Pre-auction or post-auction qualification?*

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We study whether an auctioneer generates higher revenues from a standard auction if he requires bidder qualification before or after the bidding process. There are two opposing effects: as proof of qualification is costly for bidders, there is less bidder participation under pre-auction qualification which tends to reduce revenues compared to post-auction qualification (exclusion effect). At the same time, however, bidders in an environment with post-auction qualification need to take into account that they have to incur qualification costs in case of successful bidding. As a consequence, bidders shade their bids (bid-shading effect). Contrary to what one may expect, we show that although post-auction qualification is more efficient, an auctioneer prefers pre-auction qualification if bidders' qualification costs are high.

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1 Introduction

Bidder qualification plays an important role in real-life auctions and procurement procedures. In many auction settings, auctioneers are not only interested in the price dimension but also in the quality provided. In this context, bidders often have to pass a quality threshold (i.e., a specific type of qualification requirement) to be able to execute the contract.[1] Verifying the qualification of a bidder is costly for the buyer and the potential sellers; for example, it may involve the testing of the products, the inspection of the production facilities, and the provision of industry-norm qualifications.[2]

In this paper, we analyze the question when an auctioneer should demand proof of bidders' qualification: before or after the auction. If the auctioneer demands pre-auction qualification, all bidders have to become qualified before participating in the auction. If the auctioneer demands post-auction qualification, only the winner has to undergo qualification. Even though typically both the buyer and the sellers bear the qualification cost, it is convenient to assume that all cost of qualification are on the bidders side.[3] Hence, if the auctioneer wants all bidders in an auction to proof their qualification before the auction, this means that placing a bid is associated with bidding costs for all participating bidders. If the auctioneer only requires the winning bidder to proof his qualification after the auction, all other bidders incur no bidding costs. This has important implications for potential buyers' bidding behavior and thus the auctioneer's revenues.

More precisely, there are two opposing effects. First, there is an exclusion effect under pre-auction qualification: given that the cost of qualification has to be paid prior to participation and qualified bidders may fail to win the auction, the expected surplus of bidders with valuations above but close to the qualification cost is negative. Thus, those bidders refrain from participation even if their valuation is above qualification cost. This is different for the case with post-auction qualification where all bidders with valuations above qualification cost earn a positive expected surplus and thus participate in the auction. Second, with post-auction qualification, there is a bid-shading effect: during the bidding procedure, bidders will keep in mind that upon winning they have to pay the cost of qualification. Thus, bidders in a post-auction qualification scenario shade their bids by the appropriate amount. This is different for pre-auction qualification where qualification cost are sunk at the time of bidding. The exclusion effect reduces revenue with pre-auction qualification whereas the bid-shading effect reduces revenues with post-auction qualification. A

[1]A prominent example for quality specifications that are formulated as threshold are so-called ppm (parts per million) requirements. Such requirements specify that only a certain number out of one million parts may be faulty. In the automotive industry, requirements of 10 ppm are the standard (see, e.g., `volvogroup.com`) and there is little additional benefit from suppliers with smaller failure rates.

[2]See Wan and Beil (2009) for an in-depth account of the importance of supplier qualification for procurement processes.

[3]This is without loss of generality as due to independence of physical and economic incidence, it does not matter which of the two sides in an auction incurs the costs for the qualification procedure (see, e.g., Weyl and Fabinger (2013)).

priori it is not clear which of these effects dominates and which qualification regime an auctioneer would favor.

Interestingly, we show that for a sufficiently high level of the qualification cost, pre-auction qualification is always more profitable for the auctioneer than post-auction qualification. This can be explained as follows: with pre-auction qualification, each bidder decides whether to enter the auction based on the incurred cost and his winning probability. If qualification cost rise, the cost of participation increases. However, less bidders participate and thus the winning probability increases. This increase in winning probability dampens the increase of the exclusion effect and the marginal increase goes to zero. With post-auction qualification, the bid-shading effect increases linearly with the increase in qualification cost and the marginal increase is one for all cost levels. Thus, the bid-shading effect becomes more important and pre-auction qualification yields higher revenues.

