Blue Team
Modeling & Simulation Final Presentation

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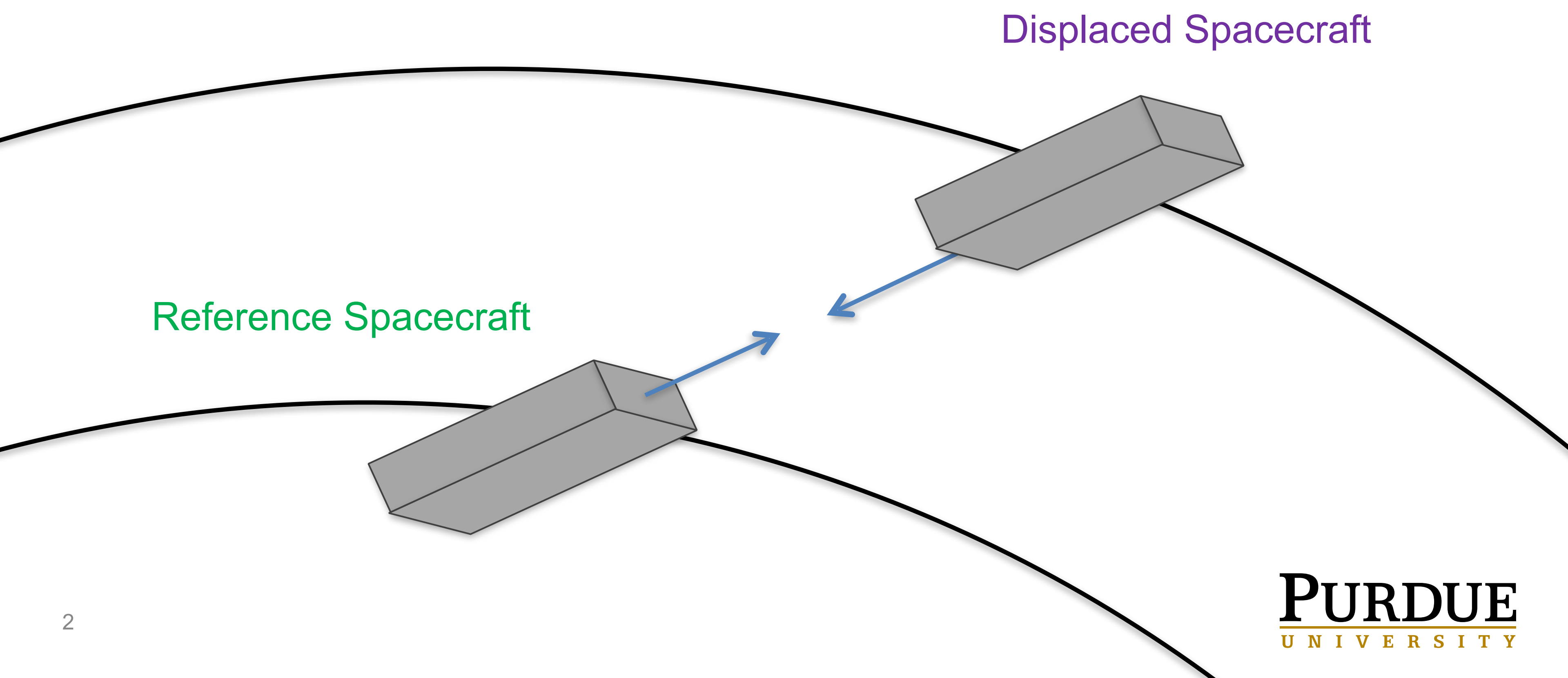
Project Motivation

- To simulate rendezvous of two spacecraft in LEO
- Spacecraft named Reference Spacecraft and Displaced Spacecraft



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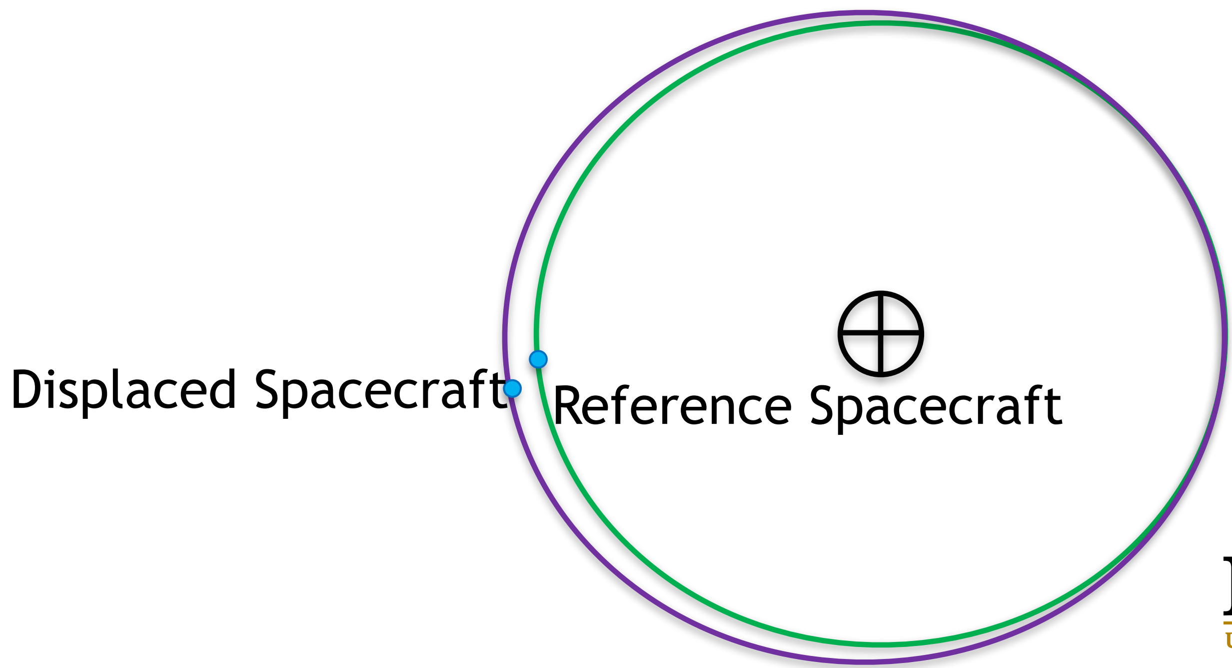
Orbit Model Assumptions

- Earth treated as a point mass
- Two circular, coplanar orbits about Earth
  - Reference Spacecraft -> Inverse Square Law
  - Displaced Spacecraft -> Hill-Clohessy-Wiltshire (HCW) equations for relative motion (assumed to be valid)
- Other natural forces not incorporated



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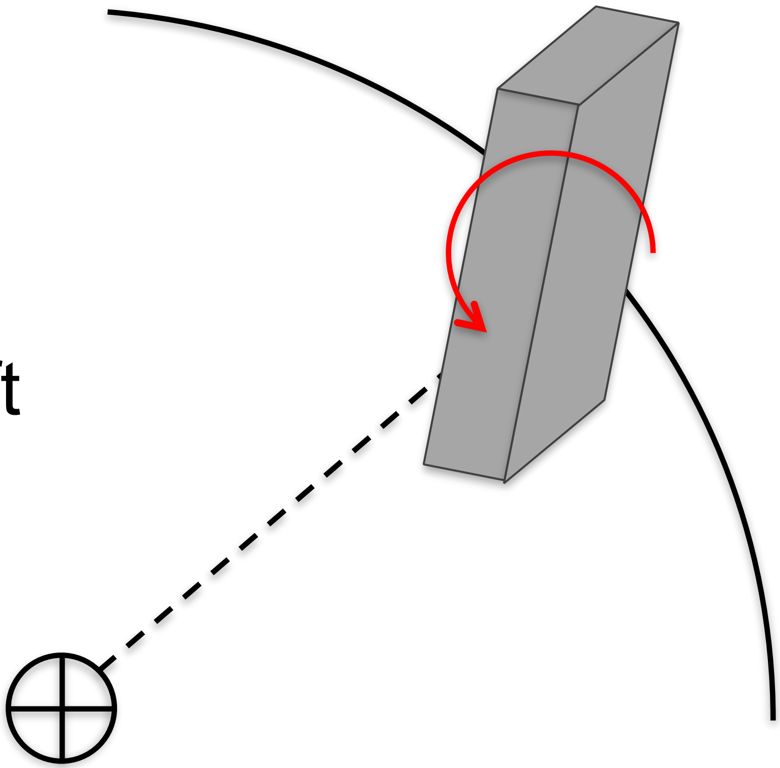
Attitude Model Assumptions

- Gravity torque affects orientations, no effect on orbits
- Other natural forces not incorporated
- Thrusters control spacecraft attitude
- Mass remains constant
- No gyroscopes



