Maneuvering Spacecraft with Unknown Mass Properties

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The goal of this paper is to investigate measurement techniques for finding the mass properties of two spacecraft - one of knwon mass properties and one of unknown mass properties (e.g. a small asteroid) - in orbit. External forces and moments are neglected and the spacecraft are assumed to be rigid bodies. The problem is at first formulated as a one dimensional problem and further expanded to three dimensions. The principal moments of inertia are found and a reference fram coincident with the principal moments of inertia is used to determine how thrusters and control moment gyroscopes should be used with the new docked configuration. Only the spacecraft with known mass properties has any control authority (although a control system is not designed). Mass properties are assumed to be constant for both bodies.

I. Introduction

In this paper, measuring the mass properties of an arbitrary body after docking with a spacecraft are investigated. Mass properties of spacecraft are easily measured on Earth [1], but here in flight measurements are investigated. Previous work has used estimation techniques such as Kalman filtering [2, 3] to estimate the principal moments of inertia (PMOI) of a single satellite. Other work has used other control algorithms to measure PMOIs of a single satellite to account for fuel consumption [4]. Previous work has investigated the possibility of using measured moments of inertia and reaction wheel actuators to measure a spacecraft's mass properties after fuel consumption or deploying some appendage. This paper uses similar techniques to find the mass properties of a spacecraft docked with an arbitrary body such as an asteroid or other body with mass properties different from its original configuration.

II. One Dimensional Formulation

Suppose there is a spacecraft in orbit around Earth with mass m_c and inertia tensor $[I^C]$ with point C as the center of mass of the spacecraft. Fig. 1 shows the spacecraft in two dimensions (note the spacecraft is not translating in inertial space, i.e. this is not an orbit mechanics problem).

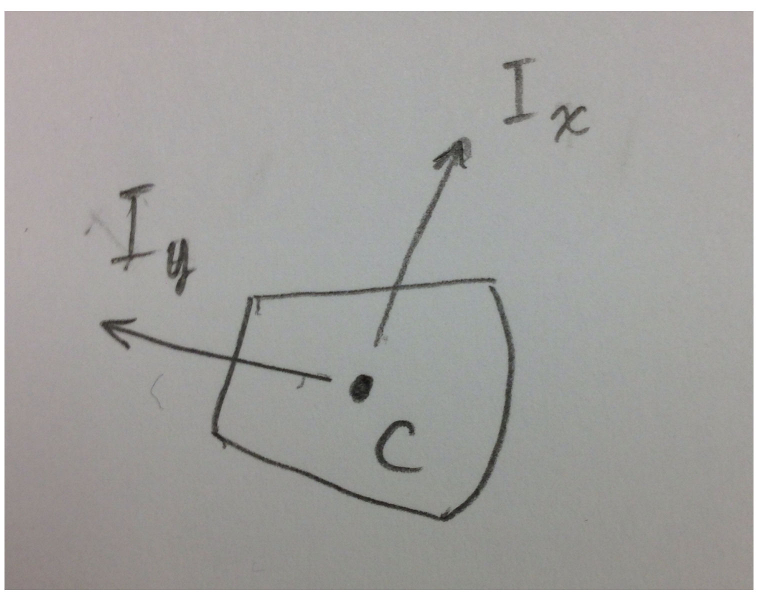

Figure 1. Spacecraft with known mass properties $[I^C]$

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The spacecraft C also has an angular velocity ω_z. The angular velocity can change at a rate of $\dot{\omega}_z$ if a moment acts on the spacecraft. The angular momentum of the spacecraft is given by

$$\{H_z^C\} = [I_z^C]\{\omega_z\} \tag{1}$$

For the one dimensional case, this is a scalar equation $H_z^C = I_z^C \omega_z$. The time derivative of the angular momentum for constan moment of inertia is

$$\dot{H}_z^C = I_z^C \dot{\omega}_z \tag{2}$$

Setting $\dot{H}^C$ equal to the moment provided by thrusters or control moment gyros (CMG) yields Euler's Equation of Motion:

$$M_z = I_z^C \dot{\omega} \tag{3}$$

Now suppose another body of arbitrary mass m_s and shape $[I_z^S]$ joins the spacecraft (i.e. rendezvous) as shown in Fig. 2.