This result is somewhat surprising as previous studies always explained the occurrence of pre-auction qualification in real-life procurement settings with the risk of project failure (due to bankruptcy, lack of expertise, etc.) after awarding the contract to the firm that won the auction.[4] Without the risk of project failure, as in our model, the superiority of post-auction qualification seemed self evident. However, our analysis highlights pre-auction qualification may be beneficial for the auctioneer even in the absence of risk of project failure.

Our study contributes to the literature on bidder qualification in the context of auctions. Wan and Beil (2009) analyze a setup where a manufacturer uses a request-for-quotes (RFQ) reverse auction together with costly supplier qualification to determine which supplier will be awarded a contract. The manufacturer may choose to either perform a pre-qualification or a post-qualification screening. The authors show that despite the fact that post qualification results in higher expected payments, post-qualification screening may be the better choice than pre-qualification screening which is the standard industrial practice. The result is driven by the fact that the winning supplier may fail to become qualified as the likelihood of this event increases the compensations to the bidders with post-auction qualification.

Wan, Beil, and Katok (2012) also analyze a procurement setting where a buyer may award the contract either to a qualified incumbent supplier or to an entrant of unknown qualification. The buyer runs a price-only, open-descending reverse auction between the incumbent and the entrant and must decide whether to perform a pre-auction or post-auction qualification screening on the entrant. With post-auction qualification, the incumbent may lose the auction but still win the contract if the entrant does not pass the qualification screening. The authors show that the incumbent's optimal bidding strategy under post-auction qualification depends on the costs: if the incumbent has high costs, then holding back on bidding or even boycotting the auction altogether is optimal; only an incumbent with lower costs bids to win the auction. The authors also test their theoretical results experimentally and find that qualitatively their theoretical analysis offers reasonable predictions. Nevertheless, incumbent suppliers bid somewhat more aggressively than predicted which means that buyers tend to use post-auction qualification.

[4]See, e.g., Wan and Beil (2009) and Wan, Beil, and Katok (2012).

Our paper is also related to auctions with participation or entry costs as our case with pre-auction qualification can obviously be interpreted as such setups (see, e.g., Menezes and Monteiro, 2000; Kaplan and Sela, 2006; Tan and Yilankaya, 2006; Celik and Yilankaya, 2009).

2 Model

There are N risk-neutral bidders competing in a second-price sealed-bid auction for one indivisible object.[5] Before the auction starts, each bidder privately observes his valuation $v_i \in [0, 1]$ where $i \in \{1, \ldots, N\}$. The valuations represent the maximum price the bidders are willing to pay.[6] They are identically and independently distributed according to a common distribution function F which is assumed to be absolutely continuous and to have a density f that is away from zero everywhere on its support with $f(1) < \infty$.[7] In order to qualify as a winner of the auction, the bidders have to be qualified. Qualification comes at a cost c which has to be borne by bidders and which is common knowledge among all bidders (with $c \in [0, 1]$). We assume that the outcome of the qualification procedure is certain, i.e., any bidder that undergoes qualification will eventually be quailfied.[8]

We compare the following two scenarios:

Pre-auction qualification: under pre-auction qualification, the bidders have to be qualified before the auction starts in order to be able to submit bids. Hence, the timing of the auction game is the following:

1. Bidders privately learn their valuations.
2. Each bidder decides whether to invest qualification costs c to qualify for participation in the auction.[9]
3. The qualified bidders submit a bid in a sealed envelope.[10]
4. The bidder with the highest bid wins the auction and pays the second-highest bid

[5]Revenue equivalence is preserved in our setting. Thus, the results also apply to all standard auction formats such as first-price auction, English and Dutch auction.

[6]We frame the setting as a selling rather than as a procurement auction. This is without loss of generality and has the advantage that most readers are more familiar with the selling auction notation.

