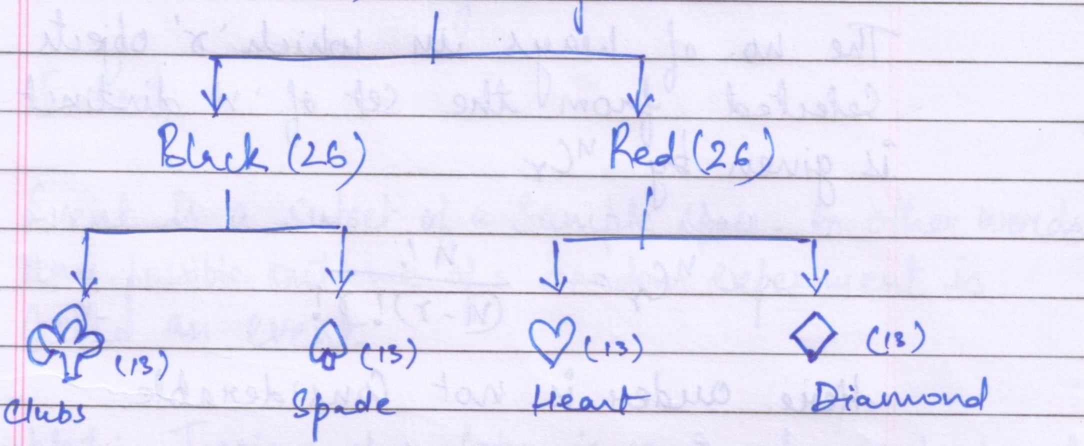
Probability and statistics

Coins:- A coin contains two sides. In that one side has "head" (H) and other side has "tail" (T).

Die :- A die is a small cube which contains six faces. These faces are labelled by 1, 2, 3, 4, 5, 6.

Pack :- Pack (52) without jockers



i.e A pack of 52 cards consists 4 suits namely club suit, spade suit, Diamond suit and heart suit. Also each suit consist 13 cards. They are A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K. Here 9 cards are numbered cards (2, 3, ... 7) and 4 cards are faced cards or are figured cards (A, J, Q, K).

Permutation

The no. of arrangements of 'r' objects from a set of 'n' distinct objects is given by ^nP_r.

$$^nP_r = \frac{n!}{(n-r)!}$$

Here order is considerable.

Combinations

The no. of ways in which 'r' objects can be selected from the set of 'n' distinct objects is given by ^nC_r

$$^nC_r = \frac{n!}{(n-r)!\,r!}$$

Here order is not considerable.

Note

~~If a work~~ If an event can happen in 'm' ways and when this has happened another event can happen in 'n' ways then the no. of ways in which both events can happen in the specified order is $m \times n$.

Random experiment

Let an experiment be repeated under essentially the same conditions and let it result be in any one of the several possible outcomes then the experiment is called a random experiment.

Sample Space

The set of all possible outcomes of a random experiment is called Sample Space. In general which is denoted by S.

Events

Event is a subset of a Sample space. In other words Any possible outcome of a random experiment is called an event.

Example 1: Tossing of a coin is a random experiment

Here the Sample Space 'S' = {H, T} also
{H} {T}, {H, T}, { } are events

Example 2: Throwing a die is a random experiment.

Here Sample Space = {1, 2, 3, 4, 5, 6}

Here E_1 = {2, 4, 6} E_5 = {2}
E_2 = {1, 3, 5} E_6 = {1, 2, 3, 4, 5, 6}
E_3 = {3, 6} E_7 = { } To get '8 number when throwing a die'
E_4 = {1}

Exhaustive events

A total no. of possible outcomes individually in any random experiment is known as exhaustive events.

Example 1: In tossing a coin there are two exhaustive events. There are {H}, {T}
Since here S = {H, T}

lly: When tossing two coins, we have
S = {TT, TH, HT, HH}
∴ The no. of exhaustive events are 4 they are {TT}, {TH}, {HT}, {HH}

So clearly, the total no. of exhaustive events when tossing 'n' coins is given by 2^n.

Example 2: In throwing a single die we have
S = {1, 2, 3, 4, 5, 6}
here the no. of exhaustive events are 6.
they are {1}, {2}, {3}, {4}, {5}, {6}

lly: When we are throwing 2 dice, the total no. of exhaustive events are 36.
Since here S = {(1,1), (1,2) - - - (1,6)
(2,1), (2,2) - - - (2,6)
(3,1), (3,2) - - - (3,6)
(4,1), (4,2) - - - (4,6)
⋮ ⋮
(6,1) (6,2) - — (6,6)}

Clearly, when we are throwing 'n' dice, the total no. of exhaustive events is 6^n.

Example 3: The total no. of ways (Exhaustive events) to draw 'n' cards from a well shuffeled pack of 52 cards is ${}^{52}C_n$.