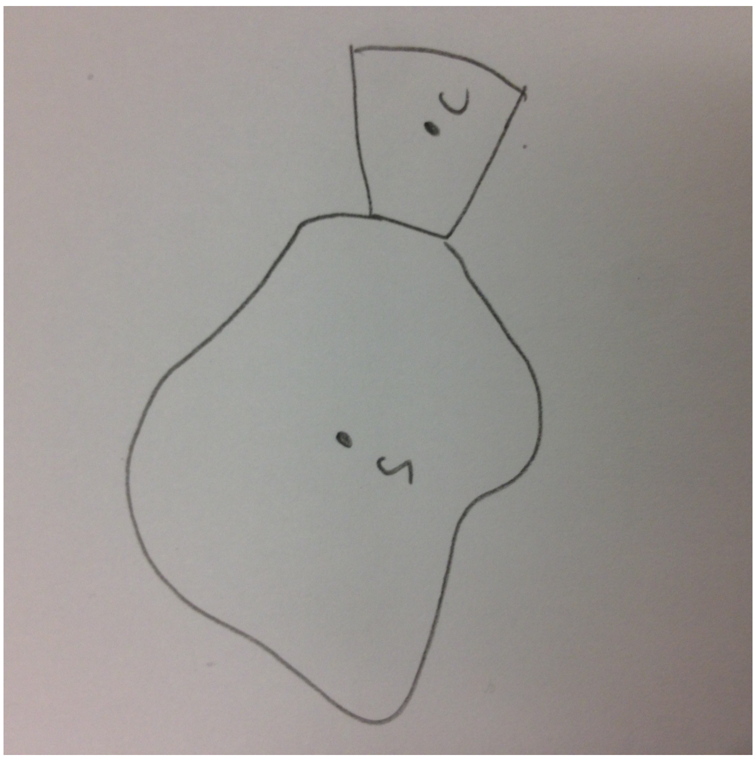

Figure 2. Spacecraft with known mass properties $[I^C]$ joined with arbitrary body of unknown mass properties $[I^S]$

The spacecraft-arbitrary body system angular momentum is now given by

$$H_z^Q = (I_z^S + m_s \left|\overline{R}^{SQ}\right|^2 + I_z^C + m_c \left|\overline{R}^{CQ}\right|^2)\omega_z \tag{4}$$

where S is the center of mass of the arbitray body and Q is the new system center of mass. The new moment of inertia about Q is found from the Parallel Axis Theorem. Taking the derivative, we obtain a new form for Euler's Equation of Motion

$$M = (I_z^S + m_s \left|\overline{R}^{SQ}\right|^2 + I_z^C + m_c \left|\overline{R}^{CQ}\right|^2)\dot{\omega}_z \tag{5}$$

It is clear that this is one equation with four unknowns, I_z^S, m_s, $\left|\overline{R}^{SQ}\right|$, and $\left|\overline{R}^{CQ}\right|$.

The easiest variable to solve is m_s. A small force in any direction imparted to the spacecraft will produce an acceleration in that direction proportional to the total mass of the system $m_c + m_s$. In this way, the mass of the system is known and thus, the mass of the arbitrary body is known.

$$m_s = \frac{F}{A} - m_c \tag{6}$$

where F is the magnitude of the force acting on the spacecraft and A is the magnitude of the acceleration.

The rotational rate ω and the centripetal acceleration $\left|\ddot{R}^{CQ}\right|$ are known from the spacecraft accelerometers and the centripetal acceleration due to the system spinning is derived in the Appendix using the Basic Kinematic Equation for clarity. The distance from the spacecraft center of mass to the spacecraft-arbitrary body center of mass is found to be

$$\left|\overline{R}^{CQ}\right| = \frac{\left|\ddot{\overline{R}}^{CQ}\right|}{\omega_z^2} \tag{7}$$

The distance between the centers of gravity of the abirtrary body and the spacecraft-arbitrary body system is given by $\left|\overline{R}^{SQ}\right|$, and can be evaluated from Equation _.

$$\left|\overline{R}^{CQ}\right| = \frac{m_c\left|\overline{R}^{CC}\right| + m_s\left|\overline{R}^{SQ}\right|}{m_c + m_s} \tag{8}$$

which reduces to

$$\left|\overline{R}^{SQ}\right| = \frac{(m_c + m_s)}{m_s}\left|\overline{R}^{CQ}\right| = \frac{(m_c + m_s)}{m_s}\frac{\left|\ddot{\overline{R}}^{CQ}\right|}{\omega_z^2} \tag{9}$$

Now we are left with one unknown and have complete knowledge of the system mass properties. Equations _ and _ are combined and rearranged into an expression for I_z^S.