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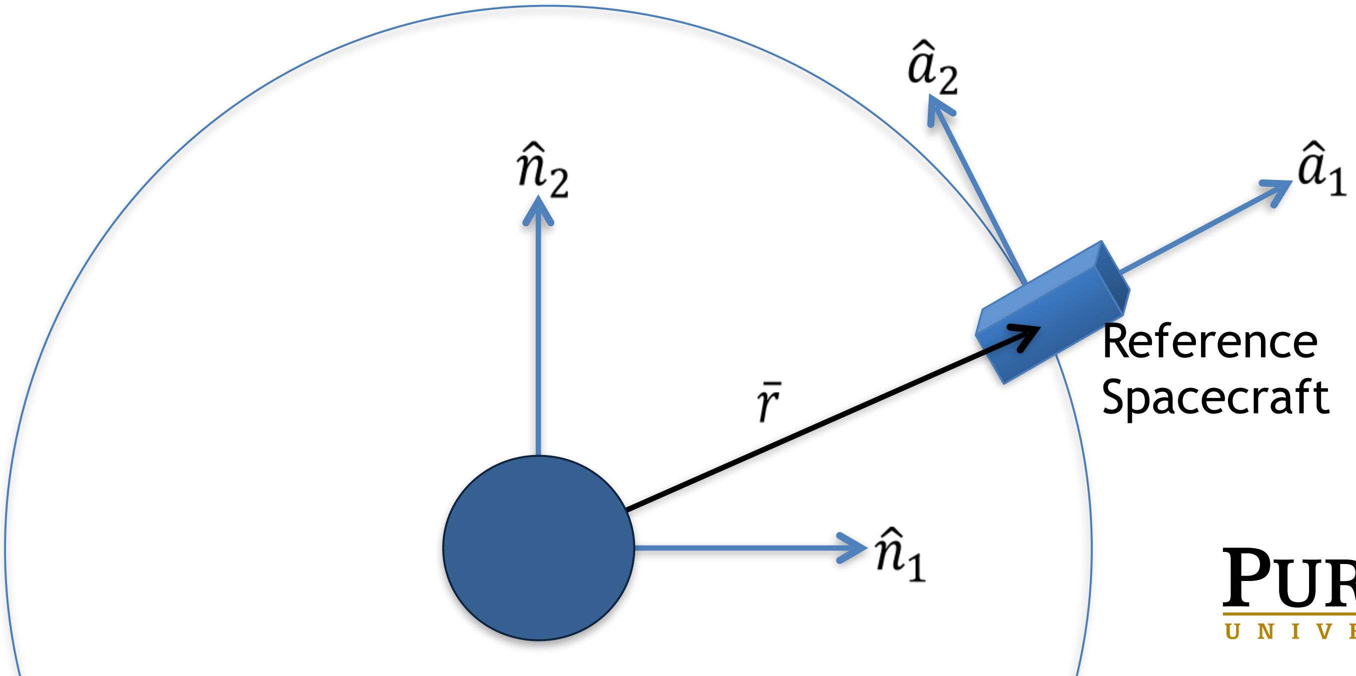
Inverse Square Law

Newton's Inverse Square Law governs motion of the Reference Spacecraft:

$$\ddot{\bar{r}} = -\frac{\mu \bar{r}}{|\bar{r}|^3}$$

Orbiting frame $\hat{a}_i$ rotates around inertial frame $\hat{n}_i$ with ${}^N\overline{\omega}^A = \Omega \hat{a}_3 = \Omega \hat{n}_3$



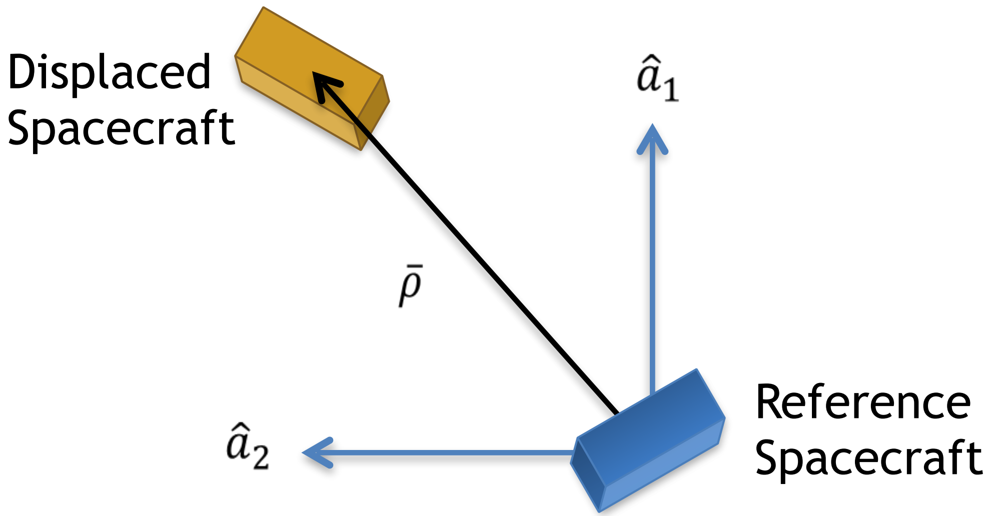
Hill-Clohessy-Wiltshire Model

Hill-Clohessy-Wiltshire Equations for relative motion govern the Displaced Spacecraft:

$$\ddot{x} = 2n\dot{y} + 3n^2x + f_x$$

$$\ddot{y} = -2n\dot{x} + f_y$$
$$\ddot{z} = -n^2z + f_z$$

where $\bar{\rho} = x\hat{a}_1 + y\hat{a}_2 + z\hat{a}_3$ and n is mean motion



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HCW Control Law

HCW in State Space Form

$$\dot{u} = Au + Bv$$

$$\dot{u} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} \quad A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 2n \\ 0 & 0 & -2n & 0 \end{bmatrix} \quad u = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad v = \begin{bmatrix} f_x \\ f_y \end{bmatrix}$$

Position error integral incorporated to change final location

Acceleration inputs become functions of state variables

$$v = -Ku$$

$$\dot{u} = A_i u + B_i v - W u_d \longrightarrow \dot{u} = (A_i - B_i K)u - W u_d$$

Use MATLAB to generate gain matrix K to stabilize the system

Nonlinear Attitude Model

Models gravitational torques and moment inputs m_i

Dependent Variables:

- Euler parameters from orbital frame A to body frame B
- Angular velocities from inertial frame N to body frame B

$$\dot{\varepsilon}_1 = \frac{1}{2}(\varepsilon_4\omega_1 + \varepsilon_2(\omega_3 + \Omega) - \varepsilon_3\omega_2)$$

$$\dot{\varepsilon}_2 = \frac{1}{2}(\varepsilon_3\omega_1 - \varepsilon_1(\omega_3 + \Omega) + \varepsilon_4\omega_2)$$

$$\dot{\varepsilon}_3 = \frac{1}{2}(-\varepsilon_2\omega_1 + \varepsilon_4(\omega_3 - \Omega) + \varepsilon_1\omega_2)$$

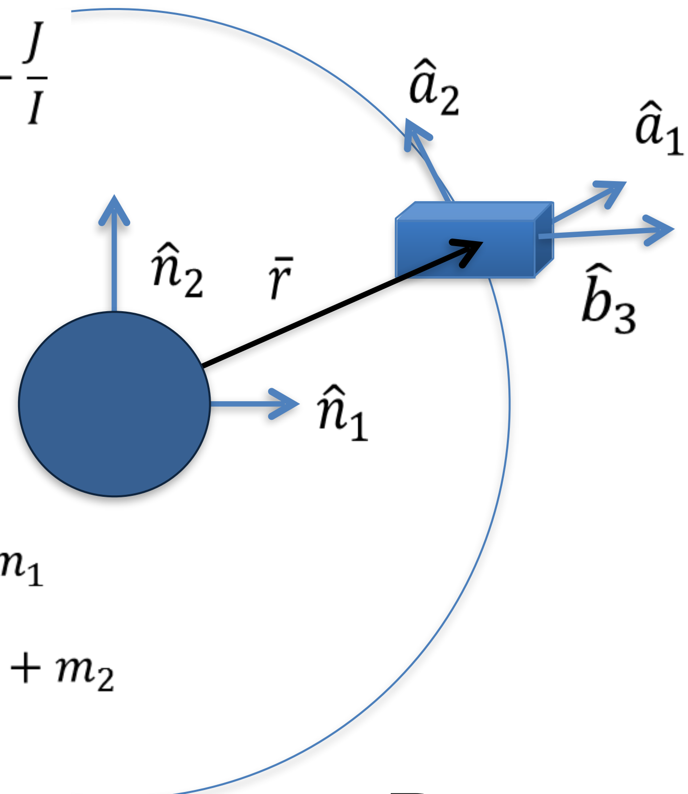
$$\dot{\varepsilon}_4 = -\frac{1}{2}(\varepsilon_1\omega_1 + \varepsilon_3(\omega_3 - \Omega) + \varepsilon_2\omega_2)$$

$$\dot{\omega}_1 = 12\Omega^2 X(\varepsilon_1\varepsilon_2 - \varepsilon_3\varepsilon_4)(\varepsilon_1\varepsilon_3 + \varepsilon_2\varepsilon_4) - X\omega_2\omega_3 + m_1$$

$$\dot{\omega}_2 = -6\Omega^2 X(1 - 2\varepsilon_2^2 - 2\varepsilon_3^2)(\varepsilon_1\varepsilon_3 + \varepsilon_2\varepsilon_4) + X\omega_3\omega_1 + m_2$$

$$\dot{\omega}_3 = m_3$$

$$X = 1 - \frac{J}{I}$$



Linearized Attitude Model

Linearized so axis of symmetry $\hat{b}_3$ is parallel to the radial direction $\hat{a}_1$ $\quad Y = 3\sqrt{2}\Omega^2 X \qquad X = 1 - \frac{J}{I}$

$$\dot{\tilde{\varepsilon}}_1 = \frac{\Omega}{2}\tilde{\varepsilon}_2 - \frac{\Omega}{2}\tilde{\varepsilon}_4 + \frac{\sqrt{2}}{4}\tilde{\omega}_1 + \frac{\sqrt{2}}{4}\tilde{\omega}_3$$

$$\dot{\tilde{\varepsilon}}_2 = -\frac{\Omega}{2}\tilde{\varepsilon}_1 - \frac{\Omega}{2}\tilde{\varepsilon}_3 + \frac{\sqrt{2}}{4}\tilde{\omega}_2$$

