Sequential Convex Programming Approach to Powered Descent Guidance of Multiple Vehicles

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The powered descent guidance problem is presented as a second order cone program (SOCP) with a nonconvex constraint on each vehicle's thrust (which is relaxed and cast as convex constraints) and the proximities between each pair of vehicles, also nonconvex constraints. A sequential convex programming (SCP) approach is appplied to the powered descent guidance problem for multiple vehicles. The powered descent guidance problem has been solved as a convex problem for a single vehicle via a relaxation of the thrust constraint. While a solution to the convex problem with relaxed convex thrust constraints is also a solution to the original problem with a nonconvex constraint, additional constraints on the proximities between vehicles cannot be cast as a set of convex constraints, so the problem for multiple vehicles cannot be cast as a convex problem. In order to solve the problem as a SOCP, the proximities are approximated by a second order Taylor series approximation and the solution is found iteratively, starting with a heurisitic and ending at a prescribed stopping point. The problem here includes both relaxed convex constraints and nonconvex constraints, which requires SCP, resulting in an approximation of the globally optimal solution.

I. Introduction

A convex programming framework for the powered descent guidance problem for Mars pinpoint landing is augmented by imposing additional constraints on the proximity between multiple vehicles. The main contribution of[1] is a formulation of the trajectory optimization problem with nonconvex control constraints as a convex optimization problem. The problem is a Second Order Cone Program (SOCP), which finds the minimum fuel optimal trajectory over a set of constraints. The powered descent guidance problem for pinpoint landing is defined as a fuel minimization problem trajectory that takes a lander with a given initial state (position and velocity) to a prescribed final state in a uniform gravity field, with magnitude constraints on the available net thrust, and various state constraints. Historically, robots have carried out exploration missions on the surface of Mars. These missions only require one vehicle to complete the Entry Descent Landing (EDL) phase of the mission before completing the science objectives. In the future, larger scale missions may be necessary for more ambitious science objectives as well as supplying astronauts with supplies on long duration missions or maintaining a permanent colony on Mars. Due to launch constraints on the mass and volume of a payload on a launch vehicle, future missions to Mars may require multiple vehicles to be deployed from orbit simultaneously. Simultaneous coordination of multiple vehicles will require collision avoidance constraints between the vehicles in addition to the aldready proven capability of precision pinpoint landing.

II. Precision Pinpoint Landing of a Single Spacecraft

The solution to the powered descent guidance problem [1] is a trajectory which minimizes the fuel from some initial condition to the surface of the planet. The final state of the vehicle is such that the velocity is zero. The trajectory is constrained to remain above the surface and the thrust is to have a minimum and maximum bound in all three dimensions of space. That is, the thrusters have a physical design lmit beyond which they cannot produce more thrust, and they cannot be turned off during flight. Additional glide slope and thrust pointing constraints are not included in this paper.

The Hamiltonian representing the original single vehicle system is given by [a]

[a]Proof that this satisfies the necessary and sufficient conditions for optimality, and that a solution to the convex problem is

$$H(\Gamma, x, \lambda_0, \lambda) = \lambda_0 \Gamma + \lambda_1^T x_2 + \lambda_2^T \frac{\|T_c(t)\|}{m(t)} + \lambda_2^T g - \alpha \lambda_3 \Gamma \tag{1}$$

The necessary conditions are as follows

$$\dot{\lambda} = \frac{\partial H}{\partial y}(y^*(t), v^*(t), \lambda_0, \lambda) \tag{2}$$

Pontryagin's pointwise maximum principle:

$$H(y^*(t), v^*(t), \lambda_0, \lambda) \geq H(y^*(t), v(t), \lambda_0, \lambda) \tag{3}$$

The optimization problem is defined as follows. [12]

P1

$$\begin{aligned} \min \int_0^{t_f} \|T_c(t)\| dt \text{ subject to} \\ \ddot{r}(t) = g + T_c(t)/m(t),\ \dot{m}(t) = -\alpha \|T_c\|, \\ 0 \leq \rho_1 \leq \rho_2,\ r_1 \geq 0, \\ m(0) = m_{wet},\ r(0) = r_0,\ \dot{r}(0) = \dot{r}_0,\ r(t_f) = \dot{t_f} = 0 \end{aligned}$$

where α, ρ_1, ρ_2, r_0, $\dot{r}_0$ are constant parameters.