$$I_z^S = \frac{M_z}{\dot{\omega}_z} - I_z^C - m_c\frac{\left|\ddot{\overline{R}}^{CQ}\right|}{\omega_z} - m_s\left|\overline{R}^{SQ}\right|^2 \tag{10}$$

and

$$I_z^Q = \frac{M_z}{\dot{\omega}_z} - m_c\frac{\left|\ddot{\overline{R}}^{CQ}\right|}{\omega_z} - m_s\left|\overline{R}^{SQ}\right|^2 \tag{11}$$

Note that for a body that is not accelerating ($\dot{\omega}_z = 0$) or not rotating in inertial space ($\omega_z = 0$), I_z^S is undefined, meaning that I_z^S cannot be measured without the system rotating.

III. Three Dimensional Formulation

In the previous section, expressions for the mass properties for a two dimensional arbitrary shape rotating about one dimension along a principal axis were derived. In this section, a more general solution for the inertia tensor are derived.

If the z -axis, or axis of rotation is assumed to be parallel to ω and $\dot{\omega}$, Equation 11 and the preceding work can still give useful information about the mass properties of the system.

The parallel axis theorem for $[I'^S]$ (the inertia matrix of the arbitrary body with respect to the center of gravity of the system) in three dimensions is given by

$$[I'^S] = \begin{bmatrix} I_{xx}^S + m_s(R_y^{SQ2} + R_z^{SQ2}) & I_{xy}^S - m_s R_x^{SQ} R_y^{SQ} & I_{xz}^S - m_s R_x^{SQ} R_z^{SQ} \\ I_{yx}^S - m_s R_y^{SQ} R_x^{SQ} & I_{yy}^S + m_s(R_x^{SQ2} + R_z^{SQ2}) & I_{yz}^S - m_s R_y^{SQ} R_z^{SQ} \\ I_{zx}^S - m_s R_z^{SQ} R_x^{SQ} & I_{zy}^S - m_s R_z^{SQ} R_y^{SQ} & I_{zz}^S + m_s(R_x^{SQ2} + R_y^{SQ2}) \end{bmatrix} \tag{12}$$

Note that for an interia matrix $[I^Q] = [I'^S] + [I^C]$ relative to the body of the spacecraft (C), $[I'^S]$ remains fixed for an infinite number of center of mass location $\overline{R}^{SQ}$ and mass distribution ($[I^S]$) combinations. That is, a point mass at the end of a long rod or if there is a large mass at the edge of the spacecraft C can give the same values for $[I'^S]$. Therefore, $\overline{R}^{SQ}$ and $[I^S]$ are not necessary to determine $[I^Q]$.

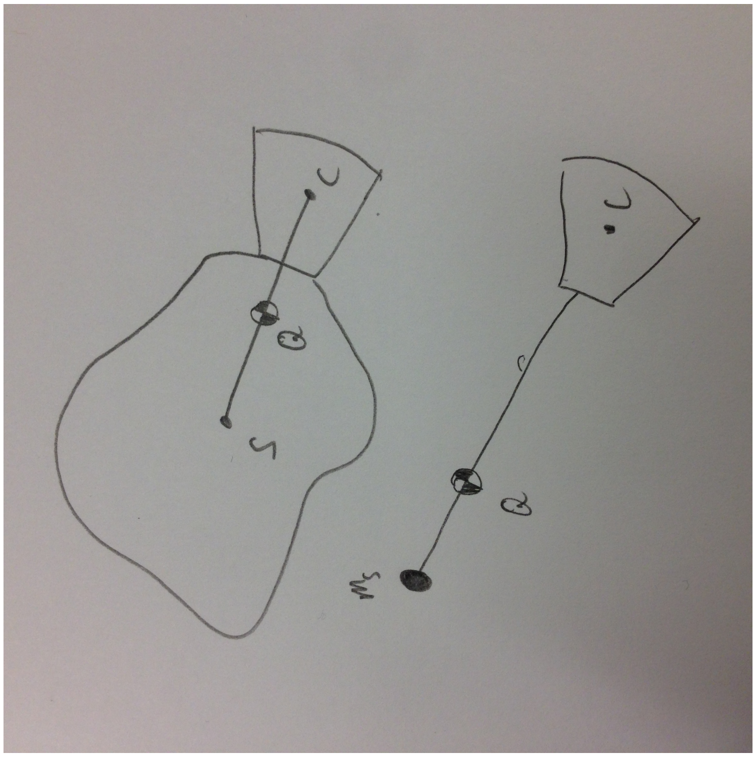

Figure 3. Spacecraft-arbitrary mass system with unknown mass properties $[I^Q]$ showing different cinfigurations

