Practical - 01

Bisection Method

Q.1 Find the roots of $f(x) = x^3 + 4x^2 - 10$ using Bisection method.

```
f[x_] := x³ + 4 x² - 10;
a = 1;
b = 2;
E = 0.01;
Nmax = 50;                        "Maximum number of iterations";
If[f[a] x f[b] > 0,
   Print["These values do not satisfy the IVP so change the initial value"],
   For[i = 1, i ) Nmax, i++,

      c = (a + b) ;
          ───────
             2
      If[Abs[ (b - a) ] ) E, Return[c],
              ───────
                 2
       Print[i, "th iteration value is ", c];
                                                              ───────
                                                              (b - a)
        Print["Estimated error in " , i, "th iteration is: ",         ];
                                                                 2

        If[f[a] x f[c] ) 0, b = c, a = c]]]];
Print["The approximate root is: ", N[c]];
Plot f x , x, 1, 2
```

1th iteration value is $\dfrac{3}{2}$

Estimated error in 1th iteration is: $\dfrac{1}{2}$

2th iteration value is $\dfrac{5}{4}$

Estimated error in 2th iteration is: $\dfrac{1}{4}$

3th iteration value is $\dfrac{11}{8}$

Estimated error in 3th iteration is: $\dfrac{1}{8}$

4th iteration value is $\dfrac{21}{16}$

Estimated error in 4th iteration is: $\dfrac{1}{16}$

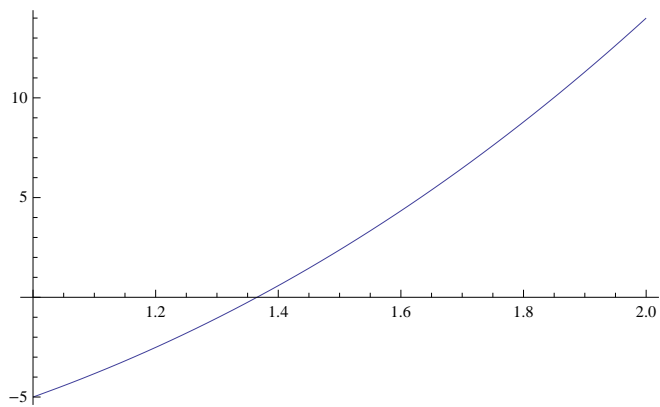5th iteration value is $\dfrac{43}{32}$

Estimated error in 5th iteration is: $\dfrac{1}{32}$

6th iteration value is $\dfrac{87}{64}$

Estimated error in 6th iteration is: $\dfrac{1}{64}$

Return[$\dfrac{175}{128}$]

The approximate root is: 1.36719

Q.2 Find the roots of $f x = \cos x$ using Bisection method.

```
f[x_] := Cos[x];
a = 1;
b = 2;
E = 0.001;
Nmax = 20;                                  "Maximum number of iterations";


If[f[a] x f[b] > 0,

   Print["These values do not satisfy the IVP so change the initial value"],
                                         (        )
     For[i = 1, i ) Nmax, i++, c = ─────────── ;
                                         2

                    (b - a)
      If[Abs[ ─────────── ] ) E, Return[c],
                       2

       Print[i, " th iteration value is : ", c];
                                                              ───────
                                                              (b - a)
         Print["Estimated error in ", i, "th iteration is: ",        ];
                                                                 2

       If[f[a] x f[c] ) 0, b = c, a = c]]]];
Print["The approximate root is : " , N[c]];
Plot f x ,  x, 0, 3.14
```

```
1 th iteration value is : 3/2

Estimated error in 1th iteration is: 1/2

2 th iteration value is : 7/4

Estimated error in 2th iteration is: 1/4

3 th iteration value is : 13/8

Estimated error in 3th iteration is: 1/8

4 th iteration value is : 25/16

Estimated error in 4th iteration is: 1/16

5 th iteration value is : 51/32

Estimated error in 5th iteration is: 1/32

6 th iteration value is : 101/64

Estimated error in 6th iteration is: 1/64

7 th iteration value is : 201/128

Estimated error in 7th iteration is: 1/128

8 th iteration value is : 403/256

Estimated error in 8th iteration is: 1/256

9 th iteration value is : 805/512

Estimated error in 9th iteration is: 1/512

Return[ 1609/1024 ]

The approximate root is : 1.57129
```
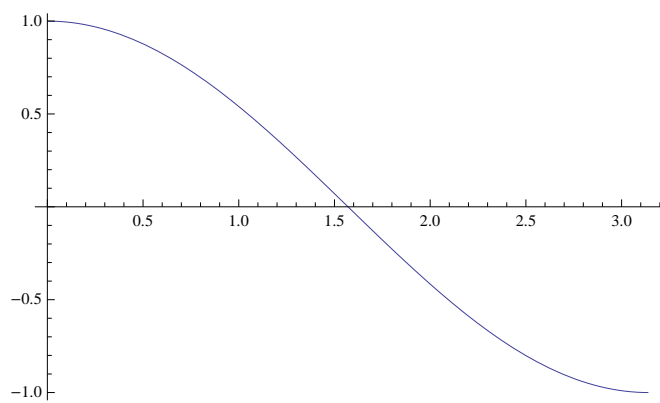
Practical-2(b)

Secant Method

Q.1 Find the root of the function $f(x) = x^3 + 2 x^2 - 3x - 1$ using Secant Method.

```
f[x_] := x³ + 2 x x² - 3 x x - 1
x₀ = 1;
x₁ = 2;
E = 0.000001;
Nmax = 50;
For[n = 2, n c Nmax, n++,

    xₙ = N[xₙ₋₁ - f[xₙ₋₁] x (xₙ₋₁ - xₙ₋₂) ];
                      f[xₙ₋₁] - f[xₙ₋₂]
    If[Abs[xₙ - xₙ₋₁] ) E, Return[xₙ]];
    Print[n - 1, "th iteration value is: ", x ₙ];
    Print["Estimated error is: ", Abs[xₙ - xₙ₋₁]]];
Plot[f[x], {x, -2, 2}]
```

```
1th iteration value is: 1.1

Estimated error is: 0.9

2th iteration value is: 1.15174

Estimated error is: 0.0517436

3th iteration value is: 1.20345

Estimated error is: 0.0517062

4th iteration value is: 1.19848

Estimated error is: 0.00496995

5th iteration value is: 1.19869

Estimated error is: 0.000210411

Return 1.19869
```
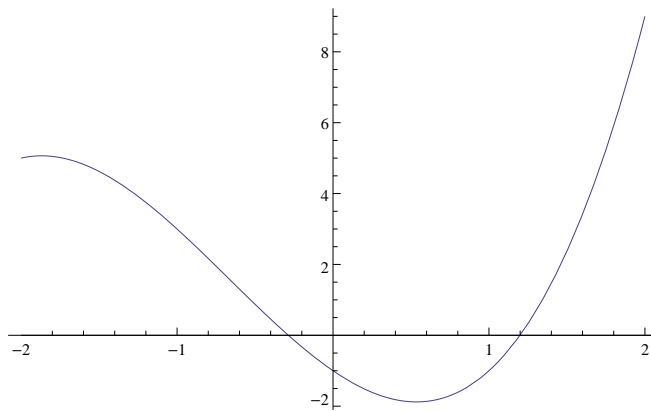