Mutually Exclusive events (Disjoint events)

The events are said to be mutually exclusive if the happening of any one event rulled out the happening of all others.
i.e if no two or more than two events can happen simultaneously in the same random experiment

In other words, two events E_1 & E_2 are said to be mutually exclusive are disjoint if $E_1 \cap E_2 = \phi$ or $E_1 \cap E_2 = \{\}$

Example :- Consider a random experiment as throwing a die here $S = \{1, 2, 3, 4, 5, 6\}$
let $E_1 = \{2, 4, 6\}$
$E_2 = \{1, 3, 5\}$
$E_3 = \{3, 6\}$

Clearly E_1 & E_2 are mutually exclusive events but E_1 & E_3 are not mutually exclusive events. Since $E_1 \cap E_3 = \{6\} \neq \{\}$.

Equally likely events

Events are said to be equally likely if there is no reason to expect any one in preference to others.

Example:- In throwing a die, all the 6 faces are equally likely to come out. therfore $\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}$ all are equally likely events.

Independent and dependent events

Two or more events are said to be independent if the happening or non happening of any one event doest not depend by the happening or non happening of any other event.

Otherwise they are said to be dependent.

Example: Suppose a box contains 4 Black balls and 3 white balls.

Let E_1 be the event that first ball drawn is black and let E_2 be the event that second ball drawn is black.

Then

(i) when the balls are not replaced after the being drawn then E_1 and E_2 are dependent events

(ii) If the balls are replaced after the bieng drawn then E_1 and E_2 are independent events.

$\frac{7!}{3!\,4!}$ $\quad \frac{^7C_4}{^7C_1} \Rightarrow \frac{35}{7} = 5$

$\frac{5\times6\times7}{3\times2}$

Probability

In a Random experiment let there be 'n' exhaustive, mutually exclusive, equally likely events and 'm' of them are favourable to the happening of an event 'E' then the probability of happening of E is given by

$$P(E) = \frac{\text{no. of favourable outcomes for } E}{\text{Total no. of outcomes}} = \frac{m}{n}$$

$$\Rightarrow \quad \therefore \boxed{P(E) = \frac{m}{n}}$$

Observations

$$P(\bar{E}) = \frac{\text{No. of favourable outcomes for } \bar{E}}{\text{Total no. of outcomes}}$$

$$= \frac{n-m}{n} \Rightarrow \frac{n}{n} - \frac{m}{n} \Rightarrow 1 - P(E)$$

$$\therefore \boxed{P(E) = 1 - P(E)}$$

Axioms of mathematical Probability

1) If E is any event then $0 \leq P(E) \leq 1$
2) If 'S' is the sample space of any random experiment then $P(S) = 1$.

3) If 'A' and 'B' are two mutually exclusive events then $P(A \cup B) = P(A) + P(B)$.

lly

If $E_1, E_2, E_3 \ldots E_n$ are mutually exclusive events then

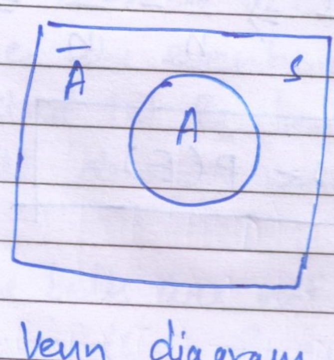
$$P(E_1 \cup E_2 \cup \ldots E_n) = P(E_1) + P(E_2) + \ldots + P(E_n)$$

Complementry Rule

If 'A' and $\bar{A}$ are Complementary events associated to the sample space 'S' then

$P(\bar{A}) = 1 - P(A)$

Proof:



From the Venn diagram it is clear that A and $\bar{A}$ are mutually exclusive (Disjoint events) and their Union is 'S'.

i.e $S = A \cup \bar{A}$

$\Rightarrow P(S) = P(A \cup \bar{A})$

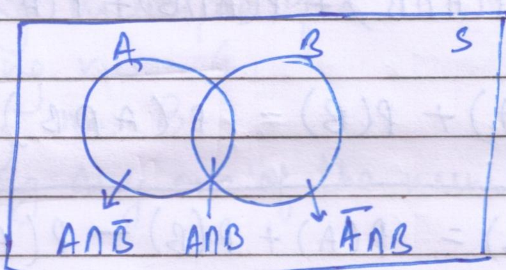
$1 = P(A) + P(\bar{A})$

$\Rightarrow \therefore \boxed{P(\bar{A}) = 1 - P(A)}$

Addition law for any two events

Let A and B be any two events then
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Proof:



Let 'A' and 'B' be any two events having common points represented by $A \cap B$. From the venn diagram it is clear that

$A \cap \bar{B}$ and $A \cap B$ are mutually exclusive events and their union is 'A'.

i.e $A = (A \cap \bar{B}) \cup (A \cap B)$

$\Rightarrow P(A) = P[(A \cap \bar{B}) \cup (A \cap B)]$

$= P(A \cap \bar{B}) + P(A \cap B)$

Ily $(A \cap B)$ and $(\bar{A} \cap B)$ are mutually exclusive events and there union is B.

