Global Optimization

Lecture 21

Lecture Outline

- Global flow analysis
- Global constant propagation
- Liveness analysis

Local Optimization

Recall the simple basic-block optimizations

- Constant propagation
- Dead code elimination

```
X := 3
Y := Z * W
Q := X + Y
```

→

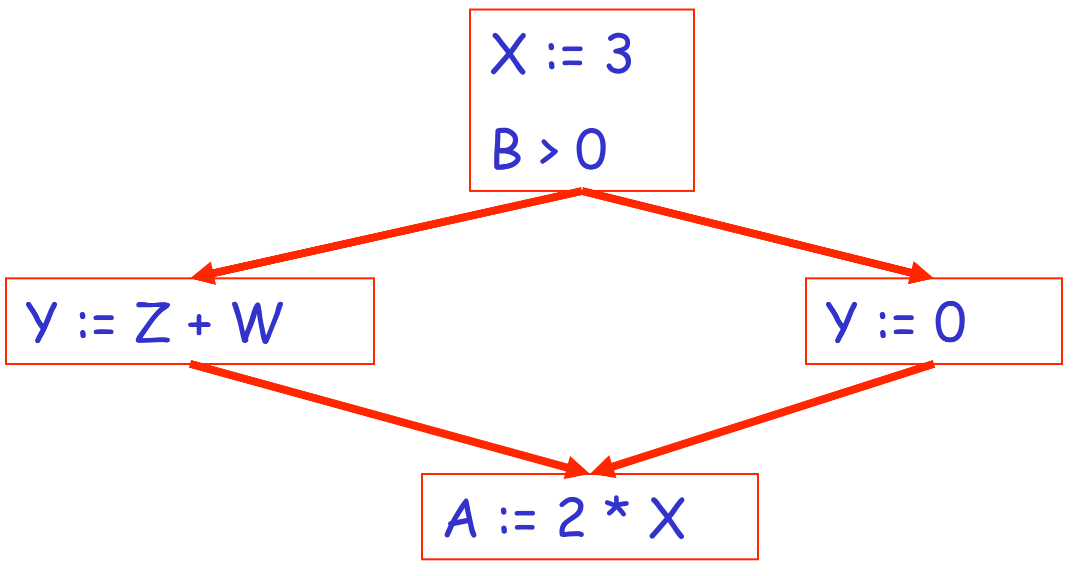
```
X := 3
Y := Z * W
Q := 3 + Y
```

→

```
Y := Z * W
Q := 3 + Y
```

Global Optimization

These optimizations can be extended to an entire control-flow graph



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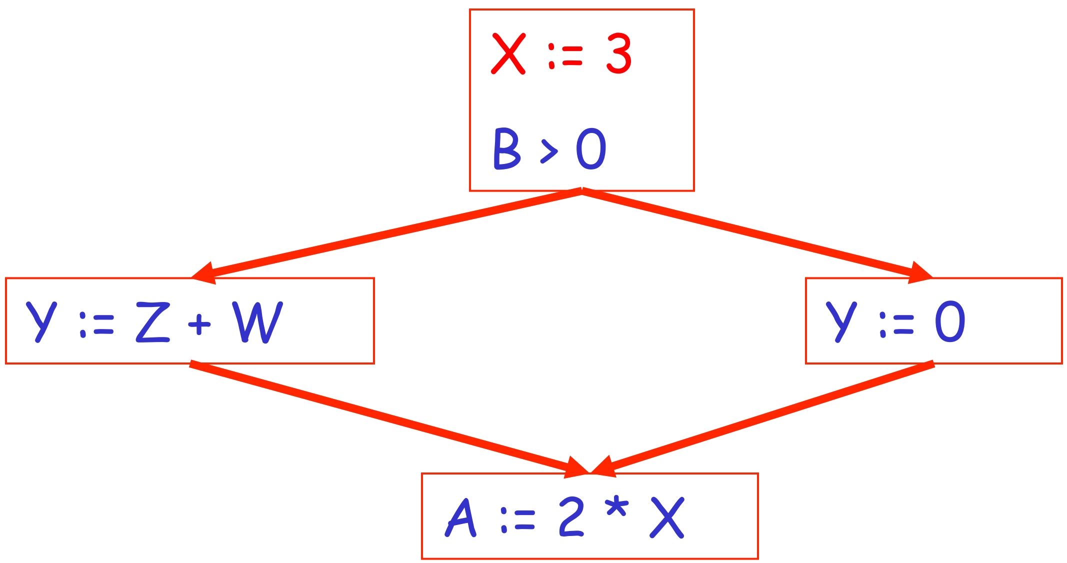

Global Optimization

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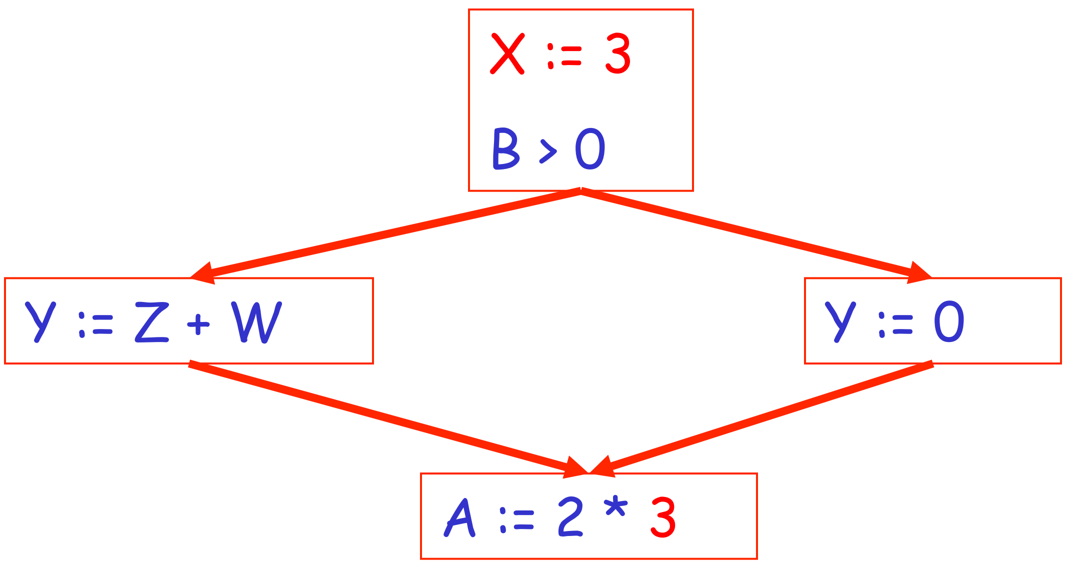
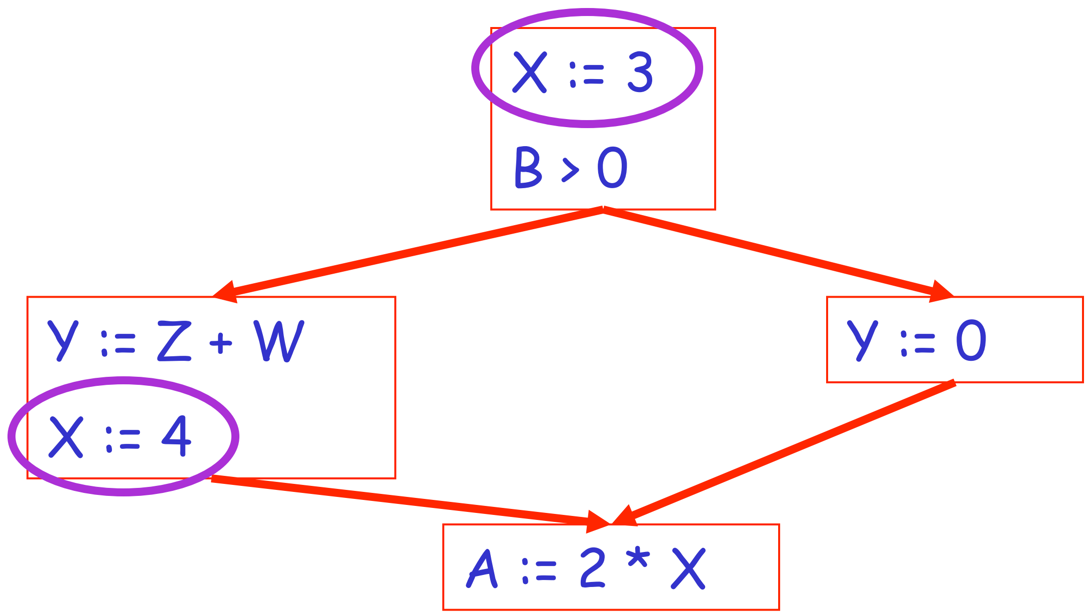


