Work Integrated Learning Programmes Division

Assignment 1

DSECL ZC416 - Mathematical Foundations for Data Science

Instructions

1. Use Python only. Do not use vectorized operations in Python, instead use 'for loops' wherever needed. Attach only the relevant data in your submission and no need to submit the code, unless asked for.

2. By random entries, we mean a system generated random number. No marks would be awarded for deterministic entries.

3. This is not a group activity. Each student should do the problems and submit individually.

4. Submissions beyond 9th of December, 2021 17.00 hrs would not be graded

5. Assignments sent via email would not be accepted

6. Copying is strictly prohibited. Adoption of unfair means would lead to disciplinary action.

7. Assignment have to be scanned as a pdf document and uploaded on canvas. File name should be your_bitsid.pdf

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Answer all the questions

Q1) Implementing Gaussian Elimination Method

(i) Find the approximate time your computer takes for a single addition by adding first 10^6 positive integers using a for loop and dividing the time taken by 10^6. Similarly find the approximate time taken for a single multiplication and division. Report the result obtained in the form of a table. (0.5)
Deliverable(s): A tabular column indicating the time taken for each of the operations

(ii) Write a function to implement Gauss elimination with and without pivoting. Also write the code to count the number of additions, multiplications and divisions performed during Gaussian elimination. *Ensure that the Gauss elimination performs 5S arithmetic which necessitates 5S arithmetic rounding for every addition, multiplication and division performed in the algorithm. If this is not implemented correctly, the rest of the answers will be considered invalid.* Note that this is not same as simple 5 digit rounding at the end of the computation. Do not hardwire 5S arithmetic in the code and use dS instead. The code can then be run with various values of d. (0.5 + 0.5)
Deliverable(s): The code for the Gaussian elimination with and without partial pivoting with the rounding part

(iii) Generate random matrices of size $n \times n$ where $n = 100, 200, \ldots, 1000$. Also generate a random $b \in \mathbb{R}^n$ for each case. Each number must be of the form $m.dddd$ (Example : 4.5444) which means it has 5 Significant digits in total. Perform Gaussian elimination with and without partial pivoting for each n value (10 cases) above. Report the number of additions, divisions and multiplications for each case in the form of a table. No need of the code and the matrices / vectors. (0.5 + 0.5)
Deliverable(s): Two tabular columns indicating the number of additions, multiplications and divisions for each value of n, for with and without pivoting

(iv) Using the code determine the actual time taken for Gaussian elimination with and without partial pivoting for the 10 cases and compare this with the theoretical time. Present this data in a tabular form. Assuming $T_1(n)$ is the actual time calculated for an $n \times n$ matrix, plot a graphs of $\log(T_1(n))$ vs $\log(n)$ (for the 10 cases) and fit a straight line to the observed curve and report the slope of the lines. Ensure that separate graphs are to be plotted for the method with and without partial pivoting. (0.5 + 1 + 1)
Deliverable(s):

(a) A table

S. No.	n	Actual time with pivoting	Actual time without pivoting	Theoretical time

(b) two log log plots and the slope in both the cases

Q2) Implementing Gauss Seidel and Gauss Jacobi Methods

(i) Write a function to check whether a given square matrix is diagonally dominant or not. If not, the function should indicate if the matrix can be made diagonally dominant by interchanging the rows? Code to be written and submitted. (1)
Deliverable(s): The code

(ii) Write a function to generate Gauss Seidel iteration for a given square matrix. The function should also return the values of $1, \infty$ and Frobenius norms of the iteration matrix. Generate a random 4×4 matrix. Report the iteration matrix and its norm values returned by the function along with the input matrix. (1)
Deliverable(s): The input matrix, iteration matrix and the three norms obtained

(iii) Repeat part (ii) for the Gauss Jacobi iteration. (1)
Deliverable(s): The input matrix, iteration matrix and the three norms obtained

(iv) Write a function that perform Gauss Seidel iterations. Generate a random 4×4 matrix A and a suitable random vector $b \in \mathbb{R}^4$ and

report the results of passing this matrix to the functions written above. Write down the first ten iterates of Gauss Seidel algorithm. Does it converge? Generate a plot of $\|x_{k+1} - x_k\|_2$ for the first 10 iterations. Take a screenshot and paste it in the assignment document. (1)
Deliverable(s): The input matrix and the vector, the 10 successive iterates and the plot

(v) Repeat part (iv) for the Gauss Jacobi method. (1)
Deliverable(s): The input matrix and the vector, the 10 successive iterates and the plot