Q.2 Find the root of the function $f(x) = e^{-x}\text{-}x$ using Secant Method.

```
f[x_] := Exp[-x] - x
x₀ = 0;
x₁ = 1;
E = 0.000001;
Nmax = 50;

For[n = 2, n c Nmax, n++,

      xₙ = N[xₙ₋₁ -  f[xₙ₋₁] x (xₙ₋₁ - xₙ₋₂) ];
                     ─────────────────────────
                        f[xₙ₋₁] - f[xₙ₋₂]

  If[Abs[xₙ - xₙ₋₁] ) E, Return[xₙ]];
  Print[n - 1, "th iteration value is: ", x ₙ];
  Print["Estimated error is: ", Abs[xₙ - xₙ₋₁]]];

Plot[f[x], {x, -2, 2}]
```

```
1th iteration value is: 0.6127
```

Estimated error is: 0.3873

```
2th iteration value is: 0.563838
```

Estimated error is: 0.0488614

```
3th iteration value is: 0.56717
```

Estimated error is: 0.00333197

```
4th iteration value is: 0.567143
```

Estimated error is: 0.0000270518

Return 0.567143
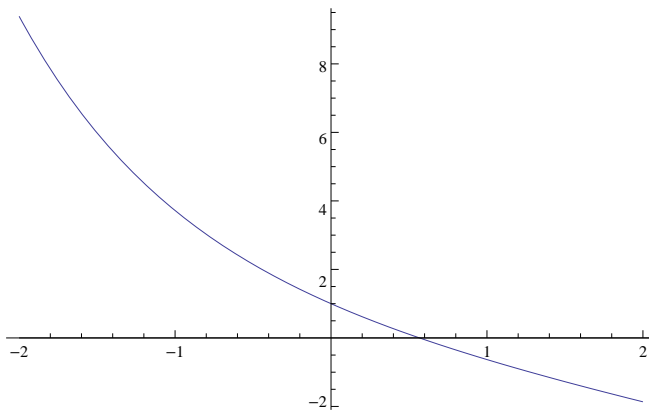
Q.3 Find the root of the function f(x) = Cos(x) using Secant Method.
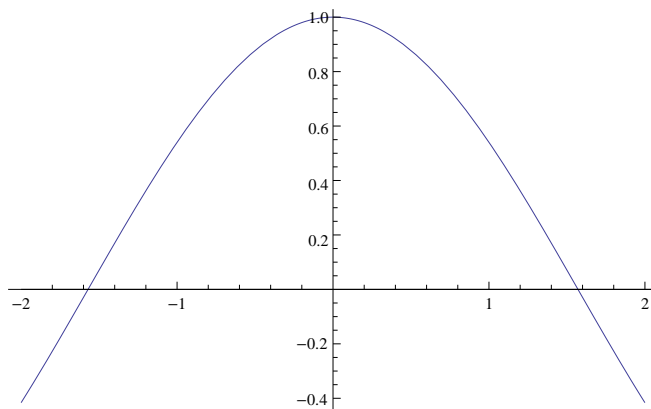
```
f[x_] := Cos[x]
x₀ = 1;
x₁ = 2;
E = 0.000001;
Nmax = 50;
For[n = 2, n c Nmax, n++,

      xₙ = N[xₙ₋₁ - f[xₙ₋₁] x (xₙ₋₁ - xₙ₋₂) / (f[xₙ₋₁] - f[xₙ₋₂])];
   If[Abs[xₙ - xₙ₋₁] ) E, Return[xₙ]];
   Print[n - 1, "th iteration value is: ", x ₙ];
   Print["Estimated error is: ", Abs[xₙ - xₙ₋₁]]];
Plot[f[x], {x, -2, 2}]
```

```
1th iteration value is: 1.5649

Estimated error is: 0.435096

2th iteration value is: 1.57098

Estimated error is: 0.0060742

3th iteration value is: 1.5708

Estimated error is: 0.000182249

Return 1.5708
```


Q.4   Find the root of the function $f(x) = x^3\text{-}13$ using Secant Method.

```
f[x_] := x³ - 13
x₀ = 1;
x₁ = 2;
E = 0.000001;
Nmax = 50;
For[n = 2, n c Nmax, n++,

    xₙ = N[xₙ₋₁ - f[xₙ₋₁] x (xₙ₋₁ - xₙ₋₂)/(f[xₙ₋₁] - f[xₙ₋₂])];

  If[Abs[xₙ - xₙ₋₁] ) E, Return[xₙ]];
  Print[n - 1, "th iteration value is: ", x ₙ];
  Print["Estimated error is: ", Abs[xₙ - xₙ₋₁]]];

Plot[f[x], {x, -2, 3}]
```

```
1th iteration value is: 2.71429

Estimated error is: 0.714286

2th iteration value is: 2.29769

Estimated error is: 0.416594

3th iteration value is: 2.34374

Estimated error is: 0.0460513

4th iteration value is: 2.35151

Estimated error is: 0.00776829

5th iteration value is: 2.35133

Estimated error is: 0.000176826

Return 2.35133
```
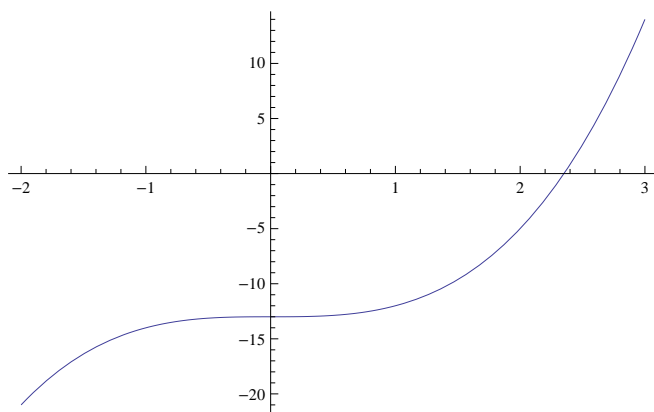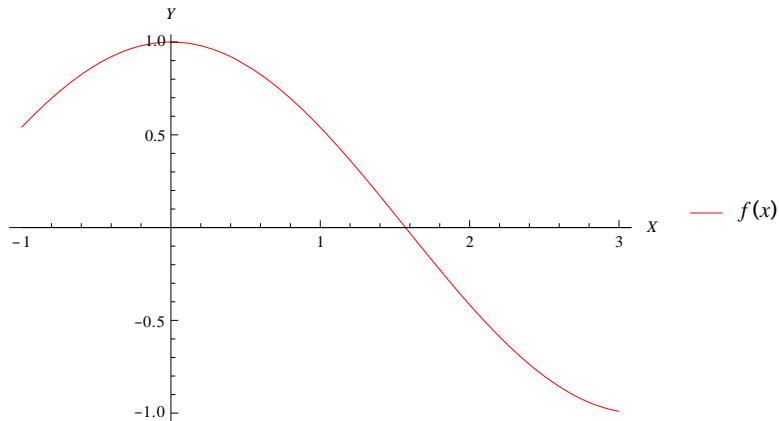
Practical-2(a)