Correctness

- How do we know it is OK to globally propagate constants?
- There are situations where it is incorrect:

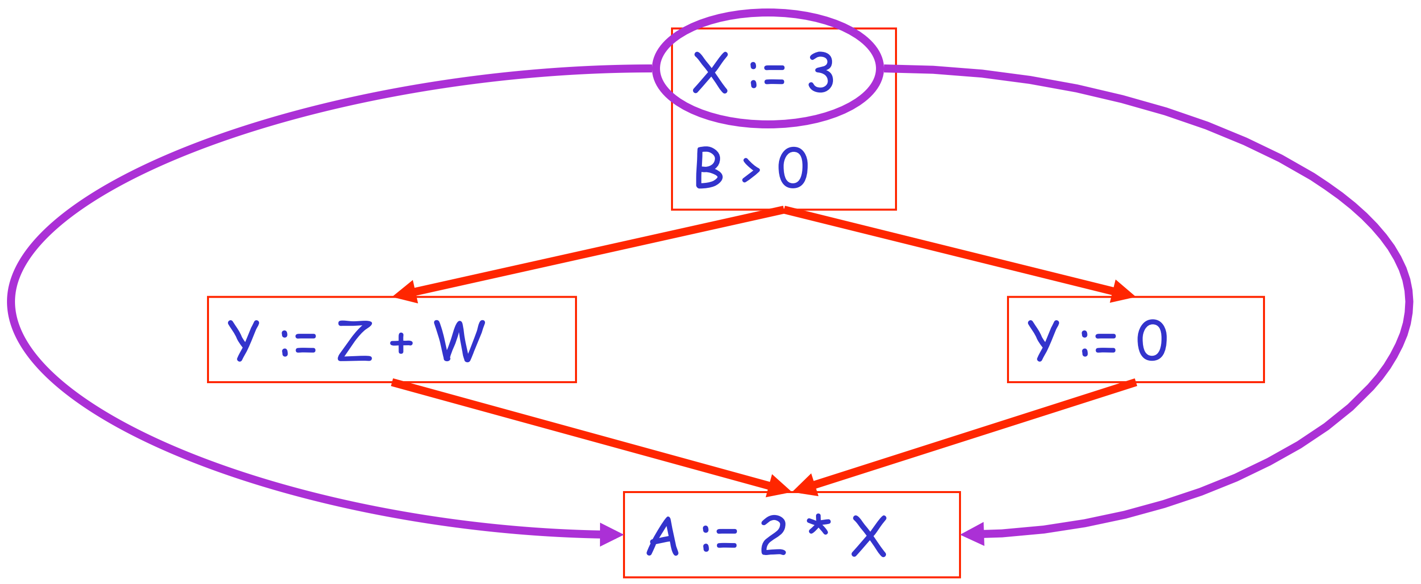


Correctness (Cont.)

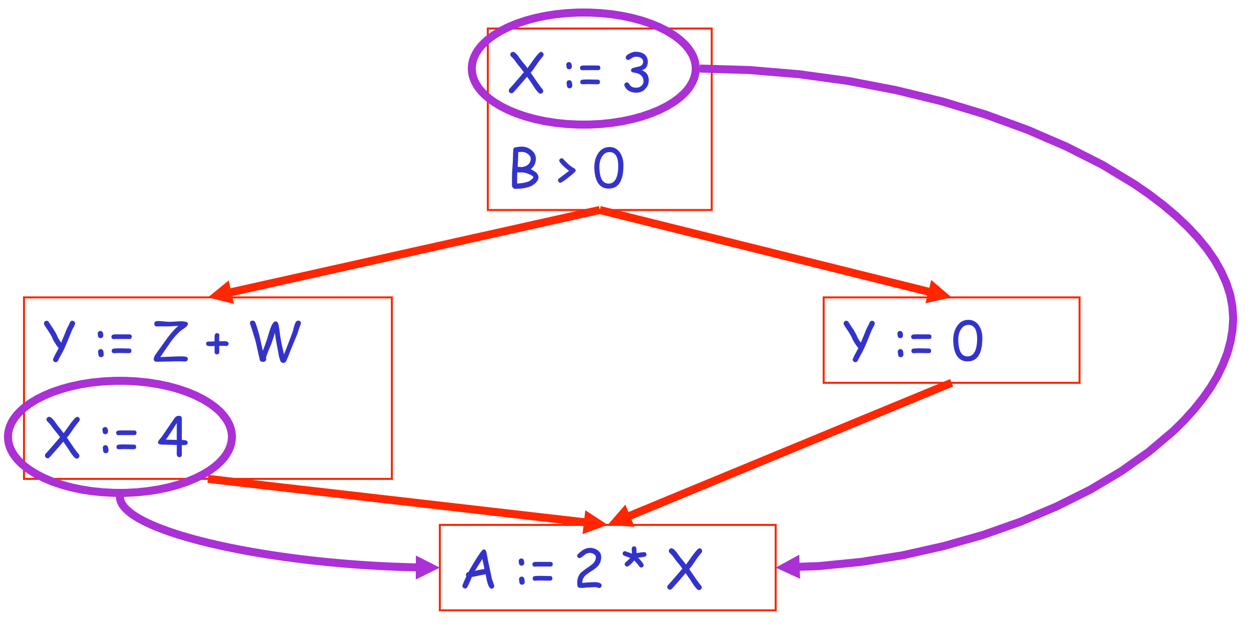
To replace a use of x by a constant k we must know that:

