6.4.3.5. Triple Exponential Smoothing

What happens if the data show trend **and** seasonality?

To handle seasonality, we have to add a third parameter

In this case double smoothing will not work. We now introduce a third equation to take care of seasonality (sometimes called periodicity). The resulting set of equations is called the "Holt-Winters" (HW) method after the names of the inventors.

The basic equations for their method are given by:

$$S_t = \alpha \frac{y_t}{I_{t-L}} + (1-\alpha)(S_{t-1} + b_{t-1}) \qquad \text{OVERALL SMOOTHING}$$

$$b_t = \gamma(S_t - S_{t-1}) + (1-\gamma)b_{t-1} \qquad \text{TREND SMOOTHING}$$

$$I_t = \beta \frac{y_t}{S_t} + (1-\beta)I_{t-L} \qquad \text{SEASONAL SMOOTHING}$$

$$F_{t+m} = (S_t + mb_t)I_{t-L+m} \qquad \text{FORECAST},$$

where

- y is the observation
- S is the smoothed observation
- b is the trend factor
- I is the seasonal index
- F is the forecast at m periods ahead
- t is an index denoting a time period

and α, β, and γ are constants that must be estimated in such a way that the MSE of the error is minimized. This is best left to a good software package.

Complete season needed

To initialize the HW method we need at least one complete season's data to determine initial estimates of the seasonal indices I_{t-L}.

L periods in a season

A complete season's data consists of L periods. And we need to estimate the trend factor from one period to the next. To accomplish this, it is advisable to use two complete seasons; that is, $2L$ periods.

Initial values for the trend factor

How to get initial estimates for trend and seasonality parameters

The general formula to estimate the initial trend is given by

$$b = \frac{1}{L}\left(\frac{y_{L+1} - y_1}{L} + \frac{y_{L+2} - y_2}{L} + \cdots + \frac{y_{L+L} - y_L}{L}\right).$$

Initial values for the Seasonal Indices

As we will see in the example, we work with data that consist of 6 years with 4 periods (that is, 4 quarters) per year.

Step 1: compute yearly averages

Step 1: Compute the averages of each of the 6 years.

$$A_p = \frac{\sum_{i=1}^{4} y_i}{4}\,, \quad p = 1,\, 2,\, \ldots,\, 6\,.$$

Step 2: divide by yearly averages

Step 2: Divide the observations by the appropriate yearly mean.

1	**2**	**3**	**4**	**5**	**6**
y_1/A_1	y_5/A_2	y_9/A_3	y_{13}/A_4	y_{17}/A_5	y_{21}/A_6
y_2/A_1	y_6/A_2	y_{10}/A_3	y_{14}/A_4	y_{18}/A_5	y_{22}/A_6
y_3/A_1	y_7/A_2	y_{11}/A_3	y_{15}/A_4	y_{19}/A_5	y_{23}/A_6
y_4/A_1	y_8/A_2	y_{12}/A_3	y_{16}/A_4	y_{20}/A_5	y_{24}/A_6

Step 3: form seasonal indices

Step 3: Now the seasonal indices are formed by computing the average of each row. Thus the initial seasonal indices (symbolically) are:

$$I_1 = (y_1/A_1 + y_5/A_2 + y_9/A_3 + y_{13}/A_4 + y_{17}/A_5 + y_{21}/A_6)/6$$

$$I_2 = (y_2/A_1 + y_6/A_2 + y_{10}/A_3 + y_{14}/A_4 + y_{18}/A_5 + y_{22}/A_6)/6$$

$$I_3 = (y_3/A_1 + y_6/A_2 + y_{11}/A_3 + y_{15}/A_4 + y_{19}/A_5 + y_{23}/A_6)/6$$

$$I_4 = (y_4/A_1 + y_6/A_2 + y_{12}/A_3 + y_{16}/A_4 + y_{20}/A_5 + y_{24}/A_6)/6\,.$$

We now know the algebra behind the computation of the initial estimates.

The next page contains an example of triple exponential smoothing.

The case of the Zero Coefficients

Zero coefficients for trend and seasonality parameters

Sometimes it happens that a computer program for triple exponential smoothing outputs a final coefficient for trend (γ) or for seasonality (β) of zero. Or worse, both are outputted as zero!

Does this indicate that there is no trend and/or no seasonality?

Of course not! It only means that the initial values for trend and/or seasonality were right on the money. No updating was necessary in order to arrive at the lowest possible MSE. We should inspect the updating formulas to verify this.