

Adjoint Method for Phase Retrieval

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Abstract: We describe an adjoint method for phase retrieval of a wavefront from measurements of intensity along the axial direction, assuming Fresnel propagation. This method allows efficient computation of gradients for iterative phase retrieval.

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1. Introduction

Thin transparent objects, such as some biological specimens, are invisible in an intensity image taken at the focal plane. However, if we move in the axial direction to a defocus plane, effects of the phase difference (present in the wavefront due to the higher refractive index of the object) can be seen, if we use coherent or partially coherent light [1]. Phase and amplitude in one plane uniquely determines intensity in any plane. From intensity measurements at defocus planes, we can attempt to reconstruct the phase and amplitude of the wavefront at the focal plane.

In an iterative method, phase retrieval is posed as an optimization problem. In an optimization problem, we iteratively update the parameters of the problem in order to minimize the Figure of Merit (*FOM*). In this case, the parameters are the magnitude and the phase of a wavefront at the focal plane. We have measurements of the intensity at certain axial planes (this is easily experimentally obtained by moving the camera axially in the optical setup). Initially, we guess the values for the magnitude and phase of the wavefront. We then Fresnel propagate this wavefront guess (W_g) to all the axial planes. The *FOM* is the squared difference of the actual and propagated fields. After calculating the *FOM*, we incrementally update W_g in a manner that will cause the *FOM* to decrease. This procedure is iterated until the *FOM* converges.

In order to update W_g , we need to know how small changes in magnitude and phase at each pixel of W_g will affect the *FOM*. Using the first order derivatives to update the parameter guesses is called the method of steepest descent. Another iterative method, the Gerchberg-Saxton algorithm, is a popular, easy-to-implement method of phase retrieval, yet it does not calculate the actual derivatives in the update step, which can lead to slow convergence [2].

A naive way to calculate the gradients of the parameters with respect to the *FOM* calls for a computational burden on the order of the number of parameters, which is computationally prohibitive. We use an adjoint method [3] to make the computational burden of these derivatives *independent* with regard to the number of parameters (i.e. pixels). A similar method is described in [4]. In this paper, we aim to give physical intuition into the reduction of computational burden with the adjoint method. We also apply the method to simulated and real data, assuming Fresnel propagation to axial planes, showing that weighting of lower order spatial frequencies of the derivative yields better convergence.

2. Adjoint Method Theory

To initialize our optimization algorithm, we start with a guess of the wavefront at the focal plane, W_g . We are trying to solve for W_g , both real and imaginary parts, at the focal plane. The *FOM* describes how well our estimate fits the measured data, and is given as:

$$FOM = \sum_i \sum_p (|E_{\text{propagated}}|^2 - |E_{\text{actual}}|^2)^2, \quad (1)$$

where we sum over p , the image pixels in every image, and i , the number of Fresnel-propagated images. $E_{\text{propagated}}$ is obtained by Fresnel propagating W_g to measurement planes p , and the corresponding intensity is given by $|E_{\text{propagated}}|^2$. The actual intensity is given as $|E_{\text{actual}}|^2$. We aim to minimize the *FOM* through an optimization procedure. To do so, we calculate the derivatives of the real and imaginary parts of W_g for every pixel p with respect to the *FOM*. Differentiating the *FOM* with respect to changes in the real and imaginary parts of W_g :

$$\delta FOM = \sum_i \sum_p \frac{\partial FOM}{\partial E_{W_g \text{ real}}} + \frac{\partial FOM}{\partial E_{W_g \text{ imag}}}. \quad (2)$$

The subscripts *real* and *imag* simply denote the real and imaginary parts of the field. Applying the chain rule to the first term of Eqn. 2 (dropping the summation over p and i , and noting that a similar derivation applies to the second term):

$$\frac{\partial FOM}{\partial E_{W_g \text{ real}}} = \frac{\partial FOM}{\partial E_{\text{propagated real}}} \frac{\partial E_{\text{propagated real}}}{\partial E_{W_g \text{ real}}} + \frac{\partial FOM}{\partial E_{\text{propagated imag}}} \frac{\partial E_{\text{propagated imag}}}{\partial E_{W_g \text{ real}}}. \quad (3)$$

Deriving an expression for the first term on the right hand side of the Eqn. 3 (again noting that a similar derivation applies to the second term):

$$\frac{\partial FOM}{\partial E_{\text{propagated real}}} \frac{\partial E_{\text{propagated real}}}{\partial E_{W_g \text{ real}}} = 4(|E_{\text{propagated}}|^2 - |E_{\text{actual}}|^2) E_{\text{propagated real}} \frac{\partial E_{\text{propagated real}}}{\partial E_{W_g \text{ real}}}. \quad (4)$$

Setting $k = 4(|E_{\text{propagated}}|^2 - |E_{\text{actual}}|^2) E_{\text{propagated real}}$, realizing that $\frac{\partial E_{\text{propagated real}}}{\partial E_{W_g \text{ real}}}$ is the Fresnel propagation of $\partial E_{W_g \text{ real}}$, and including the summation over p :

$$\sum_p \frac{\partial FOM}{\partial E_{\text{propagated real}}} \frac{\partial E_{\text{propagated real}}}{\partial E_{W_g \text{ real}}} = \sum_p k \odot h_{\text{Fresnel}} * \partial E_{W_g \text{ real}}. \quad (5)$$

Where $*$ denotes convolution, \odot is the dot product, and h_{Fresnel} is the impulse response of Fresnel propagation. Note that in order to compute this sum, we must perform one Fresnel propagation per pixel of the wavefront (denote number of pixels per image i as N). However, we can re-order the above equation, yielding:

$$\sum_p k \odot h_{\text{Fresnel}} * \partial E_{W_g \text{ real}} = \sum_p h_{\text{Fresnel}} * k \odot \partial E_{W_g \text{ real}}. \quad (6)$$

This re-ordering is the adjoint method. Note that in order to compute this sum for every pixel on the wavefront, we must still perform one Fresnel propagation each. However, the Fresnel propagation for each pixel is now the *same*, so we only have to do it once. Thus, the computational burden of our gradient calculation is *independent* of the number of pixels in the wavefront. We have gone from $N + 1$ Fresnel propagations to just two Fresnel propagations to compute Eqn. 5.