> *On every path to the use of x, the last assignment to x is x := k (Invariant #1)*

Example 1 Revisited



Example 2 Revisited



Discussion

- The correctness condition is not trivial to check
- "All paths" includes paths around loops and through branches of conditionals
- Checking the condition requires global analysis
  - An analysis of the entire control-flow graph

Global Analysis

Global optimization tasks share several traits:

- The optimization depends on knowing a property X at a particular point in program execution
- Proving X at any point requires knowledge of the entire program
- It is OK to be conservative. If the optimization requires X to be true, then want to know either
  - X is definitely true
  - Don't know if X is true
- It is always safe to say "don't know"

Global Analysis (Cont.)

- *Global dataflow analysis* is a standard technique for solving problems with these characteristics

- Global constant propagation is one example of an optimization that requires global dataflow analysis

Global Constant Propagation

- Global constant propagation can be performed at any point where Invariant #1 holds

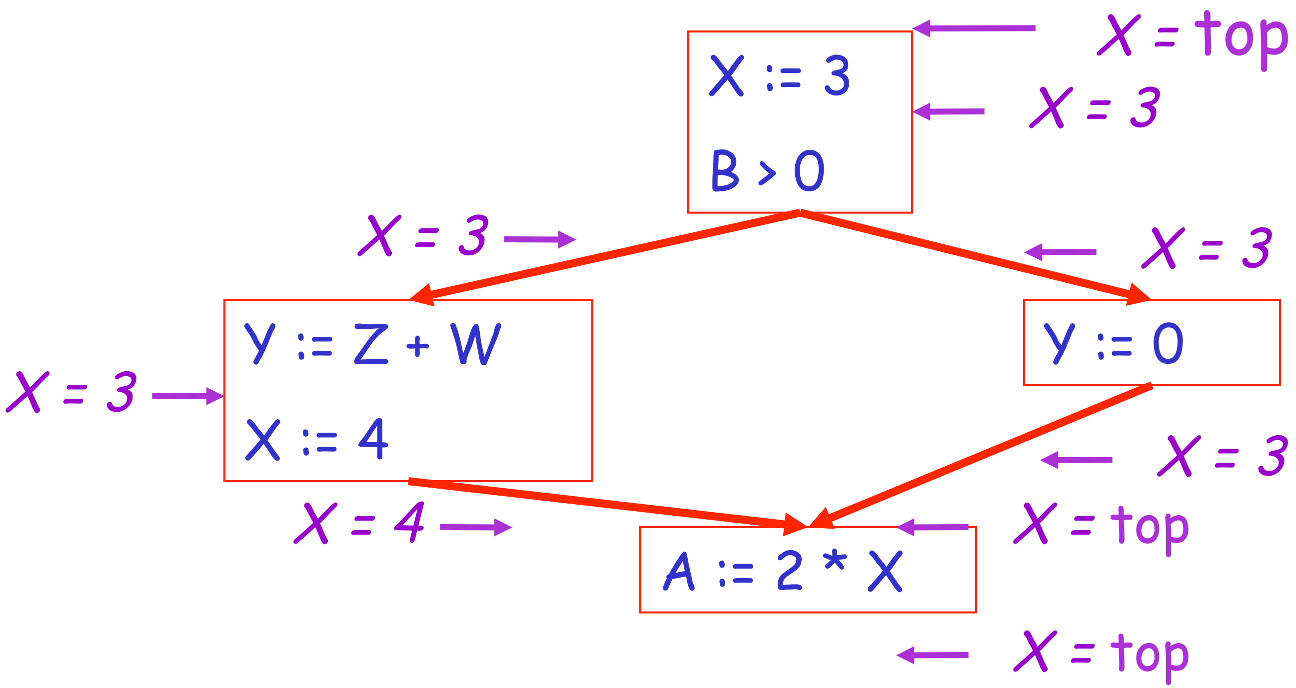
- Consider the case of computing Invariant #1 for a single variable X at all program points

Global Constant Propagation (Cont.)

- To make the problem precise, we associate one of the following values with X at every program point

value	*interpretation*
z	X=z means that analysis hasn't determined if control reaches that point
c	X = constant c
top	X is definitely not a constant

Example



Using the Information

- Given global constant information, it is easy to perform the optimization
  - Simply inspect the $x = ?$ associated with a statement using x
  - If x is constant at that point replace that use of x by the constant

- But how do we compute the properties $x = ?$

The Idea

The analysis of a complicated program can be expressed as a combination of simple rules relating the change in information between adjacent statements

Explanation

- The idea is to "push" or "transfer" information from one statement to the next
- For each statement s, we compute information about the value of x immediately before and after s

$C(x,s,in)$ = value of x before s

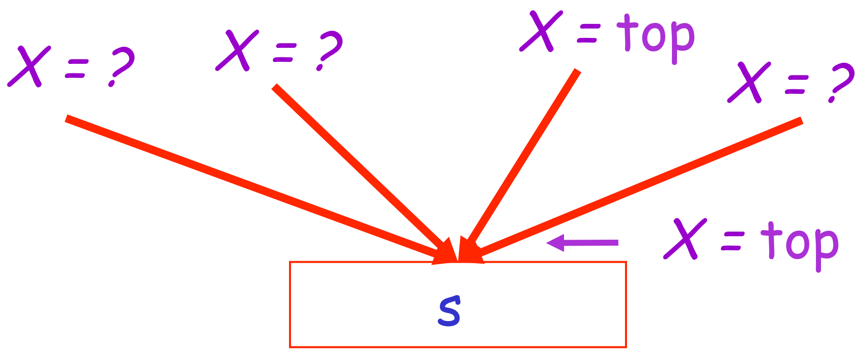
$C(x,s,out)$ = value of x after s

Transfer Functions

- Define a *transfer* function that transfers information one statement to another

