

Implementation of Lexical Analysis

Lecture 4

Tips on Building Large Systems

- KISS (Keep It Simple, Stupid!)
- Don't optimize prematurely
- Design systems that can be tested
- It is easier to modify a working system than to get a system working

Outline

- Specifying lexical structure using regular expressions
- Finite automata
 - Deterministic Finite Automata (DFAs)
 - Non-deterministic Finite Automata (NFAs)
- Implementation of regular expressions
RegExp \Rightarrow NFA \Rightarrow DFA \Rightarrow Tables

Notation

- There is variation in regular expression notation
- Union: $A \mid B \equiv A + B$
- Option: $A + \varepsilon \equiv A?$
- Range: $'a' + 'b' + \dots + 'z' \equiv [a-z]$
- Excluded range:
 $\text{complement of } [a-z] \equiv [\hat{a}-z]$

Regular Expressions in Lexical Specification

- Last lecture: a specification for the predicate
 $s \in L(R)$
- But a yes/no answer is not enough!
- Instead: partition the input into tokens
- We adapt regular expressions to this goal

Regular Expressions => Lexical Spec. (1)

1. Write a rexp for the lexemes of each token
 - Number = `digit +`
 - Keyword = `'if' + 'else' + ...`
 - Identifier = `letter (letter + digit)*`
 - OpenPar = `'('`
 - ...

Regular Expressions => Lexical Spec. (2)

2. Construct R , matching all lexemes for all tokens

$$\begin{aligned} R &= \text{Keyword} + \text{Identifier} + \text{Number} + \dots \\ &= R_1 + R_2 + \dots \end{aligned}$$

Regular Expressions \Rightarrow Lexical Spec. (3)

3. Let input be $x_1 \dots x_n$

For $1 \leq i \leq n$ check

$$x_1 \dots x_i \in L(R)$$

4. If success, then we know that

$$x_1 \dots x_i \in L(R_j) \text{ for some } j$$

5. Remove $x_1 \dots x_i$ from input and go to (3)

Ambiguities (1)

- There are ambiguities in the algorithm
- How much input is used? What if
 - $x_1 \dots x_i \in L(R)$ and also
 - $x_1 \dots x_k \in L(R)$
- Rule: Pick longest possible string in $L(R)$
 - The “maximal munch”

Ambiguities (2)

- Which token is used? What if
 - $x_1 \dots x_i \in L(R_j)$ and also
 - $x_1 \dots x_i \in L(R_k)$
- Rule: use rule listed first (j if $j < k$)
 - Treats “ if ” as a keyword, not an identifier

Error Handling

- What if
 No rule matches a prefix of input ?
- Problem: Can't just get stuck ...
- Solution:
 - Write a rule matching all “bad” strings
 - Put it last (lowest priority)

Summary

- Regular expressions provide a concise notation for string patterns
- Use in lexical analysis requires small extensions
 - To resolve ambiguities
 - To handle errors
- Good algorithms known
 - Require only single pass over the input
 - Few operations per character (table lookup)

Finite Automata

- Regular expressions = specification
- Finite automata = implementation
- A finite automaton consists of
 - An input alphabet Σ
 - A set of states S
 - A start state n
 - A set of accepting states $F \subseteq S$
 - A set of transitions $\text{state} \xrightarrow{\text{input}} \text{state}$

Finite Automata

- Transition

$$s_1 \xrightarrow{a} s_2$$

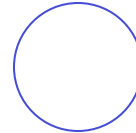
- Is read

In state s_1 on input “a” go to state s_2

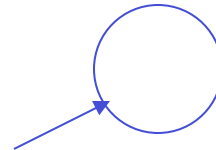
- If end of input and in accepting state => accept
- Otherwise => reject

