Mathematical description of SPLab

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Abstract. We present a brief description of the mathematical formulation and data sets of the SPLab model.

1 Stochastic optimization model

1.1 Vectorial notation

The dynamic programming formulation for the long-term operation planning problem considered in the SPLab model can be written as:

$$
\begin{aligned}
Q_t([v_t, a_{[t-\widehat{p},t-1]}], \eta_t) = \quad & \min \quad c^\top g_t + \tilde{c}^\top \mathrm{df}_t + \theta^\top f_t + \tilde{\theta}^\top s_t + \beta \mathcal{Q}_{t+1}([v_{t+1}, a_{[t+1-\widehat{p},t]}]) \\
& s.t. \quad v_{t+1} = v_t + a_t - q_t - s_t \\
& \qquad a_t = \left(\zeta_t + \textstyle\sum_{\nu=1}^{\widehat{p}} \Phi_{t,\nu} a_{t-\nu}\right) \bullet \eta_t \\
& \qquad q_t + M_I g_t + \widetilde{M_I}\,\mathrm{df}_t + M_D f_t = d_t, \\
& \qquad 0 \le v_{t+1} \le \overline{v}, \quad 0 \le q_t \le \overline{q}, \quad \underline{g} \le g_t \le \overline{g}, \\
& \qquad 0 \le \mathrm{df}_t \le \overline{\mathrm{df}}_t, \quad 0 \le f_t \le \overline{f}
\end{aligned}
$$

$$
\mathcal{Q}_{t+1}([v_{t+1}, a_{[t+1-\widehat{p},t]}]) = \begin{cases} \rho_{t+1}\left[Q_{t+1}([v_{t+1}, a_{[t+1-\widehat{p},t]}], \eta_{t+1})\right] & , t \in \{1, \ldots, T-1\} \\ 0 & , t = T \end{cases}
$$

for all $t = 1, \ldots, T$. For each stage t, the decision vector is $x_t = (v_{t+1}, a_t, q_t, s_t, g_t, \mathrm{df}_t, f_t)$. In this model the only uncertainty considered is the vector multiplicative error, that is, $\xi_t = \eta_t$.

The risk measure used considers a mean-AV@R convex combination, given by:

$$
\rho_t[Z_t] = (1-\lambda)\mathbb{E}[Z_t] + \lambda \mathrm{AV@R}_\alpha[Z_t]
$$

with $\lambda \in [0, 1]$ and $\alpha \in (0, 1]$ being chosen parameters.

1.1.1 Objective function equation

The *objective function*

$$
c^\top g_t + \tilde{c}^\top \mathrm{df}_t + \theta^\top f_t + \tilde{\theta}^\top s_t + \beta \mathcal{Q}_{t+1}([v_{t+1}, a_{[t+1-\widehat{p},t]}])
$$

represents the sum of thermal generation costs, deficit (load shedding) costs, small penalties for energy interchange, small penalties for spillage and the cost-to-go function $\mathcal{Q}_{t+1}([v_{t+1}, a_{[t+1-\widehat{p},t]}])$ where:

- c cost vector whose components are unit costs (\$/MWh) of thermal plants from each subsystem;
- g_t thermal generation vector whose components are thermal generations (MWmonth) at stage t of thermal plants from each subsystem;
- $\tilde{c}$ cost vector whose components are unit costs (\$/MWh) of energy deficits (load shedding) of each depth and from each subsystem;
- df_t deficit energy vector (load shedding) is a slack thermal generation vector whose components are energy deficits (MWmonth) at stage t of each depth and from each subsystem;
- θ penalty vector for energy flow whose components are penalties (\$/MWmonth) for energy interchange between each interconnected subsystem pair;
- f_t energy flow vector whose components are energy interchanges at stage t between each interconnected subsystem pair;
- $\tilde{\theta}$ penalty vector whose components are penalties for spillages in each subsystem;
- s_t spilled energy vector whose components are spilled energies in each subsystem during stage t;
- β discount factor constant;
- $\mathcal{Q}_{t+1}(\cdot)$ cost-to-go function of stage $t+1$, which corresponds to the total cost risk from stage $t+1$ onwards;
- v_t stored energy vector whose components are stored energies on each equivalent energy reservoir at the beginning of stage t;
- a_t energy inflow vector whose components are energy inflows on each equivalent energy reservoir during stage t;
- $a_{[t-\hat{p},t-1]}$ vector of energy inflow vectors from stage $t-\hat{p}$ to $t-1$;
- $\hat{p}$ time-series model maximum order;
- η_t vector multiplicative error whose components are the ratio between the components of energy inflows a_t during stage t and the corresponding predicted energy inflows;
- $Q_t(\cdot,\cdot)$ total cost function of stage t.

