

# Surface-Wave Propagation on Non-Hermitian Metasurfaces With Extreme Anisotropy

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**Abstract**—Electromagnetic metasurfaces enable the advanced control of surface-wave propagation by spatially tailoring the local surface reactance. Interestingly, tailoring the surface resistance distribution in space provides new, largely unexplored degrees of freedom. In this article, we show that suitable spatial modulations of the surface resistance between positive (i.e., loss) and negative (i.e., gain) values can induce peculiar dispersion effects, far beyond mere compensation. Taking inspiration from the parity-time symmetry concept in quantum physics, we put forward and explore a class of non-Hermitian metasurfaces that may exhibit extreme anisotropy mainly induced by the gain-loss interplay. Through analytical modeling and full-wave numerical simulations, we illustrate the associated phenomenon of surface-wave canalization, explore nonlocal effects and possible departures from the ideal conditions, and address the feasibility of the required constitutive parameters. Our results suggest intriguing possibilities to dynamically reconfigure the surface-wave propagation and are of potential interest for applications to imaging, sensing, and communications.

**Index Terms**—Anisotropic materials, extreme parameters, metasurfaces, non-Hermitian, surface waves.

## I. INTRODUCTION

**S**URFACE electromagnetics is a research topic of long-standing interest in microwave and antenna engineering, which is experiencing a renewed vitality (see [1] for a recent review) in view of the widespread applications of artificial (metasurfaces) [2] and natural (e.g., graphene) [3] low-dimensional (2-D) materials.

In addition to enabling advanced wavefront manipulations [4], metasurfaces can support the propagation of tightly bound surface waves [5]–[8], which can be finely controlled via transformation-optics approaches [9]–[14] conceptually similar to those applied for volumetric metamaterials [15], [16]. Likewise, exotic phenomena observed in

volumetric metamaterials can be transposed to “flatland” scenarios. These include, for instance, hyperbolic propagation (characterized by open dispersion characteristics) [17]–[22], topological transitions (from closed elliptic-like to open hyperbolic-like dispersion characteristics) [18], extreme anisotropy (i.e., very elongated dispersion characteristics) [23], and canalization (i.e., diffractionless propagation of subwavelength beams) [18]–[20], [23]. Interestingly, new intriguing concepts and effects are emerging that are specific to 2-D materials. Among these, it is worth mentioning the “line waves,” localized both in-plane and out-of-plane around a surface reactance discontinuity with dual character (capacitive/inductive) [24], [25], and the rich moiré physics observed in rotated, evanescently coupled metasurfaces [26], [27].

The above studies have focused on *passive* scenarios, in which unavoidable losses are undesired and their effects need to be minimized. In fact, it has been shown that, in certain parameter regimes, losses can be beneficially exploited to enhance the canalization effects [23]. In this study, we further leverage and generalize this concept by exploring a class of metasurfaces characterized by tailored spatial modulations of loss and gain. Inspired by quantum-physics concepts such as “parity-time” ( $\mathcal{PT}$ ) symmetry [28], these non-Hermitian configurations are garnering a growing interest in several branches of physics [29], including electromagnetics [30]. In quantum mechanics, a  $\mathcal{PT}$ -symmetric operator is characterized by a potential function that satisfies the condition  $V(-x) = V^*(x)$ , with  $x$  and  $*$  denoting a spatial coordinate and complex conjugation, respectively [28]. In view of the well-known analogies, in electromagnetic scenarios, this translates into a refractive-index distribution  $n(-x) = n^*(x)$ , which implies an imaginary part with *odd* symmetry, corresponding to alternated loss and gain regions. Likewise, when referred to metasurfaces, of specific interest in our study, this condition translates as  $Z(-x) = -Z^*(x)$  and  $\sigma(-x) = -\sigma^*(x)$  for the surface impedance and conductivity, respectively. Within this framework, several non-Hermitian metasurface scenarios have been explored, with possible applications to negative refraction [31], cloaking [32], imaging [33], [34], sensing [35]–[37], low-threshold laser and coherent perfect absorbers [38], line waves [39], and unconventional lattice resonances [40]. However, in most studies, gain and loss are distributed on separate metasurfaces, with *out-of-plane* coupling, and there are only a few examples of metasurfaces featuring *in-plane* modulation of gain and loss. In fact, the effect of the gain-loss interplay in the surface-wave propagation remains largely unexplored.

Manuscript received October 2, 2020; revised December 12, 2020; accepted January 12, 2021. Date of publication March 2, 2021; date of current version April 2, 2021. This work was supported by the University of Sannio FRA program. The work of Andrea Alù was supported in part by the Simons Foundation and in part by the Air Force Office of Scientific Research. (Corresponding author: Vincenzo Galdi.)

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Color versions of one or more figures in this article are available at <https://doi.org/10.1109/TMTT.2021.3057632>.

Digital Object Identifier 10.1109/TMTT.2021.3057632

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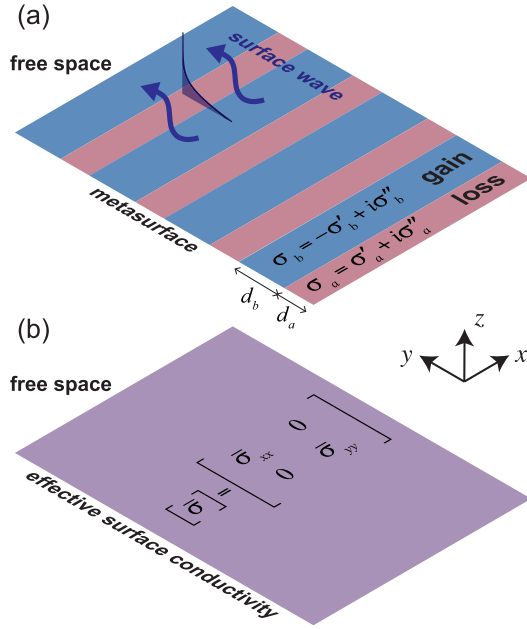


Fig. 1. (a) Problem geometry (details in the text). (b) Effective-parameter representation.

Here, inspired by recent findings for volumetric metamaterials [41], we show that the judicious tailoring of the gain-loss interplay can induce *extreme-anisotropy* responses in non-Hermitian metasurfaces. The anisotropy is particularly pronounced under  $\mathcal{PT}$ -symmetry conditions and yields strong surface-wave canalization effects that significantly depend on the gain-loss level. The possibility to control the gain (e.g., via solid-state amplifiers or optical pumping, depending on the operational frequency) opens the door to intriguing strategies for dynamic reconfiguration of the response.

The rest of this article is structured as follows. In Section II, we outline the problem formulation and its geometry. In Section III, we derive the parameter regimes of interest and illustrate some representative results, obtained via an effective-medium theory (EMT) and full-wave simulations, with details relegated in two appendixes. Specifically, we study the extreme anisotropy and associated canalization phenomena that can occur under  $\mathcal{PT}$ -symmetry conditions, as well as the effects of nonlocality and possible departures from these conditions. In Section IV, we explore the practical feasibility of the required gain levels, also addressing the stability issues. Finally, in Section V, we draw some brief conclusions and discuss possible perspectives.