Finite Automata State Graphs

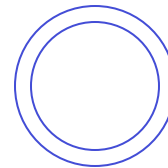
- A state



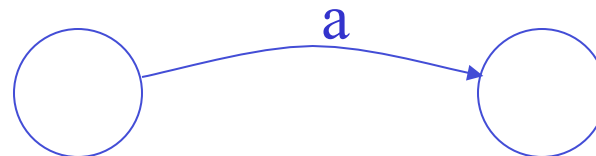
- The start state



- An accepting state

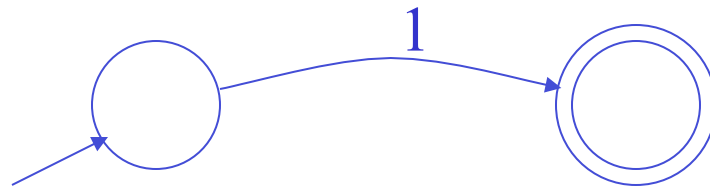


- A transition



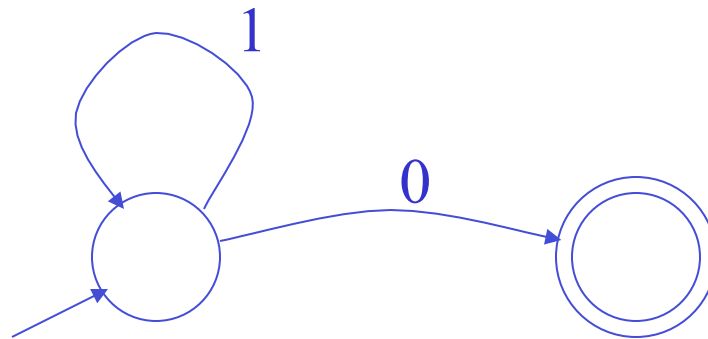
A Simple Example

- A finite automaton that accepts only “1”



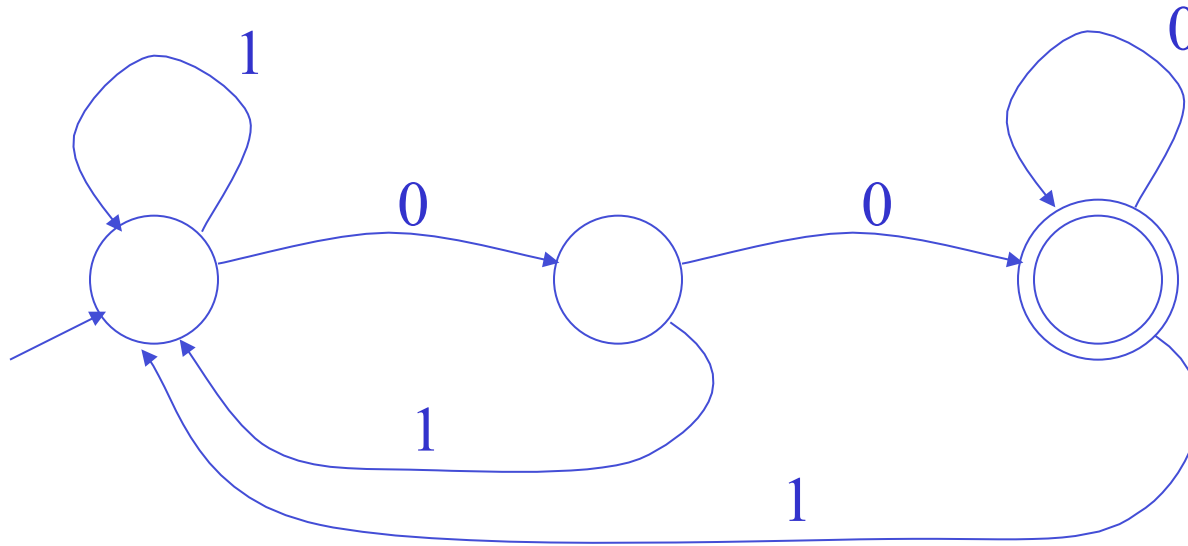
Another Simple Example

- A finite automaton accepting any number of **1**'s followed by a single **0**
- Alphabet: **{0,1}**



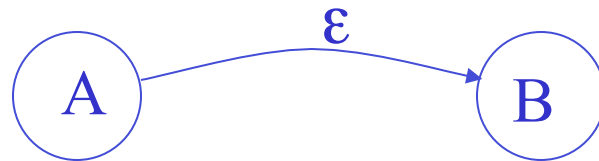
And Another Example

- Alphabet $\{0,1\}$
- What language does this recognize?