[7]We will denote by $F_1^{(N)}$ the first order statistic of N draws and by $F_2^{(N)}$ the second order statistic of N draws from F.

[8]We explicitly abstract from the possibility that one of the bidders may fail to become qualified. First, we are interested in whether the requirement of pre-auction qualification has further positive effects other than minimizing the risk that a bidder may fail to become qualified. Second, our assumption is equivalent to the assumption that bidders know whether they will pass or fail qualification. Thus, in both scenarios, bidders who know that they will fail qualification will not show up for the auction. However, our model allows for the situation where qualification may take longer for some bidders. If, in this case, costs are stochastic, we can interpret costs c as the expected costs from qualification.

[9]Whether a bidder has chosen to be qualified is not observed by the other bidders.

[10]Whether a bidder submits a bid or not is not observed by the other bidders.

Post-auction qualification: under post-auction qualification, a bidder only has to be qualified if he wins the auction. Hence, the timing of the auction game is as follows:

1. Bidders privately learn their valuations.
2. Each bidder submits a bid in a sealed envelope.[11]
3. The bidder with the highest bid wins the auction and pays the second-highest bid.
4. The winning bidder is required to invest c to get qualified.

We next turn to the analysis of the equilibrium bidding strategies.

3 Equilibrium bidding

The problem of finding a symmetric equilibrium bidding strategy with pre-auction qualification is equivalent to finding a symmetric equilibrium bidding strategy in an auction with participation or bidding costs. The qualification costs can be interpreted as bidding costs whose implications for equilibrium bidding have been exhaustively studied before.[12] For the case of post-auction qualification, the decision problem of the bidder with valuation v is equivalent to the decision problem of a bidder in an auction without qualification costs and an ex-post valuation of $v - c$. Thus, equilibrium bidding in the the second-price auction (subscript S) amounts to:

Proposition 1. *In the pre-auction scenario, only bidders with $v \geq \underline{v}(c)$ are qualified where $\underline{v}(c)$ implicitly is defined by*

$$\underline{v}(c)F^{N-1}(\underline{v}(c)) = c. \tag{1}$$

The symmetric and increasing equilibrium bidding function $\hat{\beta}_S(v) : [\underline{v}(c), 1] \to [0, 1]$ is then given by

$$\hat{\beta}_S(v) = v. \tag{2}$$

In the post-auction scenario, only bidders with $v \geq c$ undergo qualification. The symmetric and increasing equilibrium bidding function $\check{\beta}_S(v) : [c, 1] \to [0, 1]$ is given by

$$\check{\beta}_S(v) = v - c. \tag{3}$$

Bidding under pre-auction qualification is driven by what we will call the exclusion effect: as $\underline{v}(c) \geq c$, there are bidders who have valuations that are larger than the qualification costs but who nevertheless do not participate in the auction.[13] However, once a bidder decided to participate in the auction, the qualification costs are sunk

[11] Again, whether a bidder submits a bid or not is not observed by the other bidders.

[12] See, e.g., Menezes and Monteiro (2000), Kaplan and Sela (2006), Tan and Yilankaya (2006), and Celik and Yilankaya (2009).

[13] $\underline{v}(c) \geq c$ follows from the fact that $F^{N-1}(\underline{v}(c)) \leq 1$ for all $c \in [0, 1]$.

and thus there is no further bid shading. With post-auction qualification, bidders take into account that upon winning they have to pay the qualification cost. Thus, bidders shade their bids. This is what we call the bid-shading effect. However, as bidders only have to pay for qualification if they win the object, all bidders with a valuation larger than the qualification costs c participate in the auction and there is no further exclusion effect. The trade-off between the bid-shading effect and the exclusion effect is what will drive our results concerning revenue.

4 Revenue and efficiency

In this section, we analyze the expected revenue in the two qualification scenarios. The comparison of revenues in both cases helps to investigate whether there is a potential conflict of interest between the auctioneer's preferred type of qualification and the socially optimal scheme.