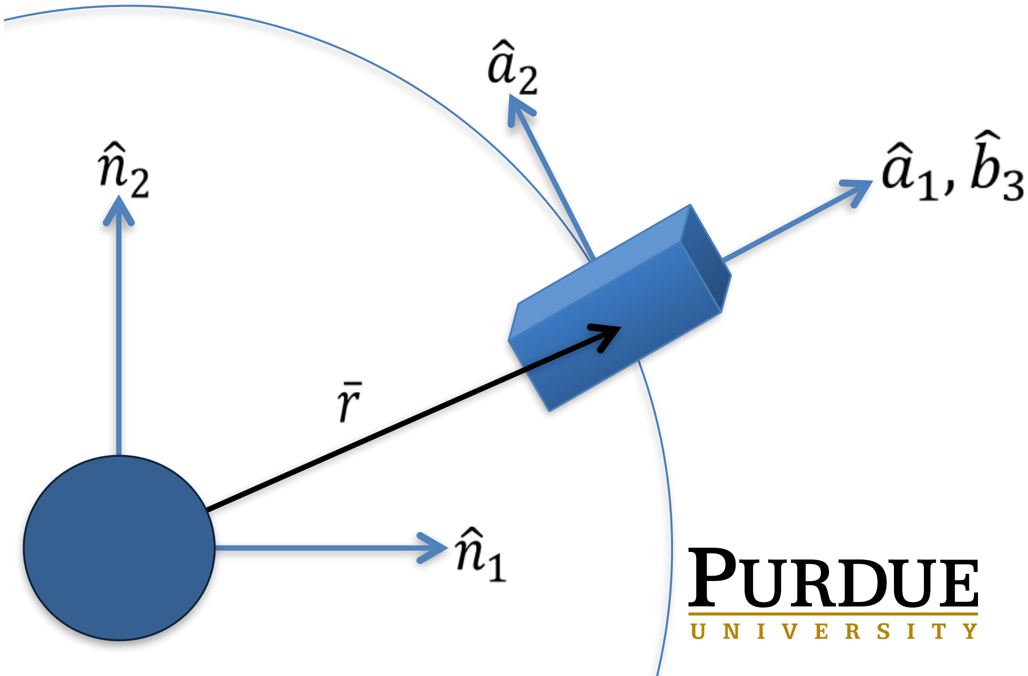
$$\dot{\tilde{\varepsilon}}_3 = \frac{\Omega}{2}\tilde{\varepsilon}_2 - \frac{\Omega}{2}\tilde{\varepsilon}_4 - \frac{\sqrt{2}}{4}\tilde{\omega}_1 + \frac{\sqrt{2}}{4}\tilde{\omega}_3$$

$$\dot{\tilde{\varepsilon}}_4 = \frac{\Omega}{2}\tilde{\varepsilon}_1 + \frac{\Omega}{2}\tilde{\varepsilon}_3 - \frac{\sqrt{2}}{4}\tilde{\omega}_2$$

$$\dot{\tilde{\omega}}_1 = Y\tilde{\varepsilon}_1 - Y\tilde{\varepsilon}_3 + \tilde{m}_1$$

$$\dot{\tilde{\omega}}_2 = -Y\tilde{\varepsilon}_2 - \Omega X\tilde{\omega}_3 + \tilde{m}_2$$

$$\dot{\tilde{\omega}}_3 = \tilde{m}_3$$



Attitude Control Law

Similar to HCW control, incorporate integral terms for the Euler parameters and stabilize the linear system with state feedback control:

$$\dot{u} = A_i u + B_i v - W u_d = (A_i - B_i G)u - W u_d$$

For each time step, solve for linear moment inputs and apply them to the nonlinear system as torques:

$$v = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = -Gu$$

**Full nonlinear equations in Appendix*

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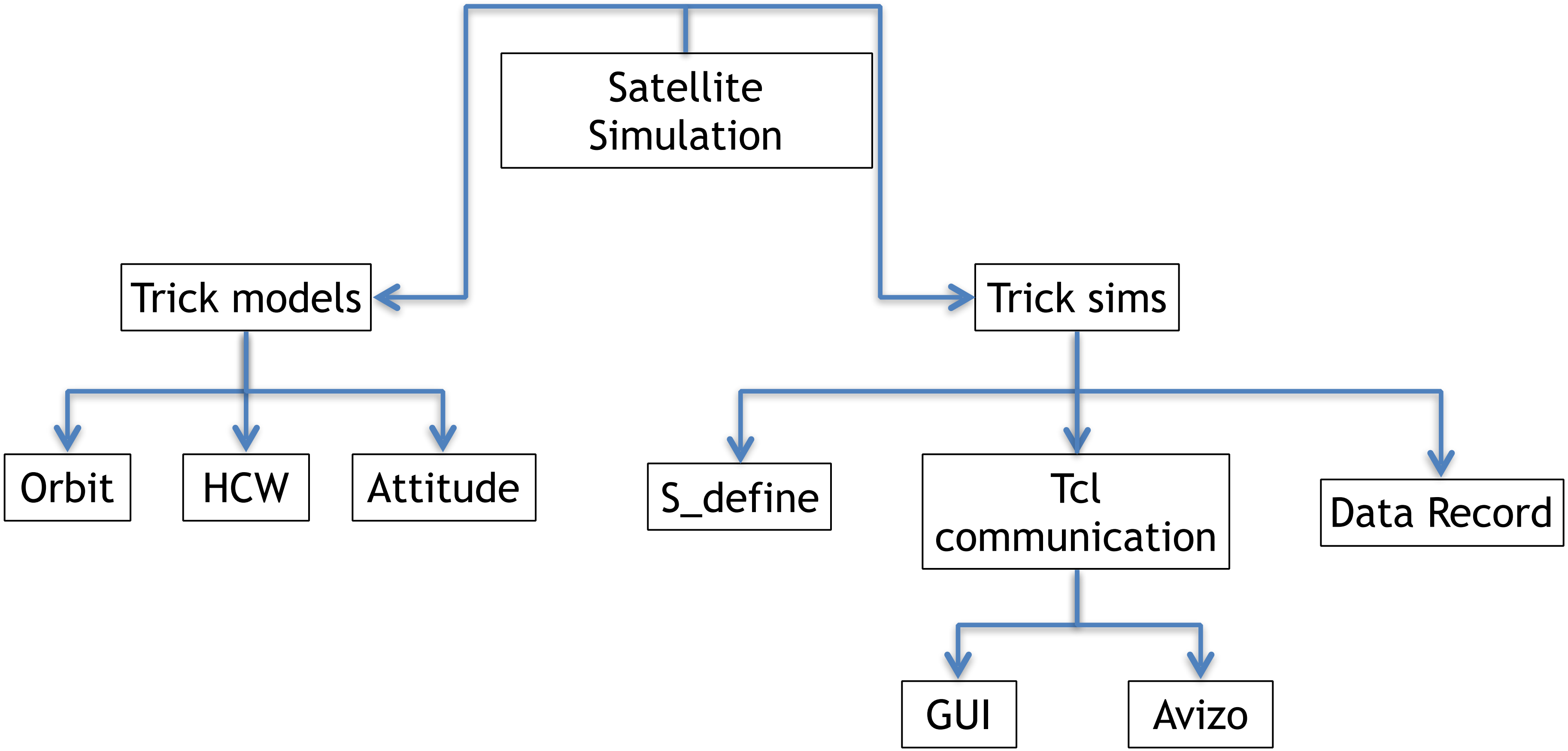
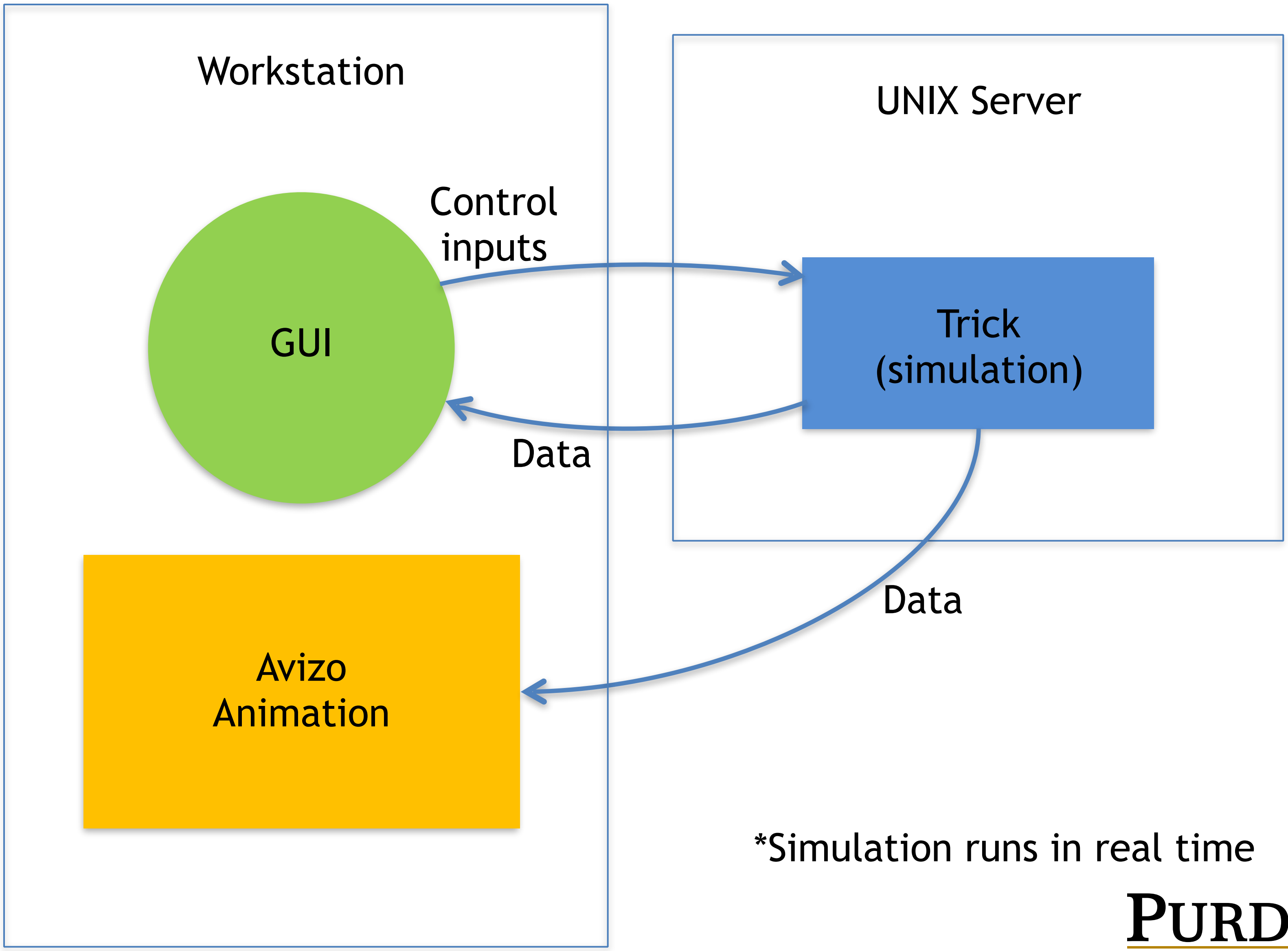
Trick Simulation Hierarchy