## II. PROBLEM GEOMETRY AND FORMULATION

### A. Geometry

The problem geometry is schematically shown in Fig. 1(a). We consider, in free space, a metasurface of infinite extent in the  $xy$  plane, featuring a 1-D periodic modulation of the surface conductivity along the  $y$ -direction, with alternating values  $\sigma_a$  and  $\sigma_b$  (and subperiods  $d_a$  and  $d_b$ , respectively). Assuming an implicit  $\exp(-i\omega t)$  time-harmonic dependence,

the generally complex-valued conductivities are written as

$$\sigma_a = \sigma'_a + i\sigma''_a, \quad \sigma_b = -\sigma'_b + i\sigma''_b \quad (1)$$

with the prime and double-prime symbols tagging the real and imaginary parts, respectively. Throughout the study, we focus on the parameter regime

$$\sigma'_{a,b} > 0, \quad \sigma''_a \sigma''_b > 0. \quad (2)$$

In view of the assumed time dependence, the first condition implies that the “ $a$ ”- and “ $b$ ”-type constituents exhibit loss and gain, respectively. The second condition instead guarantees that both constituents are either of *inductive* ( $\sigma''_{a,b} > 0$ ) or *capacitive* ( $\sigma''_{a,b} < 0$ ) nature; this rules out the possibility of a *hyperbolic* response, which is not of interest here since it has already been studied [19], [20].

We highlight that the surface-conductivity parameterization in (1) and (2) is especially suited for 2-D materials (e.g., graphene) and can be readily related to more conventional constitutive parameters. For instance, it can be obtained from the surface impedance as [42]

$$\sigma = \frac{2}{Z} \quad (3)$$

with the factor 2 accounting for the two-faced character of the sheet. Moreover, for a very thin dielectric layer of thickness  $\Delta \ll \lambda$  (with  $\lambda = 2\pi c/\omega$  denoting the free-space wavelength and  $c$  the corresponding wavespeed), it can be approximately related to the relative permittivity via [9], [43]

$$\sigma \approx \frac{i(1 - \epsilon)k\Delta}{\eta} \quad (4)$$

where  $k = \omega/c = 2\pi/\lambda$  and  $\eta = (\mu_0/\epsilon_0)^{1/2}$  denote the free-space wavenumber and characteristic impedance, respectively.

### B. Formulation

In [44], it was shown that, in the limit of *deeply subwavelength* modulation periods  $d = d_a + d_b \ll \lambda$ , a structure like the one in Fig. 1(a) could be effectively modeled by a homogeneous, uniaxially anisotropic effective surface conductivity with relevant components

$$\bar{\sigma}_{xx} = f_a \sigma_a + f_b \sigma_b, \quad \bar{\sigma}_{yy} = \frac{\sigma_a \sigma_b}{f_b \sigma_a + f_a \sigma_b} \quad (5)$$

with  $f_a = d_a/d$  and  $f_b = 1 - f_a$  denoting the filling fractions. These mixing formulas closely resemble those occurring in the EMT modeling of multilayered volumetric metamaterials [45].

We are interested in studying the propagation of surface waves along this metasurface. In mathematical terms, this entails finding nontrivial source-free solutions  $\propto \exp[i(k_x x + k_y y + k_z z)]$  that are evanescent along the  $z$ -direction and propagating in the  $xy$  plane (see Fig. 1). By assuming for now the EMT model in (5), it can be shown (see [5]–[7] for details) that the wavenumbers must satisfy the dispersion equation

$$kk_z(4 + \eta^2 \bar{\sigma}_{xx} \bar{\sigma}_{yy}) + 2\eta[k^2(\bar{\sigma}_{xx} + \bar{\sigma}_{yy}) - \bar{\sigma}_{xx} k_x^2 - \bar{\sigma}_{yy} k_y^2] = 0 \quad (6a)$$

with the constraints

$$k_x^2 + k_y^2 + k_z^2 = k^2, \quad \text{Im}(k_z) \geq 0. \quad (6b)$$

The numerical solution of the nonlinear system in (6) can be efficiently carried out following the approach described in [19]. The resulting modes are generally hybrid and contain as special cases the transverse-electric and transverse-magnetic modes that can be supported by (isotropic) inductive and capacitive metasurfaces, respectively. Moreover, it is readily verified that the dispersion equation in (6a) is invariant under the duality transformations

$$\bar{\sigma}_{xx} \rightarrow \frac{4}{\eta^2 \bar{\sigma}_{yy}}, \quad \bar{\sigma}_{yy} \rightarrow \frac{4}{\eta^2 \bar{\sigma}_{xx}}. \quad (7)$$

The above relationships somehow resemble those observed for self-complementary metasurfaces [46]–[48], which exploit Babinet's principle. In our case, rather than geometrical self-complementarity, we rely on  $\mathcal{PT}$  symmetry.

In what follows, we will study the effects of the gain-loss interplay in the surface-wave propagation, via the approximate EMT modeling and full-wave numerical simulations.

### III. MODELING AND RESULTS

#### A. Loss-Gain Compensation in Effective Parameters

Looking at the mixing formulae in (5), an interesting question is whether there exist specific combinations of the constituents (in terms of  $\sigma_a$ ,  $\sigma_b$ , and  $f_a$ ) that render the effective parameters *purely reactive*. Within the limitations of the EMT, this would imply a perfect balancing of the loss and gain effects, which in the language of  $\mathcal{PT}$ -symmetry, corresponds to the symmetric phase [28]. By substituting the complex-valued conductivities (1) in the mixing formulae (5) and enforcing the zeroing of the effective-parameter real parts, we obtain the conditions (see Appendix A for details)

$$f_a = \frac{\sigma'_b}{\sigma'_a + \sigma'_b}, \quad (8a)$$

$$\sigma''_a = \pm \sqrt{|\sigma_b|^2 - (\sigma'_a)^2}. \quad (8b)$$

In view of the assumption  $\sigma'_{a,b} > 0$ , the condition in (8a) is always feasible ( $0 \leq f_a \leq 1$ ). On the other hand, we must enforce the additional constraint

$$|\sigma_b| > \sigma'_a \quad (9)$$

in order to ensure that  $\sigma''_a$  in (8b) is consistently real-valued. Moreover, in view of our assumption  $\sigma''_a \sigma''_b > 0$ , the sign determination in (8b) must be chosen consistently with the sign of  $\sigma''_b$ . By enforcing the conditions (8) in (5), after some algebra, we obtain

$$\bar{\sigma}_{xx} = i \frac{\sigma'_a \sigma''_b \pm \sigma'_b \sqrt{|\sigma_b|^2 - (\sigma'_a)^2}}{\sigma'_a + \sigma'_b} \quad (10a)$$

$$\bar{\sigma}_{yy} = i \frac{\sigma'_a \sigma''_b \mp \sigma'_b \sqrt{|\sigma_b|^2 - (\sigma'_a)^2}}{\sigma'_a - \sigma'_b} \quad (10b)$$

which identify a class of parameter combinations that yield *purely imaginary* effective parameters, i.e., loss-gain balance.

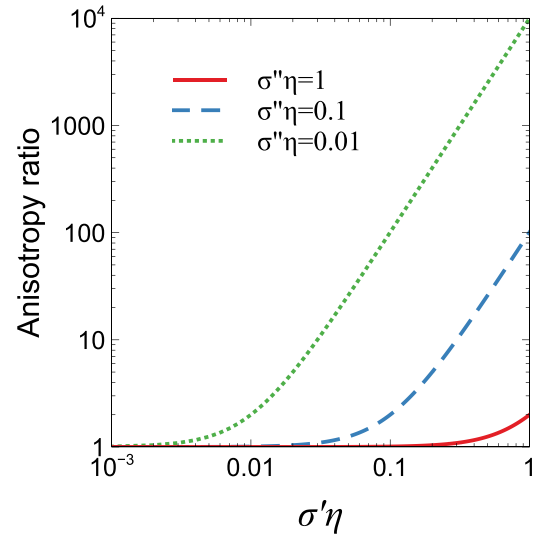


Fig. 2. Anisotropy ratio  $|\bar{\sigma}_{yy}|/|\bar{\sigma}_{xx}|$  for  $\mathcal{PT}$ -symmetric configurations, as a function of the gain-loss parameter  $\sigma'\eta$ , for representative values of the normalized susceptance:  $\sigma''\eta = 1$  (red solid curve),  $\sigma''\eta = 0.1$  (blue solid), and  $\sigma''\eta = 0.01$  (green dotted). Note the log-log scale.