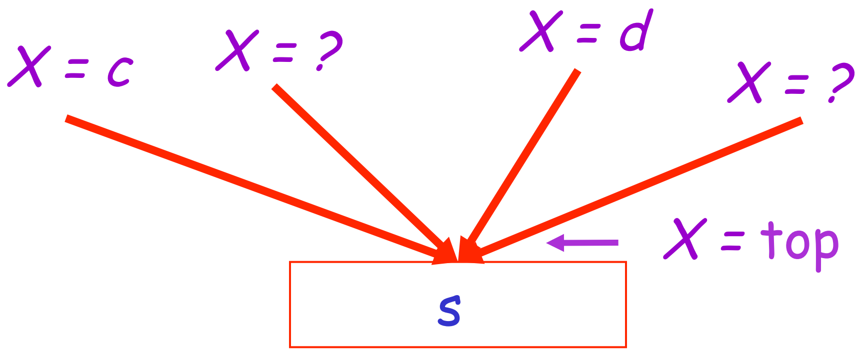
- In the following rules, let statement s have immediate predecessor statements $p_1,\ldots,p_n$

Rule 1



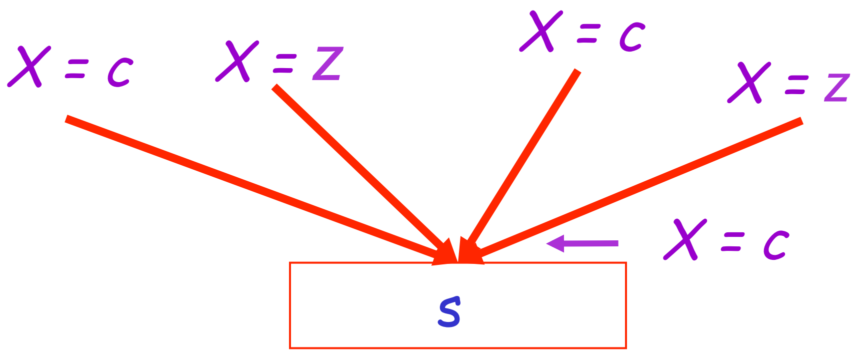
if $C(p_i, x, out) = top$ for any i, then $C(s, x, in) = top$

Rule 2



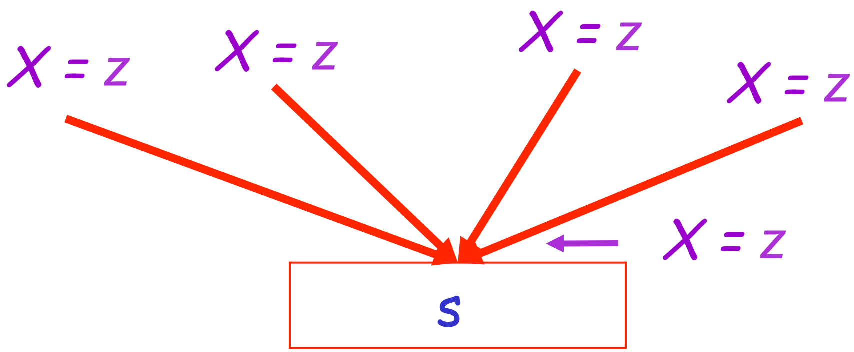
$C(p_i, x, out) = c$ & $C(p_j, x, out) = d$ & $d <> c$ then
$C(s, x, in) = top$

Rule 3



if $C(p_i, x, out) = c$ or Z for all i, then $C(s, x, in) = c$

Rule 4



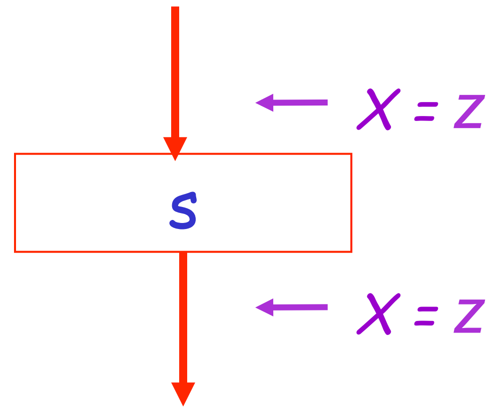
if $C(p_i, x, out) = z$ for all i, then $C(s, x, in) = z$

The Other Half

- Rules 1-4 relate the *out* of one statement to the *in* of the next statement

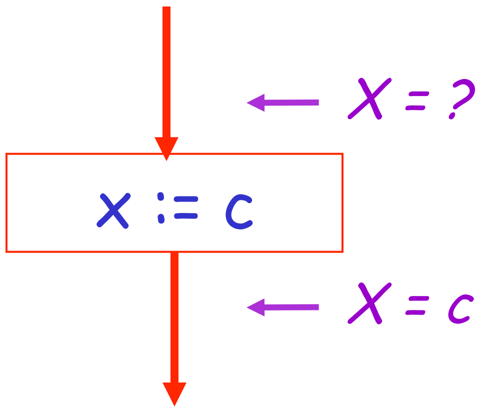
- Now we need rules relating the *in* of a statement to the *out* of the same statement

Rule 5



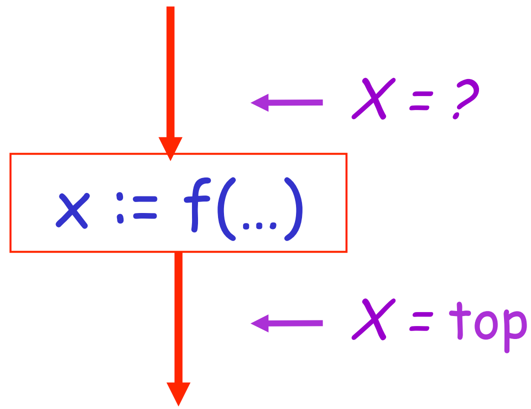
$C(s, x, out) = z$ if $C(s, x, in) = z$

Rule 6



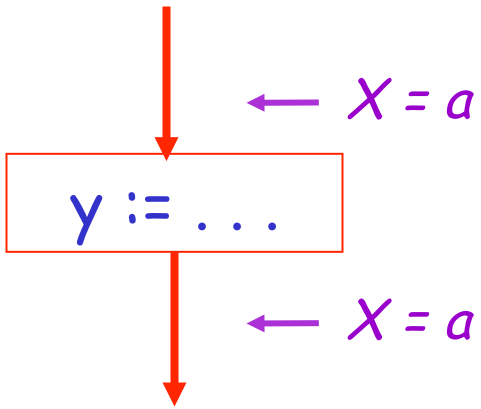
C(x := c, x, out) = c if c is a constant