Regula - Falsi method

Q.1 Find the root of function   f(x)=Cos(x)   by Regula Falsi method.

```
f[x_] := Cos[x];
a = 0;
b = 2;
E = 0.00000001;
Nmax = 50;
If[f[a] x f[b] > 0,
   Print["These values do not satisfy the IVP so change the initial value"],

   For[i = 1, i c Nmax, i++,

     x = N[ a x f[b] - b x f[a]
            ─────────────────── ];
                f[b] - f[a]
    If[f[x] x f[b] > 0, b = x, a = x];
    If[Abs[b - a] ) E, Return[x]];
    Print[i, "th iteration value is :", N[x]];

    Print["Estimated error is :", N[b - a]]]];
Print["Approximate root is :", N[x]];
Print["Estimated error is :", N[b - a]];
Plot[f[x], {x, -1, 3}, PlotStyle c Red,
 PlotLegends c "Expressions", AxesLabel c {X, Y}]
```

```
1th iteration value is :1.41228

Estimated error is :0.587717

2th iteration value is :1.57391

Estimated error is :0.161623

3th iteration value is :1.57078

Estimated error is :0.0031228

4th iteration value is :1.5708

Estimated error is :0.0000128049

Return 1.5708

Approximate root is :1.5708

Estimated error is :2.05567 x 10^{-11}
```

Q.2 Find the root of function   f(x)=e^-x - x by Regula Falsi method.

```
f[x_] := Exp[-x] - x;
a = 0;
b = 2;
E = 0.00000001;
Nmax = 50;
If[f[a] x f[b] > 0,
   Print["These values do not satisfy the IVP so change the initial value"],

   For[i = 1, i c Nmax, i++,
        a x f[b] - b x f[a]
   x = N[ ─────────────────── ];
           f[b] - f[a]
    If[f[x] x f[b] > 0, b = x, a = x];
    If[Abs[b - a] ) E, Return[x]];
    Print[i, "th iteration value is :", N[x]];

    Print["Estimated error is :", N[b - a]]]];
Print["Approximate root is :", N[x]];
Print["Estimated error is :", N[b - a]];
Plot[f[x], {x, -1, 3}, PlotStyle c Red,
 PlotLegends c "Expressions", AxesLabel c  X, Y
```

1th iteration value is :0.698162

Estimated error is :0.698162

2th iteration value is :0.58148

Estimated error is :0.58148

3th iteration value is :0.568735

Estimated error is :0.568735

4th iteration value is :0.56732

Estimated error is :0.56732

5th iteration value is :0.567163

Estimated error is :0.567163

6th iteration value is :0.567145

Estimated error is :0.567145

7th iteration value is :0.567144

Estimated error is :0.567144

8th iteration value is :0.567143

Estimated error is :0.567143

9th iteration value is :0.567143

Estimated error is :0.567143

10th iteration value is :0.567143

Estimated error is :0.567143

11th iteration value is :0.567143

Estimated error is :0.567143

12th iteration value is :0.567143

Estimated error is :0.567143

13th iteration value is :0.567143

Estimated error is :0.567143

14th iteration value is :0.567143

Estimated error is :0.567143

15th iteration value is :0.567143

Estimated error is :0.567143

16th iteration value is :0.567143

Estimated error is :0.567143

17th iteration value is :0.567143

Estimated error is :0.567143

18th iteration value is :0.567143

Estimated error is :0.567143

19th iteration value is :0.567143

Estimated error is :0.567143

20th iteration value is :0.567143

Estimated error is :0.567143

21th iteration value is :0.567143

Estimated error is :0.567143

22th iteration value is :0.567143

```
Estimated error is :0.567143
23th iteration value is :0.567143
Estimated error  is  :0.567143
24th iteration value is :0.567143
Estimated error  is  :0.567143
25th iteration value is :0.567143
Estimated error  is  :0.567143
26th iteration value is :0.567143
Estimated error  is  :0.567143
27th iteration value is :0.567143
Estimated error  is  :0.567143
28th iteration value is :0.567143
Estimated error  is  :0.567143
29th iteration value is :0.567143
Estimated error  is  :0.567143
30th iteration value is :0.567143
Estimated error  is  :0.567143
31th iteration value is :0.567143
Estimated error  is  :0.567143
32th iteration value is :0.567143
Estimated error  is  :0.567143
33th iteration value is :0.567143
Estimated error  is  :0.567143
34th iteration value is :0.567143
Estimated error  is  :0.567143
35th iteration value is :0.567143
Estimated error  is  :0.567143
36th iteration value is :0.567143
Estimated error  is  :0.567143
37th iteration value is :0.567143
Estimated error  is  :0.567143
38th iteration value is :0.567143
Estimated error  is  :0.567143
39th iteration value is :0.567143
Estimated error  is  :0.567143
40th iteration value is :0.567143
```

```
Estimated error is :0.567143

41th iteration value is :0.567143

Estimated error  is  :0.567143

42th iteration value is :0.567143

Estimated error  is  :0.567143

43th iteration value is :0.567143

Estimated error  is  :0.567143

44th iteration value is :0.567143

Estimated error  is  :0.567143

45th iteration value is :0.567143

Estimated error  is  :0.567143

46th iteration value is :0.567143

Estimated error  is  :0.567143

47th iteration value is :0.567143

Estimated error  is  :0.567143

48th iteration value is :0.567143

Estimated error  is  :0.567143

49th iteration value is :0.567143

Estimated error  is  :0.567143

50th iteration value is :0.567143

Estimated error is :0.567143

Approximate root is :0.567143

Estimated error is :0.567143
```
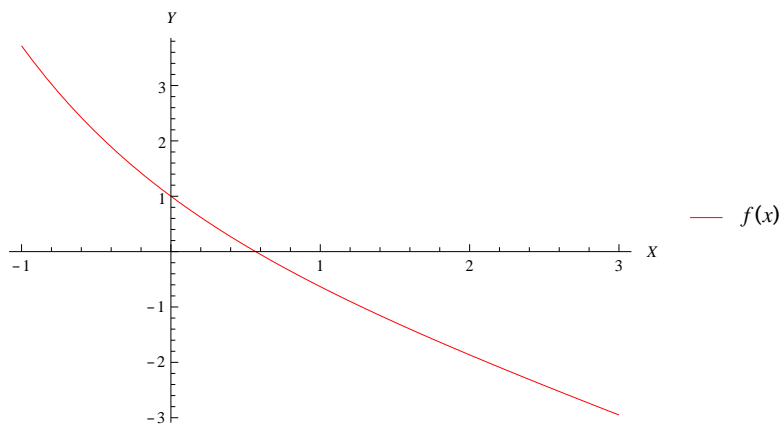