Which implies $B = (A \cap B) \cup (\bar{A} \cap B)$

$P(B) = P(A \cap B) + P(\bar{A} \cap B)$

Also $(A \cap \bar{B}), (A \cap B), (\bar{A} \cap B)$ are mutually exclusive events and there union is $A \cup B$.

ie. $A \cup B = (A \cap \bar{B}) \cup (A \cap B) \cup (\bar{A} \cap B)$

$\Rightarrow P(A \cup B) = P[(A \cap \bar{B}) \cup (A \cap B) \cup (\bar{A} \cap B)]$

$P(A \cup B) = P(A \cap \bar{B}) + P(A \cap B) + P(\bar{A} \cap B)$

$= P(A \cap \bar{B}) + P(A \cap B) + P(\bar{A} \cap B) + P(A \cap B) - P(A \cap B)$

$\Rightarrow P(A) + P(B) - P(A \cap B)$

$\therefore \boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$

Result

If A, B, C are any three events then
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

Consider

$P(A \cup B \cup C) = P(A \cup D)$, where $D = B \cup C$
$= P(A) + P(D) - P(A \cap D)$ [$\because$ Addition law]
$= P(A) + P(B \cup C) - P[A \cap (B \cup C)]$

$= P(A) + P(B \cup C) - P[(A \cap B) \cup (A \cap C)]$
$= P(A) + P(B) + P(C) - P(B \cap C) - [P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)]$

$$= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$$

$$\Rightarrow P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$$

Example 1: In throwing a single die, find the probability of

i) Getting an even no.
ii) Getting an odd no.
iii) Getting multiple of 3
(iv) Getting no. 4.
(v) Getting no. 8
(vi) Getting any one of the number (integer) from 1 to 6.

Solution:

The sample space 'S', when throwing a single die is given by $S = \{1, 2, 3, 4, 5, 6\}$

i) Let E_1 be an event to get an even number then

$E_1 = \{2, 4, 6\}$

$\Rightarrow P(E_1) = \dfrac{3}{6} = \dfrac{1}{2}$

(ii) Let E_2 be an event to get odd number

$E_2 = \{1, 3, 5\}$

$P(E_2) = \dfrac{3}{6} = \dfrac{1}{2}$

(iii) Let E_3 be an event to get multiple of 3
then $E_3 = \{3, 6\}$
$\Rightarrow P(E_3) = \frac{2}{6} = \frac{1}{3}$

(iv) Let E_4 be an event to get number 4.
then $E_4 = \{4\}$
$P(E_4) = \frac{1}{6}$

(v) Let E_5 be an event to get number 8
then $E_5 = \{\}$
$P(E_5) = \frac{0}{6} = 0$

(vi) Let E_6 be an event to get any one of the integer from 1 to 6.
then $E_6 = \{1, 2, 3, 4, 5, 6\}$
$P(E_6) = \frac{6}{6} = 1$

Observation

Obs 1 : If the probability of any event in a sample space is equal to 1

i) then that event is called sure event or certain event.
In the above example $P(E_6) = 1 = P(S)$
$\Rightarrow E_6$ is sure or certain event.

(iii) If the probability of any event in a sample space is equal to 0, then that event is called impossible event.

In the above example $P(E_5) = 0$

$\Rightarrow E_5$ is impossible event.

Example 2: In a single of through of two unbiased dice, find the probability of getting sum of two faces is i) 10. ii) 8

Here the random experiment is throwing two unbiased dies.

In this case, the sample space 'S' is given by

$$S = \left\{ \begin{array}{llll} (1,1) & (1,2) & \cdots & (1,6) \\ (2,1) & (2,2) & & (2,6) \\ (3,1) & (3,2) & \cdots & (3,6) \\ \vdots & & & \\ (6,1) & (6,2) & \cdots & (6,6) \end{array} \right\}$$

Clearly the total no. of outcomes is 36

i) Let E_1 be an event to obtain sum of the two faces is 10 then $E_1 = \{(4,6)\ (5,5)\ (6,4)\}$

$\Rightarrow P(E_1) = \frac{3}{36} = \frac{1}{12}$

(ii) Let E_2 be an event to obtain sum of the two faces is 8 then $E_2 = \{(2,6)\ (3,5)\ (4,4)\ (5,3)\ (6,2)\}$

$\Rightarrow P(E_2) = \frac{5}{36}$

Example 3: 4 cards are drawn at random from the pack of 52 cards then find the probability that

i) There are two Kings and two Ace
ii) There is one card of each suit
(iii) All are diamonds

Solution: We know that, the total no. of ways or outcomes to draw 4 cards from a pack of 52 cards is given by $^{52}C_4$

i) Let E_1 be an event of getting two Kings and two Ace.