- Simulation uses two integrators with different time steps.

Software Communication Overview



*Simulation runs in real time

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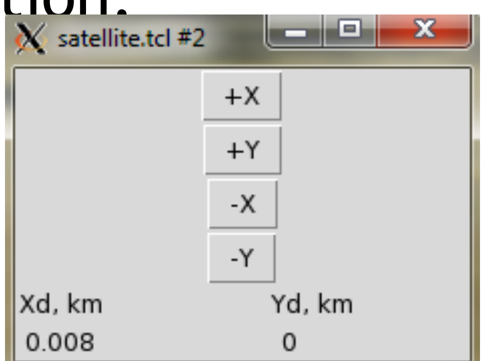
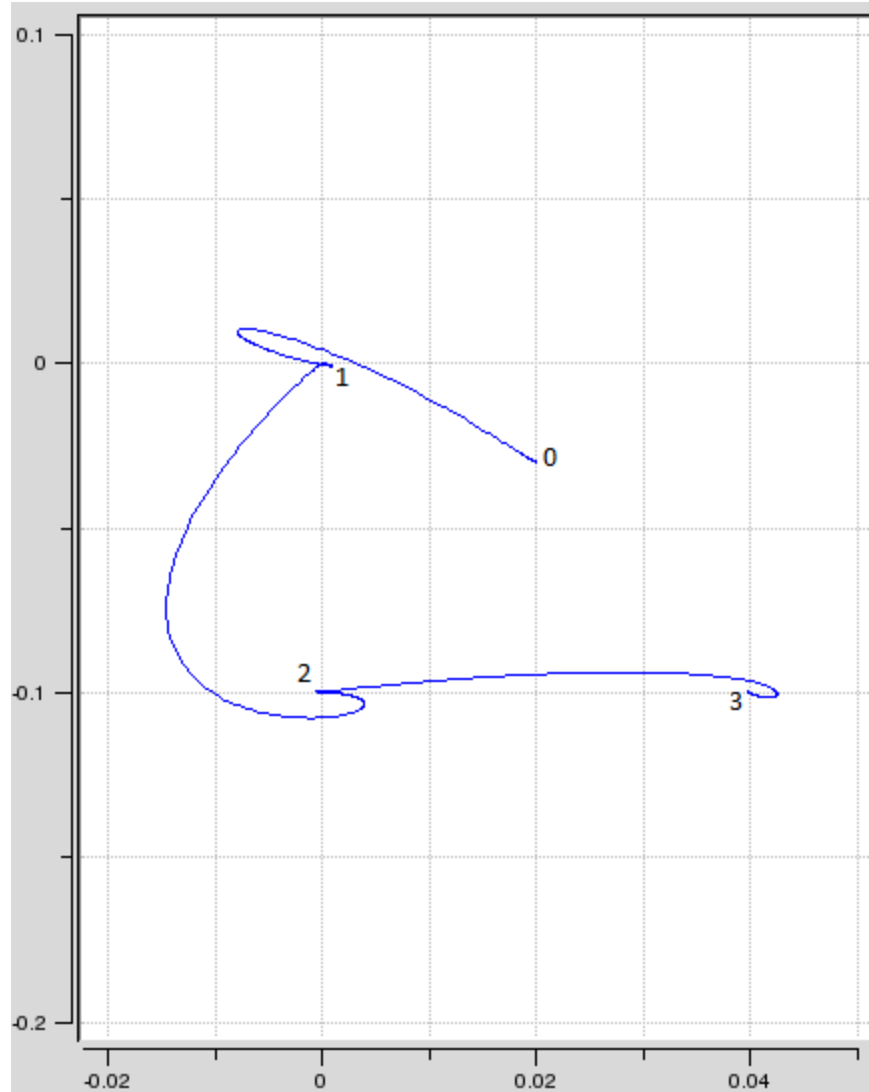
Trick GUI: HCW Controller

- GUI allows for human in the loop capability with the HCW controller.
- Uses the variable server to directly modify variables within the simulation.
- When a button is clicked, the desired relative location of the Displaced Spacecraft increments in the corresponding direction.





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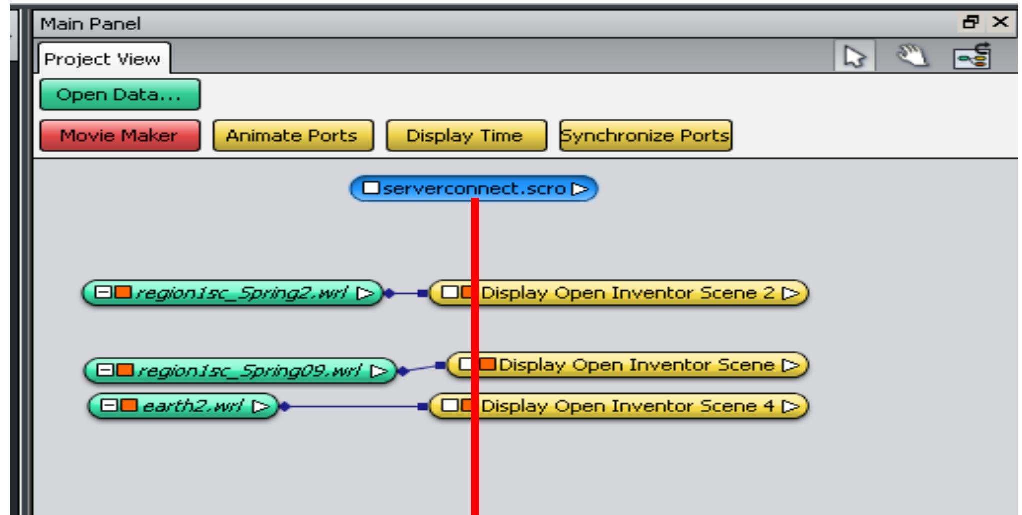
Avizo-Trick Communications

‘Serverconnect’ is the .tcl file used within Avizo to connect/listen to a remote server containing the Trick simulation.

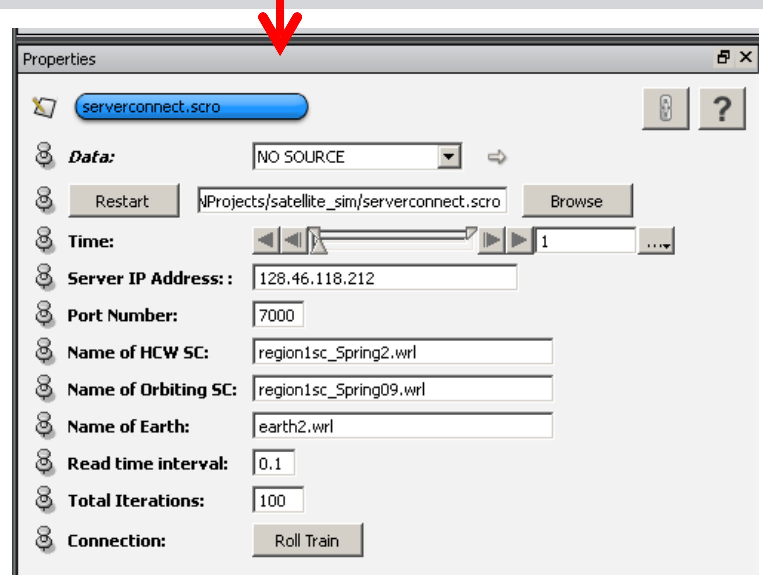
The remote server’s IP address is input by the user.

Avizo then finds Trick on the remote server via Trick’s variable server.



The Serverconnct file now interfaces between Trick and Avizo, telling Avizo what to move and how.

The inputs seen to the right are customizable within the Serverconnect file according to the user’s preferences.