Q.3 Find the root of function   f(x)=x^5+2*x-1 by Regula Falsi method.

```
f[x_] := x^5 + 2 x x - 1
a = 0;
b = 2;
E = 0.00000001;
Nmax = 50;
If[f[a] x f[b] > 0,
   Print["These values do not satisfy the IVP so change the initial value"],

   For[i = 1, i c Nmax, i++,

       a x f[b] - b x f[a]
   x = N[ ─────────────────── ];
             f[b] - f[a]
   If[f[x] x f[b] > 0, b = x, a = x];
   If[Abs[b - a] ) E, Return[x]];
   Print[i, "th iteration value is :", N[x]];
   Print["Estimated error is :", N[b - a]]]];
Print["Approximate root is :", N[x]];
Print["Estimated error is :", N[b - a]];
Plot[f[x], {x, -3, 3}, PlotStyle c Red,
  PlotLegends c "Expressions", AxesLabel c  X, Y
```

1th iteration value is :0.0555556

Estimated error is :1.94444

2th iteration value is :0.103715

Estimated error is :1.89628

3th iteration value is :0.145705

Estimated error is :1.8543

4th iteration value is :0.182497

Estimated error is :1.8175

5th iteration value is :0.214875

Estimated error is :1.78513

6th iteration value is :0.243471

Estimated error is :1.75653

7th iteration value is :0.268806

Estimated error is :1.73119

8th iteration value is :0.291311

Estimated error is :1.70869

9th iteration value is :0.311347

Estimated error is :1.68865

10th iteration value is :0.329219

Estimated error is :1.67078

11th iteration value is :0.345185

Estimated error is :1.65482

12th iteration value is :0.359468

Estimated error is :1.64053

13th iteration value is :0.372261

Estimated error is :1.62774

14th iteration value is :0.383728

Estimated error is :1.61627

15th iteration value is :0.394017

Estimated error is :1.60598

16th iteration value is :0.403254

Estimated error is :1.59675

17th iteration value is :0.411551

Estimated error is :1.58845

18th iteration value is :0.419009

Estimated error is :1.58099

19th iteration value is :0.425714

Estimated error is :1.57429

20th iteration value is :0.431744

Estimated error is :1.56826

21th iteration value is :0.43717

Estimated error is :1.56283

22th iteration value is :0.442053

Estimated error is :1.55795

23th iteration value is :0.446448

Estimated error is :1.55355

24th iteration value is :0.450404

Estimated error is :1.5496

25th iteration value is :0.453967

Estimated error is :1.54603

26th iteration value is :0.457176

Estimated error is :1.54282

27th iteration value is :0.460065

Estimated error is :1.53993

28th iteration value is :0.462668

Estimated error is :1.53733

29th iteration value is :0.465013

Estimated error is :1.53499

30th iteration value is :0.467125

Estimated error is :1.53287

31th iteration value is :0.469028

Estimated error is :1.53097

32th iteration value is :0.470743

Estimated error is :1.52926

33th iteration value is :0.472288

Estimated error is :1.52771

34th iteration value is :0.47368

Estimated error is :1.52632

35th iteration value is :0.474935

Estimated error is :1.52507

36th iteration value is :0.476066

Estimated error is :1.52393

37th iteration value is :0.477084

Estimated error is :1.52292

38th iteration value is :0.478003

Estimated error is :1.522

39th iteration value is :0.47883

Estimated error is :1.52117

40th iteration value is :0.479576

Estimated error is :1.52042

41th iteration value is :0.480248

Estimated error is :1.51975

42th iteration value is :0.480854

Estimated error is :1.51915

43th iteration value is :0.4814

Estimated error is :1.5186

44th iteration value is :0.481892

Estimated error is :1.51811

45th iteration value is :0.482336

Estimated error is :1.51766

46th iteration value is :0.482735

Estimated error is :1.51726

47th iteration value is :0.483096

Estimated error is :1.5169

48th iteration value is :0.483421

Estimated error is :1.51658

49th iteration value is :0.483713

Estimated error is :1.51629

50th iteration value is :0.483977

Estimated error is :1.51602

Approximate root is :0.483977
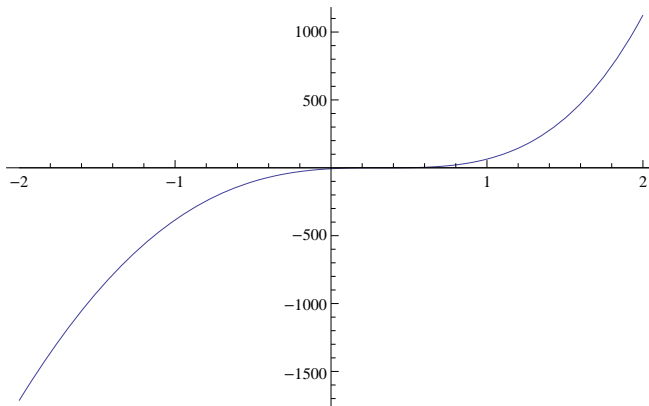
Estimated error is :1.51602
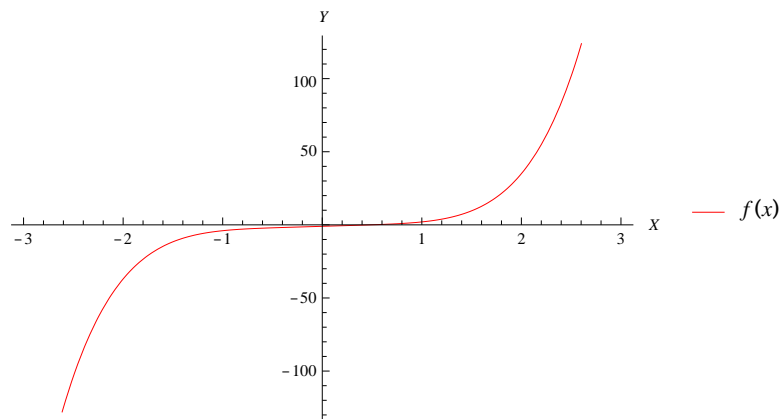
Practical-3

Newton's Raphson Method

Q.1 Find the roots of the function $f(x) = 27 x^4 + 162 x^3 - 180 x^2 + 62 x - 7$ using Newton Raphson