$\Rightarrow$ The no. of favourable outcomes for $E_1 = {}^4C_2 \times {}^4C_2$

$$P(E_1) = \frac{{}^4C_2 \times {}^4C_2}{{}^{52}C_4}$$

(ii) Let E_2 be an event of getting 1 card of each suit.

$\Rightarrow$ The no. of favourable outcomes for $E_2 = {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$

$$\therefore P(E_2) = \frac{({}^{13}C_1)^4}{{}^{52}C_4}$$

(iii) Let E_3 be an event of getting 4 diamonds

$\Rightarrow$ The no. of favourable outcomes for $E_3 = {}^{13}C_4$

$$\therefore P(E_3) = \frac{{}^{13}C_4}{{}^{52}C_4}$$

2, 4, 6, 8, 10, 12, 14, 16, 18, 20
5, 10, 15, 20 $\frac{12}{20}$

Example 4: A bag contains 20 tickets marked with no.'s 1 to 20. One ticket drawn at random. find the probability that it is a multiple of

i) 2 or 5

(ii) 3 or 5

Solution:- Here the total no. of outcomes (that is the total no. of ways to draw 1 ticket from 20 tickets) $= {}^{20}C_1 = 20$

(i) Let E_1 be the event of getting multiple of 2 or 5.

then the favourable outcomes of E_1 are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 5, 15

that is the no. of favourable outcomes of $E_1 = 12$

$\therefore P(E_1) = \frac{12}{20} = \cancel{\frac{6}{10}}\ \frac{3}{5}$

(ii) Let E_2 be the event of getting multiple of 3 or 5. then the favourable outcomes of E_2 are 3, 6, 9, 12, 15, 18, 5, 10, 20

that is the no. of favourable outcome of $E_2 = 9$

$P(E_2) = \cancel{\frac{9}{20}}\ \frac{9}{20}$

Q. 20 Books are placed at random in a shelf. Find the probability that a particular pair of books Shall be

(i) Always together
(ii) Never together.

Solution: Here the total no. of outcomes (i.e the total no. of ways to arrange the 20 books) = $^{20}P_{20} = 20!$

1) Let E_1 be the event that a particular pair of books is always together then the no. of favourable outcomes of E_1 = $19! \times 2!$

$$\therefore P(E_1) = \frac{19! \times 2!}{20!} = \frac{2}{20} = \frac{1}{10}$$

(ii) Let E_2 be the event that a particular pair of books is never together.

Clearly E_2 is complement of E_1

$$\therefore P(E_2) = \overline{P(E_1)} = 1 - P(E_1) \Rightarrow 1 - 0.1 \Rightarrow 0.8$$

Q. What is the probability that a leap year selected at random will contain 53 Sundays.

$\frac{366}{7}$ = 52 weeks and 2 days

These two days are
S & M
M & T
T & W
W & thu
Thu & Frid
Frid & Satur
Saturd & Sunday

total out comes

$\therefore \frac{2}{7}$

We know that a leap year consist 366 days.
i.e 52 weeks and 2 succesive days.
i.e In a leap year there are 52 complete weeks and 2 days.

The following are the likely cases for these two successive days

$\therefore$ The required probability = $\frac{2}{7}$

Q. The probabilities that a new year post will get an award for its design, efficient use of materials or both are 0.16, 0.24, 0.11 respectively what is the probability that it will get at least one award.

D → design
E → Efficient use of material

$P(D) = 0.16 \quad P(E) = 0.24 \quad P(D \cap E) = 0.11$

$P(D \cup E) = ?$

$$P(D \cup E) = P(D) + P(E) - P(D \cap E)$$
$$= 0.16 + 0.24 - 0.11$$
$$= 0.40 - 0.11 \Rightarrow 0.29$$

Conditional probability and multiplication law of probability.

For any two events A and B, we have

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, \text{ where } P(B) \neq 0$$

and $$P(B/A) = \frac{P(A \cap B)}{P(A)}, \text{ where } P(A) \neq 0$$

Here $P(A/B)$ represents the conditional probability of happening of A under the condition that B has already happened.

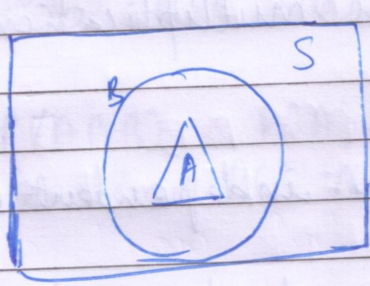
$$\begin{array}{r} 0.16 \\ 0.24 \\ \hline .40 \end{array}$$

Similarly $P(B/A)$ represents the conditional probability of happening of 'B' under the condition that 'A' is already happened.