This result is not necessarily surprising, as a balanced amount of gain and loss may be expected to compensate each other, but we will show hereafter that the gain-loss interplay may have much deeper implications on the modes supported by this metasurface.

#### B. $\mathcal{PT}$ -Symmetry-Induced Extreme Anisotropy

By inspection of (10), it can be observed that the conductivity components can differ substantially in the limit  $\sigma'_a \rightarrow \sigma'_b$ . From (8), this yields  $f_a = f_b = 0.5$  and (with the sign determination of interest here)  $\sigma''_a = \sigma''_b$ . In other words, by removing irrelevant subscripts, we obtain

$$\sigma_a = \sigma' + i\sigma'', \quad \sigma_b = -\sigma' + i\sigma'' \quad (11)$$

which, with a suitable choice of the reference-system origin, corresponds to the aforementioned  $\mathcal{PT}$ -symmetry condition

$$\sigma(-y) = -\sigma^*(y). \quad (12)$$

It can be shown (see Appendix A for details) that, under these conditions, the effective parameters reduce to

$$\bar{\sigma}_{xx} = i\sigma'', \quad \bar{\sigma}_{yy} = i \frac{|\sigma|^2}{\sigma''}. \quad (13)$$

The remarkably simple expressions in (13) clearly show that the gain-loss interplay can significantly affect the effective-parameter anisotropy. In particular, it is evident that the conditions

$$|\sigma''|\eta \ll 1, \quad \sigma' \gg |\sigma''| \quad (14)$$

would lead to  $|\bar{\sigma}_{xx}| \ll |\bar{\sigma}_{yy}|$ , i.e., *extreme anisotropy*. For a quantitative illustration, Fig. 2 shows the anisotropy ratio  $|\bar{\sigma}_{yy}|/|\bar{\sigma}_{xx}|$  as a function of the gain-loss parameter  $\sigma'\eta$  for various values of the normalized susceptance  $\sigma''\eta$ ; note that results do not depend on the sign of  $\sigma''$ , i.e., on the

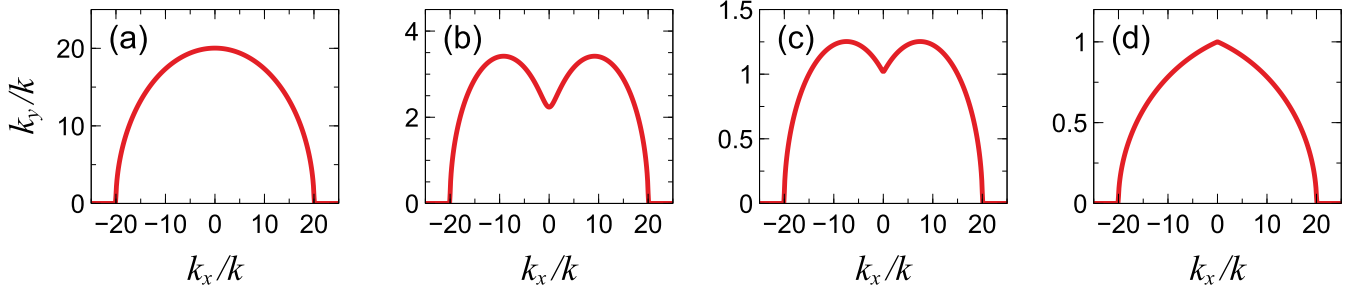


Fig. 3. Examples of EFCs for  $\mathcal{PT}$ -symmetric inductive configurations with  $\sigma''\eta = 0.1$ . (a)  $\sigma' = 0$  ( $\bar{\sigma}_{xx}\eta = \bar{\sigma}_{yy}\eta = i0.1$  and  $|\bar{\sigma}_{yy}|/|\bar{\sigma}_{xx}| = 1$ ). (b)  $\sigma'\eta = 0.3$  ( $\bar{\sigma}_{xx}\eta = i0.1$ ,  $\bar{\sigma}_{yy}\eta = i$ , and  $|\bar{\sigma}_{yy}|/|\bar{\sigma}_{xx}| = 10$ ). (c)  $\sigma'\eta = 1$  ( $\bar{\sigma}_{xx}\eta = i0.1$ ,  $\bar{\sigma}_{yy}\eta = i10.1$ , and  $|\bar{\sigma}_{yy}|/|\bar{\sigma}_{xx}| = 101$ ). (d)  $\sigma'\eta = 3$ , ( $\bar{\sigma}_{xx}\eta = i0.1$ ,  $\bar{\sigma}_{yy}\eta = i90.1$ , and  $|\bar{\sigma}_{yy}|/|\bar{\sigma}_{xx}| = 901$ ). In view of the inherent symmetry, only the  $k_y > 0$  branches are shown. Note the different scales on the vertical axes.

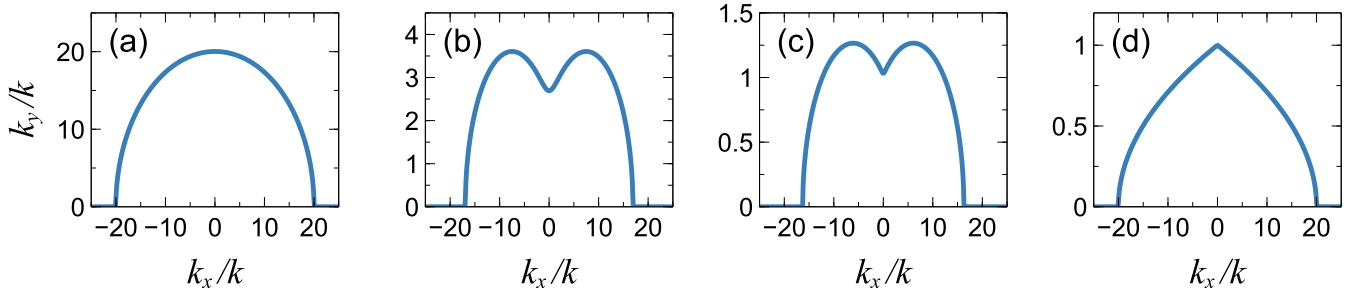


Fig. 4. Examples of EFCs for  $\mathcal{PT}$ -symmetric capacitive configurations. (a)  $\sigma' = 0$  and  $\sigma''\eta = -40$  ( $\bar{\sigma}_{xx}\eta = \bar{\sigma}_{yy}\eta = -i40$  and  $|\bar{\sigma}_{yy}|/|\bar{\sigma}_{xx}| = 1$ ). (b)  $\sigma'\eta = 12$  and  $\sigma''\eta = -5$  ( $\bar{\sigma}_{xx}\eta = -i5$ ,  $\bar{\sigma}_{yy}\eta = -i33.8$ , and  $|\bar{\sigma}_{yy}|/|\bar{\sigma}_{xx}| = 6.76$ ). (c)  $\sigma'\eta = 4$  and  $\sigma''\eta = -0.5$  ( $\bar{\sigma}_{xx}\eta = -i0.5$ ,  $\bar{\sigma}_{yy}\eta = -i32.5$ , and  $|\bar{\sigma}_{yy}|/|\bar{\sigma}_{xx}| = 65$ ). (d)  $\sigma'\eta = 0.2$  and  $\sigma''\eta = -0.001$  ( $\bar{\sigma}_{xx}\eta = -i0.001$ ,  $\bar{\sigma}_{yy}\eta = -i40$ , and  $|\bar{\sigma}_{yy}|/|\bar{\sigma}_{xx}| = 4000$ ). In view of the inherent symmetry, only the  $k_y > 0$  branches are shown. Note the different scales on the vertical axes.

inductive/capacitive character. However, for a given anisotropy ratio, the dispersion characteristics do depend on the reactive character of the metasurface.