Method.

```
f[x_] := 27 x⁴ + 162 x³ - 180 x² + 62 x - 7
x₀ = 0;
Nmax = 10;
E = 0.0000001;
For[n = 1, n c Nmax, n++,

   xₙ = N[xₙ₋₁ -  f[xₙ₋₁]  ];
                 f'[xₙ₋₁]

   If[Abs[xₙ - xₙ₋₁] ) E, Return[xₙ]];
   Print[n, "th iteration value is: ", x ₙ];
   Print["Estimated error is: ", N[xₙ - xₙ₋₁]]];

Plot f x ,  x, -2, 2
```

```
1th iteration value is: 0.112903

Estimated error is: 0.112903

2th iteration value is: 0.187147

Estimated error is: 0.0742436

3th iteration value is: 0.236208

Estimated error is: 0.0490615

4th iteration value is: 0.268729

Estimated error is: 0.0325205

5th iteration value is: 0.290328

Estimated error is: 0.0215988

6th iteration value is: 0.304691

Estimated error is: 0.0143635

7th iteration value is: 0.314251

Estimated error is: 0.0095599

8th iteration value is: 0.320617

Estimated error is: 0.00636631

9th iteration value is: 0.324858

Estimated error is: 0.00424112

10th iteration value is: 0.327685

Estimated error is: 0.00282605
```
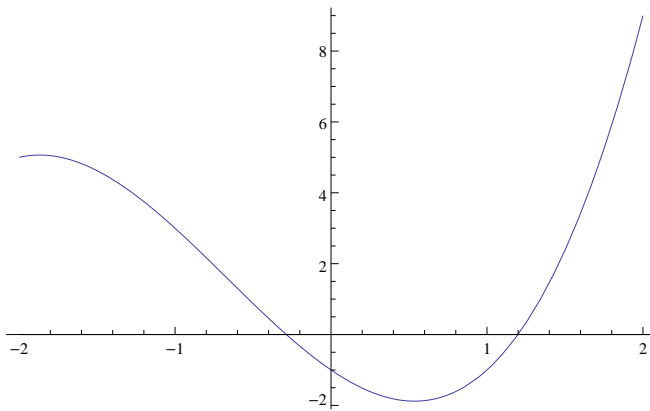

Q.2 Find the roots of the function $f(x) = x^3 + 2x^2 - 3x - 1$ using Newton Raphson Method.

```
f[x_] := x³ + 2 x x² - 3 x x - 1
x₀ = 1;
Nmax = 10;
E = 0.000001;
For[n = 1, n c Nmax, n++,
```

$$x_n = N\left[x_{n-1} - \frac{f[x_{n-1}]}{f'[x_{n-1}]}\right];$$

```
  If[Abs[xₙ - xₙ₋₁] ) E, Return[xₙ]];
  Print[n, "th iteration value is: ", x ₙ];
  Print["Estimated error is: ", N[xₙ - xₙ₋₁]]];
Plot f x ,  x, -2, 2
```

1th iteration value is: 1.25

Estimated error is: 0.25

2th iteration value is: 1.20093

Estimated error is: -0.0490654

3th iteration value is: 1.1987

Estimated error is: -0.00223874

4th iteration value is: 1.19869

Estimated error is: -4.59753×10^{-6}
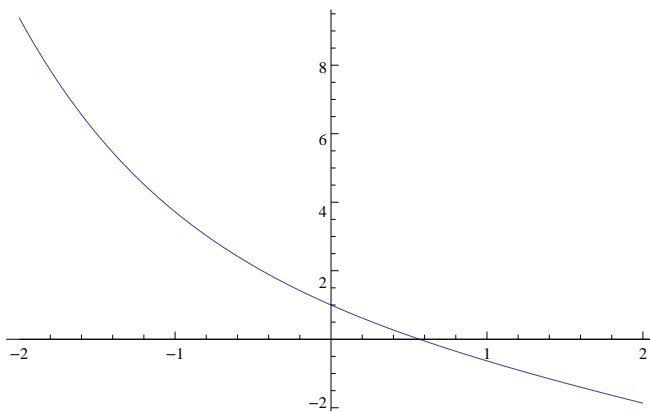
Return 1.19869

Q.3 Find the roots of the function $f(x) = e^{-x} - x$ using Newton Raphson Method.

```
f[x_] := Exp[-x] - x
x₀ = 0;
Nmax = 10;
E = 0.0000001;
For[n = 1, n c Nmax, n++,
```

$$x_n = N[x_{n-1} - \frac{f[x_{n-1}]}{f'[x_{n-1}]}];$$

```
    If[Abs[xₙ - xₙ₋₁] ) E, Return[xₙ]];
    Print[n, "th iteration value is: ", x ₙ];
    Print["Estimated error is: ", N[xₙ - xₙ₋₁]]];
Plot f x ,  x, -2, 2
```

1th iteration value is: 0.5

Estimated error is: 0.5

2th iteration value is: 0.566311

Estimated error is: 0.066311

3th iteration value is: 0.567143

Estimated error is: 0.000832162

4th iteration value is: 0.567143

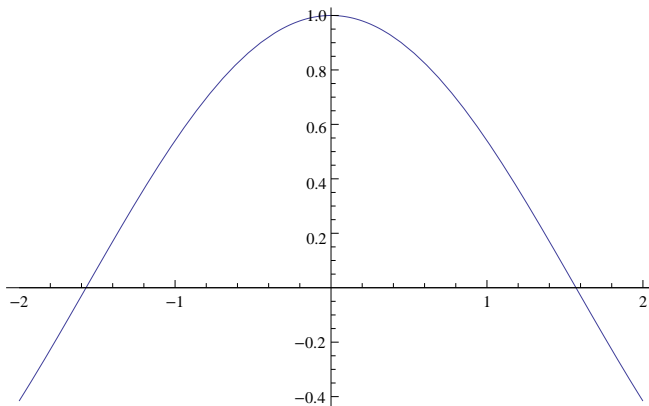Estimated error is: 1.25375×10^{-7}

Return 0.567143

Q.4 Find the roots of the function $f(x) = Cos(x)$ using Newton Raphson Method.

```
f[x_] := Cos[x]
x₀ = 1;
Nmax = 10;
E = 0.000001;
For[n = 1, n c Nmax, n++,

              f[xₙ₋₁]
   xₙ = N[xₙ₋₁ - ─────── ];
              f'[xₙ₋₁]

   If[Abs[xₙ - xₙ₋₁] ) E, Return[xₙ]];
   Print[n, "th iteration value is: ", x ₙ];
   Print["Estimated error is: ", N[xₙ - xₙ₋₁]]];
Plot f x ,  x, -2, 2
```

1th iteration value is: 1.64209

Estimated error is: 0.642093

2th iteration value is: 1.57068

Estimated error is: -0.0714173

3th iteration value is: 1.5708

Estimated error is: 0.00012105

Return 1.5708

Q.5   Find the roots of the function $f(x) = x^3 - 13$ using Newton Raphson Method.

```
f[x_] := x^3 - 13
x_0 = 1;
Nmax = 10;
E = 0.000001;
For[n = 1, n c Nmax, n++,

    x_n = N[x_{n-1} - f[x_{n-1}] / f'[x_{n-1}] ];

    If[Abs[x_n - x_{n-1}] ) E, Return[x_n]];
    Print[n, "th iteration value is: ", x_n];
    Print["Estimated error is: ", N[x_n - x_{n-1}]]];
Plot f x , x, -2, 2
```

1th iteration value is: 5.

Estimated error is: 4.