P2

$$\begin{aligned} \min \int_0^{t_f} \|\Gamma(t)\| dt \text{ subject to} \\ \dot{m}(t) = -\alpha \Gamma(t), \\ \|T_c(t)\| \leq \Gamma(t), \\ \ddot{r}(t) = g + T_c(t)/m(t), \\ \dot{m}(t) = -\alpha \|T_c\| \\ 0 \leq \rho_1 \leq \Gamma(t) \leq \rho_2, \\ r_1 \geq 0, \\ m(0) = m_{wet},\ r(0) = r_0,\ \dot{r}(0) = \dot{r}_0,\ r(t_f) = \dot{t_f} = 0 \end{aligned}$$

where α, ρ_1, ρ_2, r_0, $\dot{r}_0$ are constant parameters.

III. Proximity Constraints on Multiple Spacecraft

When simulating multiple vehicles, an additional constraint must be imposed on the system to avoid collisions. Here, the distance between any two vehicles must be above a certain minimum distance R defined by

$$\|p_i[k] - p_j[k]\|_2 \geq R, \forall i \neq j \tag{4}$$

where $p[k]$ represent the position vectors of the spacecraft at the k^{th} time step. The position vectors are expressed relative to a reference frame fixed in the surface. There is no maximum distance enforced between

also a solution to the nonconvex problem is provided in [1] and .[2]

the vehicles, only a minimum distance. In order for the problem to be formulated as an SOCP, the constraint must be convex. A Taylor series expansion of 4 results in a quatratic constraint which approximates the nonconvex constraint. This new constraint is convex, but the solution is found in multiple iterations .[3] The Taylor series is approximated about the previous solution, denoted by the superscript q:

$$\|p_i^q[k] - p_j^q[k]\|_2 + \kappa^T \left[(p_i[k]p_j[k]) - \left(p_i^q[k]p_j^q[k]\right)\right] \geq R \tag{5}$$

$$\kappa = \frac{p_i^q[k] - p_j^q[k]}{\|p_i^q[k] - p_j^q[k]\|_2} \tag{6}$$

The constraint is not applied on the first iteration. That is, the original problem for each vehicle is solved without regard for the other vehicles' relative positions. This is because there is no previous solution to provide information for the Taylor series approximation. After a solution to the origianl convex problem for each vehicle is found, the problem is modified by imposing the approximated quadratic constraint 5 and then solved. This process continues until a stopping condition is met. The new problem is not convex as before ,[3] and the optimal solution to the problem with the quadratic constraint is not necessarily the optimal solution to the problem with nonconvex constraint, but it is very close to optimal.

IV. Sequential Convex Programming Approach

It is possible that multiple vehicles can maintain this constraint naturally, e.g. if they have similar initial positions and velocities and their respective targets are all in different directions. In real world missions, this may not be the case, and a solution that ensures collision avoidance is necessary for all missions. Given that a collision-free solution may be possible by default, the approach is the following:

1. Find the optimal trajectory for each vehicle without imposing proximity constraint.
2. Given the trajectories of all other vehicles, impose the proximity constraints on the trajectory of each vehicle and solve for the optimal solution for each vehicle.
3. Repeat step 2 until stopping condition is met.

Since the problem is a fuel minimization problem with fixed endpoints, the final time is kept free. As in the original problem for a single vehicle [1] ,[2] a line search is performed to find the optimal time of flight for each vehicle.

The SCP method linearizes the nonconvex constraint about a previous solution, solving iteratively until a stop condition is satisfied. The constraint is linearized using a first-order Taylor series expansion and each solution is found until all solutions meet one of the following conditions:[3]

- The solution is an optimal solution of the approximate convex problem
- The solution fulfills the non-convex avoidance constraint
- The total fuel consumption is less than some prescribed value $\sum_{k=1}^{4K} \eta_{4k} \leq \epsilon$.