For example, Fig. 3 shows some representative dispersion characteristics, in terms of equi-frequency contours (EFCs), for an inductive ( $\bar{\sigma}'' > 0$ ) scenario, by fixing  $\sigma''\eta = 0.1$  and varying the gain-loss parameter  $\sigma'\eta$ . As can be observed, the field exhibits a propagating character within a spectral wavenumber region that can be estimated analytically from (6) (see Appendix A for details), namely,

$$|k_x| \leq k_x^{(\max)} = k \sqrt{1 - \frac{4}{\bar{\sigma}_{xx}^2 \eta^2}} \approx \frac{2k}{\sigma''\eta} \quad (15)$$

with the approximate equality holding in the limit  $\sigma''\eta \ll 1$ . Quite interestingly, this spatial bandwidth is essentially controlled by the normalized susceptance  $\sigma''\eta$ . Outside this region, the field is evanescent, with a purely imaginary  $k_y$  (not shown for brevity). The gain-loss parameter  $\sigma'\eta$  controls instead the anisotropy degree. Specifically, starting from the trivial ( $\sigma' = 0$ ) isotropic case [Fig. 3(a)], the anisotropy becomes increasingly pronounced by increasing  $\sigma'$  [Fig. 3(b) and (c)], and the EFCs approach a limiting curve for  $\sigma'\eta \gg 1$  [Fig. 3(d)].

The responses for capacitive ( $\bar{\sigma}'' > 0$ ) scenarios can be in principle obtained via the duality transformations in (7). However, for completeness, they are also exemplified in Fig. 4 directly in terms of the constituent parameters  $\sigma'$  and  $\sigma''$ . In this case, the propagating spectral region is given by (see

Appendix A for details)

$$|k_x| \leq k_x^{(\max)} = k \sqrt{1 - \frac{\bar{\sigma}_{yy}^2 \eta^2}{4}} \approx \frac{k\eta(\sigma')^2}{2|\sigma''|} \quad (16)$$

where the approximate equality holds in the asymptotic limit  $\eta(\sigma')^2 \gg |\sigma''|$ . We observe that, unlike the inductive case, the spatial bandwidth now depends on both  $\sigma'$  and  $\sigma''$ ; the representative values considered in Fig. 4 are chosen so as to maintain the spatial bandwidth  $k_x^{(\max)} \approx 20k$ , progressively moving from perfect isotropy [Fig. 4(a)] to extreme anisotropy [Fig. 4(d)].

### C. Canalization Effects

Canalization effects, intended as the diffractionless transfer of subwavelength features over distances of several wavelengths, have been observed in hyperbolic [18], [19] and extreme-anisotropy [23] metasurfaces, and loss-induced canalization has also been demonstrated [49].

These effects can be intuitively understood by looking at the examples in Figs. 3 and 4 with higher anisotropy [see Figs. 3(c) and (d) and 4(c) and (d)]. It is apparent that a significant fraction of high- $k_x$  spectral components can propagate as unattenuated surface waves in the  $xy$  plane and, in view of the pronounced flatness of the EFCs, their group velocities (normal to the EFCs) are predominantly  $y$ -directed.

For illustration, Fig. 5 shows the surface-wave propagation along an inductive, non-Hermitian metasurface [with parameters as in Fig. 3(c)], excited by a  $z$ -directed elementary dipole.



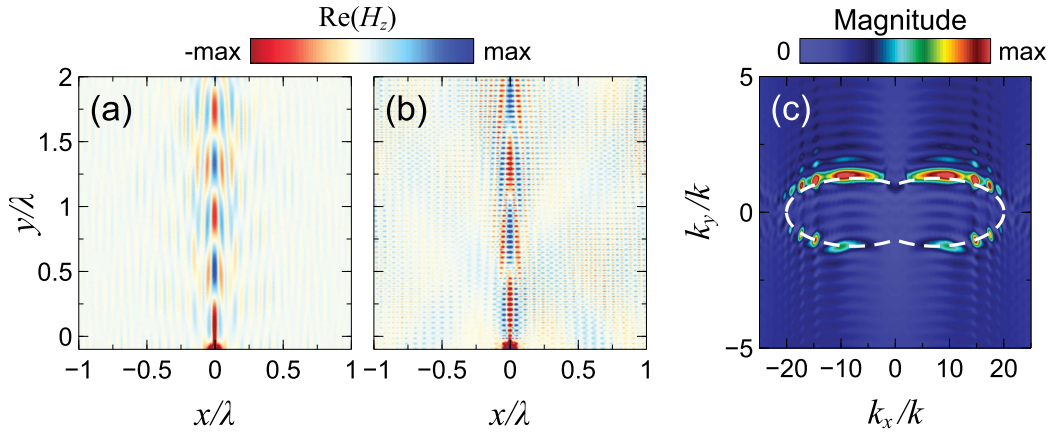


Fig. 5. Examples of canalization effects. (a) Numerically computed in-plane field map  $[\text{Re}(H_z)]$ , in false-color scale, pertaining to a metasurface with effective parameters  $\bar{\sigma}_{xx}\eta = i0.1$  and  $\bar{\sigma}_{yy}\eta = i10.1$ . (b) Corresponding results for the actual  $\mathcal{PT}$ -symmetric conductivity modulation, with  $\sigma_a\eta = 1 + i0.1$ ,  $\sigma_b\eta = -1 + i0.1$ ,  $f_a = f_b = 0.5$ , and  $d = 0.025\lambda$ . Fields are excited by a  $z$ -directed elementary magnetic dipole located at  $x = 0$ ,  $y = -0.1\lambda$ , and  $z = 0.001\lambda$  and are computed at  $z = 0.01\lambda$ . (c) Spatial spectrum (2-D Fourier transform) magnitude, in false color scale, of the field map in (a). Also shown (white dashed curve), as a reference, is the theoretical EFC from Fig. 3(c).

Specifically, Fig. 5(a) and (b) shows the numerically computed (see Appendix B for details) field maps at a close distance from the metasurface, by considering the EMT model and actual conductivity modulation, respectively. Results are in fair agreement, with the differences attributable to nonlocal effects (see Section III-E). In particular, they clearly display the aforementioned canalization effect, with unattenuated, diffractionless propagation of subwavelength features. This is markedly different from the conventional cylindrical wavefronts that would be observed in a homogeneous, isotropic case. As a further quantitative evidence, Fig. 5(c) shows the spatial spectrum (2-D Fourier transform) of the EMT field map, which is essentially peaked around the theoretical EFC. Similar results, not shown for brevity, are observed for capacitive scenarios as well.