Epsilon Moves

- Another kind of transition: ϵ -moves



- Machine can move from state A to state B without reading input

Deterministic and Nondeterministic Automata

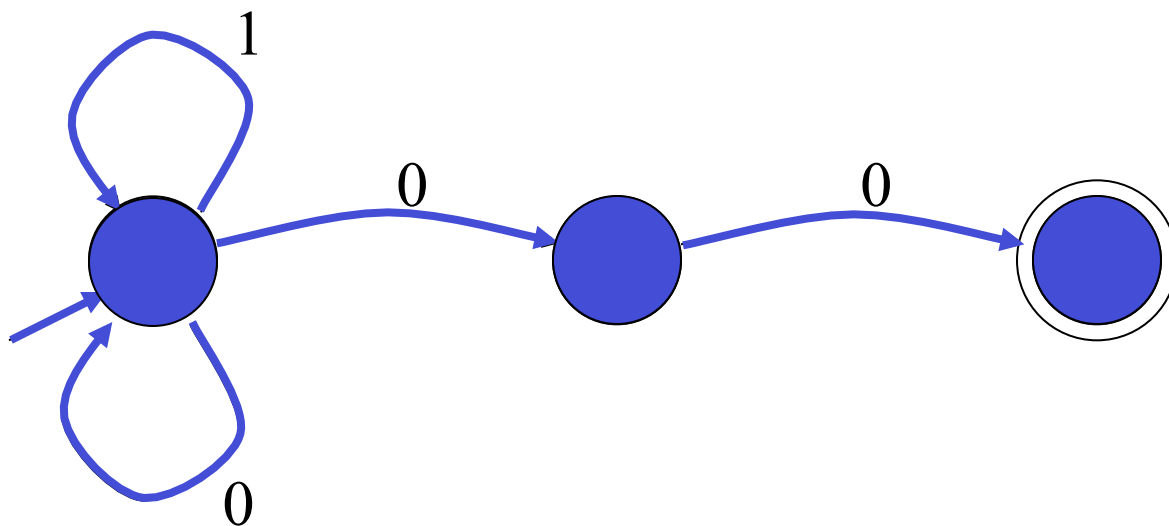
- Deterministic Finite Automata (DFA)
 - One transition per input per state
 - No ϵ -moves
- Nondeterministic Finite Automata (NFA)
 - Can have multiple transitions for one input in a given state
 - Can have ϵ -moves

Execution of Finite Automata

- A DFA can take only one path through the state graph
 - Completely determined by input
- NFAs can choose
 - Whether to make ε -moves
 - Which of multiple transitions for a single input to take

Acceptance of NFAs

- An NFA can get into multiple states



- Input: 1 0 0

Rule: NFA accepts if it can get to a final state

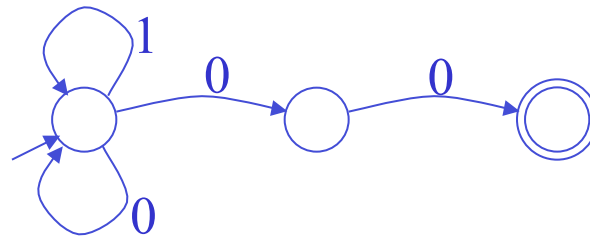
NFA vs. DFA (1)

- NFAs and DFAs recognize the same set of languages (regular languages)
- DFAs are faster to execute
 - There are no choices to consider

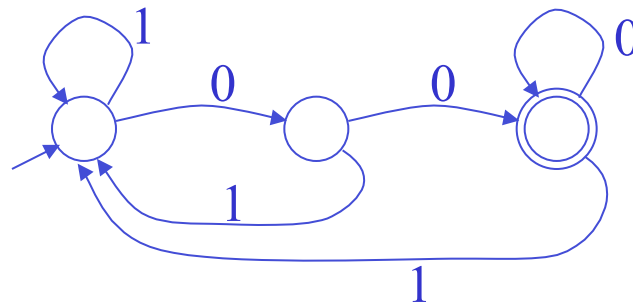
NFA vs. DFA (2)