V. Algorithm Design

The simulations are carried out using SeDuMi [4] [5] with a MATLAB interface.[6] The problem is set up as an SOCP and solved in MATLAB CVX with the same discretized objective function and constraints as Problem 4 from [1] (Fig. 4) with the additional linearized proximity constraint from [3]. The problem is set up twice: (1) as a convex optimization program, with the original norm from Equation 1, restricting the position vectors to , and as an SCP with Equations (7, 8) as the constraints to implement at each iteration.

Figure 5 Problem 4 from [1] with discretized parameters for a single vehicle

For the SCP framework, the problem is set up using CVX SCP Interface [6]. Again, the same constraints for a single vehicle are applied to all vehicles with the difference that the linearized approximation from Equations (7, 8) is added as constraints for each vehicle.

In discrete time, the problem for a single vehicle takes the following form:

P4

$$\min \omega^T \eta \text{ subject to}$$
$$\|E_u \Upsilon_k \eta\| \leq e_\sigma^T \Upsilon_k \eta, k = 0 \ldots K$$
$$\mu_1(t_k)\left[1 - (F[\xi_k + \Psi_k \eta] - z_0(t_k)) + \frac{(F[\xi_k + \Psi_k \eta] - z_0(t_k))^2}{2}\right] \leq$$
$$e_\sigma^T \Upsilon_k \eta \leq \mu_2(t_k)[1 - (F[\xi_k + \Psi_k \eta] - z_0(t_k))], k = 0 \ldots K$$
$$\ln(m_{wet} - \alpha\rho_2 t_k) \leq F[\xi_k + \Psi_k \eta] \leq \ln(m_{wet} - \alpha\rho_1 t_k), k = 0 \ldots K$$

where α, ρ_1, ρ_2, r_0, $\dot{r}_0$ are constant parameters.

VI. Optimizing for Collision Avoidance

Incorporating multiple proximity constraints requires solving P4 for N vehicles. In addition, the proximity constraints must be approximated as in 5. Since P4 is defined for a single vehicle, the state and control vectors must be modified to account for N vehicles. The problem becomes very large very quickly.

P5

$$\min \omega^T \eta \text{ subject to}$$
$$\|E_u \Upsilon_k \eta\| \leq e_\sigma^T \Upsilon_k \eta, k = 0 \ldots K$$
$$\mu_1(t_k)\left[1 - (F[\xi_k + \Psi_k \eta] - z_0(t_k)) + \frac{(F[\xi_k + \Psi_k \eta] - z_0(t_k))^2}{2}\right] \leq$$
$$e_\sigma^T \Upsilon_k \eta \leq \mu_2(t_k)[1 - (F[\xi_k + \Psi_k \eta] - z_0(t_k))], k = 0 \ldots K$$
$$\ln(m_{wet} - \alpha\rho_2 t_k) \leq F[\xi_k + \Psi_k \eta] \leq \ln(m_{wet} - \alpha\rho_1 t_k), k = 0 \ldots K$$

where α, ρ_1, ρ_2, r_0, $\dot{r}_0$ are constant parameters.

VII. Results

A. to do

The current strategy is as follows:

1. Solve the original problem for a single vehicle. (!!)
2. Solve the problem once for each of two vehicles.
3. For each vehicle, define the proximity constraint by Taylor series approximation, turning the constraint into a quadratic constraint, which is convex. The Taylor series for each time step of vehicle 1 is taken about the position of vehicle 2 at the same time step.
4. If one vehicle takes less time to complete the trajectory, lift the constraint for $K_2 \geq K_{1f}$ or $K_1 ge K_{2f}$, where K_{if} is the final time of the vehicle with the shortest time of flight.
5. Repeat the process until both solutions meet one of the following conditions:
   - The solution is an optimal solution of the approximate convex problem
   - The solution fulfills the non-convex avoidance constraint
   - The total fuel consumption is less than some prescribed value ϵ

VIII. Conclusion

The framework for a globally optimal solution to the powered descent guidance problem, which was cast as an SOCP, and solved in real time, has been adapted to find a solution for multiple vehicles. Finding an optimal solution for multiple vehicles in real time is a greater challenge which could result in greater scientific capability. The problem is recast in an SCP framework, which should yield a close approximation to the globally optimal solution in real time. The results can be compared to the convex approach in order to evaluate the accuracy of the algorithm.

References

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