1.1.2 Energy balance equation

The *energy balance* equation, in MWmonth, for each equivalent energy reservoir and stage t is:

$$v_{t+1} = v_t + a_t - q_t - s_t$$

where:

- v_t stored energy vector whose components are stored energies on each equivalent energy reservoir at the beginning of stage t;
- a_t energy inflow vector whose components are energy inflows on each equivalent energy reservoir during stage t;
- q_t turbined energy (hydro generation) vector whose components are turbined energies on each equivalent energy reservoir during stage t;
- s_t spilled energy vector whose components are spilled energies on each equivalent energy reservoir during stage t;

1.1.3 Time-series equation

The *time-series* equation of the PVAR$_m$ model, in MWmonth, for each equivalent energy reservoir and stage t is:

$$a_t = \left(\zeta_t + \sum_{\nu=1}^{\widehat{p}} \Phi_{t,\nu} a_{t-\nu} \right) \bullet \eta_t$$

where:

- a_t energy inflow vector whose components are energy inflows on each equivalent energy reservoir during stage t;
- ζ_t intercept vector of stage t;
- $\Phi_{t,\nu}$ lag ν coefficient matrix of stage t;
- $\widehat{p}$ time-series model maximum order;
- $\bullet$ Hadamard vector product, $(\boldsymbol{a} \bullet \boldsymbol{b})_i = a_i b_i$;
- η_t vector multiplicative error whose components are ratios between the components of energy inflows a_t during stage t and the corresponding predicted energy inflows;

1.1.4 Load balance equation

The *load balance* equation, in MWmonth, for each subsystem and stage t is:

$$q_t + M_I g_t + \widetilde{M}_I \, \mathrm{df}_t + M_D f_t = d_t$$

where:

d_t energy demand (load) vector on each subsystem at stage t;

q_t turbined energy (hydro generation) vector whose components are turbined energies on each equivalent energy reservoir during stage t;

M_I zero-one indicator matrix whose columns contain all deficit positions and lines contain all subsystems positions;

$M_I g_t$ total thermal generation on each subsystem at stage t;

g_t thermal generation vector whose components are thermal generations at stage t of thermal plants from each subsystem;

$\widetilde{M}_I$ zero-one indicator matrix whose columns contain all thermal generation positions and lines contain all subsystems positions;

$\widetilde{M}_I \,\mathrm{df}_t$ total deficit on each subsystem at stage t;

df_t deficit energy vector (load shedding) is a slack thermal generation vector whose components are energy deficits (MWmonth) at stage t of each depth and from each subsystem;

M_D flow direction matrix whose columns contain all energy flow positions and lines contain all subsystems positions;

$M_D f_t$ net energy interchange;

f_t energy flow vector whose components are energy interchanges at stage t between each interconnected subsystem pair;

1.1.5 Bounds

Bounds on decision variables are:

$0 \le v_{t+1} \le \overline{v}$ stored energy upper bound;

$0 \le q_t \le \overline{q}$ hydro generated energy upper bound;

$\underline{g} \le g_t \le \overline{g}$ thermal generation lower and upper bound;

$0 \le f_t \le \overline{f}$ energy flow lower and upper bound; and

$0 \le \mathrm{df}_t \le \overline{\mathrm{df}}_t$ deficit (slack thermal generation) lower and upper bound.

1.2 Indicial notation

The dynamic programming formulation for the operation planning problem considered in the SPLab model can be written as:

$$
\begin{aligned}
Q_t([v_t, a_{[t-\widehat{p},t-1]}], \eta_t) = \quad & \min \quad \Bigg\{ \sum_{k=1}^{n_s} \Big(\sum_{j \in T_k} c_j g_{t,j} + \sum_{i=1}^{n_\text{d}} \tilde{c}_i \, \text{df}_{t,ki} + \sum_{l \in \Omega_k} \theta_{lk} f_{t,lk} + \tilde{\theta}_k s_{t,k} \Big) \\
& \qquad\qquad + \ \beta \mathcal{Q}_{t+1}([v_{t+1}, a_{[t+1-\widehat{p},t]}]) \Bigg\} \\
& s.t. \quad v_{t+1,k} = v_{t,k} + a_{t,k} - q_{t,k} - s_{t,k}, \\
& \qquad\;\; a_{t,k} = \Big(\zeta_{t,k} + \sum_{\nu=1}^{\widehat{p}} \sum_{l=1}^{n_s} \phi_{t\nu,kl} a_{t-\nu,l} \Big) \cdot \eta_{t,k}, \\
& \qquad\;\; q_{t,k} + \sum_{j \in T_k} g_{t,j} + \sum_{i=1}^{n_\text{d}} \text{df}_{t,ki} + \sum_{l \in \Omega_k} \Big(r_{lk} f_{t,lk} - f_{t,kl} \Big) = d_{t,k}, \\
& \qquad\;\; 0 \le v_{t+1,k} \le \overline{v}_k, \quad 0 \le q_{t,k} \le \overline{q}_k, \quad \underline{g}_j \le g_{t,j} \le \overline{g}_j, \\
& \qquad\;\; 0 \le \text{df}_{t,ki} \le \overline{\text{df}}_{t,ki}, \quad 0 \le f_{t,lk} \le \overline{f}_{lk}, \\
& \qquad\;\; \forall k = 1, \ldots, n_s, \quad \forall j \in T_k, \quad \forall l \in \Omega_k, \quad \forall i = 1, \ldots, n_d.
\end{aligned} \tag{1}
$$

$$
\mathcal{Q}_{t+1}([v_{t+1}, a_{[t+1-\widehat{p},t]}]) = \begin{cases} \rho_{t+1} \left[Q_{t+1}([v_{t+1}, a_{[t+1-\widehat{p},t]}], \eta_{t+1}) \right] & , t \in \{1, \ldots, T-1\} \\ 0 & , t = T \end{cases} \tag{2}
$$

for all $t = 1, \ldots, T$. For each stage t, the decision vector is $x_t = (v_{t+1}, a_t, q_t, s_t, g_t, \text{df}_t, f_t)$. In this model the only uncertainty considered is the multiplicative error, that is, $\xi_t = \eta_t$.