We start by pointing out the following result which is immediate from the fact that $\underline{v}(c) \geq c$.

Proposition 2. *The auction with post-auction qualification is always more efficient than the one with pre-auction qualification.*

Conditional on the object being sold, the allocation in for both qualification regimes is fully efficient in a symmetric, increasing equilibrium. However, due to the fact that $\underline{v}(c) \geq c$, more potential bidders choose not to participate in the auction with pre-auction qualification. As a consequence, the probability of selling the object is higher with post-auction qualification. Moreover, total qualification costs are higher under pre-auction qualification as all participating bidders have to incur them.[14]

When it comes to the comparison of revenues, there is no unambiguously dominating qualification regime and which regime should be preferred by the auctioneer depends on the situation at hand. Interestingly, although post-auction qualification is more efficient and more bidders enter in this case compared to pre-auction qualification, revenues can be lower in the scenario with post-auction qualification:

Proposition 3. *Post-auction qualification yields higher revenue than pre-auction qualification whenever*

$$c < \Pr(Y_{(2)}^N \leq \underline{v}(c) | Y_{(2)}^N \geq c) \mathbb{E}[Y_{(2)}^N | c \leq Y_{(2)}^N \leq \underline{v}(c)]. \tag{4}$$

Proof. The proof is relegated to the appendix. □

Inequality (4) has a straightforward interpretation: revenue in both scenarios depends on the second-highest value of the bidders. The bid-shading effect in the post-auction qualification regime lowers the second-highest value by c. The exclusion

[14] Cao and Tian (2010) show that the auction with upfront qualification may have equilibria with asymmetric cut-offs. These equilibria are not even weakly efficient (see their Proposition 2). Hence, if pre-auction qualification exhibits asymmetric equilibria, our efficiency result is even reinforced.

effect in the pre-auction qualification regime implies that if the second-highest value is between $\underline{v}(c)$ and c, the bidder with this value chooses not to participate and thus the revenue is zero in the pre-auction qualification scenario. This bidder would have participated in the auction with post-auction qualification and yielded a positive revenue. Thus, the left-hand side of inequality (4) corresponds to the revenue loss due to the bid-shading effect in the post-auction qualification regime whereas the right-hand side corresponds to the revenue loss due to the exclusion effect in the pre-auction qualification regime.

Unfortunately, the right-hand side of inequality (4) is neither monotone in the number of bidders N nor in the qualification costs c. This makes comparative statics a rather hard exercise. However, some general results can still be derived:

Proposition 4. *For each N, there exists a cutoff c' such that revenue from pre-auction qualification is higher than revenue from post-auction qualification for all $c \geq c'$.*

Proof. The proof is relegated to the appendix. □

This is a rather surprising result indicating that if the qualification costs are high, the seller benefits from asking each bidder to undergo qualification before entering the auction. This is due to the fact that with pre-auction qualification, each bidder decides whether to enter the auction based on the incurred cost and his winning probability. If qualification cost rise, the cost of participation increases. However, less bidders participate and thus the winning probability increases. This increase in winning probability dampens the increase of the exclusion effect and the marginal increase goes to zero. With post-auction qualification, the bid-shading effect increases linearly with the increase in qualification cost and the marginal increase is one for all cost levels. Thus, the bid-shading effect becomes more important and pre-auction qualification yields higher revenues.

For lower levels of the qualification costs, the revenue ranking depends on the distribution of the valuations:

Proposition 5. *Suppose the following limit exists and it holds that*

$$\lim_{x \to 0} \frac{F(x)}{x f(x)} < N^2 - 2N + 1 =: k(N). \tag{5}$$

In this case, there exists c'' such that revenue from post-auction qualification is higher than revenue from pre-auction qualification for all $c \leq c''$. If

$$\lim_{x \to 0} \frac{F(x)}{x f(x)} > k(N),$$

then there exists c''' such that revenue from pre-auction qualification is higher than revenue from post-auction qualification for all $c \leq c'''$.

