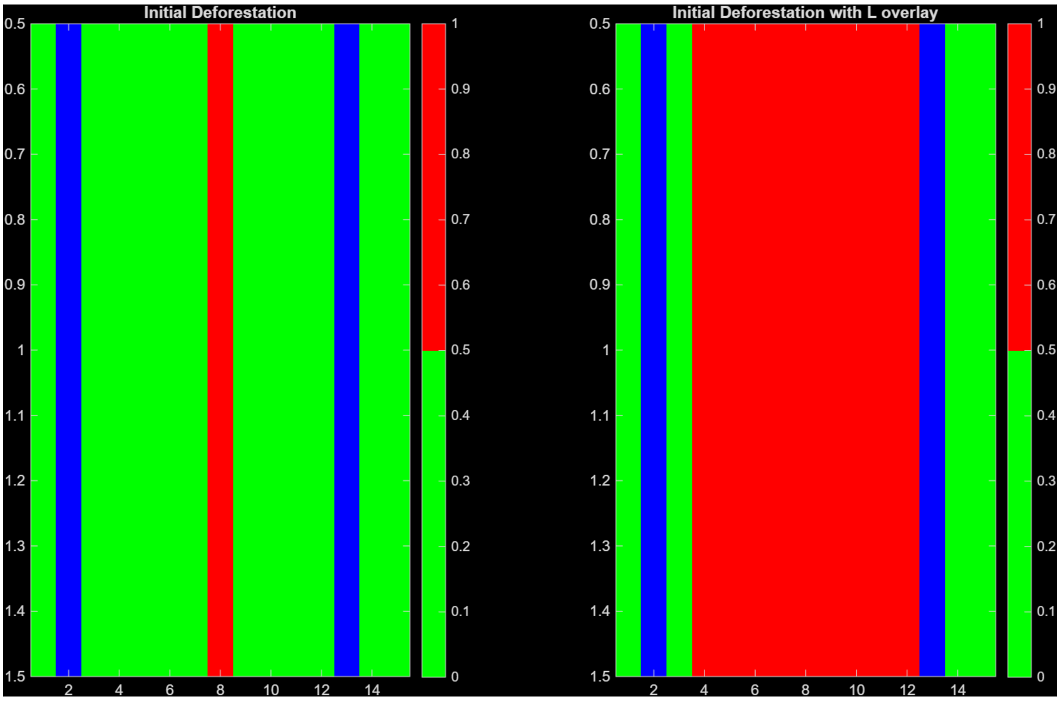


Figure 6: Agribusiness on the vertex: initial and final deforestation. 1 = deforested, 0 = not deforested

3.5 Multiple Agribusiness

To conclude, lets study a case with multiple Agribusiness in the middle and one protected area in position 7. The follwing are the relevant constants and initialization

```
    A = 2.1.*ones(1,N);
    F = 1.65*ones(1,N);
    L = zeros(1,N); L(1,7) = 1;
    w = wage * ones(1,N);
    D = zeros(1, N, Tmax); D(1,20,1) = 1; D(1,40,1) = 1;
                           D(1,60,1) = 1; D(1,80,1) = 1
    O = zeros(1, N, Tmax); O_aux = rand(1,N); O(:,:,1) =
       D(1,1,1).*O_aux;
    R = zeros(1, N, Tmax);
    Agribusiness = zeros(1, N, Tmax); Agribusiness(1,20,1) = 1;
       Agribusiness(1,40,1) = 1;
                                      Agribusiness(1,60,1) = 1;
                                         Agribusiness(1,80,1) = 1;
```