Canalization effects such as those in Fig. 5 are of great interest for applications to high-resolution imaging. Although the above phenomena may appear qualitatively similar to the canalization effects observed in hyperbolic metasurfaces [18], [19], we stress that the underlying physics is completely different. While in the hyperbolic case, these effects are induced by the dual character (inductive/capacitive) of the two constituents, in our case, there is no contrast between the reactive parts, and the extreme anisotropy is solely induced by the gain-loss interplay.

#### D. Effects of Departure From $\mathcal{PT}$ Symmetry

To better understand the crucial role played by  $\mathcal{PT}$  symmetry in establishing the extreme-anisotropy response, it is insightful to explore different configurations that do not fulfill such condition. Within this framework, it is also worth highlighting that the unavoidable material dispersion dictates (via causality) that the  $\mathcal{PT}$ -symmetry condition may occur only at isolated frequencies [35], [50], and therefore, the above phenomena are inherently narrowband.

Fig. 6 shows the EFCs pertaining to four representative non- $\mathcal{PT}$ -symmetric configurations of interest. Specifically,

Fig. 6(a) and (b) shows the (real and imaginary, respectively) results for the inductive parameter configuration in Fig. 3(c), but in the absence of gain ( $\sigma_a\eta = 1 + i0.1$ ,  $\sigma_b\eta = i0.1$ ). As also evident from the effective parameters ( $\bar{\sigma}_{xx}\eta = 0.5 + i0.1$  and  $\bar{\sigma}_{yy}\eta = 0.019 + i0.196$ ), the extreme anisotropy is now lost and, most importantly, the propagation is severely curtailed in view of the substantial values of  $\text{Im}(k_y)$ . This is clearly visible in the corresponding field map shown in Fig. 7(a). Such a stark difference from the  $\mathcal{PT}$ -symmetric case suggests interesting possibilities for dynamically reconfiguring the surface-wave response, e.g., switching between a propagating, canalized regime, and strong damping by (de)activating the gain.

Fig. 6(c) and (d) shows another interesting example with parameters  $\sigma_a\eta = 0.8 + i0.608$ ,  $\sigma_b\eta = -1 + i0.1$ ,  $f_a = 0.556$ , and  $f_b = 0.444$  satisfying the gain-loss balance conditions in (8) but not the  $\mathcal{PT}$ -symmetry condition. As expected, since the effective parameters are purely reactive (inductive), the EFCs exhibit propagating and evanescent regions as in Fig. 3. However, although the gain constituent is the same as in Fig. 3(c), the spatial bandwidth and the anisotropy are less pronounced. As can be observed in the corresponding field map shown in Fig. 7(b), this results in a general deterioration of the canalization effects. Generally speaking, it can be verified numerically that, for a fixed gain constituent, among the infinite parameter configurations that fulfill the gain-loss balance conditions in (8), the  $\mathcal{PT}$ -symmetry condition in (12) guarantees the maximum anisotropy.

As previously mentioned, when material dispersion is considered, the  $\mathcal{PT}$ -symmetry condition can only be perfectly satisfied at isolated frequencies. It is therefore instructive to look at the effects of moderate mismatches that can occur at close-by frequencies. For illustration, Fig. 6(e) and (f) shows the EFCs for parameters as in Fig. 3(c) when the susceptance are perfectly balanced, but there is a moderate gain-loss mismatch ( $\sigma_a\eta = 1.2 + i0.1$  and  $\sigma_b\eta = -1 + i0.1$ ). Interestingly, the resulting effective parameters ( $\bar{\sigma}_{xx}\eta = 0.1 + i0.1$  and  $\bar{\sigma}_{yy}\eta = -5.95 + i6.15$ ) exhibit simultaneously

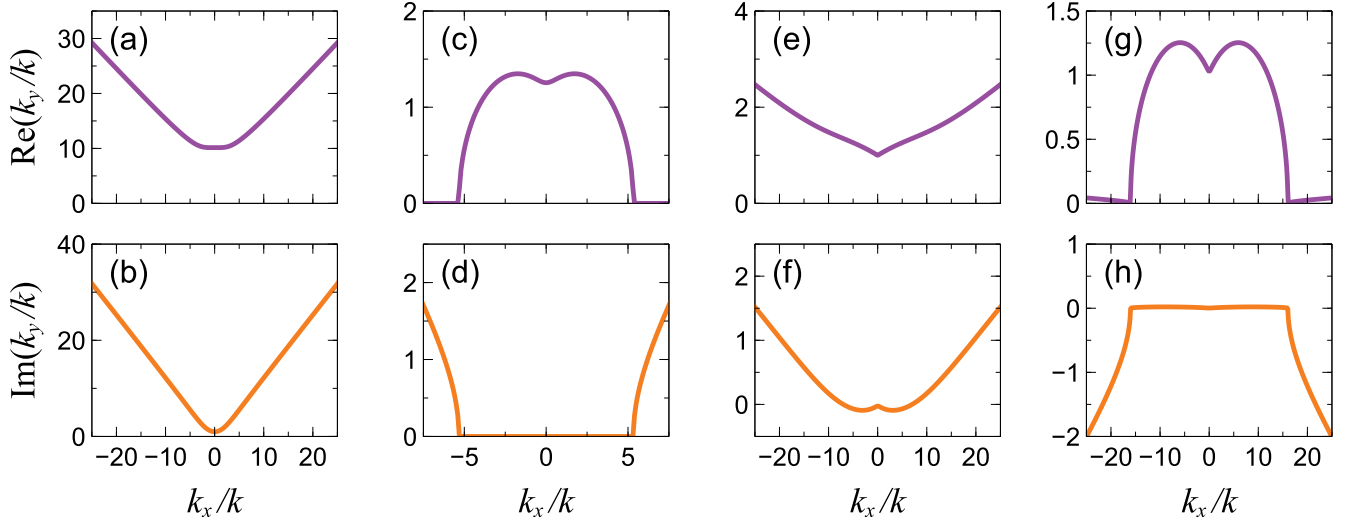


Fig. 6. Representative EFCs (top: real parts; bottom: imaginary parts) for non- $\mathcal{PT}$ -symmetric configurations. (a) and (b) No gain:  $\sigma_a\eta = 1 + i0.1$ ,  $\sigma_b\eta = i0.1$ , and  $f_a = f_b = 0.5$  ( $\bar{\sigma}_{xx}\eta = 0.5 + i0.1$  and  $\bar{\sigma}_{yy}\eta = 0.019 + i0.196$ ). (c) and (d) Gain-loss-balanced:  $\sigma_a\eta = 0.8 + i0.608$ ,  $\sigma_b\eta = -1 + i0.1$ ,  $f_a = 0.556$ , and  $f_b = 0.444$  ( $\bar{\sigma}_{xx}\eta = i0.382$  and  $\bar{\sigma}_{yy}\eta = i2.641$ ). (e) and (f) Imperfect  $\mathcal{PT}$  symmetry:  $\sigma_a\eta = 1.2 + i0.1$ ,  $\sigma_b\eta = -1 + i0.1$ , and  $f_a = f_b = 0.5$  ( $\bar{\sigma}_{xx}\eta = 0.1 + i0.1$  and  $\bar{\sigma}_{yy}\eta = -5.95 + i6.15$ ). (g) and (h) Imperfect  $\mathcal{PT}$  symmetry:  $\sigma_a\eta = 1 + i0.1$ ,  $\sigma_b\eta = -1 + i0.15$ , and  $f_a = f_b = 0.5$  ( $\bar{\sigma}_{xx}\eta = i0.125$  and  $\bar{\sigma}_{yy}\eta = 0.4 + i8.12$ ). In view of the inherent symmetry, only the  $\text{Re}(k_y) > 0$  branches are shown. Note the different scales on the vertical axes.

loss and gain along different directions. Such “indefinite” non-Hermitian character is also observed in volumetric metamaterials [41], [51]. The EFCs differ substantially from the perfectly  $\mathcal{PT}$ -symmetric reference case in Fig. 3(c), especially in light of an imaginary part which assumes small negative values at lower wavenumbers and increasingly positive values for higher wavenumbers. Fig. 7(c) shows the corresponding field map, from which we still observe some canalization effects, though with a more visible spreading and attenuation by comparison with Fig. 5.