Rule 7



$$C(x := f(\ldots), x, out) = top$$

Rule 8



$$C(y := \ldots, x, out) = C(y := \ldots, x, in) \text{ if } x \mathrel{<>} y$$

An Algorithm

1. For every entry s to the program, set $C(s, x, in) = top$

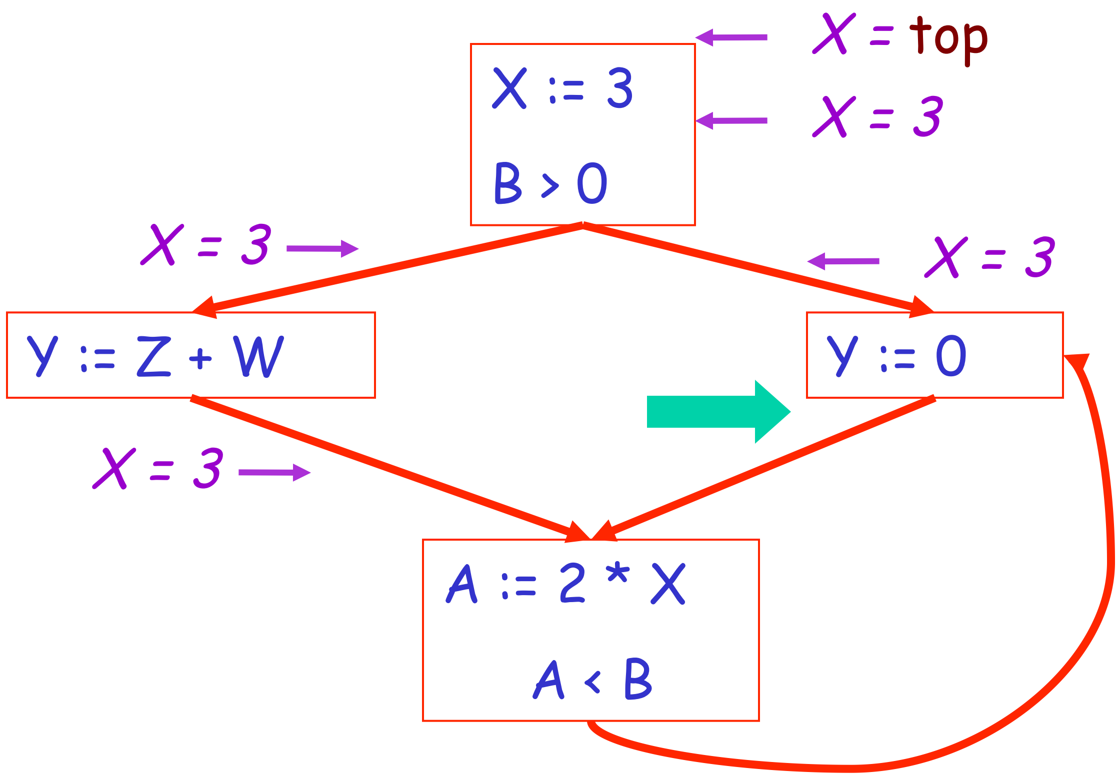
2. Set $C(s, x, in) = C(s, x, out) = z$ everywhere else

3. Repeat until all points satisfy 1-8:

   Pick s not satisfying 1-8 and update using the appropriate rule

The Value Z

- To understand why we need z, look at a loop



Discussion

- Consider the statement Y := 0
- To compute whether X is constant after this statement, we need to know whether X is constant at the two predecessors
  - X := 3
  - A := 2 * X
- But info for A := 2 * X depends on its predecessors, including Y := 0!

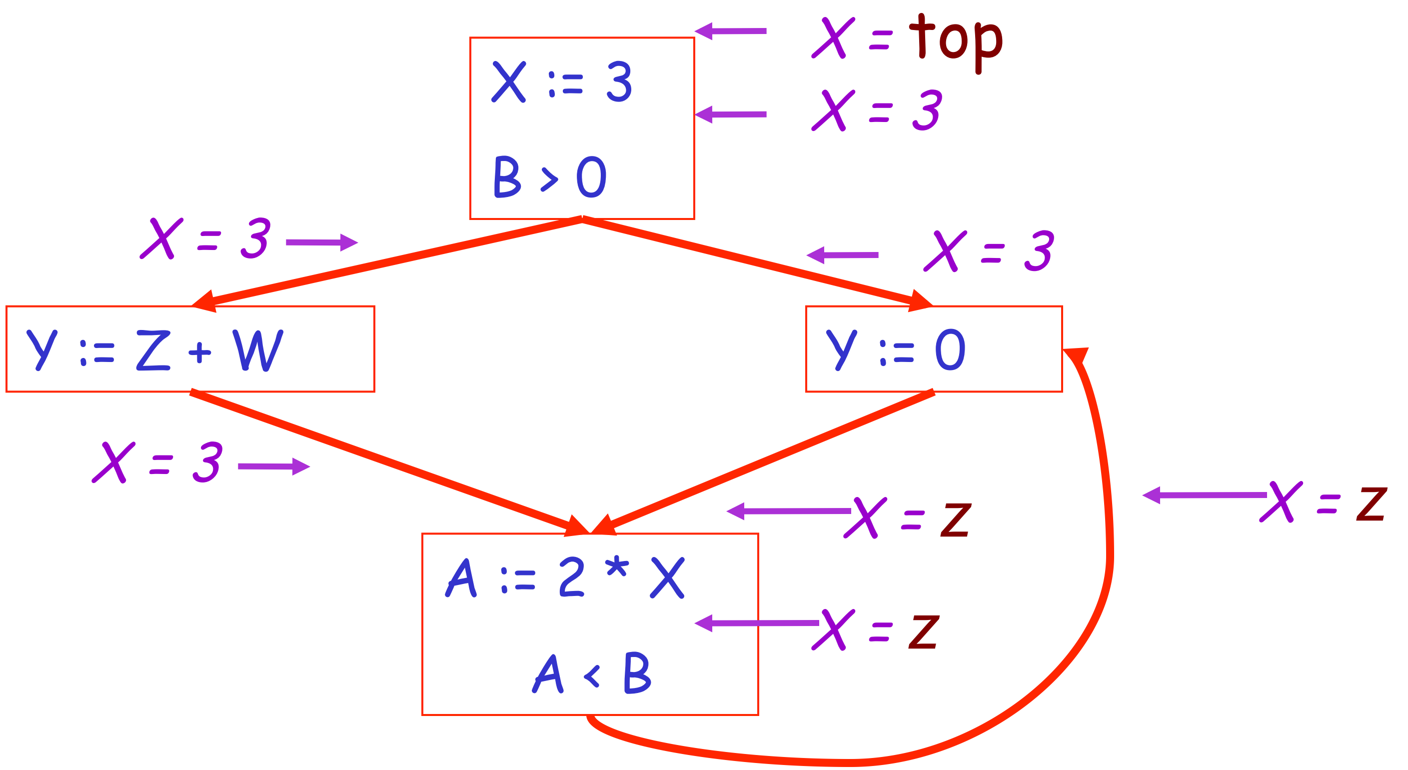
The Value Z (Cont.)