The risk measure used considers a mean-AV@R convex combination, given by:

$$
\rho_t[Z_t] = (1 - \lambda)\mathbb{E}[Z_t] + \lambda \text{AV@R}_\alpha[Z_t]
$$

with $\lambda \in [0, 1]$ and $\alpha \in (0, 1]$ being chosen parameters.

1.2.1 Objective function equation

The *objective function*

$$
\sum_{k=1}^{n_s} \Big(\sum_{j \in T_k} c_j g_{t,j} + \sum_{i=1}^{n_\text{d}} \tilde{c}_i \, \text{df}_{t,ki} + \sum_{l \in \Omega_k} \theta_{lk} f_{t,lk} + \tilde{\theta}_k s_{t,k} \Big) + \beta \mathcal{Q}_{t+1}([v_{t+1}, a_{[t+1-\widehat{p},t]}])
$$

represents the sum of thermal generation costs, deficit (slack thermal generation) costs, small penalties for energy interchange, small penalties for spillage and the cost-to-go function $\mathcal{Q}_{t+1}([v_{t+1}, a_{[t+1-\widehat{p},t]}])$ where:

n_s	number of subsystems;
T_k	thermal set of subsystem k;
Ω_k	set of subsystems directly connected to subsystem k;
c_j	unit cost (\$/MWmonth) associated to the thermal plant j;
$g_{t,j}$	thermal generation (MWmonth) at stage t of thermal plant j;
n_d	number of deficit (slack thermal generation) depths;
$\tilde{c}_i$	unit cost (\$/MWh associated to the deficit depth i;
$\mathrm{df}_{t,ki}$	deficit (MWmonth) at stage t in subsystem k of depth i;
θ_{lk}	penalty for energy flow from subsystem l to subsystem k;
$f_{t,lk}$	energy flow (MWmonth) at stage t from subsystem l to subsystem k;
$\tilde{\theta}_k$	penalty for spillage in subsystem k;
$s_{t,k}$	spilled energy (MWmonth) during stage t in subsystem k
β	discount factor;
$\mathcal{Q}_{t+1}(\cdot)$	cost-to-go function of stage $t+1$, which corresponds to the total cost risk from stage $t+1$ onwards;
$v_{t,k}$	stored energy at the beginning of stage t in subsystem k;
$a_{t,k}$	energy inflow (MWmonth) during stage t in subsystem k;
$a_{[t-\hat{p},t-1]}$	energy inflow (MWmonth) vector from stage $t-\hat{p}$ to $t-1$;
$\hat{p}$	time-series model maximum order;
$\eta_{t,k}$	time-series multiplicative error of stage t in subsystem k; and
$Q_t(\cdot,\cdot)$	total cost of stage t.

1.2.2 Energy balance equation

The *energy balance* equation for each subsystem k is:

$$v_{t+1,k} = v_{t,k} + a_{t,k} - q_{t,k} - s_{t,k}$$

where:

$v_{t,k}$	stored energy at the beginning of stage t in subsystem k;
$a_{t,k}$	energy inflow during stage t in subsystem k;
$q_{t,k}$	generated energy during stage t in subsystem k; and
$s_{t,k}$	spilled energy during stage t in subsystem k.

1.2.3 Time-series equation

The *time-series* equation of the PVAR_m model, in MWmonth, for each subsystem k is:

$$a_{t,k} = \left(\zeta_{t,k} + \sum_{\nu=1}^{\widehat{p}} \sum_{l=1}^{n_s} \phi_{t\nu,kl} a_{t-\nu,l}\right) \cdot \eta_{t,k}$$

where:

n_s	number of subsystems;
$a_{t,k}$	energy inflow during stage t in subsystem k;
$\zeta_{t,k}$	intercept of stage t and subsystem k;
$\phi_{t\nu,kl}$	lag ν coefficient of stage t representing the influence of subsystem l in subsystem k;
$\widehat{p}$	time-series model maximum order;
$\eta_{t,k}$	multiplicative error of stage t in subsystem k;

1.2.4 Load balance equation

The *load balance* equation, in MWmonth, for each subsystem k and stage t is:

$$q_{t,k} + \sum_{j \in T_k} g_{t,j} + \sum_{i=1}^{n_\text{d}} \text{df}_{t,ki} + \sum_{l \in \Omega_k} \left(r_{lk} f_{t,lk} - f_{t,kl}\right) = d_{t,k}$$

where:

T_k	thermal set of subsystem k;
Ω_k	set of subsystems directly connected to subsystem k;
$d_{t,k}$	energy demand (load) at stage t in subsystem k;
$q_{t,k}$	hydro generation at stage t in subsystem k;
$\sum_{j \in T_k} g_{t,j}$	total thermal generation at stage t in subsystem k ;
$g_{t,j}$	thermal generation at stage t of thermal plant j;
n_d	number of deficit (slack thermal generation) depths;
$\sum_{i=1}^{n_\text{d}} \text{df}_{t,ki}$	total deficit in subsystem k at stage t;
$\text{df}_{t,ki}$	deficit at stage t in subsystem k of depth i;
$\sum_{l \in \Omega_k} (r_{lk} f_{t,lk} - f_{t,kl})$	net energy interchange;
r_{lk}	transmission loss factor from subsystem l to subsystem k; and
$f_{t,lk}$	energy flow at stage t from subsystem l to subsystem k.