For the same parameter configuration, Fig. 6(g) and (h) shows instead the effects of the imbalance in the susceptances (while maintaining the inductive character), but assuming now perfect gain-loss balance ( $\sigma_a = 1 + i0.1$ ,  $\sigma_b = -1 + i0.15$ ). Despite the sizable imbalance, the EFC departures from the perfectly  $\mathcal{PT}$ -symmetric reference case in Fig. 3(c) seem less dramatic. As can be observed, there is still an extended spectral region characterized by a small positive imaginary part [ $\text{Im}(k_y) \lesssim 0.02k$ ] and pronounced anisotropy, although the spatial bandwidth is moderately reduced. Quite interestingly, outside this region, we observe  $\text{Im}(k_y) < 0$  which, assuming the conventional choice  $\text{Re}(k_y) > 0$  for the branch-cut, would imply amplification. However, this is not necessarily the case, as in structures mixing gain and loss an *a priori* choice of the branch-cut is not obvious in the presence of unbounded domains. In fact, a counterintuitive sign flip in the propagation constant has been observed at certain critical incidence conditions [52] (see also the discussion in [41]). In our specific case, the numerical simulations in Fig. 7(d) indicate the presence of canalization effects accompanied by a mild attenuation, thereby suggesting that  $\text{Re}(k_y) < 0$  may be the proper choice for the branch-cut in the regions where  $\text{Im}(k_y) < 0$ .

To sum up, the above examples indicate that, for moderate departures from the  $\mathcal{PT}$ -symmetry conditions, canalization effects are still attainable but with reduced resolution and propagation distance. In this respect, the gain-loss (im)balance turns out to be more critical than that in the susceptance.

### E. Nonlocal Effects

As previously mentioned, the EMT approximation in (5) is generally accurate for deeply subwavelength modulation periods  $d \ll \lambda$ . For a more quantitative assessment, Fig. 8 compares its predictions with the rigorous EFCs obtained via full-wave numerical simulations (see Appendix B), for different values of the modulation period  $d$ . We observe that the results for  $d = 0.001\lambda$  are hardly distinguishable from the EMT prediction, whereas some visible differences progressively appear for  $d = 0.01\lambda$  and  $d = 0.02\lambda$ , with changes in the local curvature and moderate reductions in the spatial bandwidth. Interestingly, the propagation constant remains purely real, thereby indicating that the  $\mathcal{PT}$ -symmetric-induced gain-loss compensation extends beyond the range of validity of the EMT approximation.

The observed departures from the EMT predictions indicate that nonlocal effects (i.e., spatial dispersion) are no longer negligible. In principle, these effects can be captured by introducing in the effective parameters some wavevector-dependent correction terms, e.g., along the lines of [53].

## IV. FEASIBILITY ISSUES

### A. Possible Implementation Strategies

Although this study is essentially focused on exploring the basic phenomenology and a practical implementation requires further investigations, one might wonder

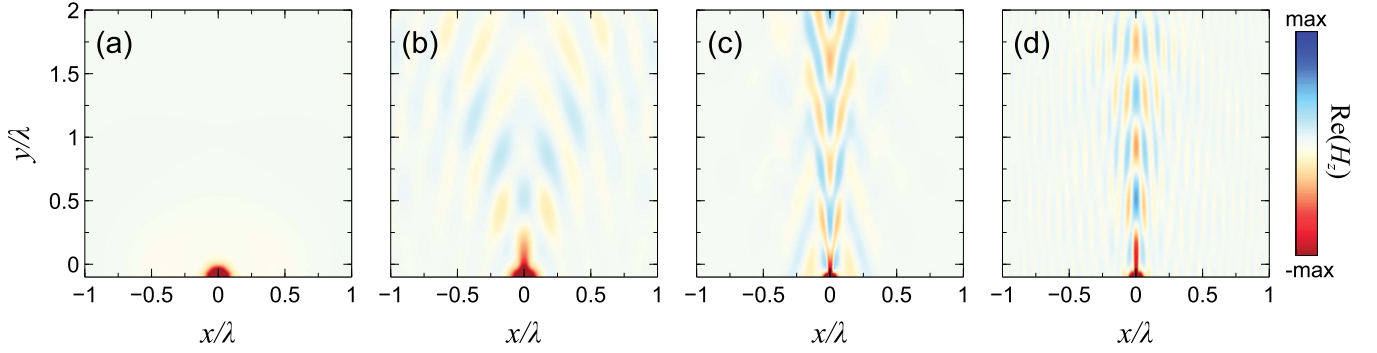


Fig. 7. Numerically computed in-plane field maps  $[\text{Re}(H_z)]$ , in false-color scale, pertaining to the non- $\mathcal{PT}$ -symmetric configurations in Fig. 6. (a) No-gain, as in Fig. 6(a) and (b). (b) Gain-loss-balanced, as in Fig. 6(c) and (d). (c) Imperfect  $\mathcal{PT}$  symmetry, as in Fig. 6(e) and (f). (d) Imperfect  $\mathcal{PT}$  symmetry, as in Fig. 6(g) and (h). Fields are excited by a  $z$ -directed elementary magnetic dipole located at  $x = 0$ ,  $y = -0.1\lambda$ , and  $z = 0.001\lambda$  and are computed at  $z = 0.01\lambda$ .

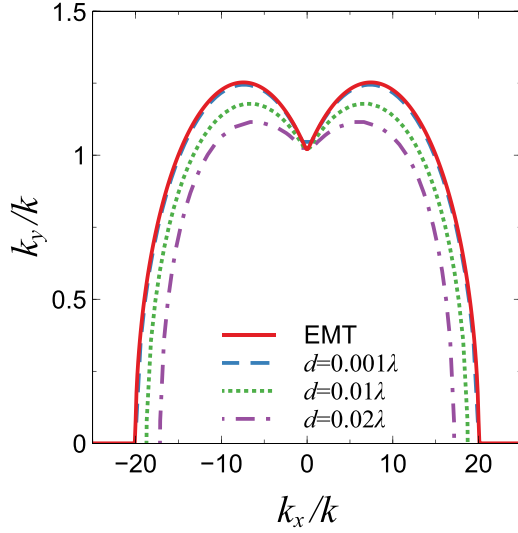


Fig. 8. EFCs pertaining to a  $\mathcal{PT}$ -symmetric parameter configuration with  $\sigma_a = 1 + i0.1$ ,  $\sigma_b = -1 + i0.1$ , and  $f_a = f_b = 0.5$ . Comparison between the EMT prediction (red-solid curve;  $\bar{\sigma}_{xx}\eta = 0.1$  and  $\bar{\sigma}_{yy}\eta = 10.1$ ) and full-wave numerical simulations for  $d = 0.001\lambda$  (blue-dashed),  $d = 0.01\lambda$  (green dotted), and  $d = 0.02\lambda$  (purple dashed-dotted). In view of the inherent symmetry, only the  $k_y > 0$  branches are shown.

to what extent the parameter configurations required for  $\mathcal{PT}$ -symmetry-induced extreme anisotropy are feasible. Within this framework, the most critical element is the gain constituent, whose implementation varies with the operational frequency of interest. For instance, at microwave frequencies, active metasurfaces typically rely on negative-resistance elements based on amplifiers [54] or tunnel diodes [55]. At terahertz frequencies, an optically pumped graphene monolayer may be a viable option, as it can support population inversion and a negative dynamic conductivity [56]. Specifically, for typical parameters found in the literature [35], [39], the real part can reach values  $\sigma'_g \approx -0.02/\eta$ , whereas the imaginary part  $\sigma''_g$  can be tuned between positive and negative values (ranging approximately from  $-0.05/\eta$  to  $0.05/\eta$ ) by acting on the frequency and quasi-Fermi energy. These figures allow in principle to attain the large anisotropy ratios of interest here.