Proof. The proof is relegated to the appendix. □

Condition (5) can be easily verified for a large class of distribution functions:

Corollary 1. *For all convex distribution functions F and $N > 2$, there exists c'' such that revenue from post-auction qualification is higher than revenue from pre-auction qualification for all $c \leq c''$. For strictly convex distributions, this holds true independent of N.*

Proof. The proof is relegated to the appendix. $\square$

Summing up, if F is convex or N relatively high, post-auction qualification yields higher revenues for small qualification costs. This is due to the fact that convex distribution place a relatively small probability on low valuations. In this case, a bidder who decides whether to invest in qualification faces a relatively low probability of wining the object. This is also true if the number of bidders is relatively high. Thus, in those cases the exclusion effect outweighs the bid shading effect for small qualification costs.

We close this chapter by providing two examples that illustrate our results.

Uniform distribution

Here $\lim_{x\to 0} F(x)/xf(x) = \lim_{x\to 0} x/x = 1$. Since $N^2 - 2N + 1 > 1$ for all $N > 2$, pre-auction qualification yields higher revenues for small c and $N = 2$. For $N > 2$, post-auction qualification will yield higher revenues for small qualification costs c. The left part of *Figure* **??** illustrates this finding.

$F(x) = x^2$

Here $\lim_{x\to 0} F(x)/xf(x) = \lim_{x\to 0} x^2/2x^2 = 1/2$. Since $N^2 - 2N + 1 > 1/2$ for all $N > 1.7$, post-auction qualification will yield higher revenues for all N and small c. The result for this example is illustrated in *Figure* **??** on the right.

5 Conclusion

In this paper, we address the question when the auctioneer should demand qualification: before or after an auction. We show that even though post-auction qualification always results in the more efficient allocation, pre-auction qualification may yield higher revenue. More precisely, pre-auction qualification yields a higher revenue whenever the cost of qualification is high. If the cost of qualification is low, the revenue ranking depends on the distribution of values. Thus, even if one abstracts from the risk that a bidder may fail to be qualified, pre-auction qualification may be beneficial for the auctioneer. As revenue equivalence holds in our setting, the results extend beyond the second-price auction.

If the auctioneer has more degrees of freedom in designing the auction than choosing the timing of the qualification, post-auction qualification dominates pre-auction qualification in terms of revenue for any initial parameter of the qualification costs c. This is due to the fact that the optimal auction with pre-auction qualification involves subsidies to all losing bidders.[15] This optimal mechanism can be replicated

[15] See Menezes and Monteiro (2000) for details.

with post-auction qualification without using subsidies and thus yielding higher revenue. However, the optimal mechanism is tailored to the fine details of the beliefs of the bidders which may be the reason that such optimal mechanisms are rarely observed in real-life procurement.

6 Appendix

6.1 Proof of *Proposition 3*

Proof. An auctioneer can expect to generate the revenue (denoted by R)

$$\mathbb{E}\left[\hat{R}\right] = (1 - F_2(\underline{v}(c)))\,\mathbb{E}\left[Y_{(2)}^N \,\middle|\, Y_{(2)}^N > \underline{v}(c)\right] = \int_{\underline{v}(c)}^1 y f_2(y) dy \tag{6}$$

from pre-auction qualification and revenue

$$\mathbb{E}\left[\check{R}\right] = (1 - F_2(c))\left(\mathbb{E}\left[Y_{(2)}^N \,\middle|\, Y_{(2)}^N > c\right] - c\right) = \int_c^1 y f_2(y) dy - (1 - F_2(c))\, c \tag{7}$$

from post-auction qualification. Now consider the difference of the expected revenues between the two qualification regimes.