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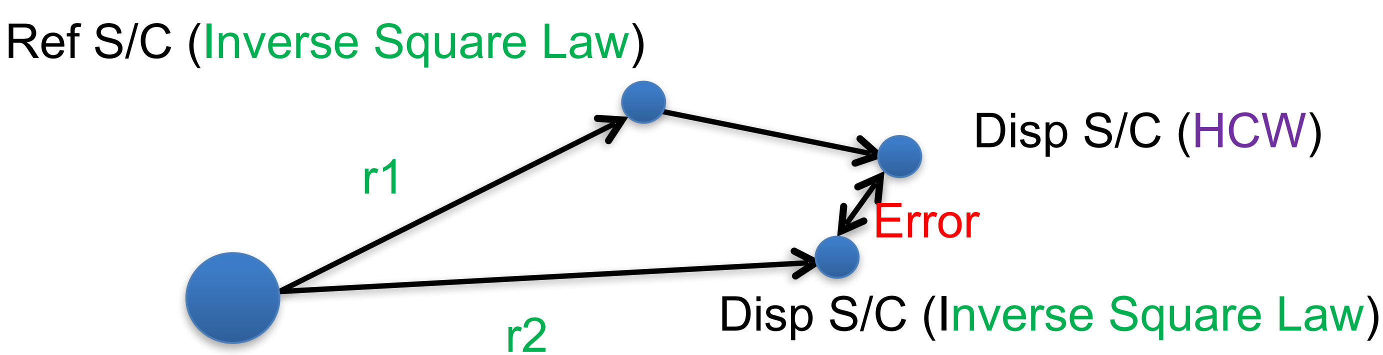
Experiment 1 Overview

Evaluate accuracy of HCW equations

All else being equal, larger initial displacements between two spacecraft will result in less time for the error in the Hill-Clohessy-Wiltshire model to exceed a prescribed threshold.

Error is the **distance** between the two Displaced Spacecraft models.



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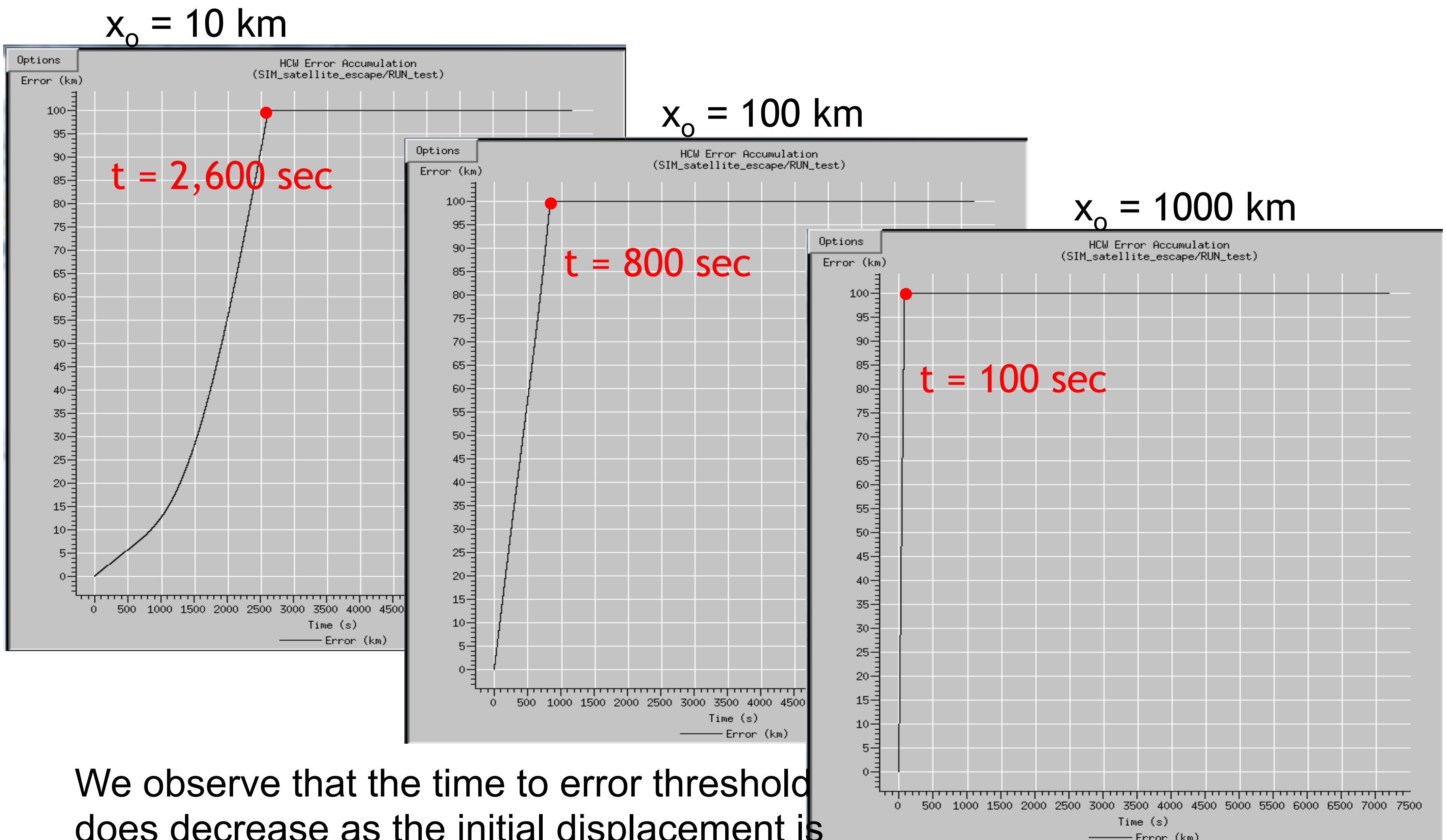
Experiment 1 Results



We observe that the time to error threshold does decrease as the initial displacement is increased. This supports the hypothesis.

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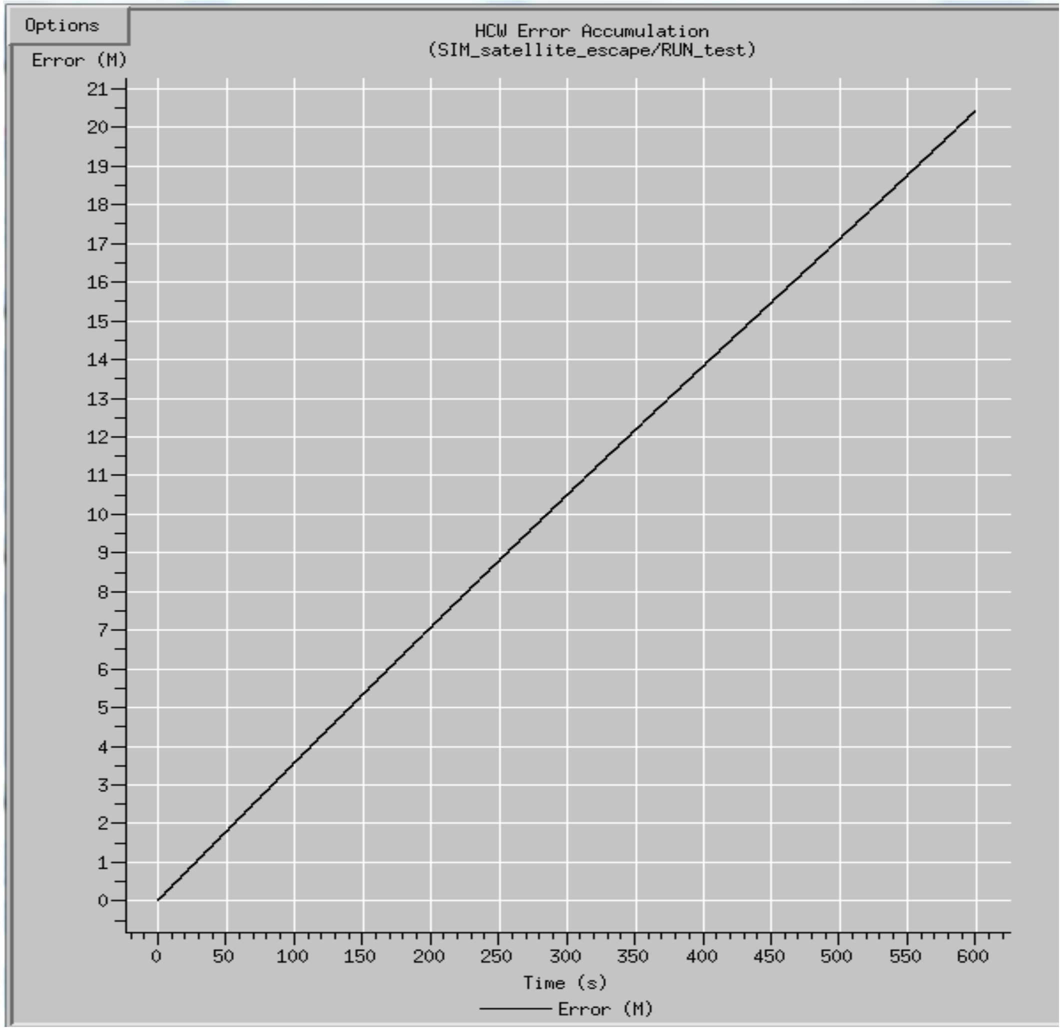
Experiment 1 Implications

Run Experiment 1 with Experiment 2 initial conditions.

$x_o = 30$ m

10 minutes of run time produces 20 meters in error.



