How many rays do I need to trace?
Applying the Rose model to computer analysis of illumination systems

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Illumination systems are analyzed by tracing large numbers of rays through computer models of lamps, reflectors, lightpipes, and diffusers. Surface or volume emitters are modeled as sources of rays that have random starting locations and directions. Many millions of rays are traced from the emitters, through the system, to a collecting plane. The irradiance is calculated by dividing the collecting plane into rectangular pixels and adding up the power of all rays in each pixel. The irradiance is then equal to the power in each pixel divided by the pixel area. This is where the trouble begins. While actual systems produce a smoothly varying irradiance, the computer simulation superimposes a grainy or noisy background. This obscures features of interest and makes it difficult to evaluate the uniformity. The best solution to this problem is to trace more rays. But how many rays are enough? Because tracing millions of rays is time consuming, we want to estimate how many rays are really needed. This estimate is provided by the Rose model.[1]

The Rose Model

Albert Rose designed television camera tubes at RCA. An early objective of this work was to develop camera tubes that could work at low light levels. At low light levels the photon nature of light becomes important, and the random distribution of arriving photons sets a limit on the achievable image quality. Albert Rose developed an equation relating image resolution, contrast, and signal-to-noise ratio to the number of photons incident on an image plane.

Illumination systems are not starved for photons. Nobody notices the arrival of individual photons on a projection screen, in a headlight field, or an LCD backlight. But computer ray trace simulations of these systems are starved for rays. In fact, the source simulations produce irradiance patterns of exactly the same sort observed in low light conditions on a camera tube, and for exactly the same reason: the rays departing from the source model are randomly distributed in both space and direction just as photons are in incoherent sources.

The objective of an illumination calculation is to simulate the irradiance distribution on an image plane of a given size. The number of pixels in the calculation is selected based on the desired image resolution (the smallest feature anticipated in the distribution) and aesthetic considerations. Less obvious, but equally important as resolution, is the need to resolve features of a given contrast. Contrast is the relative change in irradiance. If the background irradiance is E, and a bright feature is present with irradiance $E + \Delta E$, then the irradiance contrast C is

$$C = \frac{\Delta E}{E}\,. \tag{1}$$

It is the job of the analyst to select a sufficiently large number of pixels N and trace a sufficiently large number of rays to both resolve the spatial features of a distribution *and* resolve the contrast in the irradiance. If it is anticipated that the smallest feature is the same size as one pixel, then the Rose model states that the total number of rays Q incident on the image plane must exceed

$$Q_{\min} = N\,\frac{1}{C^2}\,k^2\,, \tag{2}$$

where N is the number of pixels, C is the contrast of the irradiance distribution, and k is the needed signal-to-noise ratio. We will discuss the signal-to-noise ratio later, and show that typical values for k are in the range 3 to 6.

Let us do an example. If an irradiance calculation is performed over a 100×100 square array of pixels, and the desired irradiance contrast is 10% with a signal to noise ratio of 5, then the number of rays incident on the image plane must be greater than

$$Q_{\min} = 100^2 \times \frac{1}{0.1^2} \times 5^2 = 25\ \textit{million}\,. \tag{3}$$

If, as is common in an illumination calculation, the smallest feature in an irradiance distribution covers several or even many pixels, then the number of rays given in Equation (2) is an overestimate. An alternate form of the Rose model that covers this situation is

$$Q = \frac{A_o}{A}\,\frac{1}{C^2}\,k^2\,, \tag{4}$$

where A is the area of a feature in the irradiance distribution and A_o is the full calculation area. Going back to our example of an array with 100×100 *pixels*, if the feature of interest covers an area of 10×10 *pixels*, then the number of rays is reduced from 25 *million* to

$$Q_{\min} = \frac{100^2}{10^2} \times \frac{1}{0.1^2} \times 5^2 = 250{,}000. \tag{5}$$

This is two orders of magnitude smaller than the earlier estimate in Equation (3). The reason is simply this: a feature that covers more than one pixel (100 pixels in this example) collects more rays, thereby reducing the statistical variation in its average irradiance.