- For a given language NFA can be simpler than DFA

NFA



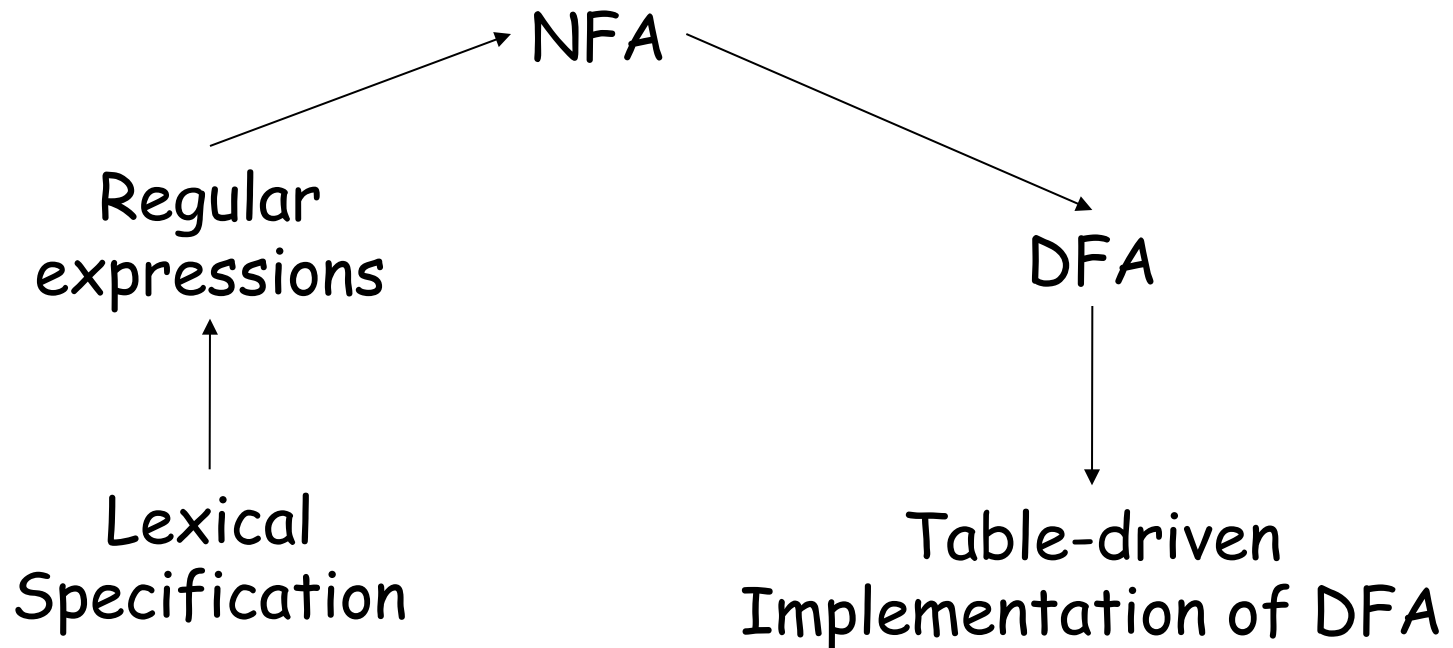
DFA



- DFA can be exponentially larger than NFA

Regular Expressions to Finite Automata

- High-level sketch

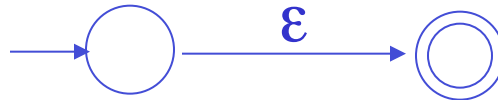


Regular Expressions to NFA (1)