Why do we get this reduction in computation, seemingly for free? In the first scenario, we calculated the change in *FOM* due to changing a mask pixel for each image pixel independently, and then summed them up. Getting $\frac{\partial FOM}{\partial W_g}$ for each image pixel is unnecessary. In the second scenario, we got the change in figure of merit due to changing an object pixel for all the image pixels already summed up, which is why we only needed one Fresnel propagation calculation instead of N Fresnel propagations. (Note that if we have i images, we need to perform i Fresnel propagations, and then sum the i gradients per pixel.)

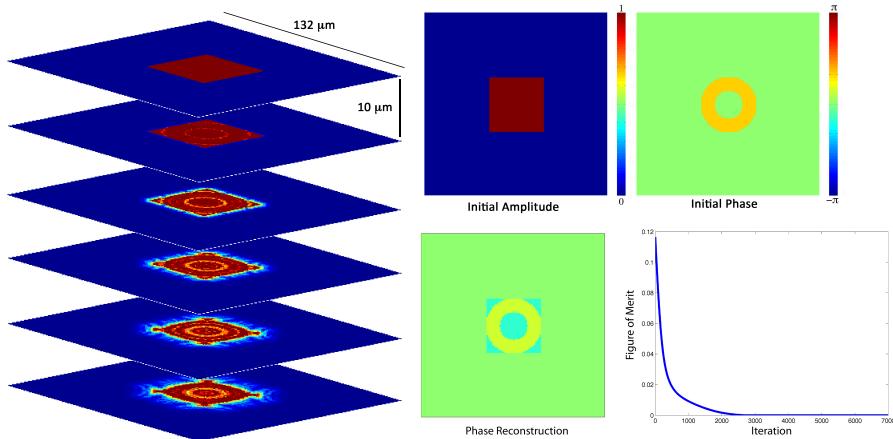


Fig. 1. The wavefront is Fresnel-propagated; the intensity image stack is shown (left), where the first image is the intensity at the focal plane. The magnitude and phase of the wavefront at the focal plane (top right), the reconstructed phase and the *FOM* as a function of iteration (bottom right). The *FOM* is normalized by the number of pixels, N , and the wavelength is 650 nm.

3. Results with Simulated Data

We simulate the phase retrieval of a wavefront with given phase and magnitude (see Fig. 2). To initialize our optimization algorithm, we start with a guess of the wavefront, W_g , that has a phase of zero. We then use the method of steepest descent with the adjoint gradient calculation described in the previous section to iterate to the final phase reconstruction. We obtain a good qualitative reconstruction of the phase.

4. Results with Real Data

The sample imaged consists of unstained cheek cells. The intensity image stack and the resulting phase reconstruction are shown. The unequal image spacing achieves optimal spatial frequency representation of the phase. In order to achieve better contrast between the cells and the background, during the update step of the algorithm, the lower spatial frequencies in the gradient were weighted more heavily. Without this weighting, the lower spatial frequencies disappear, and only the high frequency detail remains.

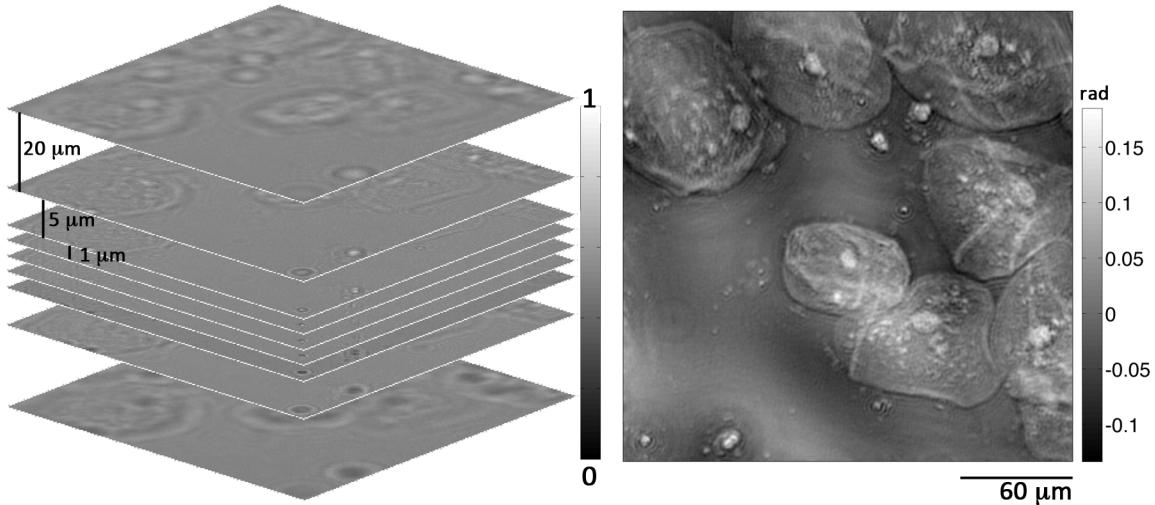


Fig. 2. The intensity image stack (left); the center image is at the focal plane. The reconstructed phase (right).

References

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