Proof: Let A and B be any two events in a random experiment associated with the sample space 'S'. then

$$P(A) = \frac{n(A)}{n(S)}, \quad P(B) = \frac{n(B)}{n(S)}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)}$$



For the conditional event A/B, the favourable outcomes must be one of the sample points of 'B'.

In other words, the sample space for the event A/B is B.

$$\therefore P(A/B) = \frac{n(A)}{n(B)} = \frac{n(A \cap B)}{n(B)} \quad \Big| \; \because A \subset B \Rightarrow A = A \cap B$$

$$= \frac{n(A \cap B)/n(S)}{n(B)/n(S)} = \frac{P(A \cap B)}{P(B)}$$

Ily $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$ —— (1)

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \quad \text{—— (2)}$$

$$(1) \Longrightarrow P(A \cap B) = P(B) \cdot P(A/B)$$
$$(2) \longrightarrow P(A \cap B) = P(A) \cdot P(B/A)$$

$$\therefore P(A \cap B) = \begin{cases} P(B) \cdot P(A/B) \\ P(A) \cdot P(B/A) \end{cases} \quad \text{—— (*)}$$

Which is known as multiplication law for dependent events

Multiplication law for independent events

If A & B are independent events then we have

$P(A/B) = P(A)$ &
$P(B/A) = P(B)$

$\therefore$ from eq *, we have $P(A \cap B) = P(A) \cdot P(B)$ ——

Which is known as multiplication law for two independent events A & B lly If A, B and C are three mutually independent events then $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$.

Result

If A and B are independent events then

(i) $\bar{A}$ and $\bar{B}$ are also independent events

(ii) $\bar{A}$ & B are also independent events

(iii) A & $\bar{B}$ are also independent events

Solⁿ: Given that A and B are independent events

$\Rightarrow \quad P(A \cap B) = P(A) \cdot P(B)$

(i) Consider $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

$= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - P(A) - P(B) + P(A \cap B)$

$= 1 - P(A) - P(B) + P(A) \cdot P(B)$

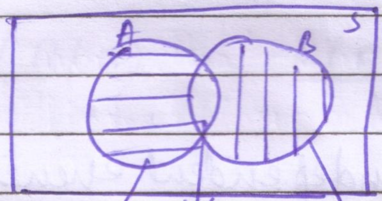
$= [1 - P(A)] - P(B)[1 - P(A)]$

$[1 - P(A)] \; [1 - P(B)]$

** $\therefore P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B})$

i.e $\bar{A}$ & $\bar{B}$ are also independent events

(iii) Consider $P(\bar{A} \cap B)$



$A \cap B$ and $\bar{A} \cap B$ are disjoint and their union is B

$$B = (A \cap B) \cup (\bar{A} \cap B)$$

$$\Rightarrow P(B) = P\left(\left[(A \cap B) \cup (\bar{A} \cap B) \right] \right)$$

$$\Rightarrow \quad = P(A \cap B) + P(\bar{A} \cap B)$$

$$P(\bar{A} \cap B) = P(B) - P(A \cap B)$$

$$= P(B) - P(A) \cdot P(B) \quad \text{(since ind}$$

$$= B \cdot \left[1 - P(A) \right] - P(B)$$

$$= P(\bar{A}) \cdot P(B)$$

$$\therefore P(\bar{A} \cap B) = P(\bar{A}) \cdot P(B)$$

$\therefore$ $\bar{A}$ and B are also independent event.

$P(\overline{\bar{A} \cup \bar{B}}) = \bar{A} \cap B$

(iii) ~~Consider~~ $P(A \cap \bar{B})$

Clearly from the above figure we have $A \cap \bar{B}$ and ~~A~~ $A \cap B$ are disjoint and their union is A

i.e $A = (A \cap \bar{B}) \cup (A \cap B)$

$\Rightarrow P(A) = [P(A \cap \bar{B}) \cup P(A \cap B)]$

$= P(A \cap \bar{B}) + P(A \cap B)$ | $\because$ Axiom ③

$\Rightarrow P(A \cap \bar{B}) = P(A) - P(A \cap B).$

$= P(A) - P(A).P(B)$

$P(A)[(1 - P(B)]$

$P(A) . P(\bar{B})$

independent) $\Rightarrow P(A \cap \bar{B}) = P(A) . P(\bar{B})$

$\therefore$ A and $\bar{B}$ are also independent events.