Signal-to-noise ratio

It remains to discuss the signal-to-noise ratio *k*. The random nature of the source model causes fluctuations in the calculated irradiance at the image plane. These fluctuations may be mistaken for real features. Conversely, statistical fluctuations in a real feature may (if the feature is brighter than the background) reduce it to the level of the background and remove it from view. To keep this from happening, we must trace enough rays to reduce the noise background to a level that is well below the signal.

To see a feature that is brighter than its surrounding background the feature's irradiance must be larger than the statistical fluctuations in the background. Suppose a feature with an area A and irradiance difference ΔE on a background that has a total area A_o. To resolve this feature unambiguously all of the statistical fluctuations A_o must be smaller than ΔE. Let σ be the standard deviation of the background fluctuations (within areas of size A) and set $\Delta E = k\sigma$ where k is the needed signal-to-noise ratio. Let $P_N(k\sigma)$ be the probability of obtaining a fluctuation that is larger than $k\sigma$ within areas A. To clearly resolve the feature and avoid a *false alarm*, in which a fluctuation is mistaken for a real feature, the probability of obtaining a fluctuation as large as $k\sigma$ must be very small. This probability is

$$P(\,false\;\;alarm) = 2\,\frac{A_o}{A}\,P_N\;\;(k\sigma) \tag{6}$$

The factor of 2 in this equation accounts for the possibility of having either bright or dark features. An important part of Equation (6) is that the required signal-to-noise depends on the feature size A. As A gets smaller (for a given fixed value of A_o) a larger signal to noise ratio is required to avoid false alarms. This is because there are more small features than large features in a given total area, and therefore more opportunities to obtain large fluctuations masquerading as real features.

The magnitude of statistical fluctuations usually follows a Normal distribution. In this case $P_N(k\sigma)$ is given by Table (1) for values of k from 1 to 6.5. To set an acceptable signal-to-noise ratio choose an acceptable false alarm probability (say 1%), and find a value of k that satisfies Equation (6). For example, a 10×10 pixel feature within a 100×100 pixel array has a ratio A_o/A of 100. Using our 1% false-alarm standard, we solve Equation (6) for $P_N(k\sigma)$ to get a value of 5×10^{-5}. Table (1) then shows that k must be just under 4 to achieve this.

k	**$P_N(k\sigma)$**	**k**	**$P_N(k\sigma)$**
1	0.16	4	3.2×10^{-5}
1.5	0.067	4.5	3.4×10^{-7}
2	0.023	5	2.9×10^{-7}
2.5	6.2×10^{-3}	5.5	1.9×10^{-8}
3	1.3×10^{-3}	6	9.9×10^{-10}
3.5	2.3×10^{-4}	6.5	4.0×10^{-11}

Table (1): Probability of obtaining a fluctuation greater than *k* standard deviations for a Normal distribution.

Application of the Rose model to the irradiance from an arc lamp

To test these ideas we apply the Rose model to the calculated irradiance distribution of an arc lamp. Irradiance calculations were done with a computer model in the Advanced Systems Analysis Program (ASAP), a product of Breault Research Organization.[2] Figure (1) shows a rendered picture of the arc lamp model. It consists of an arc source supported by three arms, a reflector, and an *integrating bar*, which is a rectangular piece of glass that is used to direct light to the desired illumination area. Figure (2) shows the calculated irradiance pattern over a 51×69 grid of pixels using as many as ten-million rays and as few as three-thousand rays. The number of rays collected on the image plane is given in the caption above the top left corner of each image. The last image shows the irradiance distribution calculated with ten-million rays. Three features of this distribution will be examined: 1. The bright region in the center of the image plane, 2. the three bright but narrow arms that extend from the bright center, and 3. the very slightly darker ring that is about halfway from the center to the top (and bottom) edge.