- For each kind of rexp, define an NFA
 - Notation: NFA for rexp M



- For ϵ

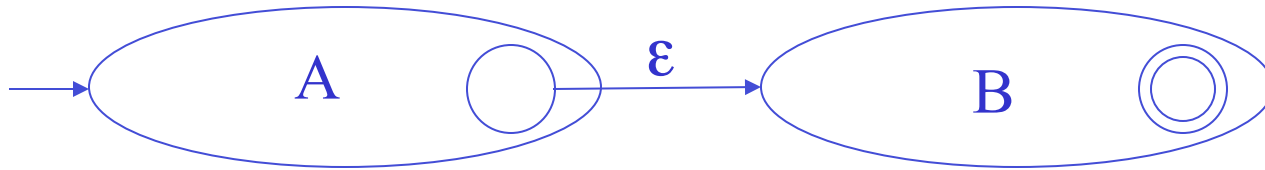


- For input a

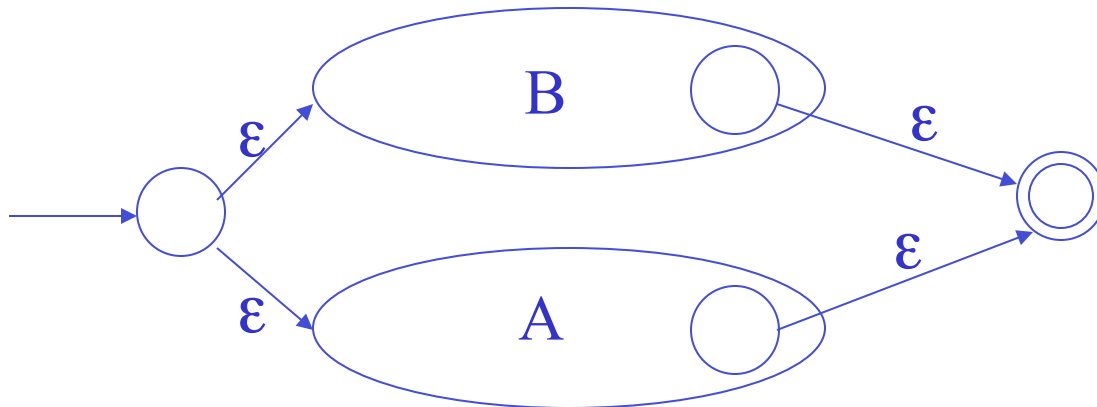


Regular Expressions to NFA (2)

