Parcial # 02 - señales y sistemas

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1) Encuentre la expresión del expectro de Fourier [función exponencial], con $A, F_0 \in \mathbb{R}^+$ para la señal $x(t) = |A\,sen(2\pi F_0 t)|^2$ con $t \in [-\frac{1}{2F_0}, \frac{1}{2F_0}]$.

S/ Se tiene que

$$X(t) = |A\,sen(2\pi F_0 (t))|^2 = A^2 sen^2(2\pi F_0 t)$$

$$\cdot\ Sen^2(\theta) = \frac{1-\cos(2\theta)}{2}, \quad T = t_f - t_i = \frac{1}{2F_0} - \left(-\frac{1}{2F_0}\right) = \frac{1}{F_0}$$

$$\therefore\ X(t) = A^2\left(\frac{1-\cos(4\pi f_0 t)}{2}\right) = \frac{A^2}{2} - \frac{A^2}{2}\cos(4\pi f_0 t)$$

Por serie trigonometrica

$$X(t) = a_0 + \sum_{n=-N}^{N} a_n \cos(n w_0 t) + b_n sen(n w_0 t)$$

donde=

$$a_0 = C_0 = \frac{1}{t_f - t_i}\int_{t_i}^{t_f} x(t)\,dt$$

y

$$a_n = \frac{2}{t_f - t_i}\int_{t_i}^{t_f} x(t)\cos(n w_0 t)\,dt$$

$$b_n = \frac{2}{t_f - t_i}\int_{t_i}^{t_f} x(t)\,sen(n w_0 t)\,dt$$

$$C_n = \frac{a_n - j b_n}{2}$$

$$\therefore \int_{-T/2}^{T/2} \cos(n W_0 t)\,dt = \frac{T}{2}$$

$$a_n = \frac{\langle X(t), \cos(n W_0 t)\rangle}{\|\cos(n W_0 t)\|^2}$$

$$b_n = \frac{\langle X(t), \cos(n W_0 t)\rangle}{T/2}$$

$$X(t) = \frac{A^2}{2} - \frac{A^2}{2}\cos(2 w_0 t) = a_0 + a_2 \cos(2 w_0 t)$$

$$a_0 = \frac{A^2}{2}; \quad = \frac{-A^2}{2} \qquad C_0 = a_0;\ C_2 = -A^2;\ C_1 = A^2$$