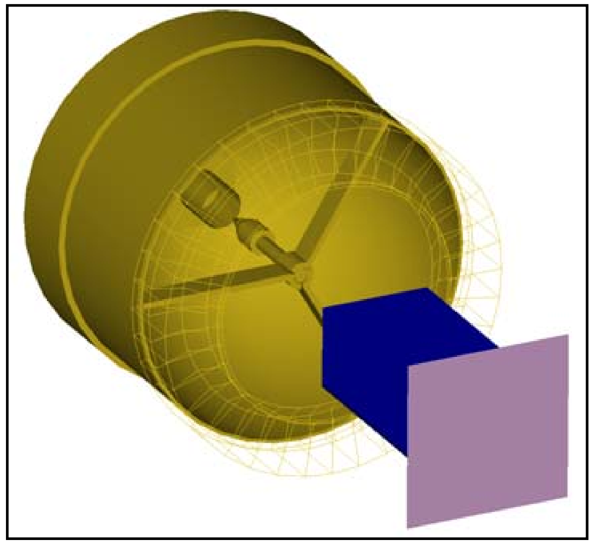

Figure (1): Picture of an arc-lamp, reflector, integrating bar, and image plane of an illumination system model that is used to demonstrate the Rose model.

The first feature of interest is the bright region in the center. How many rays are needed in the image plane to see this? This is two-step process. First the required signal-to-noise ratio is calculated with Equation (6), then the minimum number of rays is calculated with Equation (4). The bright region, including the three bright arms that extend from the center, occupies an area of about 40 pixels. There are total of 51×69 = 3519 pixels altogether, so the ratio A_o/A is 3519/40=88. Assume a tolerable false-alarm probability of 1%. Solving Equation (6) for $P_N(k\sigma)$ gives

$$P_N(k\,\sigma) = 0.01 \times \frac{1}{2} \times \frac{40}{3519} = 5.7 \times 10^{-5} \ . \qquad \textbf{(7)}$$

Interpolating from Table (1), the signal-to-noise ratio k that satisfies Equation (7) is 3.9. Now use Equation (4). The contrast C of the bright spot with its immediate surroundings is about 0.5. Substituting for the variables in Equation (4) gives the following value for the number of rays:

$$Q_{\min} = \frac{3519}{40} \times \frac{1}{0.5^2} \times 3.9^2 = 5400. \qquad \textbf{(8)}$$