1.2.5 Bounds

Bounds on decision variables are:

$$
\begin{aligned}
0 \leq v_{t+1,k} \leq \overline{v}_k \quad & \text{stored energy upper bound;} \\
0 \leq q_{t,k} \leq \overline{q}_k \quad & \text{generated energy upper bound;} \\
\underline{g}_k \leq g_{t,k} \leq \overline{g}_k \quad & \text{thermal generation lower and upper bound;} \\
0 \leq f_{t,kl} \leq \overline{f}_{kl} \quad & \text{energy flow upper bound; and} \\
0 \leq \mathrm{df}_{t,ki} \leq \overline{\mathrm{df}}_{t,ki} \quad & \text{deficit (slack thermal generation) upper bound.}
\end{aligned}
$$

1.3 Time-series model

A Periodic Vector Autoregressive Multiplicative model of period $\mathbb{S}$ and order $\mathbf{p} = (p_1, \ldots, p_{\mathbb{S}})$, $\mathrm{PVAR}_m(\mathbf{p})$, is given by

$$
a_t = \left(\zeta_t + \sum_{\nu=1}^{p_t} \Phi_{t,\nu} a_{t-\nu} \right) \bullet \eta_t,
$$

for all positive t, where $\{\eta_1, \eta_2, \ldots\}$ are independent multiplicative errors. For each time t, the model coefficients and order are equal modulo $\mathbb{S}$; the errors are identically distributed modulo $\mathbb{S}$ with unit mean and constant variance (modulo $\mathbb{S}$). Formally:

- $\zeta_t = \zeta_{t+\mathbb{S}}$ and $\Phi_{t,\nu} = \Phi_{t+\mathbb{S},\nu}$;
- $p_t = p_{t+\mathbb{S}}$;
- η_t has the same distribution as $\eta_{t+\mathbb{S}}$; and
- $\mathbb{E}[\eta_t] = \mathbb{1}$,

where $\mathbb{1} := (1, \ldots, 1)^\top$.

It is instructive to note that PVAR_m is a vector model:

$$
a_t = \begin{pmatrix} a_{t,1} \\ a_{t,2} \\ \vdots \\ a_{t,n_s} \end{pmatrix}, \quad
\zeta_t = \begin{pmatrix} \zeta_{t,1} \\ \zeta_{t,2} \\ \vdots \\ \zeta_{t,n_s} \end{pmatrix}, \quad
\Phi_{t,\nu} = \begin{pmatrix} \phi_{t\nu,11} & \phi_{t\nu,12} & \cdots & \phi_{t\nu,1n_s} \\ \phi_{t\nu,21} & \phi_{t\nu,22} & \cdots & \phi_{t\nu,2n_s} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{t\nu,n_s1} & \phi_{t\nu,n_s2} & \cdots & \phi_{t\nu,n_sn_s} \end{pmatrix}, \quad
\eta_t = \begin{pmatrix} \eta_{t,1} \\ \eta_{t,2} \\ \vdots \\ \eta_{t,n_s} \end{pmatrix},
$$

where n_s is the number of subsystems.

2 Data description

Data regarding the operation planning problem are provided by four files namely `data.txt`, `linear_models.txt`, `scenarios.txt`, `scenarios_efs.txt`. A detailed description of each input file follows.

Remark 1. The input data are described following the math formulation and *not* the order of the inputs on the files.

2.1 `data.txt` file

This file is organized in sections identified by “field descriptors” in the sense that each section contains the corresponding data values.

2.1.1 Risk averse parameters (λ, α)

The adopted risk measure considers a mean-AV@R convex combination, given by:

$$\rho_t[Z_t] = (1-\lambda)\mathbb{E}[Z_t] + \lambda \text{AV@R}_\alpha[Z_t],$$

with $\lambda \in [0,1]$ and $\alpha \in (0,1]$ being fixed parameters. The risk parameters λ and α are identified by `RISK AVERSION DATA` keyword as described below.

```
RISK AVERSION DATA
------------------
RISK AVERSE APPROACH       : 1    (0 = NO; 1 = AVAR; 2 = AVAR WITH STATE VARIABLE)
RISK AVERSE LAMBDA PARAMETER
 STAGE   LAMBDA   ALPHA   (0 <= LAMBDA <= 1, 0 <= ALPHA <= 1)
-------  ------  ------
      1    0.15    0.05
9999999
```

In this example, $\lambda = 0.15$ and $\alpha = 0.05$.

Remark 2. Flag equal to 0 (zero) in the "`RISK AVERSE APPROACH`" field corresponds to the risk neutral case.