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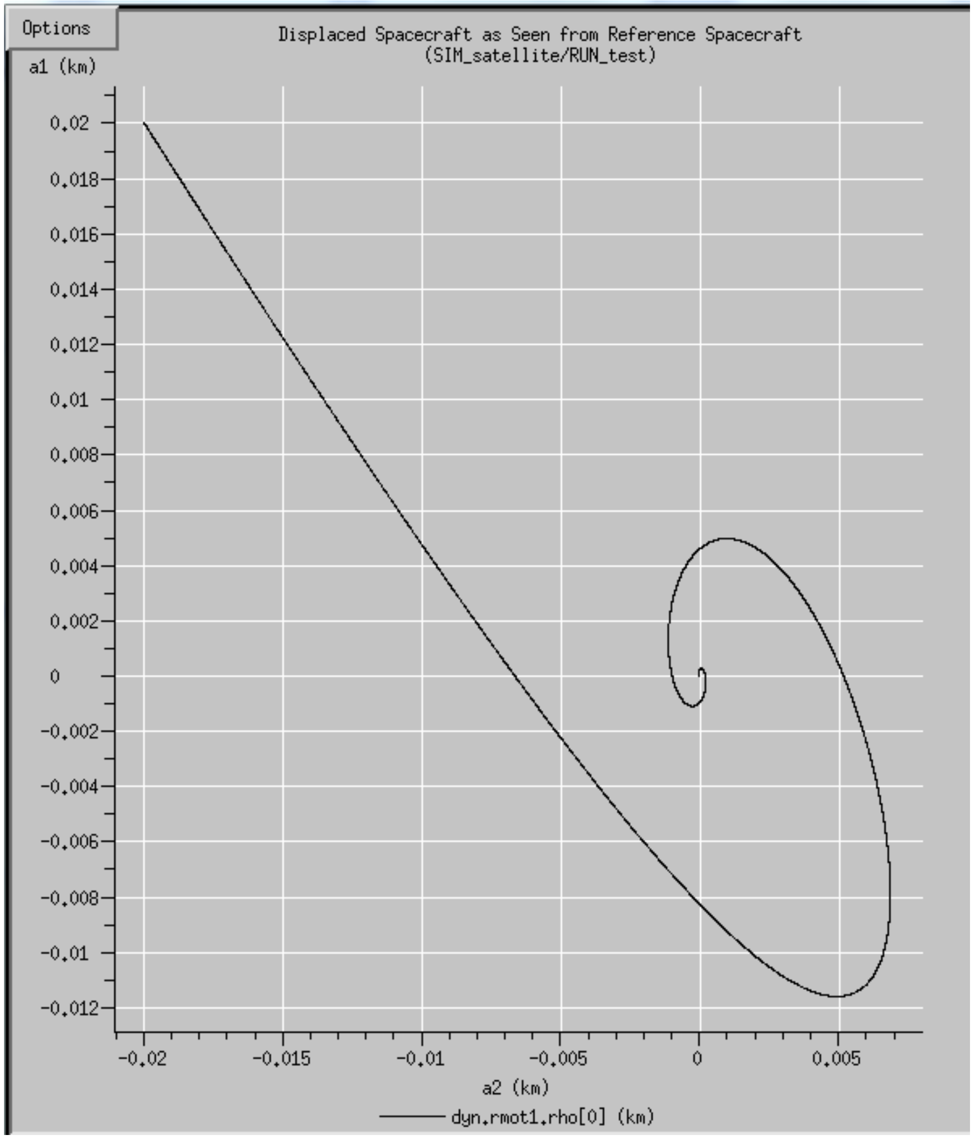
Experiment 2 Overview

Control of the HCW Equations



The response of a 6^{th} order HCW model with an integral and state feedback controller can be modeled as a 2^{nd} order system if 4 of the poles are placed relatively far away.

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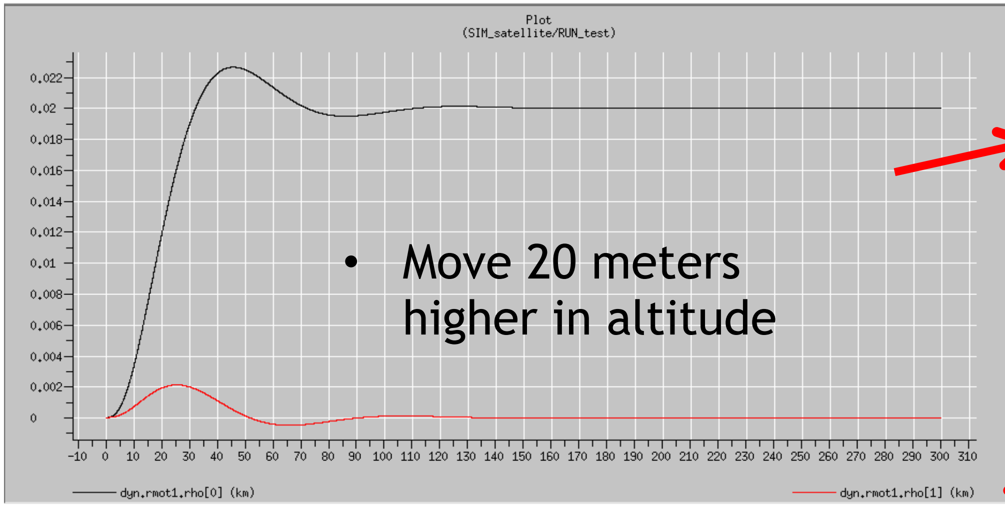
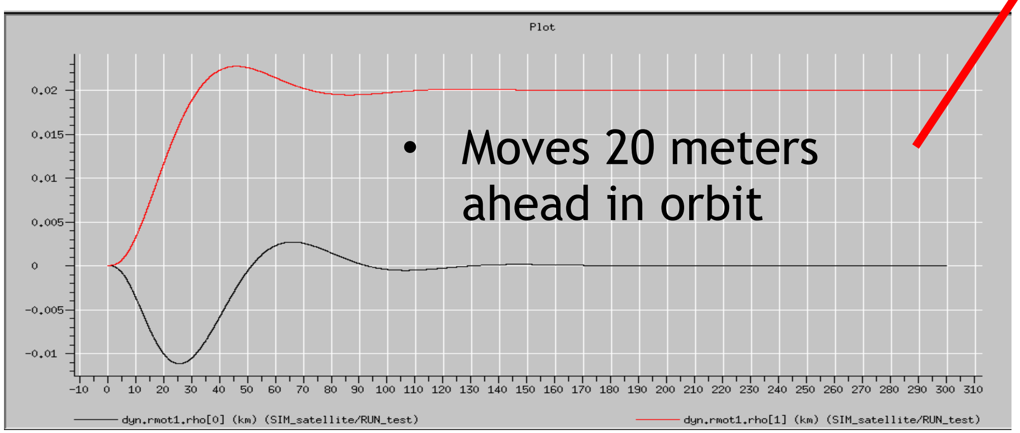
Experiment 2 Results

X = relative altitude, Y = relative position along orbit



- Move 20 meters higher in altitude



- Moves 20 meters ahead in orbit

X Step Response

settling time, s	95.3
rise time, s	***20.34***
percent overshoot, %	13.21

Y Step Response

settling time, s	96.21
rise time, s	***20.45***
percent overshoot, %	13.59

Second Order Performance

settling time, s	75
rise time, s	***23.18***
percent overshoot, %	0.1867

We observe that 2nd order approximations do not accurately predict the behavior of the HCW equations under control of a 6th order state-feedback controller.

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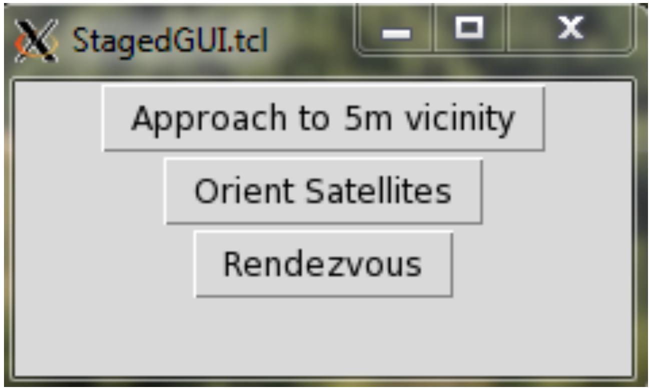
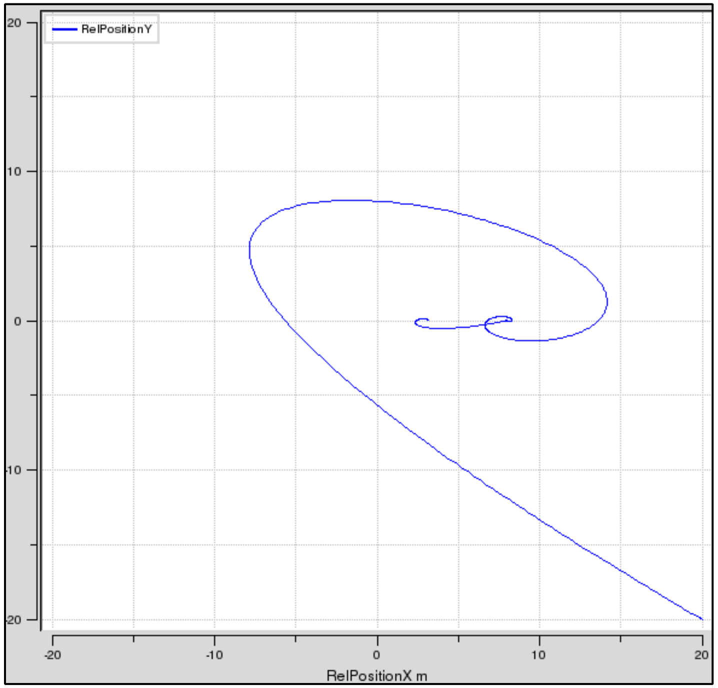
Trick GUI: Staged Rendezvous

- GUI created to control staged events of rendezvous
  - Bring Displaced Spacecraft to within 5m of Reference Spacecraft
  - Stabilize the orientation of both spacecrafts
  - Bring Displaced Spacecraft to rendezvous with Reference Spacecraft