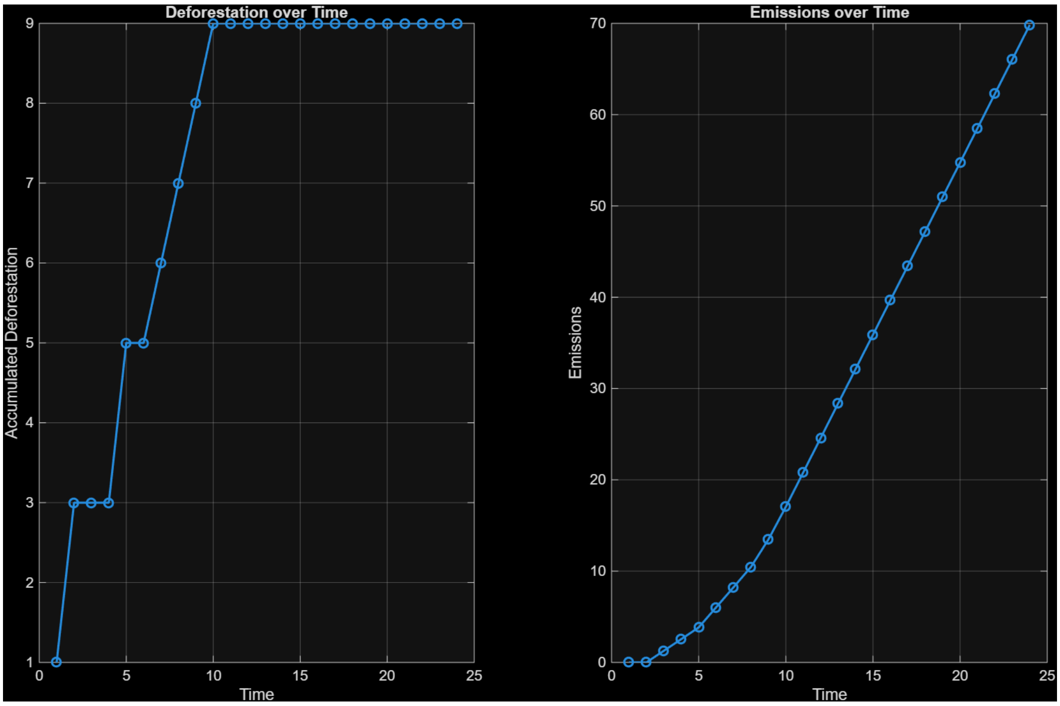


Figure 7: Agribusiness on the vertex: accumulated deforestation and emissions

Table 4: Agribusiness in one vertex

Variable	Value	Description
N	100	Number of spatial plots
$T_{\max}$	20	Number of time periods
β	0.10	Discount factor)
α	0.75	Land productivity parameter
π	0.55	Probability of acquiring property rights
T_{lag}	2	Time producing at low productivity
δ	1	Demand shifter
τ	1.2	Transport cost
w_n	1	Wage level
σ	1.5	Elasticity parameter
$\bar{S}$	10	Σ parameter
p_I	1	Investment price parameter
ρ	1.5	Interest rate