- Because of cycles, all points must have values at all times

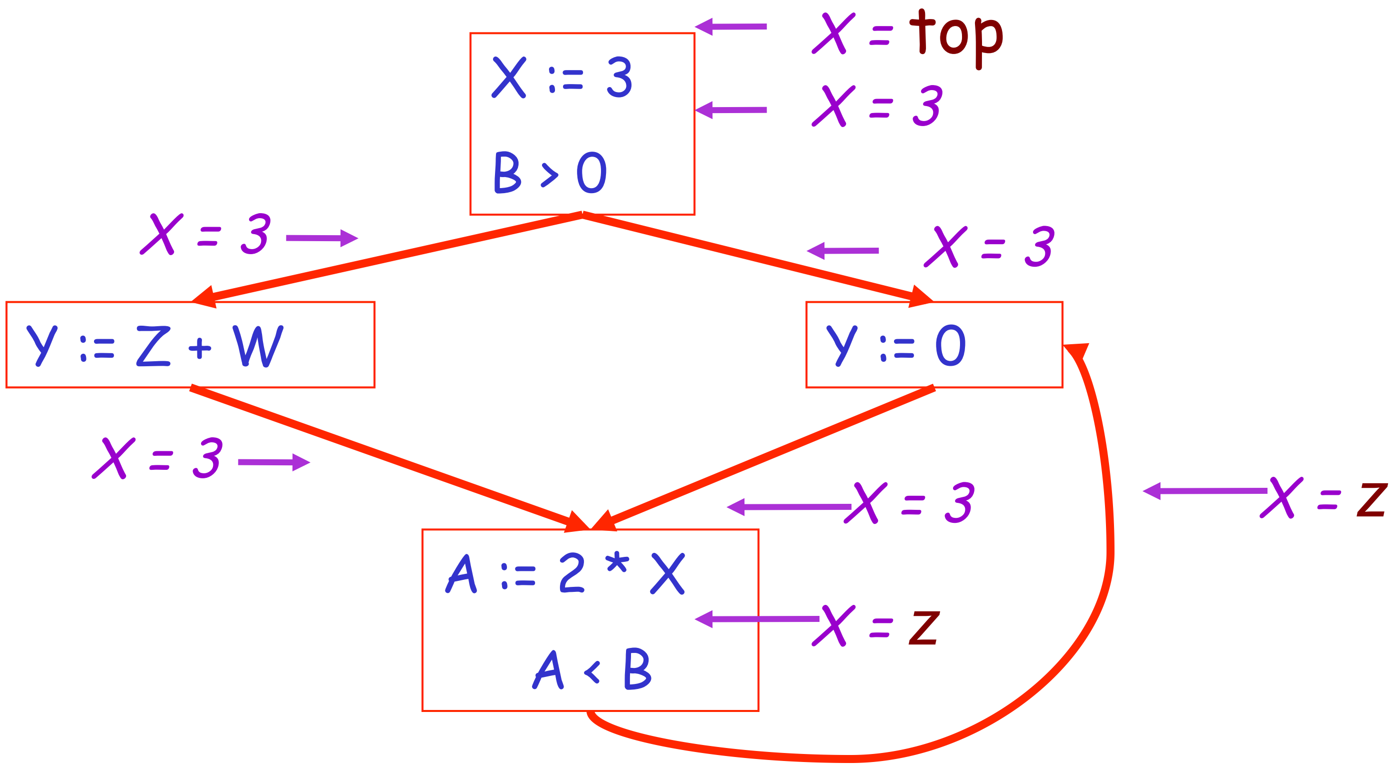
- Intuitively, assigning some initial value allows the analysis to break cycles

- The initial value z means "So far as we know, control never reaches this point"

