

For a Bayesian analysis comparing two groups we use the following sampling model (Hoff p. 127):

$$\begin{aligned} Y_{i,1} &= \mu + \delta + \epsilon_{i,1} \\ Y_{i,2} &= \mu + \delta + \epsilon_{i,2} \\ \{\epsilon_{i,j}\} &\sim \text{i.i.d. normal}(0, \sigma^2). \end{aligned}$$

Letting $\theta_1 = \mu + \delta$ and $\theta_2 = \mu - \delta$ we see that $\delta = (\theta_1 - \theta_2) / 2$ is half the population difference in means, and $\mu = (\theta_1 + \theta_2) / 2$ is the pooled average. We'll assume conjugate prior distributions

$$\begin{aligned} p(\mu, \delta, \sigma^2) &= p(\mu) \times p(\delta) \times p(\sigma^2) \\ \mu &\sim \text{normal}(\mu_0, \gamma_0^2) \\ \delta &\sim \text{normal}(\delta_0, \tau_0^2) \\ \sigma^2 &\sim \text{inverse-gamma}(\nu_0/2, \nu_0\sigma_0^2/2), \end{aligned}$$

from which the full conditional distributions follow:

$$\{\mu | \mathbf{y}_1, \mathbf{y}_2, \delta, \sigma^2\} \sim \text{normal}(\mu_n, \gamma_n^2), \text{ where}$$

$$\mu_n = \gamma_n^2 \times \left[\frac{\mu_0}{\gamma_0^2} + \frac{\sum_{i=1}^{n_1} (y_{i,1} - \delta) + \sum_{i=1}^{n_2} (y_{i,2} + \delta)}{\sigma^2} \right]$$

$$\gamma_n^2 = \left[\frac{1}{\gamma_0^2} + \frac{(n_1 + n_2)}{\sigma^2} \right]^{-1}$$

The math scores were based on results of a national exam in the United States, standardized to produce a nationwide mean of 50 and a standard deviation of 10. Unless these two schools were known in advance to be extremely exceptional, reasonable prior parameters can be based on this information. For the prior distributions of μ and σ^2 , we'll take $\mu_0 = 50$ and $\sigma_0^2 = 10^2 = 100$, although this latter value is likely to be an overestimate of the within-school sampling variability. We'll make these prior distributions somewhat diffuse, with $\gamma_0^2 = 25^2 = 625$ and $\nu_0 = 1$. For the prior distribution on δ , choosing $\delta_0 = 0$ represents the prior opinion that $\theta_1 > \theta_2$ and $\theta_2 > \theta_1$ are equally probable. Finally, since the scores are bounded between 0 and 100, half the difference between θ_1 and θ_2 must be less than 50 in absolute value, so a value of $\tau_0^2 = 25^2 = 625$ seems reasonably diffuse.