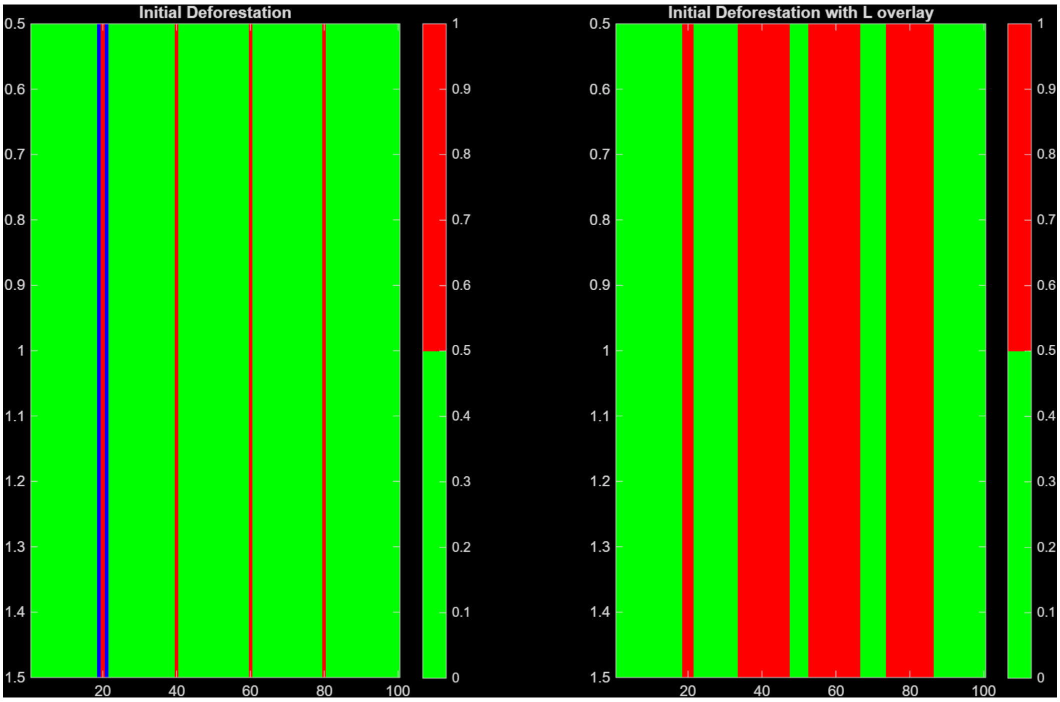


Figure 8: Agribusiness on the vertex: initial and final deforestation. 1 = deforested, 0 = not deforested

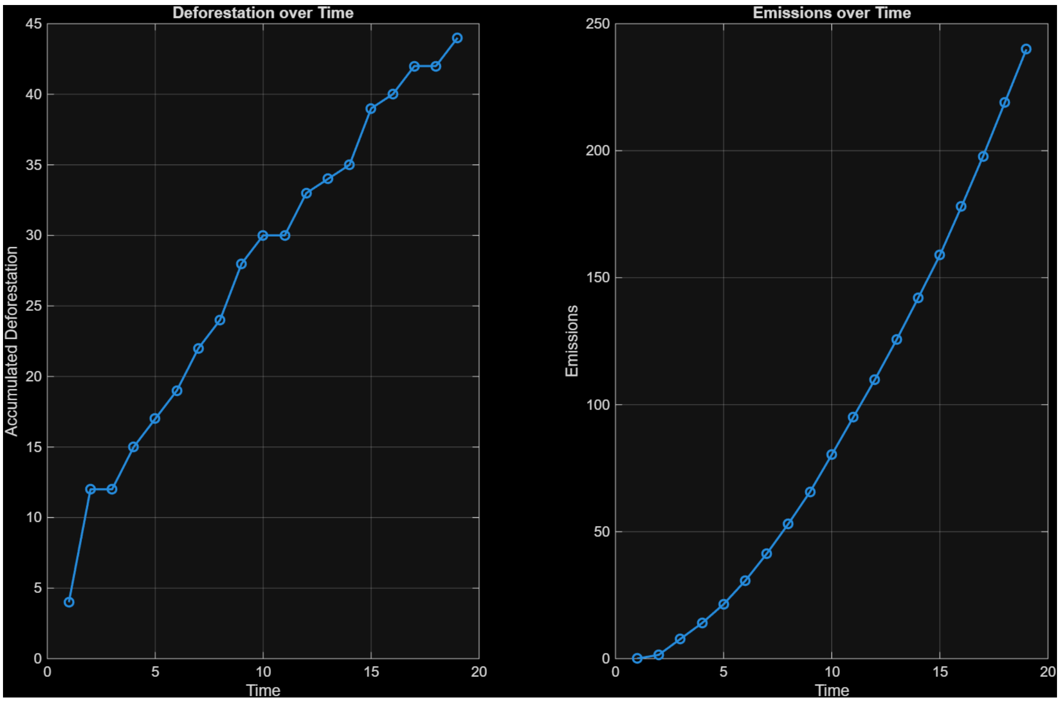


Figure 9: Agribusiness on the vertex: accumulated deforestation and emissions