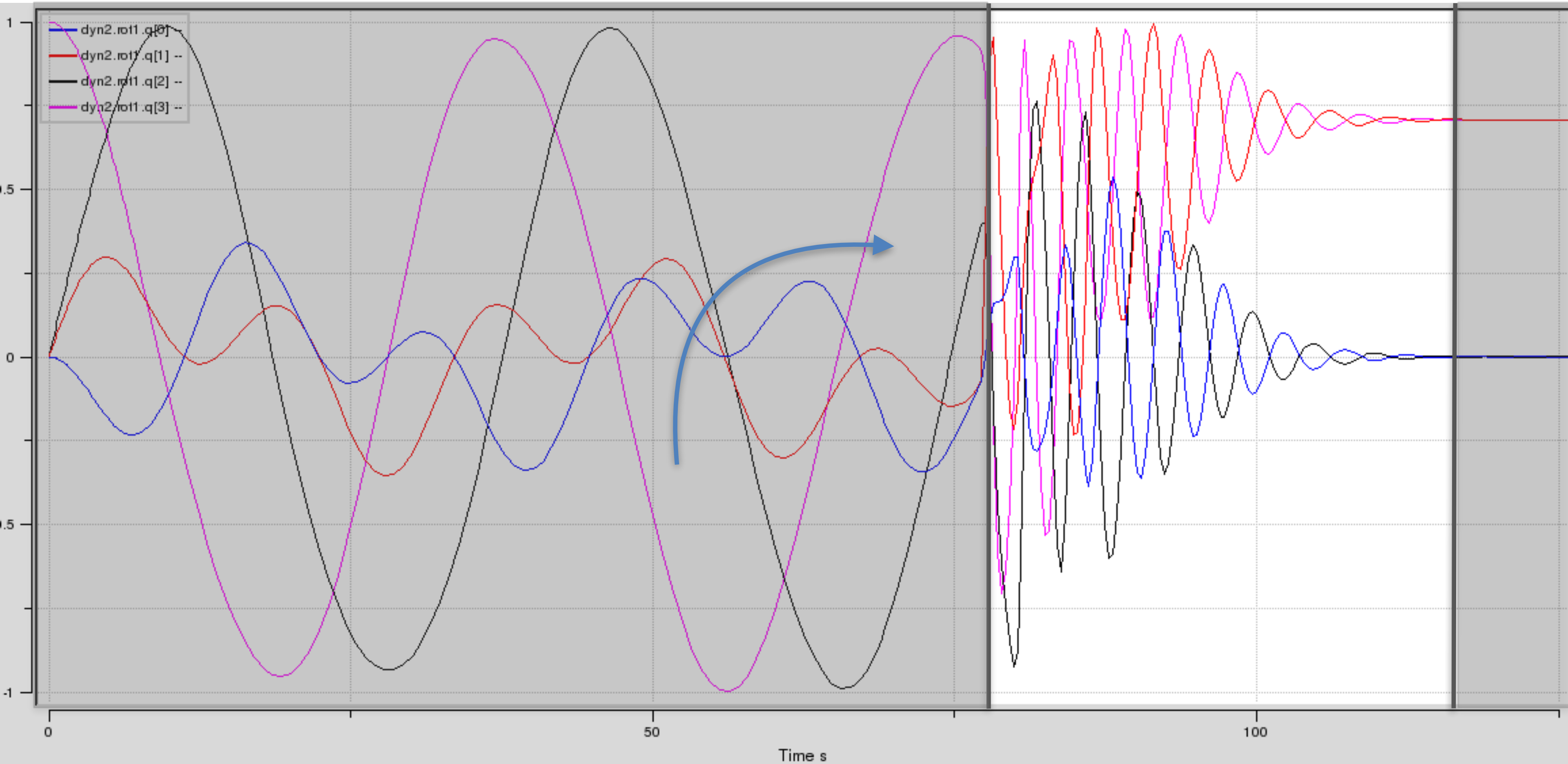


Relative Position Time History



Staged Controller Demonstration

Euler Parameters Time History



Stage 1: Approach

Stage 2: Orient

Stage 3: Rendezvous

Staged Controller Demonstration

Staged
Video

Appendix Slides

HCW Control Law

$$\dot{u} = A_i u + B_i v - W u_d$$

$$\dot{u} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \\ x - x_d \\ y - y_d \end{bmatrix} \quad A_i = \begin{bmatrix} \begin{matrix} & A & \end{matrix} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \end{bmatrix} \quad u = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \\ \int (x - x_d) dt \\ \int (y - y_d) dt \end{bmatrix} \quad B_i = \begin{bmatrix} B \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \end{bmatrix} \quad v = \begin{bmatrix} f_x \\ f_y \end{bmatrix}$$

$$W = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad u_d = \begin{bmatrix} x_d \\ y_d \end{bmatrix}$$

Linearized Attitude Model

Linearized where axis of symmetry is parallel to radial direction:

$$\varepsilon_1 = 0 + \tilde{\varepsilon}_1$$

$$\varepsilon_2 = \frac{\sqrt{2}}{2} + \tilde{\varepsilon}_1$$

$$\varepsilon_3 = 0 + \tilde{\varepsilon}_3$$

$$\varepsilon_4 = \frac{\sqrt{2}}{2} + \tilde{\varepsilon}_4$$

$$\omega_1 = -\Omega + \widetilde{\omega}_1$$

$$\omega_2 = 0 + \widetilde{\omega}_2$$

$$\omega_3 = 0 + \widetilde{\omega}_3$$

$$m_1 = 0 + \widetilde{m}_1$$

$$m_2 = 0 + \widetilde{m}_2$$

$$m_3 = 0 + \widetilde{m}_3$$

$$Y = 3\sqrt{2}\Omega^2 X$$

$$X = 1 - \frac{J}{I}$$

$$\dot{\tilde{\varepsilon}}_1 = \frac{\Omega}{2}\tilde{\varepsilon}_2 - \frac{\Omega}{2}\tilde{\varepsilon}_4 + \frac{\sqrt{2}}{4}\widetilde{\omega}_1 + \frac{\sqrt{2}}{4}\widetilde{\omega}_3$$

$$\dot{\tilde{\varepsilon}}_2 = -\frac{\Omega}{2}\tilde{\varepsilon}_1 - \frac{\Omega}{2}\tilde{\varepsilon}_3 + \frac{\sqrt{2}}{4}\widetilde{\omega}_2$$

$$\dot{\tilde{\varepsilon}}_3 = \frac{\Omega}{2}\tilde{\varepsilon}_2 - \frac{\Omega}{2}\tilde{\varepsilon}_4 - \frac{\sqrt{2}}{4}\widetilde{\omega}_1 + \frac{\sqrt{2}}{4}\widetilde{\omega}_3$$

$$\dot{\tilde{\varepsilon}}_4 = \frac{\Omega}{2}\tilde{\varepsilon}_1 + \frac{\Omega}{2}\tilde{\varepsilon}_3 - \frac{\sqrt{2}}{4}\widetilde{\omega}_2$$

$$\dot{\widetilde{\omega}}_1 = Y\tilde{\varepsilon}_1 - Y\tilde{\varepsilon}_3 + \widetilde{m}_1$$

$$\dot{\widetilde{\omega}}_2 = -Y\tilde{\varepsilon}_2 - \Omega X\widetilde{\omega}_3 + \widetilde{m}_2$$

$$\dot{\widetilde{\omega}}_3 = \widetilde{m}_3$$

Nonlinear Attitude Model

$$\dot{\varepsilon}_1 = \frac{1}{2}(\varepsilon_4\omega_1 + \varepsilon_2(\omega_3 + \Omega) - \varepsilon_3\omega_2)$$

$$\dot{\varepsilon}_2 = \frac{1}{2}(\varepsilon_3\omega_1 - \varepsilon_1(\omega_3 + \Omega) + \varepsilon_4\omega_2)$$

$$\dot{\varepsilon}_3 = \frac{1}{2}(-\varepsilon_2\omega_1 + \varepsilon_4(\omega_3 - \Omega) + \varepsilon_1\omega_2)$$

$$\dot{\varepsilon}_4 = -\frac{1}{2}(\varepsilon_1\omega_1 + \varepsilon_3(\omega_3 - \Omega) + \varepsilon_2\omega_2)$$

$$\dot{\omega}_1 = 12\Omega^2 X(\varepsilon_1\varepsilon_2 - \varepsilon_3\varepsilon_4)(\varepsilon_1\varepsilon_3 + \varepsilon_2\varepsilon_4) - X\omega_2\omega_3 + m_1$$

$$\dot{\omega}_2 = -6\Omega^2 X(1 - 2\varepsilon_2^2 - 2\varepsilon_3^2)(\varepsilon_1\varepsilon_3 + \varepsilon_2\varepsilon_4) + X\omega_3\omega_1 + m_2$$

$$\dot{\omega}_3 = m_3$$

$$m_1 = -G_{1,1}\varepsilon_1 - G_{1,2}\varepsilon_2 - G_{1,3}\varepsilon_3 - G_{1,4}\varepsilon_4 - G_{1,5}\omega_1 - G_{1,6}\omega_2 - G_{1,7}\omega_3 - G_{1,8}e_1 - G_{1,9}e_2 - G_{1,10}e_3 - G_{1,11}e_4$$

$$m_2 = -G_{2,1}\varepsilon_1 - G_{2,2}\varepsilon_2 - G_{2,3}\varepsilon_3 - G_{2,4}\varepsilon_4 - G_{2,5}\omega_1 - G_{2,6}\omega_2 - G_{2,7}\omega_3 - G_{2,8}e_1 - G_{2,9}e_2 - G_{2,10}e_3 - G_{2,11}e_4$$

$$m_3 = -G_{3,1}\varepsilon_1 - G_{3,2}\varepsilon_2 - G_{3,3}\varepsilon_3 - G_{3,4}\varepsilon_4 - G_{3,5}\omega_1 - G_{3,6}\omega_2 - G_{3,7}\omega_3 - G_{3,8}e_1 - G_{3,9}e_2 - G_{3,10}e_3 - G_{3,11}e_4$$

$$\bar{\bar{I}}^B = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & J \end{bmatrix}$$

$$X = 1 - \frac{J}{I}$$

$$e_i = \int (\varepsilon_i - \varepsilon_{i,d})dt$$

Pole Placement and Gains

Experiment 2 6th Order Model

Poles:

$$\left.\begin{aligned} \lambda_1 &= -0.04 + 0.02i \\ \lambda_2 &= -0.04 - 0.02i \end{aligned}\right\} \text{Poles that govern 2nd order response}$$

$$\left.\begin{aligned} \lambda_3 &= -0.15 \\ \lambda_4 &= -0.16 \\ \lambda_5 &= -0.18 \\ \lambda_6 &= -0.2 \end{aligned}\right\} \text{Poles that are relatively far away}$$