The angular momentum can be written as

$$\overline{H}^C = [I^Q]\overline{\omega} \tag{13}$$

where $[I^Q] = [I^C] + [I'^S]$, which gives Euler's Equations of Motion

$$\overline{M} = [I^Q]\dot{\overline{\omega}} + \tilde{\omega}[I^Q]\overline{\omega} \tag{14}$$

where $\tilde{\omega}$ is the omega cross operator or affinor of rotation defined by

$$\tilde{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{15}$$

Note that there are three equations of motion with three unknowns, the x, y, and z components of $\overline{R}^{SQ}$, however, $[I^Q]$ cannot be solved for directly in this form. Rearranging gives Equation 16 below.

$$\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} \dot{\omega}_x & -\omega_y\omega_z & \omega_z\omega_y \\ \omega_x\omega_z & \dot{\omega}_y & -\omega_x\omega_z \\ -\omega_x\omega_y & \omega_x\omega_y & \dot{\omega}_z \end{bmatrix} \begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix} \tag{16}$$

where $\begin{bmatrix} I_x & I_y & I_z \end{bmatrix}^T$ and are the principal moments of inertia of the spacecraft-arbitrary body system with respect to the Q frame. This yields the solution $[I^Q] * \begin{bmatrix}1 & 1 & 1\end{bmatrix}^T = W^{-1}M$ where

$$W = \begin{bmatrix} \dot{\omega}_x & -\omega_y\omega_z & \omega_z\omega_y \\ \omega_x\omega_z & \dot{\omega}_y & -\omega_x\omega_z \\ -\omega_x\omega_y & \omega_x\omega_y & \dot{\omega}_z \end{bmatrix}$$

This solution only applies when the principal moments about the Q frame are aligned with the principal moments about the C frame.

IV. Existence of More General Solutions

A more general expression of Euler's Equations of Motion is given by

$$M = [T]^{-1}[I^Q][T]\dot{\omega} + [T]^{-1}[\tilde{W}][I^Q][T]\omega \tag{17}$$

where M and ω are given in C coordinates, $[I^Q]$ is in the Q frame, $[T]$ is the transformation matrix from the C frame to the Q frame and $\tilde{W}$ is the cross operator transformed into the Q frame defined by

$$\tilde{W} = \begin{bmatrix} 0 & -(T_{31}\omega_x + T_{32}\omega_y + T_{33}\omega_z) & T_{21}\omega_x + T_{22}\omega_y + T_{23}\omega_z \\ T_{31}\omega_x + T_{32}\omega_y + T_{33}\omega_z & 0 & -(T_{11}\omega_x + T_{12}\omega_y + T_{13}\omega_z) \\ -(T_{21}\omega_x + T_{22}\omega_y + T_{23}\omega_z) & T_{11}\omega_x + T_{12}\omega_y + T_{13}\omega_z & 0 \end{bmatrix} \tag{18}$$

Note that $[T]^{-1} = [T]^T$.

This results in three equations with twelve unknowns. There exist six orthogonality contraints on $[T]$ (each row and column must have a magnitude of 1) and the principal moments of inertia in the Q frame must obey $I_i + I_j \geq I_k$, so there are nine equations of constraint in total. Minimizing $\Phi = \frac{1}{2}\cos^{-1}(tr([T]) - 1)$ subject to these constrains would give the full inertia matrix of the total spacecraft-arbirtraty body system Q, but the problem becomes an optimization problem and not so much a dynamics problem.

Fore every n distinct maneuvers, there are 3 equations of motion, but there are $12 + 9n$ unknowns due to each additional rotation matrix, so attempting to measure the mass properties with multiple maneuvers actually complicates the problem further and an optimization framework cannot be avoided.

V. Conclusion

1. The mass properties of an arbitrary body docked with a spacecraft of known mass properties has been found for the spacial case where the principal moments of inertia of the total spacecraft-arbitraty body system are aligned (and for the special one-dimensional case).
2. The problem has been cast for inertia matrices that are misaligned, but it is a complicated optimization problem and not a dynamics problem.
3. When the principal moments of inertia of the spacecraft and arbitrary body are aligned with each other, a solution for the principal moments of inertia of the compete system can be found analytically.

References

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