Example: Given that $P(A) = 1/4$, $P(B/A) = 1/2$ and $P(A/B) = 1/4$ then check whether

1. A and B are mutually exclusive
2. A and B are independent.

Solⁿ: Given that $P(A) = 1/4$, $P(B/A) = 1/2$ and $P(A/B) = 1/4$

(i) We know that from the definition of Conditional probability

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(A \cap B) = P(A) \cdot P(B/A)$$

$$= \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8} \neq 0$$

$\therefore$ A and B are not mutually exclusive events.

(ii) $$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B) = \frac{P(A \cap B)}{P(A/B)} = \frac{1/8}{1/4} = \frac{4}{8}\ \frac{1}{2}$$

$$P(B) = 0.5$$

Consider $P(A) \cdot P(B) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

and $P(A \cap B) = 1/8$

$P(A \cap B) = P(A) \cdot P(B)$

A and B are independent events.

Q An article manufactured by a Company consist of two parts A and B in the process of ~~many~~ manufactures of part A, 9 out of 100 are likely to be ~~effective~~ defective. Similarly 5 out of 100 are likely to be defective in the manufactures of part B. Calculate the probability that the assume part will not be defective.

~~$P(E_1)$~~

Let E_1 be the event that part A is defective then

$$P(E_1) = \frac{9}{100}, \quad P(\bar{E}_1) = 1 - \frac{9}{100} = \frac{91}{100}$$

Let E_2 be the event that Part B is defective then

$$P(E_2) = \frac{5}{100}, \quad P(\bar{E}_2) = 1 - \frac{5}{100} = \frac{95}{100}$$

$$P(\overline{E_1 \cup E_2}) = P(\bar{E}_1 \cap \bar{E}_2)$$

∴ the probability that the assumed part will not be defective is given by

$$P(\overline{E_1 \cup E_2}) = P(\bar{E}_1 \cap \bar{E}_2)$$

$P(\bar{E}_1 \cap \bar{E}_2) = P(\bar{E}_1) \cdot P(\bar{E}_2)$ (Since E_1 and E_2 are independent event)

$= \frac{91}{100} \times \frac{95}{100}$

Q. A problem in statistics is given to 3 students A, B and C. whose chances of solving it are $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{2}$ respectively

What is the probability that problem will be solved

Let E_1 be the event that the problem is solved by A

Let E_2 by B
Let E_3 by C

then

$P(E_1) = 1/3$, $P(\bar{E}_1) = 2/3$

$P(E_2) = 1/4$, $P(\bar{E}_2) = 3/4$

$P(E_3) = 1/2$, $P(\bar{E}_3) = 1/2$

$\therefore$ the problem will be solved is given by

$P(E_1 \cup E_2 \cup E_3) = 1 - P(\overline{E_1 \cup E_2 \cup E_3})$

$= 1 - P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3)$

$1 - P(\bar{E}_1) \cdot P(\bar{E}_2) \cdot P(\bar{E}_3)$

$1 - \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{4} \times \frac{1}{\cancel{2}} \Rightarrow 3/4$

Q. The Odds (opinions) that a book will be preferably review by three independent reviews are 5 to 2, 4 to 3. and 3 to 4 respectively. What is the probability that are the three reviews majority will be favour.

Sol^n: Let E_1 be the event that A gives a favourable review

E_2 .... " B ...

E_3 " ... C ...

$P(E_1) = \frac{5}{7}$, $P(\bar{E}_1) = \frac{2}{7}$

$P(E_2) = \frac{4}{7}$, $P(\bar{E}_2) = \frac{3}{7}$

$P(E_3) = 3/7$, $P(\bar{E}_3) = 4/7$

140
45
24
$\overline{209}$

$\therefore$ P(E) = Probability that of ~~wide~~ three reviews majority will be favours is given by

$$P(E_1 \cap E_2 \cap E_3) + P(E_1 \cap E_2 \cap \bar{E}_3) + P(E_1 \cap \bar{E}_2 \cap E_3) + P(\bar{E}_1 \cap E_2 \cap E_3)$$

$$= P(E_1) \cdot P(E_2) \cdot P(E) \cdot + P(E_1) \cdot P(E_2) \cdot P(\bar{E}_3) + P(E_1) \cdot P(\bar{E}_2) \cdot P(E_3) + P(\bar{E}_1) \cdot P(E_2) \cdot P(E_3)$$

$$= \frac{5}{7} \times \frac{4}{7} \times \frac{3}{7} + \frac{5}{7} \times \frac{4}{7} \times \frac{4}{7} + \frac{5}{7} \times \frac{3}{7} \times \frac{3}{7} + \frac{2}{7} \times \frac{4}{7} \times \frac{3}{7}$$

$$\frac{1}{7^3}(60 + 80 + 45 + 24)$$

$49 \times 7 = 343$

$$\frac{209}{343} \text{ Ans.}$$

Inverse probability theorem or Bay's theorem

Let $E_1, E_2, E_3 \ldots E_n$ are 'n' mutually exclusive and exhaustive events with $P(E_i) \neq 0$, $i = 1, 2, 3 \ldots n$. For any arbitrary event A, such that $A \subset (\bigcup_{i=1}^{n} E_i)$, $P(A) > 0$