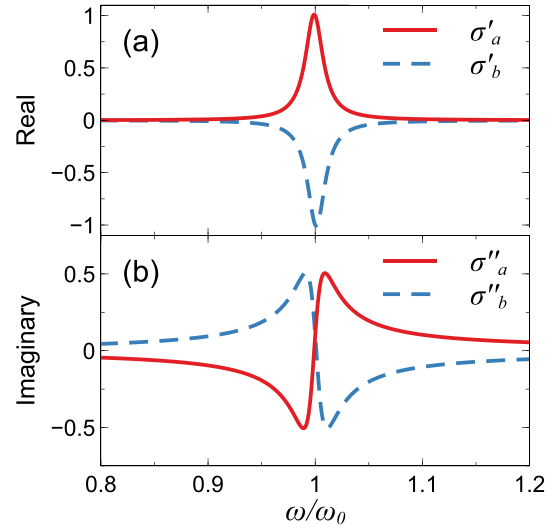


Fig. 9. (a) and (b) Real and imaginary parts, respectively, of dispersive models for loss and gain constituents, from (19). Parameters are chosen as:  $A_L = A_G = 2.02 \cdot 10^{-2}\omega_0$ ,  $\omega_L = 0.999\omega_0$ ,  $\omega_G = 1.001\omega_0$ , and  $\Gamma_L = \Gamma_G = 0.02\omega_0$ , in order to satisfy the conditions in Fig. 5,  $\sigma_a\eta = 1 + i0.1$  and  $\sigma_b\eta = -1 + i0.1$ , at the operational radian frequency  $\omega_0$ .

At optical wavelengths, gain media are typically obtained by doping host media with organic dyes [57], [58] or quantum dots [59]–[61]. To derive some basic quantitative estimates, we consider the thin-dielectric-layer model in (4) and invert for the complex-valued relative permittivity

$$\varepsilon = \varepsilon' + i\varepsilon'' = 1 + i\frac{\sigma\eta}{k\Delta}. \quad (17)$$

Accordingly, the conditions in (14) for extreme anisotropy translate into

$$|1 - \varepsilon'|k\Delta \ll 1, \quad |1 - \varepsilon'| \ll |\varepsilon''|. \quad (18)$$

For instance, assuming that  $k\Delta = 0.1$ , in the capacitive case, a relative permittivity  $\varepsilon = 1.01 - i2$  would yield a normalized conductivity  $\sigma\eta = -0.2 - i0.001$ , such as the one considered in the extreme-anisotropy case in Fig. 4(d). These permittivity values are in line with those attainable at infrared wavelengths

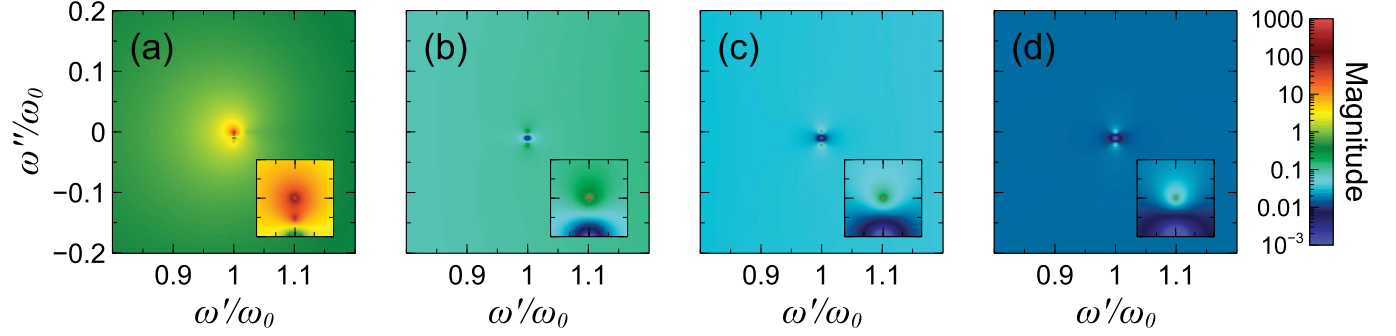


Fig. 10. Representative stability maps for the parameter configuration in Fig. 9. (a)–(d) Inverse residual (magnitude of left-hand side reciprocal) pertaining to the dispersion equation (6a) for  $k_x = 0$ ,  $k_x = 5k_0$ ,  $k_x = 10k_0$ , and  $k_x = 15k_0$ , respectively, in the complex  $\tilde{\omega} = \omega' + i\omega''$  plane;  $k_0 = \omega_0/c$  denotes the wavenumber at the operational frequency, and  $k_y$  and  $k_z$  are determined from the nominal EFC in Fig. 3(c) and from (6b), respectively. The insets show the magnified views around ( $\omega' = \omega_0$  and  $\omega'' = 0$ ). Note the presence of surface-wave poles on the real axis at  $\omega' = \omega_0$  and the absence of poles in the top half-plane  $\omega'' > 0$ .

by doping transparent conductive oxides (such as indium tin oxide) with lanthanides [41], [62]–[64]. Similar considerations hold for the inductive case too.

As for the practical realizability of the required gain-loss spatial modulation, one possibility could be to rely on high-resolution selective optical pumping [65], possibly based on digital spatial light modulators [66]. Alternatively, one could think of relying on a uniform optical pumping and patterning a thin layer of gain material with thin, lossy strips, so as to suitably overcompensate the gain in certain selected regions.

The above considerations suggest that the gain-loss configurations of interest are within reach with current or near-future technologies, although further studies are needed to develop some practical designs. In particular, the presence of a substrate should also be considered, and a simple EMT modeling may not be applicable, thereby requiring extensive numerical optimization driven by full-wave simulations. These aspects are beyond the scope of the present investigation and will be the subject of forthcoming studies.