2.1.2 Monthly discount rate β

The monthly discount rate is a constant used to rescale a future cost to the same basis of a present cost. Considering the annual rate r_{annual}, we can get the monthly discount rate by the following formula:

$$\beta = \frac{1}{(1 + r_{\text{annual}})^{\frac{1}{12}}}.$$

This block is identified by `ANUAL DISCOUNT RATE` keyword. The r_{annual} input is described below.

```
ANUAL DISCOUNT RATE (%)    : 12
```

In this example, $r_{\text{annual}} = 0.12$ and therefore $\beta = 1/(1+0.12)^{\frac{1}{12}} = 0.99060$.

2.1.3 Subsystem set K

The subsystems consist of partitions of the system which have their particular demand, thermal generation and equivalent energy reservoir. Those subsystems are connected by equivalent energy transmission lines. The subsystems set K are identified by the `SYSTEMS DATA` keyword. The input K is described below.

```
SYSTEMS DATA
 CODE       NAME
------  ------------
     1       SUDESTE
     2           SUL
     3      NORDESTE
     4         NORTE
     5       NOFICT1
9999999
```

Remark 3. In this particular case $n_s = 5$.

2.1.4 Energy demand $d_{t,k}$

The load balance equation is given by:

$$q_{t,k} + \sum_{j \in T_k} g_{t,j} + \sum_{i=1}^{n_\mathrm{d}} \mathrm{df}_{t,ki} + \sum_{l \in \Omega_k} \Big(r_{lk} f_{t,lk} - f_{t,kl} \Big) = d_{t,k},$$

with the energy demand (load) $d_{t,k}$. The energy demand block is identified by the `ENERGY DEMAND` keyword and is described below.

```
ENERGY DEMAND
 STAGE    VALUE SYS   1  VALUE SYS   2  VALUE SYS   3  VALUE SYS   4  VALUE SYS   5
-------   ------------   ------------   ------------   ------------   ------------
    001          45515          11692          10811           6507              0
    002          46611          11933          10683           6564              0
    003          47134          12005          10727           6506              0
    004          46429          11478          10589           6556              0
    005          45622          11145          10389           6645              0
    ...            ...            ...            ...            ...            ...
    116          46149          11051          10372           6772              0
    117          46336          10917          10675           6843              0
    118          46551          11015          10934           6815              0
    119          46035          11156          11004           6871              0
    120          45234          11297          10914           6701              0
9999999
```

For instance, the energy demand for stage 4 and subsystem 3 is $d_{4,3} = 10589$ MWmonth.

Remark 4. The subsystem $k = 5$ has no demand because it represents just a *transshipment* node.

2.1.5 Deficit cost $\tilde{c}_j$, upper bound $\overline{\mathrm{df}}_{t,ki}$ and number of deficit depths n_d

The deficit is a slack thermal generation which represents the load shedding. The deficit cost $\tilde{c}_i$ is associated to the economical impact of not meeting one unit of energy demand in a given depth i. The depth of a deficit is a measure of severity of the load shedding. The more severe is the load shedding, the more expensive is the economical impact to society. Below, we recall the deficit cost $\tilde{c}_i$ and number of depth n_d contribution for the objective function

$$\sum_{k=1}^{n_s} \Big(\sum_{j \in T_k} c_j g_{t,j} + \sum_{i=1}^{n_\mathrm{d}} \tilde{c}_i \, \mathrm{df}_{t,ki} + \sum_{l \in \Omega_k} \theta_{lk} f_{t,lk} + \tilde{\theta}_k s_{t,k} \Big) + \beta \mathcal{Q}_{t+1}([v_{t+1}, a_{[t+1-\widehat{p},t]}])$$

and the bound $\overline{\mathrm{df}}_t$ contribution for box constraint:

$$0 \le \mathrm{df}_{t,ki} \le \overline{\mathrm{df}}_{t,ki}.$$

The deficit block is identified by the `ENERGY DEFICITS DATA` keyword. The deficit value $\mathrm{df}_{t,kj}$ for each stage t, subsystem k and depth i is described below.

```
ENERGY DEFICITS DATA
CODE  DEPTH    COST
----  -----  --------
   1   0.05   1142.80
   2   0.05   2465.40
   3   0.10   5152.46
   4   0.80   5845.54
9999
```

The deficit depth value provide us the percentage of the load shedding that corresponds to a given economic impact. In this case, the number of deficit depth is $n_d = 4$ and the deficit cost $\tilde{c}$ (\$/MWh) and bounds $d_{t,k}$ are

$$\begin{array}{l} \tilde{c}_1 = 1142.80 \\ \tilde{c}_2 = 2465.40 \\ \tilde{c}_3 = 5152.46 \\ \tilde{c}_4 = 5845.54 \end{array}, \quad \begin{array}{l} \overline{\mathrm{df}}_{t,k1} = 0.05 d_{t,k} \\ \overline{\mathrm{df}}_{t,k2} = 0.05 d_{t,k} \\ \overline{\mathrm{df}}_{t,k3} = 0.10 d_{t,k} \\ \overline{\mathrm{df}}_{t,k4} = 0.80 d_{t,k} \end{array},$$

for all subsystem k and stage t.

Remark 5. The subsystem $k = 5$ has no deficit variable because it represents a transshipment node.