$$\begin{aligned}
\Delta R := \mathbb{E}\left[\check{R}\right] - \mathbb{E}\left[\hat{R}\right] \geq 0 \\
\Leftrightarrow (1 - F_2(c))\left(\mathbb{E}\left[Y_{(2)}^N \,\middle|\, Y_{(2)}^N > c\right] - c\right) - (1 - F_2(\underline{v}(c)))\,\mathbb{E}\left[Y_{(2)}^N \,\middle|\, Y_{(2)}^N > \underline{v}(c)\right] \geq 0 \\
\Leftrightarrow \int_c^{\underline{v}(c)} y f_2(y) dy - (1 - F_2(c))\, c \geq 0 \\
\Leftrightarrow \frac{\Pr\left(c \leq Y_{(2)}^N \leq \underline{v}(c)\right)}{\Pr\left(Y_{(2)}^N > c\right)} \mathbb{E}\left[Y_{(2)}^N \,\middle|\, c \leq Y_{(2)}^N \leq \underline{v}(c)\right] - c \geq 0 \\
\Leftrightarrow \Pr\left(Y_{(2)}^N \leq \underline{v}(c) \,\middle|\, Y_{(2)}^N > c\right) \mathbb{E}\left[Y_{(2)}^N \,\middle|\, c \leq Y_{(2)}^N \leq \underline{v}(c)\right] - c \geq 0 \\
\Leftrightarrow c \leq \Pr\left(Y_{(2)}^N \leq \underline{v}(c) \,\middle|\, Y_{(2)}^N > c\right) \mathbb{E}\left[Y_{(2)}^N \,\middle|\, c \leq Y_{(2)}^N \leq \underline{v}(c)\right] =: c'
\end{aligned}$$

Since we have that $0 \leq \Pr\left(Y_{(2)}^N \leq \underline{v}(c) \,\middle|\, Y_{(2)}^N > c\right) \mathbb{E}\left[Y_{(2)}^N \,\middle|\, c \leq Y_{(2)}^N \leq \underline{v}(c)\right] \leq 1$, this c' is well defined and the proposition holds. □

6.2 Proof of *Proposition 4*

Proof. As the revenue is equal to zero for $c = 1$ in both scenarios, we will show that the difference in revenue ΔR has a local maximum in $c = 1$. Consider the following derivatives

$$\frac{\partial}{\partial c} \Delta R = \underline{v} f_2^{(N)} \underline{v}_c + F_2^{(N)}(c) - 1$$

and

$$\frac{\partial^2}{\partial c^2} \Delta R = \underline{v}_c^2 \left(f_2(\underline{v}) + \underline{v} f_2^{(N)\prime}(\underline{v})\right) + \underline{v} f_2^{(N)}(\underline{v}) \frac{\partial^2 \underline{v}}{\partial c^2} + f_2(c)$$

with

$$\underline{v}_c := \frac{\partial \underline{v}}{\partial c} = \frac{1}{F^{N-1}(\underline{v}) + \underline{v}(n-1)F^{N-2}(\underline{v})f(\underline{v})}.$$

Note that for $\lim_{c\to 1}$, both $\underline{v}_c$ and $\partial \underline{v}_c/\partial c$ stay bounded. Also note that for all probability densities f, $f_2^{(N)}(1) = 0$ and $\lim_{x\to 1} f_2^{(N)\prime}(x) < 0$.

We have

$$\lim_{c\to 1} \frac{\partial}{\partial c}\Delta R = 0 \tag{8}$$

and

$$\lim_{c\to 1} \frac{\partial^2}{\partial c^2}\Delta R = \lim_{c\to 1} \underline{v}_c^2 \underline{v} f_2'(v) < 0. \tag{9}$$

The second limit is a strict inequality since we assumed $f(1) < \infty$ which means that $\lim_{c\to 1} \underline{v}_c > 0$. Finally note that expression (8) provides the possible extremum in $c = 1$ whereas expression (9) confirms that it is as a local maximum. This means that in a neighborhood $U = (\tilde{c}, 1)$, $\Delta R < 0$ for all $c \in [0, 1]$. □