2th iteration value is: 3.50667

Estimated error is: -1.49333

3th iteration value is: 2.69018

Estimated error is: -0.816491

4th iteration value is: 2.39222

Estimated error is: -0.297954

5th iteration value is: 2.35203

Estimated error is: -0.040192

6th iteration value is: 2.35133

Estimated error is: -0.000694633

Return 2.35133
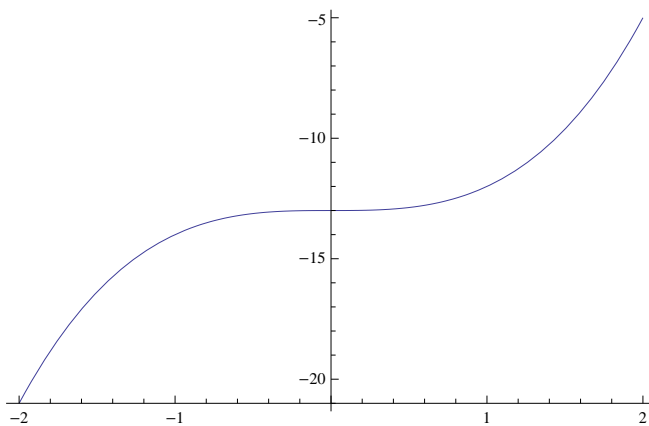
Practical-4(a)

Gauss Elimination Method

Q.1 Solve

x + 10y + 100z + 1000w = 227.04

x + 15y + 225z + 3375w = 362.78

x + 20y + 400z + 8000w = 517.35

x + 22.5y + 506.25z + 11391w = 602.97

```
A =  {1, 10, 100, 1000}, {1, 15, 225, 3375},
    {
A // MatrixForm
b = {227.04, 362.78, 517.35, 602.97};
b // MatrixForm
m1 = Length[A];
m2 = Length[b];
x = Table[0, {m1}];
If[m1  m2, Print["The system cannot be solved"],
 Table[AppendTo[A[[i]], b[[i]]], {i, m1}]; Print["[A|b]=", A // MatrixForm];
 For[i = 1, i c m1 - 1, i++, s = Abs[A[[i, i]]]; c = i;
  For[j = i + 1, j c m1, j++, If[Abs[A[[j, i]]] > s, s = A[[j, i]]; c = j;]];
  For[k = 1, k c m1 + 1, k++, d[k] = A[[i, k]]; A[[i, k]] = A[[c, k]]; A[[c, k]] = d[k]];
  Print["Step=", i, A // MatrixForm];
  For[j = i + 1, j c m1, j++, m = A[[j, i]] / A[[i, i]];
   For[k = 1, k c m1 + 1, k++, A[[j, k]] = A[[j, k]] - (m x A[[i, k]])];];
  Print[A // MatrixForm];];
 For[i = 0, i c m1 - 1, i++,
  x[[m1 - i]] = (A[[m1 - i, m1 + 1]] - Sum [A[[m1 - i, j]] x x[[j]], {j, m1 - i + 1, m1 }]) /
     A[[m1 - i, m1 - i]];];
 Print "x=", x    MatrixForm ;
```

$$\begin{pmatrix} 1 & 10 & 100 & 1000 \\ 1 & 15 & 225 & 3375 \\ 1 & 20 & 400 & 8000 \\ 1 & 22.5 & 506.25 & 11391 \end{pmatrix}$$

$$\begin{pmatrix} 227.04 \\ 362.78 \\ 517.35 \\ 602.97 \end{pmatrix}$$

$$[A|b] = \begin{vmatrix} 1 & 10 & 100 & 1000 & 227.04 \\ 1 & 15 & 225 & 3375 & 362.78 \\ 1 & 20 & 400 & 8000 & 517.35 \\ 1 & 22.5 & 506.25 & 11391 & 602.97 \end{vmatrix}$$

$$\text{Step=1} \begin{vmatrix} 1 & 10 & 100 & 1000 & 227.04 \\ 1 & 15 & 225 & 3375 & 362.78 \\ 1 & 20 & 400 & 8000 & 517.35 \\ 1 & 22.5 & 506.25 & 11391 & 602.97 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 10 & 100 & 1000 & 227.04 \\ 0 & 5 & 125 & 2375 & 135.74 \\ 0 & 10 & 300 & 7000 & 290.31 \\ 0 & 12.5 & 406.25 & 10391 & 375.93 \end{vmatrix}$$

$$\text{Step=2} \begin{pmatrix} 1 & 10 & 100 & 1000 & 227.04 \\ 0 & 12.5 & 406.25 & 10391 & 375.93 \\ 0 & 10 & 300 & 7000 & 290.31 \\ 0 & 5 & 125 & 2375 & 135.74 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 10 & 100 & 1000 & 227.04 \\ 0 & 12.5 & 406.25 & 10391 & 375.93 \\ 0. & 0. & -25. & -1312.8 & -10.434 \\ 0. & 0. & -37.5 & -1781.4 & -14.632 \end{vmatrix}$$

$$\text{Step=3} \begin{pmatrix} 1 & 10 & 100 & 1000 & 227.04 \\ 0 & 12.5 & 406.25 & 10391 & 375.93 \\ 0. & 0. & -37.5 & -1781.4 & -14.632 \\ 0. & 0. & -25. & -1312.8 & -10.434 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 10 & 100 & 1000 & 227.04 \\ 0 & 12.5 & 406.25 & 10391 & 375.93 \\ 0. & 0. & -37.5 & -1781.4 & -14.632 \\ 0. & 0. & 3.55271 \times 10^{-15} & -125.2 & -0.679333 \end{vmatrix}$$

$$x = \begin{pmatrix} -4.22796 \\ 21.2599 \\ 0.132431 \\ 0.00542599 \end{pmatrix}$$

Practical —04 (b)

Gauss — Jordan Method

Q.1

$2x + y + z - 2w = -10$

$4x + 2z + w = 8$

$3x + 2y + 2z = 7$

$x + 3y + 2z - w = -5$

```
A = {{2, 1, 1, -2, -10}, {4, 0, 2, 1, 8}, {3, 2, 2, 0, 7}, {1, 3, 2, -1, -5}}
A // MatrixForm
RowReduce A     MatrixForm
```

2, 1, 1, −2, −10 , 4, 0, 2, 1, 8 , 3, 2, 2, 0, 7 , 1, 3, 2, −1, −5

$$\begin{pmatrix} 2 & 1 & 1 & -2 & -10 \\ 4 & 0 & 2 & 1 & 8 \\ 3 & 2 & 2 & 0 & 7 \\ 1 & 3 & 2 & -1 & -5 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & -10 \\ 0 & 0 & 0 & 1 & 8 \end{vmatrix}$$

```
Solve   x == 5, y == 6, z == -10, w == 8 , x, y, z, w
```

x c 5, y c 6, z c −10, w c 8

qn -2

Q.2

$3x + 3y + 4z = 20$

$2x + y + 3z = 13$

$x + y + 3z = 6$

```
A = {{3, 3, 4, 20}, {2, 1, 3, 13 }, {1, 1, 3, 6}}
A // MatrixForm
RowReduce A     MatrixForm
```

3, 3, 4, 20 , 2, 1, 3, 13 , 1, 1, 3, 6

$$\begin{vmatrix} 3 & 3 & 4 & 20 \\ 2 & 1 & 3 & 13 \\ 1 & 1 & 3 & 6 \end{vmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & \frac{1}{5} \\ 0 & 0 & 1 & -\frac{2}{5} \end{pmatrix}$$

```
Solve[{x == 7, y == 1/5, z == -2/5}, {x, y, z}]
```

$$\{\{x \to 7, y \to \frac{1}{5}, z \to -\frac{2}{5}\}\}$$

Practical - 05(a)