2.1.6 Thermal generation set T_k, cost c_j and bounds $\underline{g}$, $\overline{g}$

Each subsystem has its own set of thermal plants T_k. Below, we recall the thermal cost c_i contribution for the objective function

$$\sum_{k=1}^{n_s} \Big(\sum_{j \in T_k} c_j g_{t,j} + \sum_{i=1}^{n_\mathrm{d}} \tilde{c}_i \, \mathrm{df}_{t,ki} + \sum_{l \in \Omega_k} \theta_{lk} f_{t,lk} + \tilde{\theta}_k s_{t,k} \Big) + \beta \mathcal{Q}_{t+1}([v_{t+1}, a_{[t+1-\widehat{p},t]}])$$

and the bounds $\underline{g}$, $\overline{g}$ contribution for box constraint:

$$\underline{g}_k \le g_{t,k} \le \overline{g}_k.$$

The thermal generation block is identified by the `THERMAL PLANTS` keyword. The thermal generation parameters are described below.

```
THERMAL PLANTS
NUM  SYS       NAME         COST    TGEN_MIN  TGEN_MAX
---  ---  ------------  --------  --------  --------
  1    1  ANGRA 1          21.49       520       657
 13    1  ANGRA 2          18.96      1080      1350
  7    1  CARIOBA            937         0        36
 88    1  CCBS_L          194.79      59.3       250
 89    1  CCBS_TC         222.22      27.1       250
...  ...           ...       ...       ...       ...
 67    3  TERMONE          329.2         0       171
 96    3  TERMOPE          70.16     348.8       533
 83    3  VALE DO ACU     287.83         0       323
 70    4  NOVA OLINDA     329.56         0       166
 73    4  TOCANTINOPO     329.56         0       166
999
```

For instance, the thermal set of subsystem 1 is $T_1 = \{1, 13, 211, 7, 219, \dots\}$ and the corresponding costs c_j (\$/MWh) and bounds $\underline{g}, \overline{g}$ (MWmonth) are

$$\begin{array}{rcr} c_1 &=& 21.49 \\ c_{13} &=& 18.96 \\ c_7 &=& 937 \\ c_{88} &=& 194.79 \\ c_{89} &=& 222.22 \\ & \vdots & \end{array}, \quad \begin{array}{rcr} \underline{g}_1 &=& 520 \\ \underline{g}_{13} &=& 1080 \\ \underline{g}_7 &=& 0 \\ \underline{g}_{88} &=& 59.3 \\ \underline{g}_{89} &=& 27.1 \\ & \vdots & \end{array}, \quad \begin{array}{rcr} \overline{g}_1 &=& 657 \\ \overline{g}_{13} &=& 1350 \\ \overline{g}_7 &=& 36 \\ \overline{g}_{88} &=& 250 \\ \overline{g}_{89} &=& 250 \\ & \vdots & \end{array}$$

Remark 6. The subsystem $k = 5$ has no thermal generation variable because it represents a transshipment node.

2.1.7 Hydro generation upper bound $\overline{q}$, storage energy upper bound $\overline{v}$ and initial condition v_1

Each subsystem has only *one* equivalent energy reservoir that aggregates all hydro power plants belonging to a homogeneous hydrological region. Below, we recall the contribution of the storage initial condition v_1 for the energy balance equation on t

$$v_{t+1,k} = v_{t,k} + a_{t,k} - q_{t,k} - s_{t,k},$$

the hydro generation upper bound $\overline{q}$ and storage energy upper bound $\overline{v}$ contribution on box constraint:

$$0 \leq q_{t,k} \leq \overline{q}_k, \quad 0 \leq v_{t+1,k} \leq \overline{v}_k.$$

The hydro generation block is identified by the `HYDRO PLANTS` keyword. The hydro generation parameters are described below.

```
# VALORES USADOS SAO DA CONFIGURACAO 1 (JAN/2011)
HYDRO PLANTS
NUM  SYS      NAME       PRODFACT  MAX TURB   MAX VOL  INIT VOL
---  ---  ------------  --------  --------  --------  --------
1      1   REE SUDESTE         1   45414.3  200717.6   59419.3
2      2       REE SUL         1   13081.5   19617.2    5874.9
3      3  REE NORDESTE         1    9900.9   51806.1   12859.2
4      4     REE NORTE         1    7629.9   12744.9    5271.5
999
```

Based on these registers, the corresponding values in the math formulation are

$$\begin{array}{lll} v_{1,1} = 59419.3 & \overline{q}_1 = 45414.3 & \overline{v}_1 = 200717.6 \\ v_{1,2} = 5874.9 & \overline{q}_2 = 13081.5 & \overline{v}_2 = 19617.2 \\ v_{1,3} = 12859.2\text{'} & \overline{q}_3 = 9900.9\text{'} & \overline{v}_3 = 51806.1 \\ v_{1,4} = 5271.5 & \overline{q}_4 = 7629.9 & \overline{v}_4 = 12744.9 \end{array}$$

Remark 7. Subsystem $k = 5$ has no hydro generation variable because it represents a transshipment node.

