(Hoff p. 152ff.)

Let $\mathbf{y}$ be the n-dimensional column vector $(y_1, ..., y_n)^T$ and let $\mathbf{X}$ be the $n \times p$ matrix whose ith row is $\mathbf{x}_i = \{x_{i,1}, x_{i,2}, ..., x_{i,p}\}$. Then the normal regression model is

$$\{\mathbf{y}|\mathbf{X}, \beta, \sigma^2\} \sim \text{multivariate normal}\left(\mathbf{X}\beta, \sigma^2\mathbf{I}\right),$$

where $\mathbf{I}$ is the $p \times p$ identity matrix and

$$\mathbf{X}\beta = \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_n \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_p \end{pmatrix} = \begin{pmatrix} \beta_1 x_{1,1} + \cdots + \beta_p x_{1,p} \\ \vdots \\ \beta_1 x_{n,1} + \cdots + \beta_p x_{n,p} \end{pmatrix} = \begin{pmatrix} E\left[Y_1|\beta, \mathbf{x}_1\right] \\ \vdots \\ E\left[Y_n|\beta, \mathbf{x}_n\right] \end{pmatrix}$$

We compute the ordinary least squares estimates

$$\hat{\beta}_{ols} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{y}$$

and

$$\hat{\sigma}^2_{ols} = \frac{SSR\left(\hat{\beta}_{ols}\right)}{(n-p)} = \frac{\sum\left(y_i - \hat{\beta}^T_{ols}x_i\right)^2}{(n-p)}.$$