then we have

$$P(E_i/A) = \frac{P(E_i).\,P(A/E_i)}{\sum_{j=1}^{n} P(E_j).\,P(A/E_j)}$$

Proof: Given that A is subset of $\bigcup_{i=1}^{n} E_i$

$$\Rightarrow \quad A = A \cap \left(\bigcup_{i=1}^{n} E_i\right)$$

$$= A \cap (E_1 \cup E_2 \cup E_3 \cup \ldots E_n)$$

$$= (A \cap E_1) \cup (A \cap E_2) \cup (A \cap E_3) \ldots \cup (A \cap E_n)$$

$$\Rightarrow P(A) = P[(A \cap E_1) \cup (A \cap E_2) \cup (A \cap E_3) \cup \ldots (A \cap E_n)]$$

$$= P(A \cap E_1) + P(A \cap E_2) + P(A \cap E_3) + \ldots P(A \cap E_n)$$

$\because E_1, E_2, E_3, \ldots E_n$ are mutually exclusive

$\Rightarrow (A \cap E_1), (A \cap E_2) \ldots (A \cap E_n) \rightarrow$ also mutually exclusive

By Axiom 3

$$= P(E_1).P(A/E_1) + P(E_2) + P(A/E_2) + P(E_3).P(A/E_3) + \ldots P(E_n).P(A/E_n)$$

(Conditional Probability)

$$\therefore P(A) = \sum_{j=1}^{n} P(E_j).P(A/E_j) \quad \text{——} \quad (1)$$

Also by definition of conditional probability we have $P(E_i/A) = \frac{P(E_i \cap A)}{P(A)}$

$$= \frac{P(E_i) \cdot P(A/E_i)}{P(A)}$$

$$\boxed{\therefore P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum_{j=1}^{n} P(E_j) \cdot P(A/E_j)}}$$

Q.1. In a bolt factory ~~machines~~ A, B and C manufacture respectively 25%, 35%, 40% of the total. Of there output 5% ~~percent~~, 4% ~~percent~~, 2% ~~per-cent~~ are deffective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it is manufactured by machines A, B and C.

Let E_1 be an event that a bolt is manufactured by machine A

Let E_2 - - - - - - - B

Let E_3 - - - - - - - C.

$P(E_1) = 25\% = \frac{25}{100}$

$P(E_2) = \frac{35}{100}$, $P(E_3) = \frac{40}{100}$

Let E be an event that a bolt is defective than we have

$P(E/E_1) = \frac{5}{100}$, $P(E/E_2) = \frac{4}{100}$, $P(E/E_3) = \frac{2}{100}$

(i) The probability that a bolt is defected which is manufactured by machine A is given by $P(E_1/E) = \frac{P(E_1) \cdot P(E/E_1)}{\sum_{j=1}^{3} P(E_j) \cdot P(E/E_j)}$

$$= \frac{P(E_1) \cdot P(E/E_1)}{P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2) + P(E_3) \cdot P(E/E_3)}$$

$$= \frac{\frac{25}{100} \times \frac{5}{100}}{\frac{25}{100} \times \frac{5}{100} + \frac{35}{100} \times \frac{4}{100} + \frac{40}{100} \times \frac{2}{100}} = \frac{25 \times 5}{25 \times 5 + 35 \times 4 + 40 \times 2}$$

$$\frac{125}{125 + 140 + 80} = \frac{\cancel{125}}{\cancel{345}} \; \frac{\cancel{25}}{69}$$

125
220
345

My $P(E_2/E) = \dfrac{P(E_2)\cdot P(E/E_2)}{\sum_{j=1}^{3} P(E_j)\cdot P(E/E_j)}$

$= \dfrac{140}{345}$

Q. The contents of boxes I, II, III are as follows:

I – 1 white, 2 black, 3 red coines (6)

II – 2 white, 1 black, 1 red Coines (4)

III – 4 white, 5 black, 3 red coines (12)

One box is chosen at random and two coins are drawn, they happened to white and red. What is the probability that they came from boxes I, II and III.

