$Y_1, ..., Y_n$ are *exchangeable*
$\Rightarrow$
$Y_1, ..., Y_n | \theta$ are conditionally i.i.d., $\theta \sim \pi(\theta)$ (de Finetti)
$\Rightarrow$

$$
\begin{aligned}
p(\tilde{Y} = \tilde{y} | Y_1 = y_1, ..., Y_n = y_n) &= \frac{\int p(\tilde{y}, y_1, ..., y_n | \theta)\pi(\theta) d\theta}{\int p(y_1, ..., y_n | \theta)\pi(\theta) d\theta} && (1) \\
& && (2) \\
&= \frac{\int p(\tilde{y}|\theta) p(y_1, ..., y_n | \theta)\pi(\theta) d\theta}{\int p(y_1, ..., y_n | \theta)\pi(\theta) d\theta} && (3) \\
& && (4) \\
&= \frac{\int p(\tilde{y}|\theta) p(\theta | y_1, ..., y_n) p(y_1, ..., y_n) d\theta}{\int p(y_1, ..., y_n | \theta)\pi(\theta) d\theta} && (5) \\
& && (6) \\
&= \frac{\cancel{p(y_1, ..., y_n)} \int p(\tilde{y}|\theta) p(\theta | y_1, ..., y_n) d\theta}{\cancel{p(y_1, ..., y_n)}} && (7) \\
& && (8) \\
&= \int p(\tilde{y}|\theta) p(\theta | y_1, ..., y_n) d\theta && (9)
\end{aligned}
$$