

A Theoretical Framework for Staged Microbubble Condensers

Vishal Murali

1. Introduction

This study presents a theoretical foundation and connects the dots with prior literature which inadvertently lays the foundation to designing a staged microbubble condenser. The study discusses prior models built around bulk condensing microbubbles and converts the models into simpler ordinary differential equations. Converting into simpler ODE, would help in building custom solvers for the researchers to test on complex custom-built Navier-Stokes with various additional and non-dimensional terms. The study poses to provide a theoretical idea backed by literature on microbubble generation and staging.

2. Model Validation

2.1. Momentum Equation

The momentum equation used to describe the flow of microbubbles is a straightforward Navier-Stokes equation which relates the internal and external forces acting on the microbubble. The momentum equation of microbubble is given by:

$$\rho \frac{\partial u}{\partial t} + \rho(u \cdot \nabla u) = -\nabla p + \mu \nabla^2 u + \rho g \quad (1)$$

Equation (3) describes the internal and external forces acting on the microbubble in 3 directions (x, y and z).

Considering flow of microbubbles in the Y- direction, the equation reduces to:

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} + \rho g \quad (2)$$

Considering similarity term η given by:

$$\eta = \frac{y}{\sqrt{vt}} \quad (3)$$

where 'v' is given by:

$$v = \frac{\mu}{\rho} \quad (4)$$

Upon simplifying and converting the PDE to an ODE using similarity method, we get the resultant ODE:

$$\frac{d^2u}{d\eta^2} + \frac{\eta}{2} \frac{du}{d\eta} - X = 0 \quad (5)$$

where,

$$X = -\frac{t}{\rho} \frac{\partial p}{\partial x} + gt \quad (6)$$

2.2. Heat Equation

The heat equation of a microbubble cloud which condenses the bulk vapour around it is given by the following equation [5]:

$$\frac{DT}{Dt} = \nabla^2 - \phi N_{Mpemba}(T) u \cdot \nabla T \quad (7)$$

where,

$$N_{Mpemba} = \frac{\phi F'(T)}{\rho C_p} \quad (8)$$

and, ϕ is the microbubble phase fraction given by:

$$\phi = \frac{\text{volume of microbubble cloud}}{\text{total working volume}}$$

In order to reduce the equation into an Ordinary Differential Equation (ODE) [2], heat flow in one direction was considered to reduce complications. The obtained results can be used for the other two dimensions as well.

Considering heat flow in x-direction, equation (4) can be written as:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial x^2} - \phi N_{Mpemba}(T) u \frac{\partial T}{\partial x} \quad (9)$$

Equation (5) can be reduced to:

$$\frac{\partial T}{\partial t} + u[1 + \phi N_{M p e m b a}(T)] \frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial x^2} \quad (10)$$

$$\frac{\partial T}{\partial t} + V \frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial x^2} \quad (11)$$

Where $V = u + [1 + \phi N_m(T)]$

In order to reduce the above partial differential equation into a simple ordinary differential equation, a similarity parameter is introduced (η) which is used to reduce the partial differential equation into a uniform ordinary differential equation.

$$\eta = \frac{x - Vt}{2\sqrt{t}} \quad (12)$$

From the η function, the components of the initial heat equation are as follows,

$$\frac{\partial T}{\partial t} = -\left(\frac{V}{2\sqrt{t}} + \frac{\eta}{2t}\right)f' \quad (13)$$

$$\frac{\partial T}{\partial x} = \frac{f'}{2\sqrt{t}} \quad (14)$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{f''}{4t} \quad (15)$$

From equations (9), (10) & (11), (7) reduces to:

$$-\left(\frac{V}{2\sqrt{t}} + \frac{\eta}{2t}\right)f' + V \frac{f'}{2\sqrt{t}} = \frac{f''}{4t} \quad (16)$$

Simplifying, it becomes,

$$2f'' + \eta f' = 0 \quad (17)$$

Where 'f' s a function of η .

Although OpenFOAM or COMSOL solves the PDE directly by using its own PDE solvers, it often takes quite a time to solve the PDE over the given geometry sometimes taking hours to get a solution. Due to the complexity of Navier-Stokes equations, a solution is not always guaranteed so in order to reduce the computational burden to test out equations and guarantee a quick solution to complex partial differential equations, similarity transformations could easily benefit in testing out equations quickly.

3. Designing a stacked Microbubble Distiller

Zimmerman [4] discussed in detail about the function of the airlift bioreactor design on how microbubbles would condense the bulk vapour around it. It has been reported that microbubbles are the carriers of latent heat from the hot zone to the cold zone which condenses the bulk vapour around it. In order to mimick the working of an airlift bioreactor [1], the microbubble has to travel a long distance to reach the top and travel down the downcomer region.