Let E_1 be an event that choosing box I
" E_2 " " " " " " II
" E_3 " " " " " III

Then $P(E_1) = P(E_2) = P(E_3) = 1/3$

1.08
1.5
1.03

10.1
1.5
5.5

4) Let E be an event that drawing 2 coins White and red then

$$P(E/E_1) = \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2} = \frac{1 \times 3}{6 \times 5/_{2 \times 1}} = 1/5$$

$$P(E/E_2) = \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2} = \frac{2 \times 1}{4 \times 3/_{2 \times 1}} = 1/3$$

$$P(E/E_3) = \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2} \quad \frac{4 \times 3}{12 \times 11/_2} = \frac{2}{11}$$

1) Probability that drawing 2 coins white and red from box 1 is given by $P(E_1/E)$

$$P(E_1/E) = \frac{P(E_1) \cdot P(E/E_1)}{\sum_{i=1}^{3} P(E_i) \cdot P(E/E_i)} \quad \text{(Bay's theorem)}$$

$$= \frac{P(E_1) \cdot P(E/E_1)}{P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2) + P(E_3) \cdot P(E/E_3)}$$

$$\frac{\frac{1}{3} \times \frac{1}{5}}{1/3 \times 1/5 + 1/3 \times 1/3 + 1/3 \times 2/11} = \frac{1/15}{1/3\left(1/5 + 1/3 + 2/11\right)}$$

$$\frac{1/15}{1/3 \left(\frac{33+55+15}{165}\right)} \Rightarrow \frac{\cancel{3} \times \cancel{165}^{\,33}}{\cancel{5}\cancel{15} \times 103} = \frac{33}{108}$$

Ily the probability that drawing 2 bolls One white and one Red box by box II is given by

$$P(E_2/E) = \frac{P(E_2) \cdot P(E/E_2)}{P(E_1) \cdot P(E_1/E) + P(E_2) \cdot P(E_2/E) + P(E_3) \cdot P(E_3/E)}$$

$$\frac{1/9}{\frac{33+55+30}{495}} = \frac{55}{118}$$

$$P(E_3/E) = \frac{P(E_3) \cdot P(E/E_3)}{P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2) + P(E_3) \cdot P(E/E_3)}$$

$$= \frac{1/3 \times 2/11}{\frac{33+55+30}{495}} = \cancel{\frac{2 \times 30}{}} \; \frac{30}{118}$$

Observation:

Here. $P(E_1/E) + P(E_2/E) + P(E_3/E)$

$$= \frac{33}{118} + \frac{55}{118} + \frac{30}{118} = \frac{118}{118}$$

$$\frac{1.2}{4.6} = \frac{6}{23} \qquad \frac{1.2}{1.2+1+2.4}$$

$$\frac{0.4}{1.2} \qquad 2.4 \qquad 3.4 \qquad \frac{1.2}{4.6}$$

Q. The Chances of ~~x, y~~ X, Y and Z becoming manager of a Certain Company are 4:2:3. the probability that bonus scheme will be introduced if X, Y and Z become managers are 0.3, 0.5, 0.8 respectiviely. If the bonus Scheme has to be introduced then what are the probabilities that X, Y and Z are appointed as a manager. Also find the probability that to introduce bonus scheme to the company.

Let E_1 be an event that X will become manager
" E_2 " " " Y " " "
" E_3 " " " Z " " "

then $P(E_1) = 4/9$
$P(E_2) = 2/9$ $\quad (\because 4:2:3)$
$P(E_3) = 3/9$

Let E be an event that to introduce bonus Scheme then
$P(E/E_1) = 0.3$, $P(E/E_2) = 0.5$, $P(E/E_3) = 0.8$

(1) The probability that 'X' will appointed as a manager when bonus Scheme has introduced. is given by

$$P(E_1/E) = \frac{P(E_1).P(E/E_1)}{P(E_1).P(E/E_1) + P(E_2).P(E/E_2) + P(E_3).P(E/E_3)}$$

$$= \frac{4/9 \times 0.3}{4/9 \times 0.3 + 2/9 \times 0.5 + 3/9 \times 0.8} = \frac{4 \times 0.3}{4 \times 0.3 + 2 \times 0.5 + 3 \times 0.8}$$

$= 6/23$

$$\frac{1.2}{4.6}$$

$$P(E_2/E) = \frac{2/9 \times 0.5}{4/9 \times 0.3 + 2/9 \times 0.5 + 3/9 \times 0.8}$$

$$\frac{1}{1.2 + 1 + 2.4} = \frac{1/10}{\cancel{4.6}} = \frac{5}{23}$$

$$P(E_3/E) = \frac{3/9 \times 0.8}{4/9 \times 0.3 + 2/9 \times 0.5 + 3/9 \times 0.8} = \frac{\cancel{2.4}\ 12}{\cancel{4.6}\ 23}$$

Also the probability that to introduce the bonus scheme to the company is given by

$$P(E) = \sum_{j=1}^{3} P(E_j) \cdot P(E/E_j) \quad \text{(Bay's theorem)}$$

$$P(E) = P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2) + P(E_3) \cdot P(E/E_3)$$

$$P(E) = \frac{4}{9} \times 0.3 + \frac{2}{9} \times 0.5 + \frac{3}{9} \times 0.8$$

$$= \frac{\cancel{46}}{\cancel{90}} \quad \frac{23}{45}$$