Gauss-Jacobi method

Q.1   Solve the system of linear equations :-
$5x_1 + 2x_2 + x_3 = 10$
$3x_1 + 7x_2 + 4x_3 = 21$
$x_1 + x_2 + 9x_3 = 12$

```
n = 3;
a = {{5, 2, 1}, {3, 7, 4}, {1, 1, 9}};
MatrixForm[a]
x = {0, 0, 0}
y = {0, 0, 0}
b = {10, 21, 12}
For[k = 1, k c 25, k++,
  For[i = 1, i c n, i++,
    y[[i]] = (b[[i]] - Sum[a[[i, j]] x x[[j]], {j, 1, i - 1}] -
        Sum[a[[i, j]] x x[[j]], {j, i + 1, n}]) / a[[i, i]]];
  For[m = 1, m c n, m++, x[[m]] = N[y[[m]]]]];
For p = 1, p c n, p++, Print "x ", p, " =", x    p
```

$$\begin{pmatrix} 5 & 2 & 1 \\ 3 & 7 & 4 \\ 1 & 1 & 9 \end{pmatrix}$$

```
0, 0, 0

0, 0, 0

10, 21, 12

x 1 =1.

x 2 =2.

x 3 =1.
```

Q.2   Solve the system of linear equations :-
$6x_1 - 2x_2 - x_3 = 4$
$x_1 + 5x_2 + x_3 = 3$
$2x_1 + x_2 + 4x_3 = 27$

```
n = 3;
a = {{6, -2, -1}, {1, 5, 1}, {2, 1, 4}};
MatrixForm[a]
x = {0, 0, 0}
y = {0, 0, 0}
b = {4, 3, 27}
For[k = 1, k c 25, k++,
  For[i = 1, i c n, i++,
    y[[i]] = (b[[i]] - Sum[a[[i, j]] x x[[j]], {j, 1, i - 1}] -
        Sum[a[[i, j]] x x[[j]], {j, i + 1, n}]) / a[[i, i]]];
  For[m = 1, m c n, m++, x[[m]] = N[y[[m]]]]];
For p = 1, p c n, p++, Print "x ", p, " =", x   p
```

$$\begin{pmatrix} 6 & -2 & -1 \\ 1 & 5 & 1 \\ 2 & 1 & 4 \end{pmatrix}$$

0, 0, 0

0, 0, 0

4, 3, 27

x 1 =1.40157

x 2 =-0.937008

x 3 =6.28346

Q.3 Solve the system of linear equations :-

$10x_1 - 5x_2 - 2x_3 = 3$

$4x_1 - 10x_2 + 3x_3 = -3$

$x_1 - 6x_2 + 10x_3 = -3$

```
n = 3;
a = {{10, -5, -2}, {4, -10, 3}, {1, -6, 10}};
MatrixForm[a]
x = {0, 0, 0}
y = {0, 0, 0}
b = {3, -3, -3}
For[k = 1, k c 25, k++,
  For[i = 1, i c n, i++,
    y[[i]] = (b[[i]] - Sum[a[[i, j]] x x[[j]], {j, 1, i - 1}] -
      Sum[a[[i, j]] x x[[j]], {j, i + 1, n}]) / a[[i, i]]];
  For[m = 1, m c n, m++, x[[m]] = N[y[[m]]]]];
For p = 1, p c n, p++, Print "x ", p, " =", x   p
```

$$\begin{pmatrix} 10 & -5 & -2 \\ 4 & -10 & 3 \\ 1 & -6 & 10 \end{pmatrix}$$

0, 0, 0

0, 0, 0

3, -3, -3

x 1 =0.538709

x 2 =0.499171

x 3 =-0.0543701

Practical - 05(b)

Gauss-Seidel method

Q.1   Solve the system of linear equations :-
$$5x_1 + 2x_2 + x_3 = 10$$
$$3x_1 + 7x_2 + 4x_3 = 21$$
$$x_1 + x_2 + 9x_3 = 12$$

```
n = 3;
a = {{5, 2, 1}, {3, 7, 4}, {1, 1, 9}};
MatrixForm[a]
x = {0, 0, 0}
y = {0, 0, 0}
b = {10, 21, 12}
For[k = 1, k c 25, k++,
  For[i = 1, i c n, i++,
    y[[i]] = (b[[i]] - Sum[a[[i, j]] x y[[j]], {j, 1, i - 1}] -
        Sum[a[[i, j]] x x[[j]], {j, i + 1, n}]) / a[[i, i]]];
  For[m = 1, m c n, m++, x[[m]] = N[y[[m]]]]];
For p = 1, p c n, p++, Print "x ", p, " =", x   p
```

$$\begin{pmatrix} 5 & 2 & 1 \\ 3 & 7 & 4 \\ 1 & 1 & 9 \end{pmatrix}$$

```
 0, 0, 0

 0, 0, 0

 10, 21, 12
```

```
x 1 =1.
```

```
x 2 =2.
```

```
x 3 =1.
```

Q.2   Solve the system of linear equations :-
$$6x_1 - 2x_2 - x_3 = 4$$
$$x_1 + 5x_2 + x_3 = 3$$
$$2x_1 + x_2 + 4x_3 = 27$$

```
n = 3;
a = {{6, -2, -1}, {1, 5, 1}, {2, 1, 4}};
MatrixForm[a]
x = {0, 0, 0}
y = {0, 0, 0}
b = {4, 3, 27}
For[k = 1, k c 25, k++,
  For[i = 1, i c n, i++,
    y[[i]] = (b[[i]] - Sum[a[[i, j]] x y[[j]], {j, 1, i - 1}] -
        Sum[a[[i, j]] x x[[j]], {j, i + 1, n}]) / a[[i, i]]];
  For[m = 1, m c n, m++, x[[m]] = N[y[[m]]]]];
For p = 1, p c n, p++, Print "x ", p, " =", x   p
```

$$\begin{pmatrix} 6 & -2 & -1 \\ 1 & 5 & 1 \\ 2 & 1 & 4 \end{pmatrix}$$

0, 0, 0

0, 0, 0

4, 3, 27

x 1 =1.40157

x 2 =-0.937008

x 3 =6.28346

Q.3   Solve the system of linear equations :-
$10x_1 - 5x_2 - 2x_3 = 3$
$4x_1 - 10x_2 + 3x_3 = -3$
$x_1 - 6x_2 + 10x_3 = -3$

```
n = 3;
a = {{10, -5, -2}, {4, -10, 3}, {1, -6, 10}};
MatrixForm[a]
x = {0, 0, 0}
y = {0, 0, 0}
b = {3, -3, -3}
For[k = 1, k c 25, k++,
  For[i = 1, i c n, i++,
    y[[i]] = (b[[i]] - Sum[a[[i, j]] x y[[j]], {j, 1, i - 1}] -
        Sum[a[[i, j]] x x[[j]], {j, i + 1, n}]) / a[[i, i]]];
  For[m = 1, m c n, m++, x[[m]] = N[y[[m]]]]];
For p = 1, p c n, p++, Print "x ", p, " =", x   p
```

$$\begin{pmatrix} 10 & -5 & -2 \\ 4 & -10 & 3 \\ 1 & -6 & 10 \end{pmatrix}$$

0, 0, 0

0, 0, 0

3, -3, -3

x 1 =0.538715

x 2 =0.499176

x 3 =-0.0543657

Practical - 06(a)

