

Probability Assignment - I

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Question:

Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X .

Solution:

Let X denote the random variable representing number of sixes.

Number of sixes when rolling two dice can be 0, 1 or 2.

Hence, Sample space of $X = \{0, 1, 2\}$

Expectation of X can be defined as:

$$E(X) = 0 \cdot P(X=0) + 1 \cdot P(X=1) + 2 \cdot P(X=2)$$

Now we have to find $P(X=0)$, $P(X=1)$ and $P(X=2)$.

Let p be the probability of getting a six on a single die, and q be the probability of not getting a six on a single die.

Since each die has 6 possible outcomes and only one of them is a six, we have $p = \frac{1}{6}$ and $q = 1 - p = \frac{5}{6}$.

Parameter	Description	Value
p	Probability of getting a six on a single die	$\frac{1}{6}$
q	Probability of not getting a six on a single die	$\frac{5}{6}$

We have two independent dice rolls, so the probability of getting no sixes, $P(X=0)$, is q^2

$$P(X=0) = q \cdot q = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

We can get exactly one 6 in two ways: either we get six on first die and not on second, or we get six on second die and not on the first. So $P(X=1)$ can be written as:

$$P(X=1) = (p \cdot q) + (q \cdot p) = 2 \cdot (p \cdot q) = 2 \cdot \left(\frac{1}{6} \cdot \frac{5}{6}\right) = \frac{10}{36}$$

The probability of getting two sixes, $P(X=2)$, is p^2 since we have two independent dice:

$$P(X=2) = p \cdot p = \left(\frac{1}{6}\right)^2 = \frac{1}{36}$$

Now we have all 3 probabilities required to calculate the expectation:

X	$P(X)$
0	$P(X=0) = \frac{25}{36}$
1	$P(X=1) = \frac{10}{36}$
2	$P(X=2) = \frac{1}{36}$

Substituting these values back into the expectation formula:

$$E(X) = 0 \cdot \left(\frac{25}{36}\right) + 1 \cdot \left(\frac{10}{36}\right) + 2 \cdot \left(\frac{1}{36}\right) = \frac{12}{36} = \frac{1}{3}$$

Therefore, the expectation of X is $\frac{1}{3}$