The theoretical assumptions made by Zimmerman [4], might be mathematically valid, coming into the design of the condenser which uses an airlift bioreactor arrangement would be heavily counter-productive. The bubble would take really long time to reach the top of the downcomer while reducing its temperature and even more time to travel down to condense the vapour around it. In order to efficiently design a condenser which would work on the theoretical assumptions made by Zimmerman [4] where microbubbles act as carriers of latent heat from one hot to cold zones, a totally new design approach should be considered as the airlift reactor configuration is counter-productive.

3.1. Functioning of an Airlift-loop Bioreactor

As discussed in [1], the design of an airlift-loop reactor is fairly simple. A sparger sparges the microbubbles from the bottom which rises through the internal column of the vessel. The bubbles travel down the downcomer once reaching the top. In Zimmerman [4], it has been discussed that microbubbles are carriers of latent heat form hot to cold zones.

Although the use of an airlift bioreactor configuration makes sense, however it is not practical enough for the microbubbles to rise and come down the downcomer to condense the vapour around it. The main reason behind this is that the microbubbles rise and travel through the liquid medium quite slowly.

In order to make the condensation process more productive, a staged microbubble condenser is introduced where physical stages distill the volatile fluid from a binary/ternary mixture and then gradually in later physical stages temperature reduces in order to slowly condense the volatile vapour.

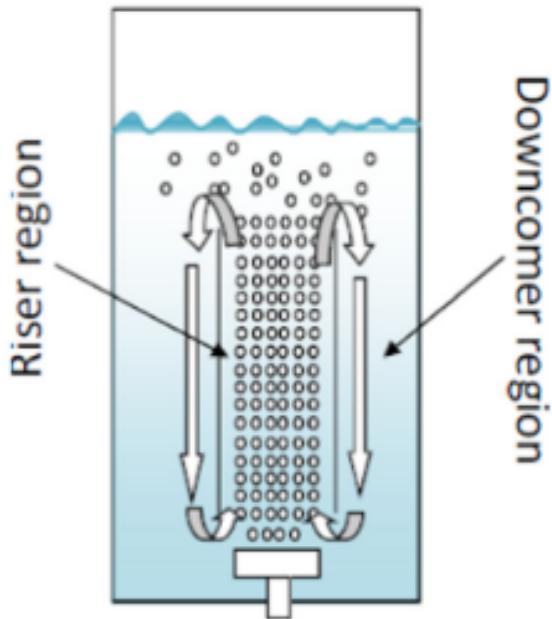


Figure 1: Airlift Bioreactor [4], [1]

4. Staged Condenser: A Theory

In Desai et.al [3], it has been discussed in detail on the various effect of staging of generating microbubbles. It has to be noted that the word "staging" according to the authors in the paper refers to the "ON or OFF" stage of the Desai-Zimmerman Fluidic Oscillator. The paper discussed and deliberated Prof Guyon's work on the role of wetted surfaces and how they impact the formation of bubbles. In the paper, series of experiments were carried out which inadvertently laid out the foundations for developing a staged microbubble condenser. The experiments/ the models discussed the ability of the microbubbles to have its memory on its flow regime after the oscillator is being switched off.

The main driving force behind this phenomena is that the pores of the membrane are wetted and are partially filled which oscillates the fluid entering through the membrane pores. By these micro-oscillations, the microbubbles retain the "memory" of the flow regime when the DZFO was turned

on after it being turned off. This might seem like an insignificant detail when considering building a staged condenser. The studies by Zimmerman [4] where microbubbles are the carriers of latent heat, carrying latent heat from hot zones to the cold and the work done by Desai et.al [3] can be used as a groundwork to design a fully functional staged microbubble condenser.

As the bubbles physically need to travel through the stages, the bubbles when moving from one stage to another, they tend to follow the phenomena laid out in Desai et.al [3]. Although this doesn't mean that the same bubble has to travel from the first stage all the way to the top. The bubble has to sufficiently travel through 'x' number of stages such that the next bubble takes in the mass and the heat from the initial bubble and carries to the next stage.

The current study does not mean to replicate the work or provide experimental or simulation studies at present but poses the ideas and postulates a solid working theory for a staged microbubble condenser. Although this could be taken into later studies where the micro-oscillations could be studied in detail and a CFD study on a two stage microbubble diffuser where the oscillated bubbles from stage 1 enters stage 2 and retains its "memory" and comes out as nearly as close as the "freshly oscillated" bubbles rather than it being unevenly distributed.

References

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