6.3 Proof of *Proposition 5*

Proof. As the difference in revenue ΔR is equal to zero for $c = 0$ (revenue equivalence), we will show that the first derivative of ΔR is positive or negative, respectively. We have

$$\begin{aligned} \frac{\partial}{\partial c}\Delta R &= \frac{N(N-1)(1-F(\underline{v}))f(\underline{v})}{\frac{F(\underline{v})}{\underline{v}} + (N-1)f(\underline{v})} - 1 - F_2^{(N)}(c) \\ &= \frac{N(N-1)(1-F(\underline{v}))}{\frac{F(\underline{v})}{\underline{v}f(\underline{v})} + N - 1} - 1 - F_2^{(N)}(c). \end{aligned} \tag{10}$$

Now there are two possible scenarios:

1. $\lim_{c\to 0} \frac{\partial}{\partial c}\Delta R \to \infty$.
2. $\lim_{c\to 0} \frac{\partial}{\partial c}\Delta R < \infty$ for all N.

If the first possibility holds, we have

$$\lim_{c\to 0} \frac{N(N-1)(1-F(\underline{v}))}{\frac{F(\underline{v})}{\underline{v}f(\underline{v})} + N - 1} \to \infty.$$

However, this means that $F(\underline{v})/\underline{v}f(\underline{v}) + N - 1 \to 0$ which implies that $F(\underline{v})/vf(\underline{v}) \to 1 - N < 0$. This is a contradiction to

$$\begin{cases} F \text{ probability distribution} \\ f \text{ density} \\ \underline{v} \in [0, 1] \end{cases}$$

which means that the second case is the relevant one. We are looking at a rational function and since the limits on both sides exist, we can write

$$\lim_{x\to 0} \frac{N(N-1)(1-F(\underline{v}))}{\frac{F(\underline{v})}{\underline{v}f(\underline{v})}+N-1} = \frac{\lim_{x\to 0} N(N-1)(1-F(\underline{v}))}{\lim_{x\to 0} \frac{F(\underline{v})}{\underline{v}f(\underline{v})}+N-1}.$$

We are looking at three possible scenarios:

1. $\lim_{c\to 0} F(\underline{v})/\underline{v}f(\underline{v}) = 0.$
2. $\lim_{c\to 0} F(\underline{v})/\underline{v}f(\underline{v}) = m > 0.$
3. $\lim_{c\to 0} F(\underline{v})/\underline{v}f(\underline{v}) = \infty.$

For the first two scenarios, we have

$$\begin{aligned}\lim_{c\to 0} \frac{\partial}{\partial c}\Delta R &= \lim_{c\to 0} \frac{N(N-1)(1-F(v))}{\frac{F(v)}{vf(v)}+(N-1)} - 1 - F_2(c)\\ &= \frac{\lim_{c\to 0}(N(N-1)(1-F(v)))}{\lim_{c\to 0}\left(\frac{F(v)}{vf(v)}+(N-1)\right)} - 1\\ &= \frac{N(N-1)}{m+(N-1)} - 1.\end{aligned}$$

For $m = 0$, this gives $\lim_{c\to 0} \frac{\partial}{\partial c}\Delta R = N-1$. For $m > 0$, this leaves us with the condition

$$m < N^2 - 2N + 1 = k(N) \tag{11}$$

for $\lim_{c\to 0} \frac{\partial}{\partial c}\Delta R > 0$. If the third possibility occurs, $\frac{\partial}{\partial c}\Delta R = -1 < 0$. □

6.4 Proof of *Corollary 1*

Proof. For a convex distribution F, it holds that $F(y) \geq F(x) + f(x)(y-x)$ for all $x, y \in [0,1]$. Choosing $y = 0$ and with $F(0) = 0$, we get $F(x)/xf(x) \leq 1$, and with $k(N) > 1$ for all $N > 2$ the corollary follows. For a strictly convex distribution F, $F(x)/xf(x) < 1$ and it follows for all N. □

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