ECE351 Sample Questions (First Set): Regular languages, Regular expressions, NFA/DFA, Context-free Languages

Question 1: True/False questions (20 points)

Please specify if the following assertions are True or False. Each sub-question is worth 2 points.

1. Deterministic Finite Automata are strictly weaker class than Non-deterministic Finite Automata (NFAs), i.e., there exists a language that is accepted by an NFA but is not accepted by any DFA.

   Answer: False

2. $a^n b^m$, where the alphabet is a, b and $n \geq 0, m \geq 0$, is a regular language.

   Answer: True

3. It is known that context-free languages are not closed under intersection. Since every regular language is context-free, it follows that the intersection of a regular language with a context-free one is also not context-free.

   Answer: False

4. Let $\Sigma = (,)$ be an alphabet. The following grammar represents the empty language.

   $$S \rightarrow (S)$$

   Answer: True

5. The following is an identity, where r, s are regular expressions, where $r = s$ means $L(r) = L(s)$

   $$(r + s)^* = r^* + s^*$$

   Answer: False

6. The following regular expressions represent the same language, where a, b are letters in an alphabet,

   $$(a^* b^*)^* = (a + b)^*$$

   Answer: True

7. The following grammar represent the language of all strings over the alphabet a, b with equal number of a's followed by equal number of b's

   $$S \rightarrow aSb \mid aabb$$

   Answer: False

8. There are regular languages that contain context-free languages as subsets.

   Answer: True

9. We say a problem P is undecidable, if there does not exist an algorithm that given any instance of P will halt and produce the correct answer. This necessarily implies that any algorithm designed to solve P will always go into an infinite loop on all possible inputs.

   Answer: False

10. Given a context-free language (CFL) A, it is undecidable whether A is ambiguous.

    Answer: True

Question 2: Short answer questions (20 points)

For each of the questions below, provide a short correct and complete answer. Each question carries a maximum of 2 points.

1. It is well-known that regular languages are closed under the following operations: union, complement, and intersection. It is also well-known that all finite languages are regular. For each of the operations, union, intersection and complement, are finite languages closed under them? If yes, prove it. Else provide couterexamples.

   Answer: Yes, for union and intesection. No, for complement.

2. Is the following set regular

$$\{0^{2n} \mid n \geq 1\}$$

   If yes, write down the corresponding regular expression. Else, prove that the language is not regular.

   Answer: Yes. $(00)^+$

3. Is the set of odd-length strings over 0 regular, where the alphabet is $\Sigma = 0$?

   Answer: Yes. $\Sigma^* - (00)^*$

4. It is known that the following set $S := \{0^n \mid n \text{ is a prime number}\}$ is not regular. Is the complement of S regular?

   Answer: No.

5. Give a context-free grammar for generating the set of palindromes over the alphabet a,b.

   Answer:

$$S \rightarrow aSa \mid bSb \mid a \mid b\epsilon$$

6. Give a context-free grammar for generating all strings over a,b such that $\mid a \mid = 2 * \mid b \mid$

   Answer:

$$S \rightarrow aSaSb \mid aSbSa \mid bSaSa \mid \epsilon$$

7. The complement of an infinite language is necessarily finite.

   Answer: False.

8. For a language L, if L^* is regular then L is regular, where $\Sigma = a, b$ is the alphabet.

   Answer: False. Let, $L = A \cup \Sigma$ where A is not regular. Clearly, L^* is regular because $L^* = \Sigma^*$, but L is not.

9. If a language is accepted by a NFA, then it is clearly context-free.

   Answer: True, simply because every language accepted by an NFA is regular, and every regular language is context-free.

10. Consider the grammar

$$E \to E + E \mid E * E \mid (E) \mid id$$

   How can the above ambiguous grammar be rewritten to make it non-ambiguous?

   Answer:

$$E \to E + T \mid T$$

$$T \to T * F \mid F$$

$$F \to (E) \mid a$$

Question 3: Regular Expressions, Regular Languages and NFAs (30 points)

Each of the following is worth 15 points.

1. Give a regular expression for the following regular languages, assuming the alphabet is $\Sigma := \{0, 1\}$.

   (a) The set of all strings, when viewed as binary representation of integers, that are divisible by 2.
   Answer: $(0 + 1)^*0$

   (b) The set of all strings containing 00.
   Answer: $(0 + 1)^*00(0 + 1)^*$

   (c) Give a closed-form regular expression for the set of all string not containing consecutive 00.
   Answer: First convert the above regular expression in answer (b) into an NFA. Then determinize it, followed by constructing its complement. Then convert it back to a regular expression. The answer here is:

$$(1 + 01)^*(0 + \epsilon)$$

2. We say that two regular languages are equal if they have the same regular expression representation or DFAs. Let L_1 and L_2 denote two regular languages, one of them is given to you as a regular expression while the other is represented as a DFA. How would you verify that they are equal? Describe your steps in detail.

   Answer: We conver the DFA into a regular expression using the generalized transition graph method given in the review notes.

Question 4: Context-free Languages and Grammars (30 points)

Each of the following is worth 10 points.

1. Prove that the following language $L = L_1 \cup L_2$ has an ambiguous grammar, where

$$L_1 = \{a^n b^n c^m\}$$
$$L_2 = \{a^n b^m c^m\}$$

   and n, m are natural numbers.

   Answer: The following is the grammar for the language L with start symbol S, where S is start symbol of the grammar for L_1, and S_2 is the start symbol of the grammar for L_2. The grammar is:

$$S \rightarrow S_1 \mid S_2$$
$$S_1 \rightarrow S_1 c \mid A$$
$$A \rightarrow aAb \mid \epsilon$$
$$S_2 \rightarrow aS_2 \mid B$$
$$B \rightarrow bBc \mid \epsilon$$

   Consider the string $a^n b^n c^n$. This string has two different parse trees in grammar S depending on which of the two productions $S \rightarrow S_1$ or $S \rightarrow S_2$ is chosen first.

2. Is the intersection of two CFGs necessarily always context-free?

   Answer: No. The intersection of the two CFGs L_1 and L_2 given above is the language $\{a^n b^n c^n\}$, which is a well-known non context-free language.

3. Draw the left and right-most derivation trees for the string $a + a * a$ belonging to the grammar $E \rightarrow E + E \mid E * E \mid (E) \mid id$ of arithmetic expressions? Discuss ways in which this grammar can be made non-ambiguous without rewriting it as above?

   Answer: Use associativity and precedence rules. Give * precedence over +.