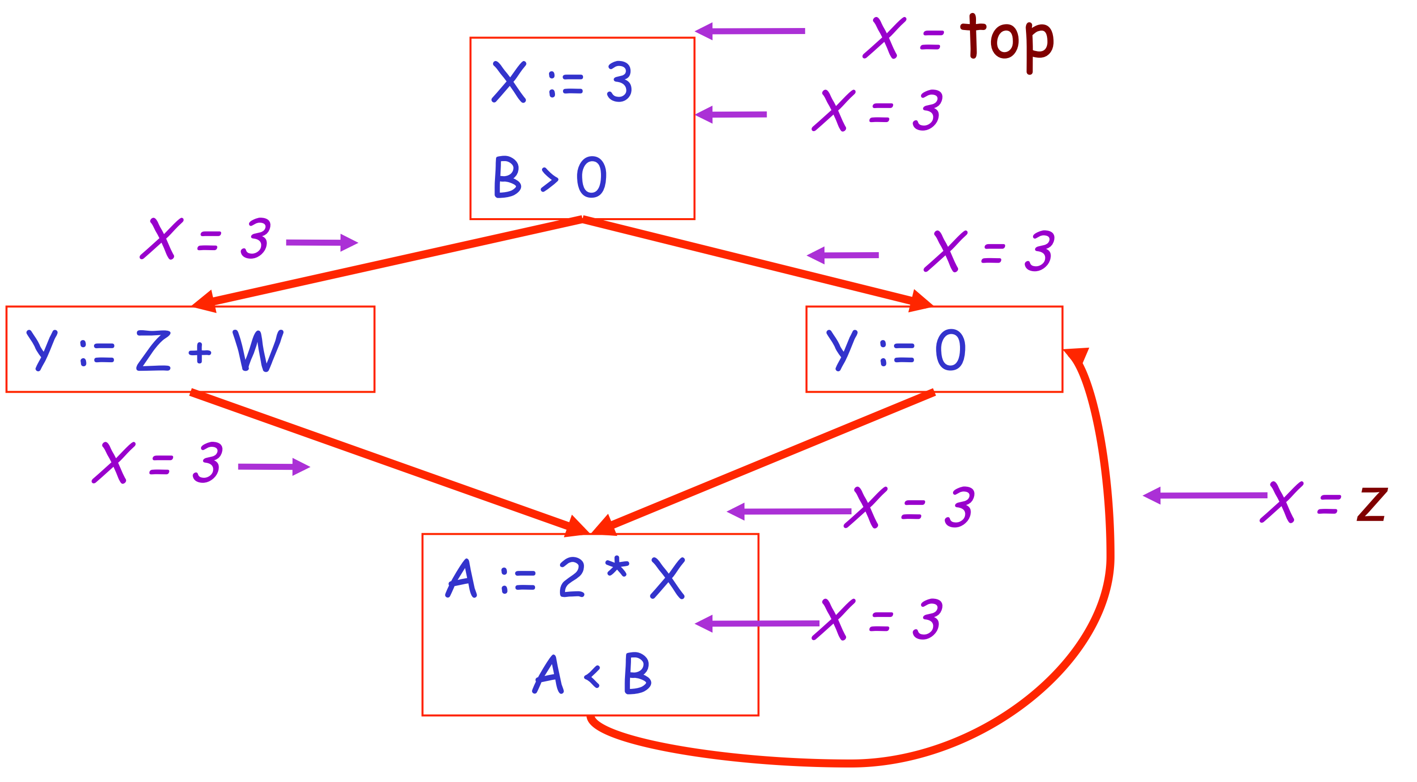
Example



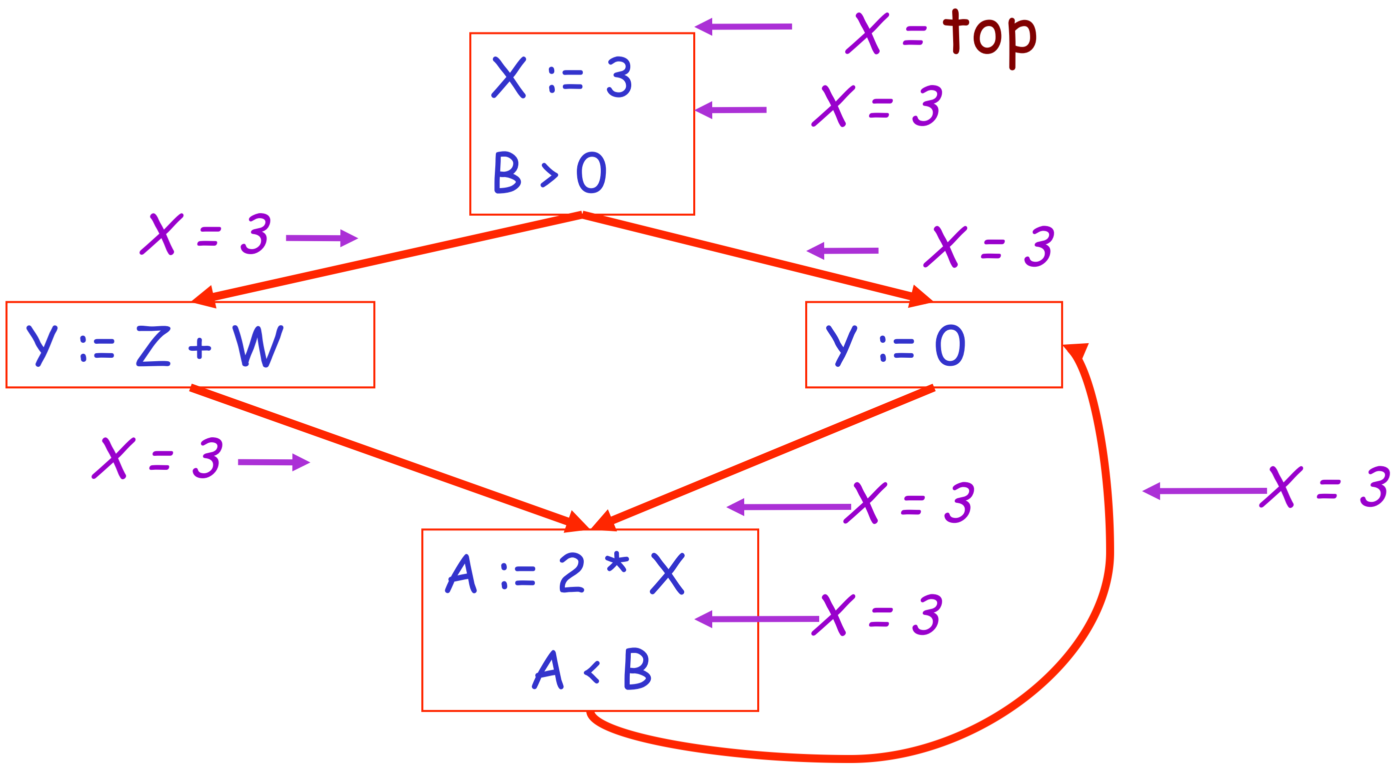
Example



Example



Example

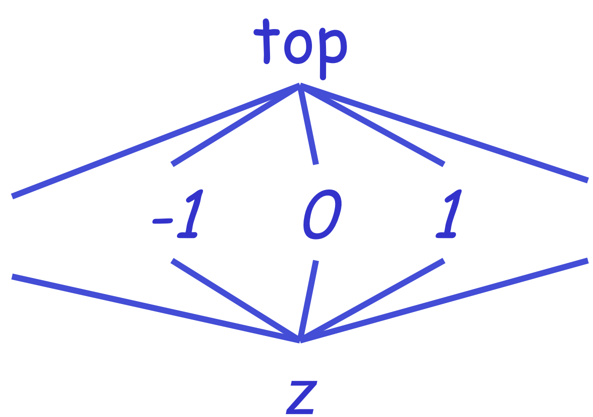


Orderings

- We can simplify the presentation of the analysis by ordering the values

$$z < c < top$$

- Drawing a picture with "lower" values drawn lower, we get



Orderings (Cont.)

- top is the greatest value, z is the least
  - All constants are in between and incomparable

- Let *lub* be the least-upper bound in this ordering

- Rules 1-4 can be written using lub:

  C(s, x, in) = lub { C(p, x, out) | p is a predecessor of s }

How do we argue that this algo terminates?

- Simply saying "repeat until nothing changes" doesn't guarantee that eventually nothing changes

- The use of lub explains why the algorithm terminates
  - Values start as z and only *increase*

    z can change to a constant, and a constant to top
  - Thus, C(s, x, _) can change at most twice

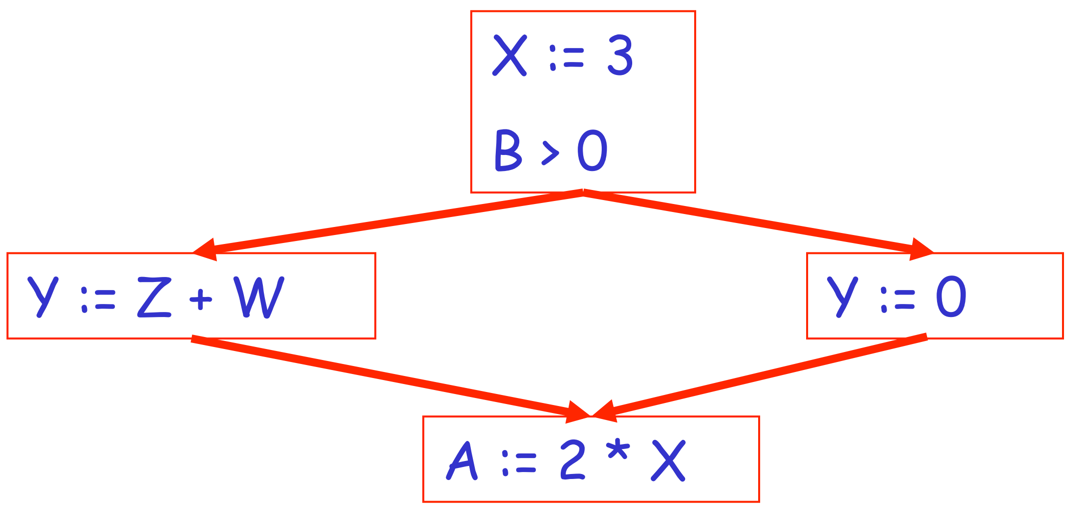
Termination (Cont.)