Gains Matrix from *place* command

$$K = \begin{bmatrix} 0.0442 & -0.0163 & 0.3901 & -0.0399 & 0.0012 & -0.0015 \\ 0.0040 & 0.0420 & 0.0124 & 0.3799 & 0.0003 & 0.0011 \end{bmatrix}$$

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2nd Order Performance

Experiment 2 2nd Order Model

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = (s - \lambda_1)(s - \lambda_2)$$

$$t_{s,5\%} = \frac{3}{\zeta\omega_n}$$

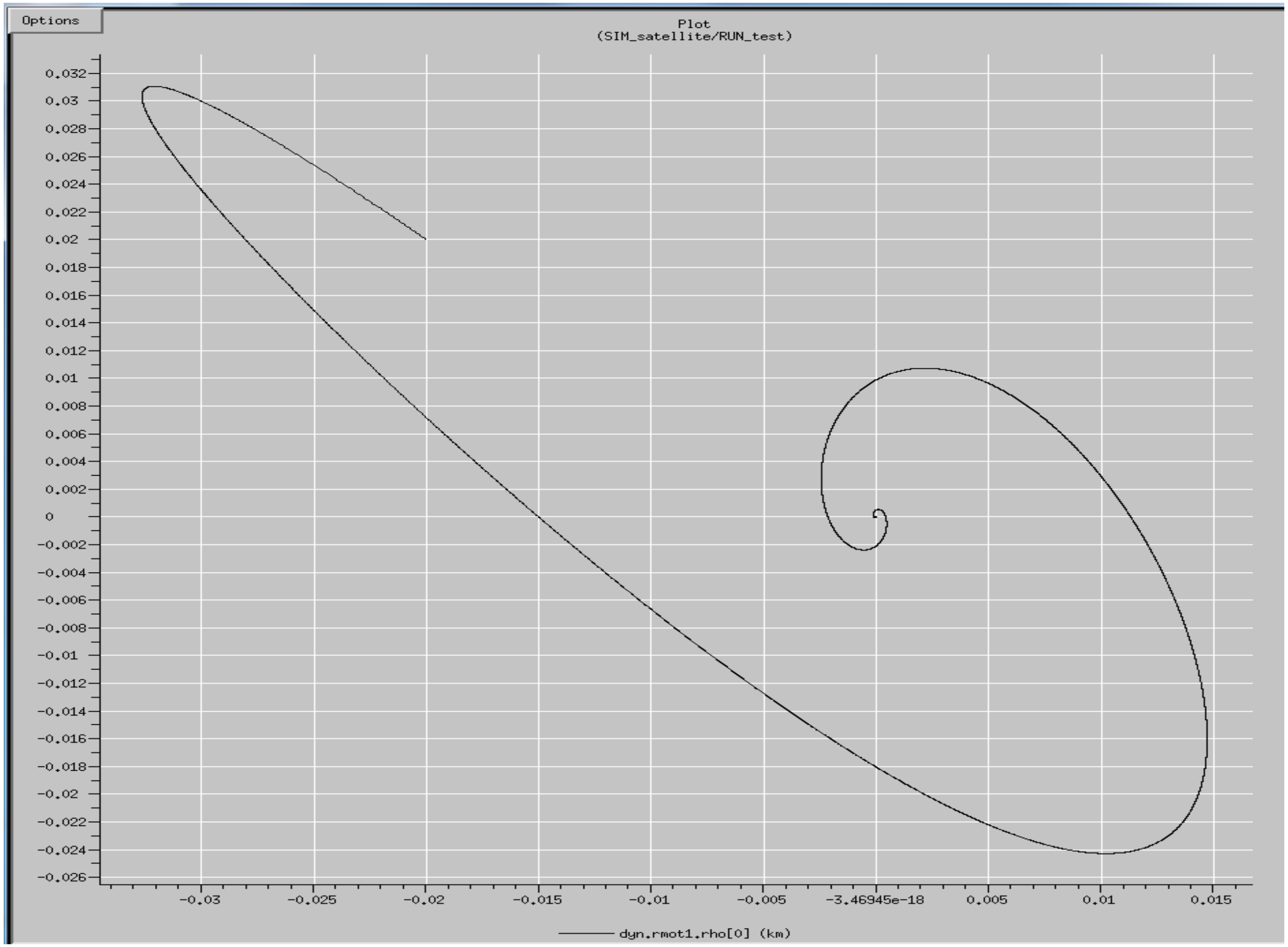
$$t_r = \frac{1}{\omega_d} \tan^{-1} \frac{\omega_d}{\zeta\omega_n}$$

$$PO = 100\% \times e^{\left(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)}$$

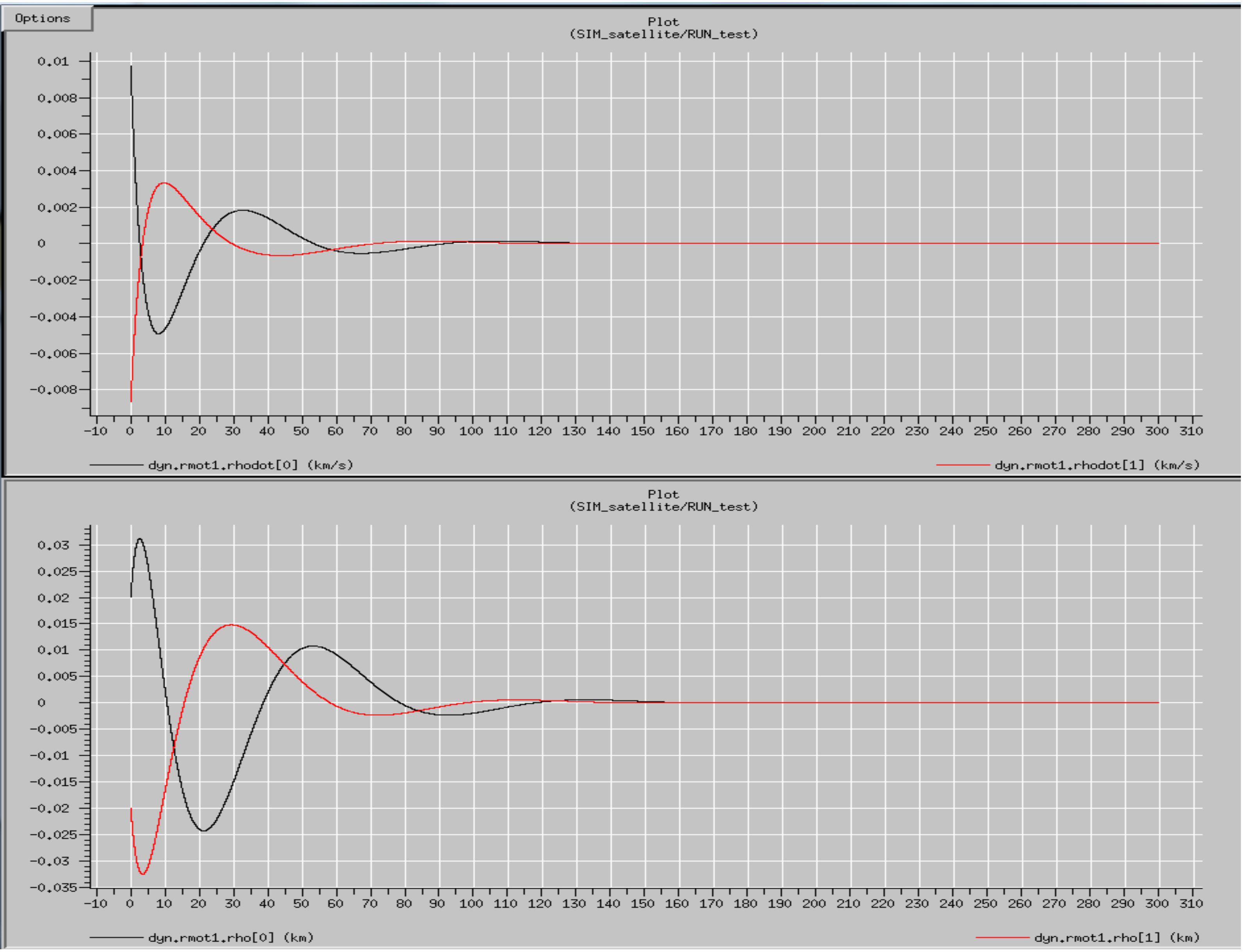
settling time, s	75
rise time, s	23.18
percent overshoot, %	0.1867

Random Initial Conditions

- Start at (0.02, -0.02) and move to (0, 0) with initial velocity 0.0096985207 km/s in the x-direction and -0.0086985207 km/s in the y-direction



Random Initial Conditions



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