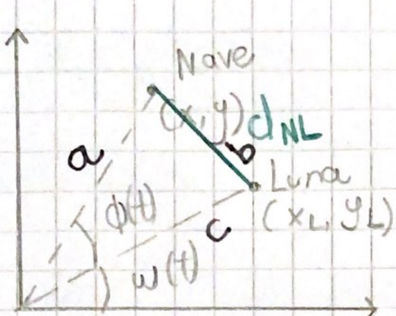
0.1. Cálculo de trayectoria para nave exploratoria lunar

c)



$$b^2 = a^2 + c^2 - 2ca\cos(\phi - \omega t)$$

$$r_L(r, \phi, t) = \sqrt{a^2 + c^2 - 2ac\cos(\phi - \omega t)}$$

$$r_L(r, \phi, t) = \sqrt{r(t)^2 + d^2 - 2r(t)d\cos(\phi - \omega t)}$$

d) La energía cinética es

$$K = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) \Longrightarrow \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2)$$ usando coordenadas polares.

La energía potencial es

$$U = \underbrace{-\frac{GmM_T}{r}}_{\text{Fuerza g. tierra}} \underbrace{- \frac{GmM_L}{r_L(r, \phi, t)}}_{\text{Fuerza g. Luna}}$$

$L = K - U$

$$p_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r}, \quad p_\phi = \frac{\partial L}{\partial \dot{\phi}} = mr^2\dot{\phi}$$

$$\Longrightarrow H = p_r\dot{r} + p_\phi\dot{\phi} - L = \frac{p_r^2}{2m} + \frac{p_\phi^2}{2mr^2} - \frac{GmM_T}{r} - \frac{GmM_L}{r_L(r, \phi, t)}$$

- $\dot{r} = \frac{\partial H}{\partial p_r} = \frac{p_r}{m}$; $\dot{\phi} = \frac{\partial H}{\partial p_\phi} = \frac{p_\phi}{mr^2}$ Momento lineal y angular

$\rightarrow \dot{p}_r = \frac{-\partial H}{\partial r} = \frac{p_\phi^2}{mr^3} - \frac{GmM_T}{r^2} - \frac{GmM_L}{r_L^3(r, \phi, t)}[r - d_{TL}\cos(\phi - \omega t)]$

$\rightarrow \dot{p}_\phi = \frac{-\partial H}{\partial \phi} = \frac{-GmM_L}{r_L^3(r, \phi, t)} r d_{TL} \sin(\phi - \omega t)$ Fuerza

f) $\tilde{r} = \frac{r}{d}$; $\tilde{p}_r = \frac{p_r}{md}$; $\tilde{p}_\phi = \frac{p_\phi}{md^2}$

$$\frac{\dot{r}}{\cancel{d}} = \frac{p_r}{m\cancel{d}} \Rightarrow \dot{\tilde{r}} = \tilde{p}_r$$

$$\dot{\tilde{\phi}} = \frac{\frac{p_\phi}{md^2}}{\left(\frac{r^2}{d^2}\right)} = \frac{p_\phi}{mr^2} = \dot{\phi}$$

$$\dot{\tilde{p}}_r = \frac{\frac{p_\phi^2}{m^2d^4}}{\frac{r^3}{d^3}} - \Delta\left\{\frac{d^2}{r^2} + \frac{\mu}{r^3}\left[r/d - \cos(\phi-\omega t)\right]\right\}$$

$$= \frac{p_\phi^2}{m^2r^3d} - \frac{GM_T}{d^3}\left\{\frac{d^2}{r^2} + \frac{M_L}{M_T(r)}\left[r/d - \cos(\phi-\omega t)\right]\right\}$$

$$= \frac{p_\phi^2}{mr^3} - \frac{GmM_T}{r^3} - \frac{GmM_L}{r_L(r,\phi,t)^3}\left[r - d\cos(\phi-\omega t)\right]$$

$$= \dot{p}_r$$

$$\dot{\tilde{p}}_\phi = \frac{-Gm_T}{d^3}\frac{r}{d}\frac{1}{\left(\sqrt{1+\tilde{r}^2-2\tilde{r}\cos}\right)^3}\sin(\phi-\omega t)$$

g) $$\tilde{p}_r = \frac{p_r}{md} = \frac{m}{md}\frac{dr}{dt} = \frac{1}{d}\frac{d}{dt}\sqrt{x^2+y^2} = \frac{x\dot{x}+y\dot{y}}{rd}$$

$$= \frac{x v\cos\theta + y v\sin\theta}{rd} = \frac{r v\cos\theta\cos(\phi) + r v\sin\theta\sin\phi}{rd}$$

$$= \tilde{v}\cos(\theta-\phi)$$

$$\tilde{p}_\phi = \frac{p_\phi}{md^2} = \tilde{r}^2\frac{d}{dt}\arctan\left(\frac{y}{x}\right) = \frac{\tilde{r}^2}{1+\frac{y^2}{x^2}}\frac{d}{dt}\left(\frac{y}{x}\right)$$

$$= \frac{\tilde{r}^2}{r^2}(\dot{y}x - y\dot{x}) = \tilde{r}\tilde{v}\sin(\theta-\phi).$$