Lagrange Interpolation

Q.1

```
xi = {-1, 0, 1, 2};
fi = {5, 1, 1, 11};
n = Length[xi];
For[k = 1, k c n, k++,
```

$$L_k[x\_] = \left(\prod_{j=1}^{k-1} \frac{x - xi[[j]]}{xi[[k]] - xi[[j]]}\right) x \left(\prod_{j=k+1}^{n} \frac{x - xi[[j]]}{xi[[k]] - xi[[j]]}\right)];$$

$$P[x\_] = \sum_{k=1}^{n} L_k[x] \times N[fi[[k]]];$$

```
Print["Lagrange Polynomial, P[x]= ", P[x]]
Print["Simplified Polynomial, P[x]= ", Simplify[P[x]]]
Print "Approximate value of f at x = 1.5 is ", P 1.5
```

Lagrange Polynomial, P[x]= $-0.833333 (1 - x) (2 - x) x +$

  $0.5 \ 1 - x \ \ 2 - x \ \ 1 + x \ + 0.5 \ 2 - x \ x \ 1 + x \ + 1.83333 \ -1 + x \ x \ 1 + x$

Simplified Polynomial, $P \ x = 1. - 3. \ x + 2. \ x^2 + 1. \ x^3$

Approximate value of f at x = 1.5 is 4.375

Q.2

```
Clear[x, k, f, l, P]
xi = {1, 2, 3};
f[x_] := Log[x];
n = Length[xi];
For[k = 1, k c n, k++,
```

$$L_k[x\_] = \left(\prod_{j=1}^{k-1} \frac{x - xi[[j]]}{xi[[k]] - xi[[j]]}\right) x \left(\prod_{j=k+1}^{n} \frac{x - xi[[j]]}{xi[[k]] - xi[[j]]}\right)];$$

$$P[x\_] = \sum_{k=1}^{n} L_k[x] \times N[f[xi[[k]]]];$$

```
Print["Lagrange Polynomial, P[x]= ", P[x]]
Print["Simplified Polynomial, P[x]= ", Simplify[P[x]]]
Print "Approximate value of f at x = 1.5 is ", P 1.5
```

Lagrange Polynomial, $P \ x = 0. + 0.693147 \ 3 - x \ \ -1 + x \ + 0.549306 \ -2 + x \ \ -1 + x$

Simplified Polynomial, $P \ x = -0.980829 + 1.12467 \ x - 0.143841 \ x^2$

Approximate value of f at x = 1.5 is 0.382534

Practical - 06(b)

Newton's Interpolation

Q.1 By using points (3,293), (5,508), (6,585), (9,764).   Evaluate at point   [2.5]

```
sum = 0;
points = {{3, 293}, {5, 508}, {6, 585}, {9, 764}};
n = Length[points]
y = points[[All, 1]]
f = points[[All, 2]]
dd[k_] :=
 Sum[(f[[i]] / Product[If[Equal[j, i], 1, (y[[i]] - y[[j]])], {j, 1, k}]), {i, 1, k}]
p[x_] = Sum[(dd[i] x Product[If[i c j, 1, x - y[[j]]], {j, 1, i - 1}]), {i, 1, n}]
Simplify[p[x]]
Evaluate p 2.5
```

4

 3, 5, 6, 9

 293, 508, 585, 764

$293 + \frac{215}{2} (-3 + x) - \frac{61}{6} (-5 + x) (-3 + x) + \frac{35}{36} (-6 + x) (-5 + x) (-3 + x)$

$\frac{1}{36} (-9702 + 9003 x - 856 x^2 + 35 x^3)$

222.288

Practical 7 -(a) - trapezoidal Rule

Find the integral

$$\int_1^5 \frac{1}{x^2} \, dx$$

```
f[x_] :=  1
          ──
          x²

n = 10
a = 1
b = 5

    b - a
h = ─────
      n

      h
sol = ─ x (f[a] + 2 (Sum[f[a + i x h], {i, 1, n - 1}]) + f[b]);
      2

       h
sol2 = ─ x (f[a] + 2 (Sum[f[i], {i, a + h, b - h, h}]) + f[b]);
       2

N[sol]
N sol2

10

1

5

2
─
5

0.825681

0.825681
```

```
f[x_] := 1/(1 + x)

n = 8

a = 0

b = 1

h = (b - a)/n

sol = h/2 x (f[a] + 2 (Sum[f[a + i x h], {i, 1, n - 1}]) + f[b]);

sol2 = h/2 x (f[a] + 2 (Sum[f[i], {i, a + h, b - h, h}]) + f[b]);

N[sol]

N sol2

8

0

1

1/8

0.694122

0.694122


f[x_] := 1/(3 x + 5)

n = 10

a = 1

b = 2

h = (b - a)/n

sol = h/2 x (f[a] + 2 (Sum[f[a + i x h], {i, 1, n - 1}]) + f[b]);

sol2 = h/2 x (f[a] + 2 (Sum[f[i], {i, a + h, b - h, h}]) + f[b]);

N[sol]

N sol2

10

1

2

1/10

0.10617

0.10617
```

```
f[x_] := Exp[-x^2]
h = 0.1
a = 0
b = 0.6
      b - a
n = ─────
        h
       h
sol = ─ x (f[a] + 2 (Sum[f[a + i x h], {i, 1, n - 1}]) + f[b]);
      2
       h
sol2 = ─ x (f[a] + 2 (Sum[f[i], {i, a + h, b - h, h}]) + f[b]);
       2
N[sol]
N sol2
```

0.1

0

0.6

6.

0.534455

0.534455

Practical 07 - (b) Simpson's Rule

Q.1) Find $\int_0^1 \frac{1}{5+3x}\, x$ using Simpson's Rule.

```
a = 0;
b = 1; n = 6;
h = (b - a) / n;
f[x_] := 1 / (5 + 3 x x);
sol = (h / 3) x (f[a] + 4 x Sum[f[i], {i, a + h, b - h, 2 x h}] +
      2 x Sum[f[i], {i, a + 2 h, b - 2 h, 2 x h}] + f[b]);
Print[" Simpson's Estimate is : ", sol]
```

$$\text{Simpson's Estimate is : } \frac{338\,743}{2\,162\,160}$$

N sol

0.156669

Practical 08

Euler's Method

```
f[x_, y_] = (y - x) / (y + x); y[1] = 1;
x[1] = 0;
h = 0.02;
For[i = 1, i ) 7, i++, x[i + 1] = x[i] + h;
 y[i + 1] = y[i] + h x f[x[i], y[i]];
 Print  x i , y i

 0, 1

 0.02, 1.02

 0.04, 1.03923

 0.06, 1.05775

 0.08, 1.0756

 0.1, 1.09283
```