2.1.8 Energy interchange capacity $\overline{f}_{kl}$

Transmission lines connect subsystems and allow the energy interchange between them. Below, we recall the contribution of the interchange capacity $\overline{f}_{kl}$ in the box constraint:

$$0 \leq f_{t,kl} \leq \overline{f}_{kl}.$$

The interchange capacity block is identified by the `ENERGY EXCHANGES CAPACITY BETWEEN SYSTEMS` keyword. The interchange capacity $\overline{f}$ and the set Ω_k of subsystems directly connected to a given subsystem k are described below.

```
# CAPACIDADES DE INTERCAMBIO DE DEZ/12 FORAM USADAS
ENERGY EXCHANGES CAPACITY BETWEEN SYSTEMS
 FROM/TO    SYS  1    SYS  2    SYS  3    SYS  4    SYS  5
     ---  --------  --------  --------  --------  --------
  SYS  1  99999999      7379      1000         0      4000
  SYS  2      5625  99999999         0         0         0
  SYS  3       600         0  99999999         0      2236
  SYS  4         0         0         0  99999999  99999999
  SYS  5      3154         0      3951      3053  99999999
99999
```

In this particular example, we have the following sets and values:

$$\begin{array}{lllll} \Omega_1 = \{2,3,5\} & \Omega_2 = \{1\} & \Omega_3 = \{1,5\} & \Omega_4 = \{5\} & \Omega_5 = \{1,3,4\} \\ \overline{f}_{12} = 7379 & \overline{f}_{21} = 5625 & \overline{f}_{31} = 600 & \overline{f}_{45} = 99999999 & \overline{f}_{51} = 3154 \\ \overline{f}_{13} = 1000 & & \overline{f}_{35} = 2236 & & \overline{f}_{53} = 3951 \\ \overline{f}_{15} = 4000 & & & & \overline{f}_{54} = 3053 \end{array}.$$

2.1.9 Energy interchange penalties θ_{lk}

Small penalties for energy interchange indicates to the model a preference for meeting the demand with generation from the same subsystem if there are multiple optimal solutions (considering only costs). Below, we recall the energy interchange penalty θ_{lk} contribution to the objective function:

$$\sum_{k=1}^{n_s} \Big(\sum_{j \in T_k} c_j g_{t,j} + \sum_{i=1}^{n_\mathrm{d}} \tilde{c}_i \,\mathrm{df}_{t,ki} + \sum_{l \in \Omega_k} \theta_{lk} f_{t,lk} + \tilde{\theta}_k s_{t,k} \Big) + \beta \mathcal{Q}_{t+1}([v_{t+1}, a_{[t+1-\widehat{p},t]}]).$$

The interchange capacity block is identified by the `ENERGY EXCHANGES PENALTIES` keyword. The energy interchange penalties θ_{lk} are described below.

```
ENERGY EXCHANGES PENALTIES
 FROM/TO     SYS   1     SYS   2     SYS   3     SYS   4     SYS   5
     ---   --------   --------   --------   --------   --------
  SYS  1          0      0.001      0.001      0.001     0.0005
  SYS  2      0.001          0      0.001      0.001     0.0005
  SYS  3      0.001      0.001          0      0.001     0.0005
  SYS  4      0.001      0.001      0.001          0     0.0005
  SYS  5     0.0005     0.0005     0.0005     0.0005          0
99999
```

In this particular example, we have the following values:

$$\begin{array}{lllll} \Omega_1 = \{2,3,5\} & \Omega_2 = \{1\} & \Omega_3 = \{1,5\} & \Omega_4 = \{5\} & \Omega_5 = \{1,3,4\} \\ \theta_{12} = 0.001 & \theta_{21} = 0.001 & \theta_{31} = 0.001 & \theta_{45} = 0.0005 & \theta_{51} = 0.0005 \\ \theta_{13} = 0.001 & & \theta_{35} = 0.0005 & & \theta_{53} = 0.0005 \\ \theta_{15} = 0.0005 & & & & \theta_{54} = 0.0005 \end{array}.$$

2.1.10 Energy interchange loss factor r_{lk}

Energy losses for energy interchange is represented by a constant factor r_{lk}. Below, we recall the energy interchange loss factor r_{lk} contribution to the load balance equation:

$$q_{t,k} + \sum_{j \in T_k} g_{t,j} + \sum_{i=1}^{n_\mathrm{d}} \mathrm{df}_{t,ki} + \sum_{l \in \Omega_k} \Big(r_{lk} f_{t,lk} - f_{t,kl} \Big) = d_{t,k}.$$

The energy interchange loss factors are generated from the block identified by the `ENERGY EXCHANGES LOSS FACTORS` keyword.

```
ENERGY EXCHANGES LOSS FACTORS (PU)
 FROM/TO     SYS   1     SYS   2     SYS   3     SYS   4     SYS   5
     ---   --------   --------   --------   --------   --------
  SYS  1          0      0.001      0.001      0.001     0.0005
  SYS  2      0.001          0      0.001      0.001     0.0005
  SYS  3      0.001      0.001          0      0.001     0.0005
  SYS  4      0.001      0.001      0.001          0     0.0005
  SYS  5     0.0005     0.0005     0.0005     0.0005          0
99999
```