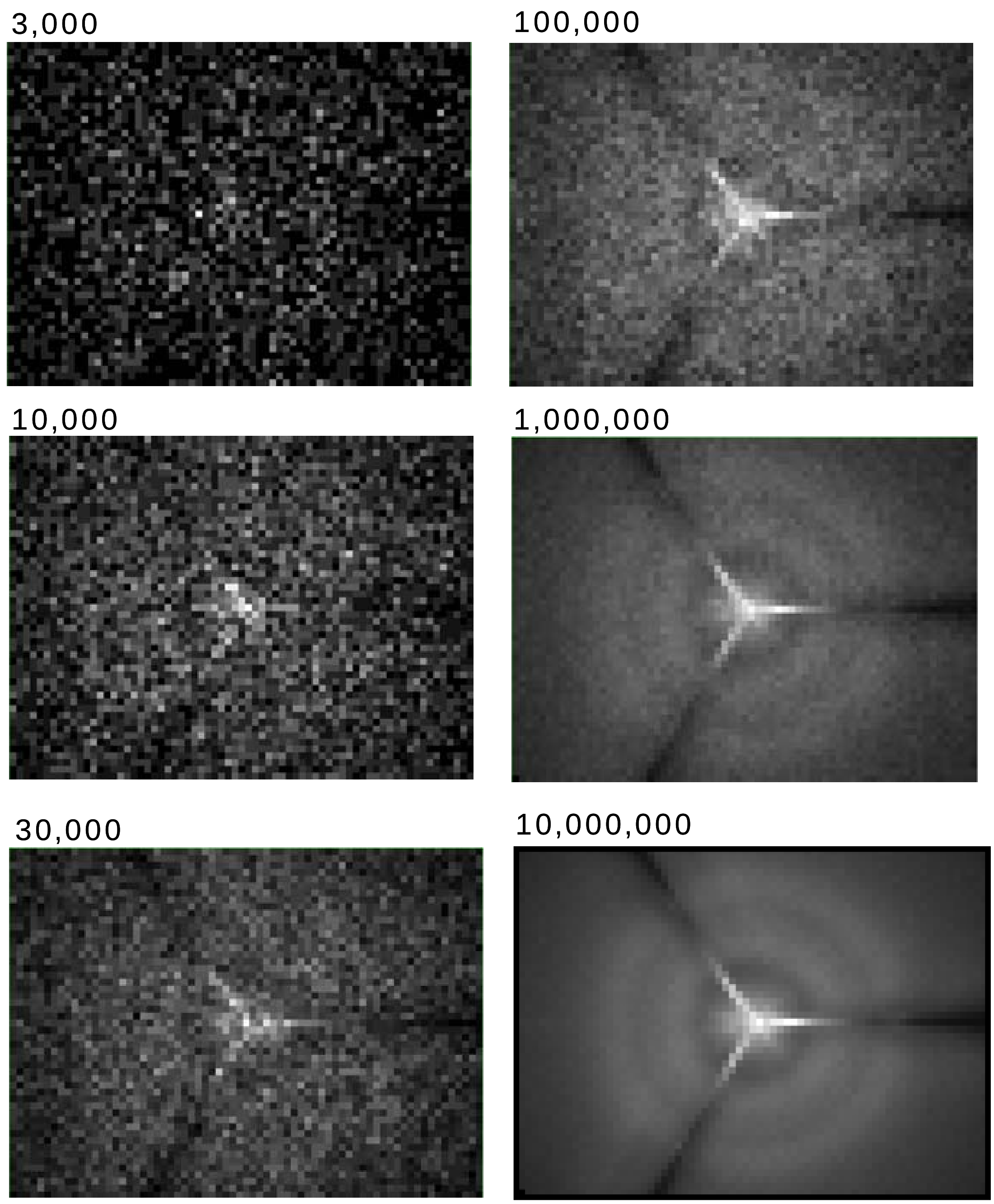

Figure (2): Irradiance patterns obtained by tracing different numbers of rays; from three-thousand to ten-million.

Inspection of Figure (2) shows that the bright spot is clearly visible when 10,000 rays are collected, but its presence is all but invisible with 3000 rays. This is consistent with the result in Equation (8).

The three arms extending from the bright center in Figure (2) are just as bright as the center, but occupy only 7 pixels each, instead of 40 pixels for the entire bright region. The ratio A_o/A is 3519/7 for each arm. Using this value in Equation (6) and solving for k gives a signal-to-noise ratio of 4.4. Applying Equation (4) for the number of rays gives

$$Q_{\min} = \frac{3519}{7} \times \frac{1}{0.5^2} \times 4.4^2 = 39{,}000\,. \tag{9}$$

Inspection of Figure (2) shows that these arms are barely visible with 30,000 rays, but that they are bright and clear with 100,000 rays; consistent with value given by Equation (9).

Finally, consider the very-slightly darker ring within the halo that surrounds the bright center. The contrast of this ring is 1%, and there are approximately 200 pixels within the ring. Solving Equation (6) for k gives a value of 3.5. The number of rays required to see the ring is then given by Equation (4):

$$Q_{\min} = \frac{3519}{200} \times \frac{1}{0.01^2} \times 3.5^2 = 2.2 \quad \textit{million}\,. \tag{10}$$

Inspection of Figures (2) shows that the ring is clearly visible with ten-million rays, barely visible with one-million, and essentially invisible with 100,000; consistent with Equation (10).

Summary

The Rose model is an effective tool for estimating the number of rays that are needed to resolve features in an illumination calculation. Given the need to resolve a feature of a given area and contrast, and given a tolerable false-alarm probability, Equations (4) and (6), along with Table (1), give quick and easy estimates for the number of rays we must collect to obtain meaningful results.

References

1. Albert Rose, Vision: Human and Electronic, (Plenum Press, New York, 1973).

2. Breault Research Organization, 6400 East Grant Road, Suite 350, Tucson, Arizona 85715.