- For AB

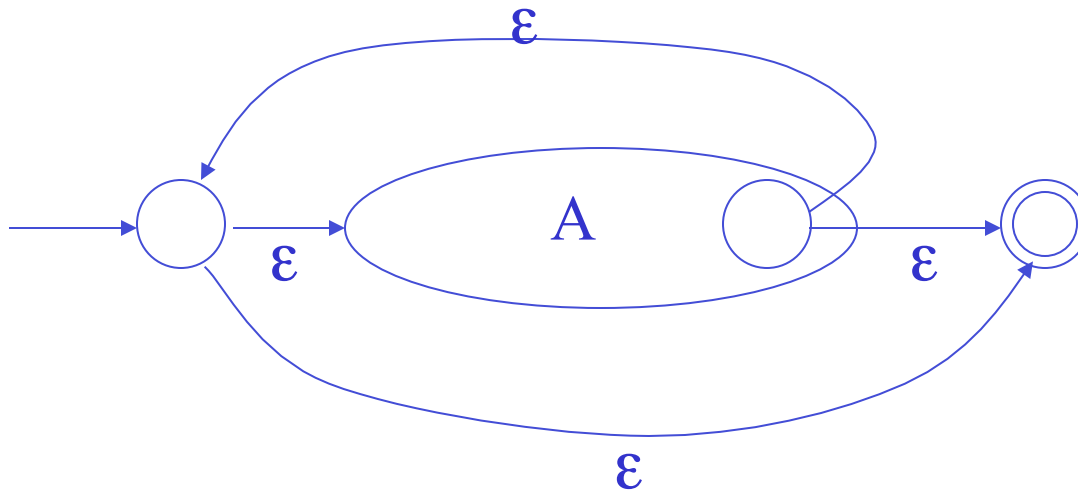


- For $A + B$



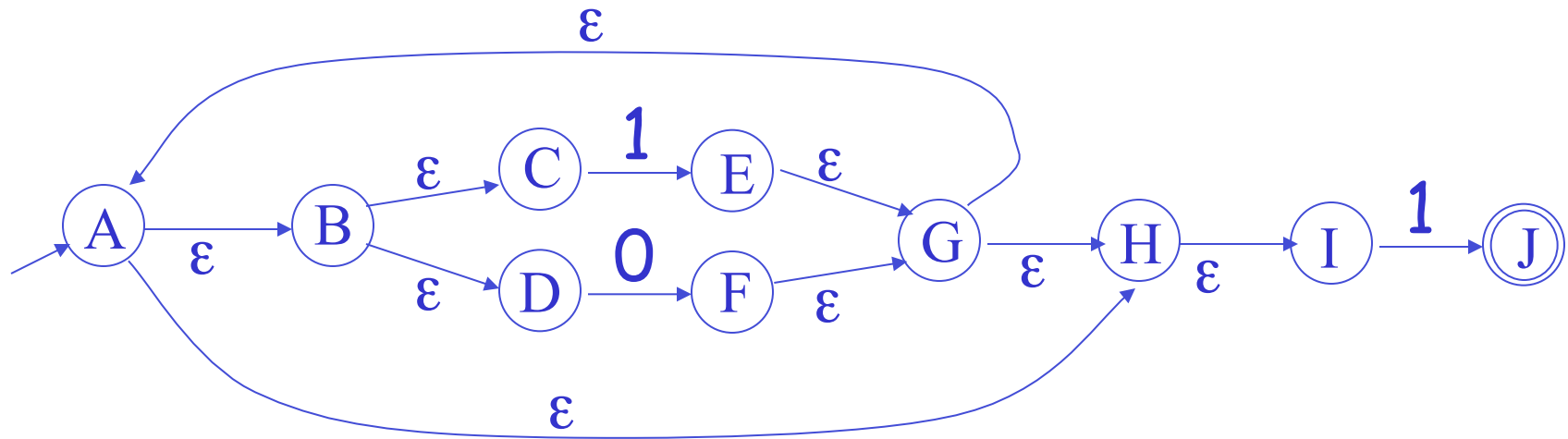
Regular Expressions to NFA (3)

- For A^*



Example of RegExp -> NFA conversion

- Consider the regular expression
 $(1+0)^*1$
- The NFA is



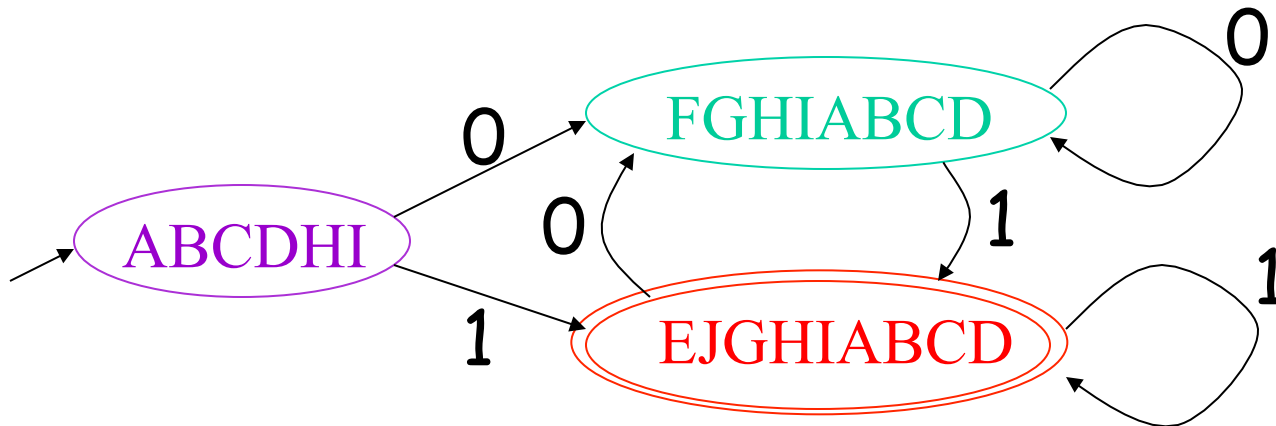
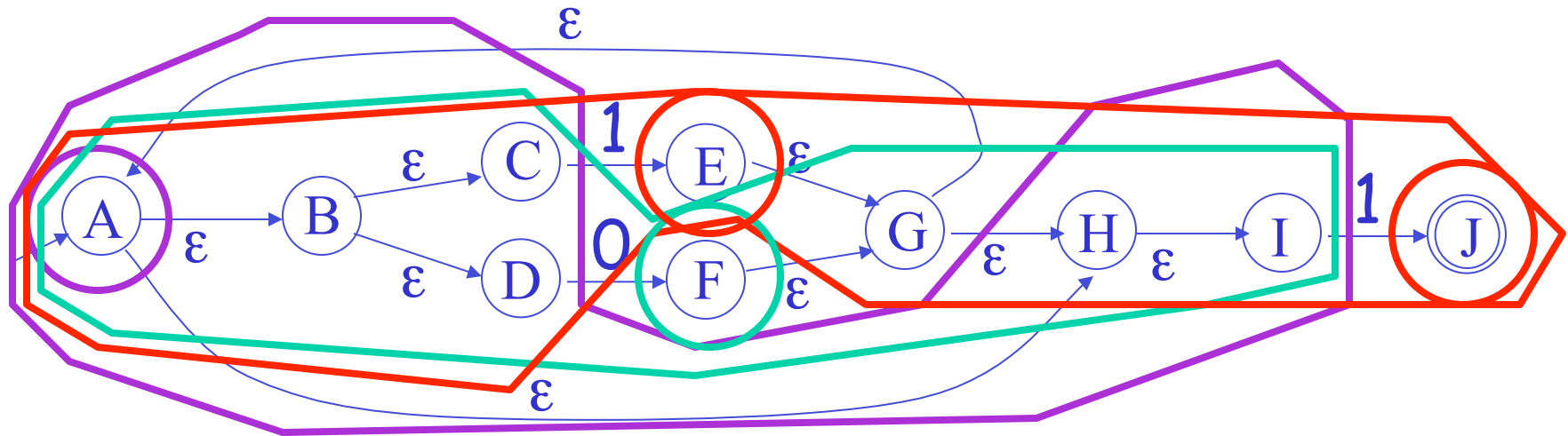
NFA to DFA: *The Trick*

