# Parcial # 1: Señales y Sistemas

- Desarrollo preguntas:

(a) $x_1(t) \in \mathbb{R}, \mathbb{C}$ y $x_2(t) \in \mathbb{R}, \mathbb{C}$

$$d^2(x_1, x_2) = \overline{P}_{x_1 - x_2} = \lim_{T \to \infty} \frac{1}{T} \int_T |x_1(t) - x_2(t)|^2 dt$$

$x_1(t) = Ae^{jw_0 t}$ $\qquad x_2(t) = Be^{j5w_0 t}$

Encontrar la distancia entre ambas señales

$|x_1(t) - x_2(t)|^2 =$ reemplazamos

$$|Ae^{jw_0 t} - Be^{j5w_0 t}|^2 = |Ae^{jw_0 t}|^2 - 2ABe^{j6w_0 t} + |Be^{j5w_0 t}|^2$$

Sabemos que $|e^j|^2 = e^j \cdot e^{-j} = 0^0 = 1$ entonces

$$|x_1(t) - x_2(t)|^2 = A^2 - 2ABe^{j6w_0 t} + B^2$$

Integramos

$$\frac{1}{T}\int_0^T |x_1(t) - x_2(t)|^2 dt = \frac{1}{T}\int_0^T A^2 - 2ABe^{j6w_0 t} + B^2 dt$$

$$= \frac{1}{T}\left[\int_0^T A^2 dt - \int_0^T 2ABe^{j6w_0 t} dt + \int_0^T B^2 dt\right]$$

$$\int_0^T A^2 dt = A^2[T]\Big|_0^T = A^2 T \qquad \int_0^T B^2 dt = B^2[T]\Big|_0^T = B^2 T$$

$$-2\int_0^T ABe^{j6w_0 t} dt \qquad u = j6w_0 t \qquad du = j6w_0 dt \qquad dt = du/j6w_0$$

$$-2AB\int_0^T e^u dt = -2AB\int_0^T \frac{e^u}{j6w_0} du \qquad w_0 = \frac{2\pi}{T}$$

$$= \frac{-2AB}{j6w_0}\left[e^{j6w_0 t}\right]\Big|_0^T = \frac{-2AB}{j6w_0}\left[e^{j6\frac{2\pi T}{T}} - e^{j6\frac{2\pi 0}{T}}\right]$$

$$= \frac{-2AB}{j6w_0}\cdot\left(e^{j62\pi} - e^0\right) = \frac{-2AB}{j6\frac{2\pi}{T}} \cdot e^{j12\pi} + \frac{2AB}{j6\frac{2\pi}{T}} = -\frac{T2AB}{j62\pi}e^{j12\pi} + \frac{2ABT}{j62\pi}$$

$$= -\frac{TAB}{j6\pi}e^{j12\pi} + \frac{TAB}{j6\pi}$$

Juntamos las tres integrales

$$= \frac{1}{T}\left[A^2T - \frac{TAB}{j6\pi}e^{j12\pi} + \frac{TAB}{j6\pi} + B^2T\right]$$

$$= \frac{A^2\cancel{T}}{\cancel{T}} - \frac{1}{\cancel{T}}\frac{\cancel{T}AB}{j6\pi}e^{j12\pi} + \frac{1}{\cancel{T}}\frac{\cancel{T}AB}{j6\pi} + \frac{B^2\cancel{T}}{\cancel{T}}$$

$$= A^2 - \frac{ABe^{j12\pi}}{j6\pi} + \frac{AB}{j6\pi} + B^2$$

(2)

$$X(t) = 3\cos(1000\pi t) + 5\operatorname{sen}(2000\pi t) + 10\cos(11000\pi t)$$

$T_s = 5000\,Hz$ $\quad F_1 = \frac{1000\pi}{2\pi} = 500 \quad F_2 = \frac{2000\pi}{2\pi} = 1000 \quad F_3 = \frac{11000\pi}{2\pi} = 5500$ X

convertimos la onda a discreta

$F_s \geq 2F_3 \geq 5000$ X

$$X(nT_s) = 3\cos(1000\pi nT_s) + 5\operatorname{sen}(2000\pi nT_s) + 10\cos(11000\pi nT_s)$$

$$= 3\cos(1000\pi n/11000) + 5\operatorname{sen}(2000\pi n/11000) + 10\cos(11000\pi n/11000) \quad T_s = \frac{1}{11000}$$

$$= 3\cos\left(\frac{\pi n}{11}\right) + 5\operatorname{sen}\left(\frac{2\pi n}{11}\right) + 10\cos(\pi n)$$

buscamos la copia de $\Omega$

$$\left.\begin{array}{l} -\pi \leq \frac{\pi}{11} \leq \pi \\ -\pi \leq \frac{2\pi}{11} \leq \pi \\ -\pi \leq \Omega \leq \pi \end{array}\right\} \text{original}$$

$$X(nT_s) = 3\cos\left(\frac{\pi}{11}n\right) + 5\operatorname{sen}\left(\frac{2\pi}{11}n\right) + 10\cos(\pi n)$$

$$\Omega_{copia} = \frac{\pi}{11} + \frac{2\pi}{11} + \pi n = \frac{14}{11}\pi \Rightarrow \Omega(\text{original}) = \frac{14\pi}{11} - 2\pi = -\frac{8\pi}{11}$$

$$\frac{\frac{-8\pi}{11}}{\frac{2\pi}{1}} = \frac{-8\cancel{\pi}}{22\cancel{\pi}} = \frac{-8}{22} = -\frac{4}{11}$$

$f_{original} = -\frac{4}{11}$

$M_{original} = 11$

c) $x(t) = 20(\cos(t/3) + \cos(t/4))$

Se comprueba la cuasi-periocidad

$W_1 = 1/3 \qquad W_2 = 1/4$

$$W_1/W_2 = \frac{1/3}{1/4} = \frac{4}{3} \in \mathbb{R}$$

miramos la periocidad

$$T_1 = \frac{2\pi}{1/3} = 6\pi \qquad T_2 = \frac{2\pi}{1/4} = 8\pi$$

$$T = 6\pi = 8\pi \qquad \frac{T}{\pi} = \frac{6\cancel{\pi}}{\cancel{\pi}} = \frac{8\cancel{\pi}}{\cancel{\pi}} \qquad \frac{T}{\pi} = 6k = 8k$$

mcm =

| | | |
|---|---|---|
| 8 | 6 | 2 |
| 4 | 3 | 2 |
| 2 | 3 | 2 |
| 1 | 3 | 3 |
| | 1 | |

$\text{mcm} = 2 \times 2 \times 2 \times 3 = \underline{24}$

$$\frac{T}{\pi} = 24 \qquad T = 24\pi$$

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