Cuaderno 2 → SLIT

Ejercicio → Respuesta impulso

- resolver la EDO con la convolución

$x(t)$ = señal de entrada

$$\frac{d}{dt} y(t) + y(t) = x(t)$$

$\mathcal{E}(t)$ = función escalón

$x(t) = e^{-2t}$

Para $t \geq 0$ y 0 para $t < 0$

$$\frac{d}{dt} y(t) + y(t) = e^{-2t} \cdot \mathcal{E}(t)$$

Para $t<0$:

$$\frac{d}{dt} y(t) + y(t) = 0$$

$$\frac{d}{dt} y(t) = -y(t)$$

$$= \int \frac{1}{y(t)} dy(t) =$$

$$\int -dt = \ln|y(t)|$$

$$= -t + c$$

$$y(t) = e^{-t+c}$$

$$y(t) = ke^{-t}$$

Para $t \geq 0$

$$\frac{d}{dt} y(t) + y(t) = e^{-2t}$$

$$y_p(t) = Ae^{-2t}$$

$$y_p'(t) = -2Ae^{-2t} = e^{-2t} \delta(t)(e^{-2t})$$

$$A = \delta(t) \Rightarrow y_p(t) = -\delta(t) e^{-2t}$$

Solución general:

$$y(t) = y_p(t) + y_h(t) = ke^{-t} - e^{-2t}$$

$$\lim_{C \to 0} (ce^{-t} - e^{-2t}) = \lim_{C \to 0} ce^{-t}$$

$$C = 1$$

$$h(t) = e^{-t}$$

→ convolución

$$x(t) = e^{-2t} \cdot \mathcal{E}(t)$$

$$y(t) = \int_0^t y(T) \cdot x(t-T) dt = \int_0^t (e^{-T} - e^{-2T})(e^{-2(t-T)}) \cdot \mathcal{E}(t-T) dt$$

$\mathcal{E}(t) = 1$

$$\int_0^t (e^{-T} - e^{2T})(e^{-2t+2T}) dt$$

$$\int_0^t e^{t-2t} dt - \int_0^t e^{-2t} dt = e^{-2t} \cdot e^{T}\Big|_0^t - e^{-2t} \cdot T\Big|_0^t$$

$$= y(t) = e^{-t} - e^{-2t}$$