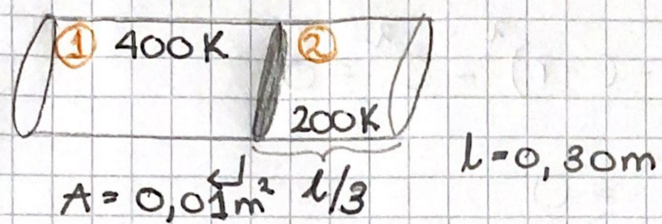
0.3 Termodinámica



a) $U_{10} + U_{20} = U_{1f} + U_{2f}$

$\Rightarrow T_{1f} + \boxed{T_{2f}} = \underline{T_{10}} + \underline{T_{20}}$

$PV = nRT \Rightarrow T_{1f} = \dfrac{P_f V_{1f}}{\cancel{nR}}$

$P_f = \cancel{nR}\,\dfrac{(T_{10} + T_{20})}{V\ total}$

$\Rightarrow T_{1f} = \dfrac{(T_{10} + T_{20})\,V_{1f}}{V\ total} = \dfrac{(200\,K + 400\,K)(0{,}1\,m \cdot \cancel{0{,}01\,m^2})}{\cancel{0{,}01\,m^2} \cdot 0{,}30\,m}$

$= 200K$

$T_{2f} = 600K - 200K = 400K$

b) Según la ley de Fourier, el flujo de calor se comporta de la forma

$$\frac{dQ}{dt} = -KS\frac{dT}{dx}$$

Como no hay trabajo, $dQ = dU = nc_v\,dT$

$\Rightarrow nC_v\dfrac{dT}{dt} = -KA\,(T_1 - T_2)$

c) $T_1' = -C\,(T_1 - T_2)$

$T_2' = C\,(T_1 - T_2)$

$$x'(t) = A\,\vec{x}(t)$$

En nuestro caso, $A = \begin{pmatrix} -C & C \\ C & -C \end{pmatrix}$; $\vec{x} = \vec{v}e^{rt}$

$(A - rI)\vec{v} = 0$

$\det\begin{pmatrix} -C-r & C \\ C & -C-r \end{pmatrix} = 0 \quad (-C-r)^2 - C^2 = 0$

$\Rightarrow \cancel{C^2} - 2Cr + r^2 - \cancel{C^2} = 0$

$r^2 = 2Cr \Rightarrow r = 2C;\ r = 0$

$r = 2C$

$\begin{pmatrix} -3C & C \\ C & -3C \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \quad , \vec{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$-3\cancel{C}(v_1) + \cancel{C}(v_2) = 0 \Rightarrow -3v_1 = -v_2$

$v_1(\cancel{C}) - 3C(v_2) = 0 \Rightarrow v_1\cancel{C} = 3\cancel{C}v_2$

$r = 0$

$\begin{pmatrix} -C & C \\ C & -C \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \quad \begin{matrix} -\cancel{C}v_1 + \cancel{C}v_2 = 0 \\ \cancel{C}v_1 - \cancel{C}v_2 = 0 \end{matrix}$

$\vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\vec{x} = C_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{2Ct} + C_2\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$T_1 = C_1 e^{2Ct} + C_2$

$T_2 = C_3 e^{2Ct} + C_4$

$\frac{T_1}{dt} = -C_1(\cancel{e^{2Ct}}^{1} \cdot 2\cancel{C}) = -\cancel{C}(T_{10} - T_{20})$

$\left.\begin{matrix} C_1 = -(T_{10} - T_{20}/2) \\ C_2 = (T_{10} - T_{20}/2) \end{matrix}\right\} = -C_1$