Thus the algorithm is linear in program size

Number of steps =
Number of C(….) value computed * 2 =
Number of program statements * 4

Liveness Analysis

Once constants have been globally propagated, we would like to eliminate dead code



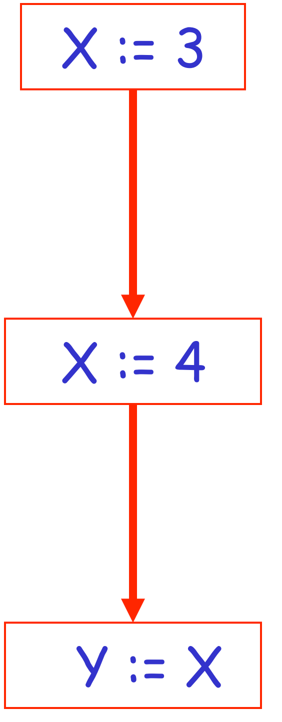
After constant propagation, X := 3 is dead (assuming X not used elsewhere)

New Example: Live and Dead

- The first value of x is *dead* (never used)

- The second value of x is *live* (may be used)

- Liveness is an important concept



Liveness

A variable x is live at statement s if

- There exists a statement s' that uses x such that
  - There is a path from s to s'
  - That path has no intervening assignment to x

Global Dead Code Elimination

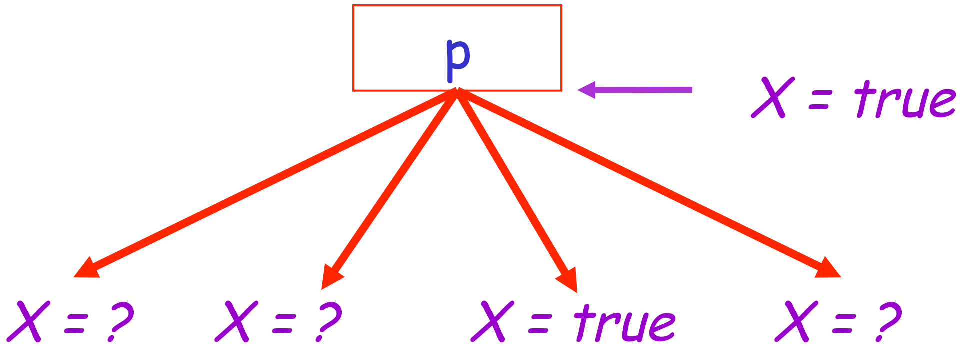
- A statement x := … is dead code if x is dead after the assignment
- Dead statements can be deleted from the program
- But we need liveness information first . . .

Computing Liveness

- We can express liveness in terms of information transferred between adjacent statements, just as in copy propagation

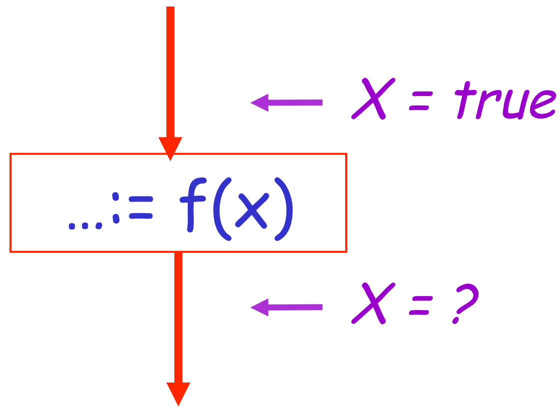
- Liveness is simpler than constant propagation, since it is a boolean property (true or false)

Liveness Rule 1



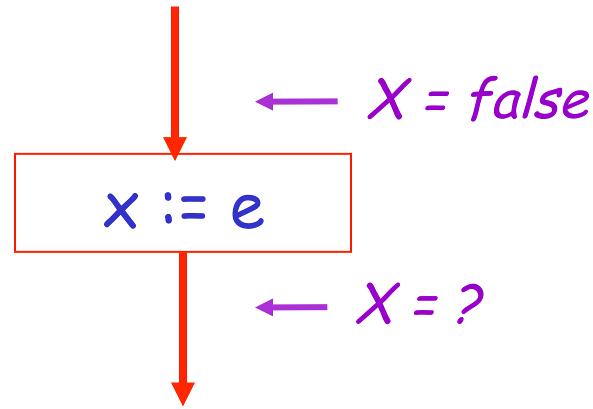
$$L(p, x, out) = \vee \{ L(s, x, in) \mid s \text{ a successor of } p \}$$

Liveness Rule 2



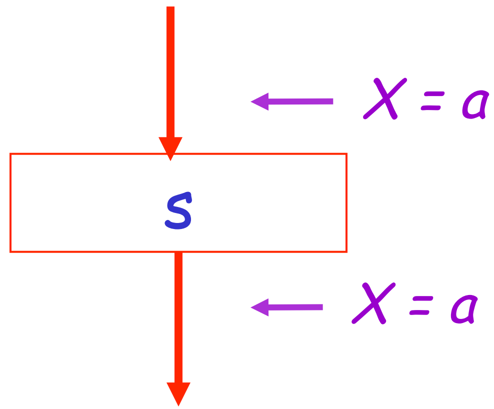
$L(s, x, in) = true$ if s refers to x on the rhs

Liveness Rule 3



$L(x := e, x, in) = false$ if e does not refer to x

Liveness Rule 4



L(s, x, in) = L(s, x, out) if s does not refer to x

Algorithm

1. Let all L(...) = false initially

2. Repeat until all statements s satisfy rules 1-4

   Pick s where one of 1-4 does not hold and update using the appropriate rule

Termination

- A value can change from false to true, but not the other way around
- Each value can change only once, so termination is guaranteed
- Once the analysis is computed, it is simple to eliminate dead code

Forward vs. Backward Analysis

We've seen two kinds of analysis:

Constant propagation is a *forwards* analysis: information is pushed from inputs to outputs

Liveness is a *backwards* analysis: information is pushed from outputs back towards inputs

Analysis

- There are many other global flow analyses
- Most can be classified as either forward or backward
- Most also follow the methodology of local rules relating information between adjacent program points