- Simulate the NFA
- Each state of DFA
 - = a non-empty subset of states of the NFA
- Start state
 - = the set of NFA states reachable through ϵ -moves from NFA start state
- Add a transition $S \xrightarrow{a} S'$ to DFA iff
 - S' is the set of NFA states reachable from any state in S after seeing the input a , considering ϵ -moves as well

NFA to DFA. Remark

- An NFA may be in many states at any time
- How many different states ?
- If there are N states, the NFA must be in some subset of those N states
- How many subsets are there?
 - $2^N - 1 = \text{finitely many}$

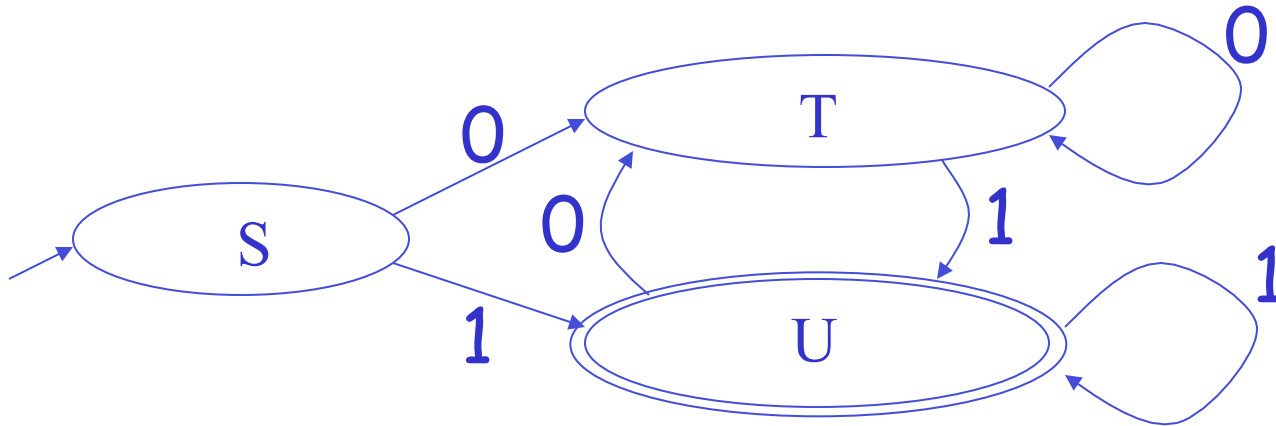
NFA -> DFA Example



Implementation

- A DFA can be implemented by a 2D table T
 - One dimension is “states”
 - Other dimension is “input symbol”
 - For every transition $S_i \xrightarrow{a} S_k$ define $T[i,a] = k$
- DFA “execution”
 - If in state S_i and input a , read $T[i,a] = k$ and skip to state S_k
 - Very efficient

Table Implementation of a DFA



	0	1
S	T	U
T	T	U
U	T	U

Implementation (Cont.)

- NFA \rightarrow DFA conversion is at the heart of tools such as flex
- But, DFAs can be huge
- In practice, flex-like tools trade off speed for space in the choice of NFA and DFA representations