### B. Stability

An important issue in non-Hermitian configurations featuring gain is the potential onset of instability, manifested as self-oscillations supported by the system [67]. In previous studies dealing with planar [33] and cylindrical [34] non-Hermitian metasurfaces, the stability issue was addressed by introducing a physical, dispersive model for the gain constituent and by looking at the poles of a relevant transfer function analytically continued in the complex frequency plane. Here, we follow a similar approach and consider for the conductivities in the loss and gain regions a Lorentzian (standard and inverted, respectively) model [68]

$$\sigma_a(\omega) = \frac{iA_L\omega}{\eta(\omega^2 - \omega_L^2 + i\Gamma_L\omega)} \quad (19a)$$

$$\sigma_b(\omega) = -\frac{iA_G\omega}{\eta(\omega^2 - \omega_G^2 + i\Gamma_G\omega)} \quad (19b)$$

where  $A_{L,G}$ ,  $\omega_{L,G}$ , and  $\Gamma_{L,G}$  denote some dimensional constants, peak radian frequencies, and damping factors, respectively. Fig. 9 shows the above dispersion laws with

parameters chosen so as to attain at an operational radian frequency  $\omega_0$  the nominal values  $\sigma_a\eta = 1 + i0.1$  and  $\sigma_b\eta = -1 + i0.1$  considered in the canalization example of Fig. 5.

We then consider the EMT dispersion equation in (6), which can be viewed as a pole condition for the reflection/transmission coefficient under plane-wave illumination from free space, and substitute the dispersive effective parameters computed from (10) with (19). Finally, for fixed values of the wavenumbers, we look for roots of (6) (i.e., poles) in the complex  $\tilde{\omega} = \omega' + i\omega''$  plane. In view of the assumed time-harmonic convention, instability corresponds to complex poles laying in the upper half-plane  $\omega'' > 0$ .

Fig. 10 shows the stability maps for representative values of the wavenumbers on the nominal EFC in Fig. 3(c). We observe the expected presence of the surface-wave pole on the real axis at the operational radian frequency  $\omega' = \omega_0$  and the absence of poles in the upper half-plane  $\omega'' > 0$ , which indicates that the system is unconditionally stable for any temporal excitation. The above examples only serve to illustrate that it is possible, in principle, to attain stability via suitable dispersion engineering. Clearly, different parameter choices in the dispersion models and/or the anisotropy ratios may induce the transition of some poles to the upper half-plane  $\omega'' > 0$ , thereby causing instabilities.

### V. CONCLUSION

In conclusion, we have studied the surface-wave propagation for a class of non-Hermitian metasurfaces based on  $\mathcal{PT}$ -symmetric modulations of the surface conductivity. Through a simple EMT model, we have shown that a suitably tailored gain-loss interplay can induce extreme anisotropy, giving rise to interesting canalization effects. These theoretical predictions are in good agreement with numerical full-wave simulations. Moreover, we have explored the effects of possible departures from perfect  $\mathcal{PT}$  symmetry, as well as of nonlocality. Finally, we have addressed some preliminary feasibility issues, including the stability.

The outcomes from this study open new perspectives in the design of metasurfaces. Within this enlarged framework, the parameter space extends over the entire complex plane of the complex conductivity, and losses are not treated as second-order, detrimental effects to be minimized or compensated.



Instead, their interplay with gain is harnessed to attain exotic dispersion effects, which can be dynamically controlled and/or reconfigured by acting on the gain level (e.g., via optical pumping) while maintaining a strong out-of-plane confinement. This can find a variety of potential applications in scenarios including sensing, subdiffractive imaging, and communications.

Current and future studies are aimed at exploring such applications and designing some practical implementations of the idealized configuration considered here. Also of great interest is the study of *exceptional points* and lasing conditions in these metasurfaces [30].

#### APPENDIX A DETAILS ON EMT MODELING

By substituting (1) in (5), we derive the real parts of the effective parameters

$$\bar{\sigma}'_{xx} = f_a \sigma'_a - (1 - f_a) \sigma'_b \quad (20a)$$

$$\bar{\sigma}'_{yy} = \frac{f_a \sigma'_a |\sigma_b|^2 - (1 - f_a) \sigma'_b |\sigma_a|^2}{D(\sigma_a, \sigma_b, f_a)} \quad (20b)$$

with

$$D(\sigma_a, \sigma_b, f_a) = (1 - f_a)^2 |\sigma_a|^2 + f_a |\sigma_b|^2 + 2f_a(1 - f_a)(\sigma'_a \sigma'_b + \sigma''_a \sigma''_b). \quad (21)$$

By recalling that, in view of the assumptions (2),  $D(\sigma_a, \sigma_b, f_a)$  in (21) is a sum of nonnegative terms and hence never vanishes, and (8)–(10) follow from zeroing (20a) and the numerator of (20b).

In the  $\mathcal{PT}$ -symmetric case, the expressions in (13) can be directly obtained by substituting (11) in (5) or, equivalently, by particularizing (10). In this last case, a limit operation is entailed in (10b), which yields a 0/0 indeterminate form that can be evaluated by means of the L'Hôpital's rule.

The spatial bandwidths in (15) and (16) are calculated by solving the dispersion equation (6) for  $k_y = 0$ . Solving with respect to  $k_z$ , we find two solutions

$$k_{zi} = -\frac{2k}{\eta \bar{\sigma}_{xx}}, \quad k_{zc} = -\frac{k \eta \bar{\sigma}_{yy}}{2}. \quad (22)$$

Recalling the assumed branch-cut  $\text{Im}(k_z) \geq 0$ , it is readily verified that  $k_{zi}$  is the proper solution in the inductive case ( $\bar{\sigma}_{xx}'' > 0$ ), whereas  $k_{zc}$  should be selected in the capacitive case ( $\bar{\sigma}_{xx}'' < 0$ ). The equalities in (15) and (16) follow from solving the dispersion equation (6a) with respect to  $k_x$ , with the proper choice of  $k_z$  in (22).

#### APPENDIX B DETAILS ON NUMERICAL SIMULATIONS

The field maps in Figs. 5 and 7, as well as the rigorous EFCs in Fig. 8, are computed via numerical (finite-element) numerical simulations via the commercial software package COMSOL Multiphysics [69].

For the dipole-excited configurations in Figs. 5 and 7, we consider a 3-D computational domain of total size  $3.2\lambda \times 3.2\lambda \times 0.35\lambda$ . The metasurface is modeled via an impedance boundary condition at  $z = 0$  enforced in terms of a surface

current density ( $J_x = \sigma_{xx} E_x$ ,  $J_y = \sigma_{yy} E_y$ , and  $J_z = 0$ ). For the configuration in Fig. 5(b), we model the actual conductivity modulation, with  $d = 0.04\lambda$  and a total of 55 periods. To minimize the finite-size effects, the metasurfaces are terminated in-plane with fictitious sections of length  $0.4\lambda$  with electrical conductivity tapered so as to match the free-space level. The domain is terminated (with the exception of the  $z = 0$  face) by perfectly matched layers of thickness  $0.25\lambda$  and is discretized with an adaptive mesh, resulting in  $\sim 2.3$  million degrees of freedom. We carry out a frequency-domain analysis, by means of the Pardiso direct solver (with default parameters).

The spatial spectrum in Fig. 5(c) is computed from the calculated field distribution at  $z = 0.01\lambda$ , by means of a  $2048 \times 512$  2-D fast Fourier transform implemented in a Python code via the routine `fft2` available in the NumPy package [70].

For the EFCs in Fig. 8, we consider instead a 2-D computational domain (assuming an infinite extent along the  $x$ -direction), including only one period of the conductivity modulation. The structure is terminated by phase shift walls along the  $y$ -direction and includes a free-space layer terminated by a perfectly matched layer, both of thickness  $0.5\lambda$ . Once again, an adaptive mesh is applied, which yields  $\sim 1.7$  million degrees of freedom. In this case, we utilize the modal analysis, with the MUMPS direct solver and default parameters. To calculate the EFCs, we scan the wavenumber  $k_y$  over the real axis and compute the corresponding eigenvalue  $k_x$ .

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