Background

This algorithm performs the Fibonacci Line Search to find the minimum of a unimodal function, $f(x)$, over an interval, $a \leq x \leq b$. The algorithm calculates the number of iterations required to insure the final interval is within the *user-specified* tolerance. This is found by solving for the smallest value of n that makes this inequality true: $F_n > \frac{(b-a)}{\text{tolerance}}$, where n is the Fibonacci number from the sequence $\{F_0, F_1, F_2, ...\}$. The Fibonacci search concept involves placing two experiments between $[a, b]$ using the ratios of Fibonacci numbers. (The limit of the ratio of Fibonacci numbers is the golden section 0.618 but the Fibonacci method converges quicker.) One experiment is placed at position, $a + \frac{F(n-k-2)(b-a)}{F(n-k)}$, and the other at position,

$a + \frac{F(n-k-1)(b-a)}{F(n-k)}$. The function, to be minimized, is evaluated at these two points and the functional values are compared. We want to keep the smaller functional value (in our minimization problem) and its corresponding opposite end -point. At the end of the required iterations, the final interval is the answer. At times when the final answer must be a single point and not an interval, the convention of selecting the midpoint is provided. This algorithm works when the function is not differentiable and we are looking for a solution. If you need to minimize a function, then multiple the function by (-1) and find the minimum.

To find a minimum solution to given a function, $f(x)$, on the interval $[a, b]$ where the function, $f(x)$, is unimodal.

Text adapted from:

https://www.maplesoft.com/applications/view.aspx?SID=4193&view=html

Instructions

Implement the Fibonacci Line Search algorithm on a programming language of your choosing. Use the following input parameters:

x = 0.3

y=0.8

n = 8

To minimize the following function:

$$f(x) = \sqrt{\frac{e^{4x^2} + \sin(x)^2}{3x^2 + x^4}}$$

For every iteration output the value of x, y, u, v, k, f(u), f(v).

Fibonacci Line Search

Step 1

Input x**,** y and n

Step 2

Compute F_2, F_3…,F_n using $F_k = F_{k-1} + F_{k-2}$ for $k \geq 2$

$$F_0 = F_1 = 1$$

Step 3

Assign $I_1 = y - x$ and compute

$$I_2 = \frac{F_{n-1}}{F_n} I_1$$

$$u = x + I_2$$

$$v = y - I_2$$

$$a = f(u)$$

$$b = f(v)$$

Set $k = 1$

Step 4

Compute I_{k+2} using $I_{k+2} = \dfrac{F_{n-k-1}}{F_{n-k}} I_{k+1}$

If $b \geq a$, then set:

$$x = v$$

$$v = u$$

$$u = x + I_{k+2}$$

$$b = a$$

$$a = f(u)$$

Otherwise set:

$$y = u$$

$$u = v$$

$$v = y - I_{k+2}$$

$$a = b$$

$$b = f(v)$$

Step 5

If $k = n - 2$ or $u > v$, then

Output $xs = u$ and $fs = f(xs)$

Otherwise:

Set $k = k + 1$ and repeat from Step 4