In this particular example, we have the following values:

$$
\begin{array}{lllll}
\Omega_1 = \{2,3,5\} & \Omega_2 = \{1\} & \Omega_3 = \{1,5\} & \Omega_4 = \{5\} & \Omega_5 = \{1,3,4\} \\
r_{12} = 1 - 0.001 & r_{21} = 1 - 0.001 & r_{31} = 1 - 0.001 & r_{45} = 1 - 0.0005 & r_{51} = 1 - 0.0005 \\
r_{13} = 1 - 0.001 & & r_{35} = 1 - 0.0005 & & r_{53} = 1 - 0.0005 \\
r_{15} = 1 - 0.0005 & & & & r_{54} = 1 - 0.0005
\end{array}.
$$

2.1.11 Spillage penalty $\tilde{\theta}_k$

Small penalties for energy spillage indicates to the model a preference for not spilling hydro energy if there are multiple optimal solutions (considering only costs). Below, we recall the energy spillage penalty $\tilde{\theta}_k$ contribution to the objective function:

$$\sum_{k=1}^{n_s} \Big(\sum_{j\in T_k} c_j g_{t,j} + \sum_{i=1}^{n_\mathrm{d}} \tilde{c}_i \,\mathrm{df}_{t,ki} + \sum_{l\in\Omega_k} \theta_{lk} f_{t,lk} + \tilde{\theta}_k s_{t,k} \Big) + \beta \mathcal{Q}_{t+1}([v_{t+1}, a_{[t+1-\widehat{p},t]}]).$$

These penalties are hard coded and have the following values:

$$
\begin{array}{l}
\tilde{\theta}_1 = 0.001 \\
\tilde{\theta}_2 = 0.001 \\
\tilde{\theta}_3 = 0.001 \\
\tilde{\theta}_4 = 0.001
\end{array}.
$$

Remark 8. Subsystem $k = 5$ has no spillage variable because it represents a transshipment node.

2.2 `linear_models.txt` file

2.2.1 PVAR$_m$ coefficients ζ_t, $\Phi_{t,\nu}$ and maximum order $\hat{p}$

The multiplicative time-series model PVAR$_m$ is used to describe the uncertainty of the inflow a_t. It is instructive to emphasize the following properties of this model:

- linearity – this property allows us to add the time-series model equation as constraints and the stochastic programming problem remains linear;
- error independence – After adding the time-series model equation as constraints, the inflow becomes a variable and the multiplicative error becomes the stochastic program uncertainty. This is interesting for computational purpose, since the stagewise independence property reduces the number of cost-to-go functions to one per stage. From an algorithm perspective, this consequence allows us to share cuts;
- positive variables – With an appropriate parameter estimation, the multiplicative model PVAR$_m$ ensure that all inflow scenarios are positive.

The first two properties are requirements of the original SDDP algorithm. The third property is an important stylized fact, since the inflows are always nonnegative. Below, we recall the coefficients ζ_t, $\Phi_{t,\nu}$ and maximum order $\hat{p}$ contributions to the PVAR$_m$ time-series equation:

$$a_{t,k} = \left(\zeta_{t,k} + \sum_{\nu=1}^{\widehat{p}} \sum_{l=1}^{n_s} \phi_{t\nu,kl} a_{t-\nu,l} \right) \cdot \eta_{t,k}.$$

In this model, the maximum order $\hat{p}$ and the periodicity $\mathbb{S}$ are hard coded and equal to 1 and 12, respectively. The coefficients ζ_t, $\Phi_{t,\nu}$ block is identified by the `MONTH` keyword followed by the month associated to the time t.

```
MONTH: 1
COEF#   PLANT   1   PLANT   2   PLANT   3   PLANT   4
-----  ----------  ----------  ----------  ----------
    0    19053.62    3458.317    6955.198    2305.304
    1   0.8658406           0           0           0
    2           0   0.3877219           0           0
    3           0           0   0.7045067           0
    4           0           0           0    1.270784
99999

MONTH: 2
COEF#   PLANT   1   PLANT   2   PLANT   3   PLANT   4
-----  ----------  ----------  ----------  ----------
    0    21290.56    2536.189    2532.467    5104.156
    1   0.6539529           0           0           0
    2           0   0.6903953           0           0
    3           0           0   0.8295804           0
    4           0           0           0   0.7327421
99999

MONTH: 3
COEF#   PLANT   1   PLANT   2   PLANT   3   PLANT   4
-----  ----------  ----------  ----------  ----------
    0    21292.61    2611.061    480.6702    4828.961
    1   0.5560207           0           0           0
    2           0    0.475443           0           0
    3           0           0   0.9531763           0
    4           0           0           0    0.763194
99999
```

In this particular example, we have the following vector and matrices:

$$\Phi_{1,1} = \begin{pmatrix} 0.8658406 & 0 & 0 & 0 \\ 0 & 0.3877219 & 0 & 0 \\ 0 & 0 & 0.7045067 & 0 \\ 0 & 0 & 0 & 1.270784 \end{pmatrix}, \qquad \zeta_1 = \begin{pmatrix} 19053.62 \\ 3458.317 \\ 6955.198 \\ 2305.304 \end{pmatrix},$$

$$\Phi_{2,1} = \begin{pmatrix} 0.6539529 & 0 & 0 & 0 \\ 0 & 0.6903953 & 0 & 0 \\ 0 & 0 & 0.8295804 & 0 \\ 0 & 0 & 0 & 0.7327421 \end{pmatrix}, \qquad \zeta_2 = \begin{pmatrix} 21290.56 \\ 2536.189 \\ 2532.467 \\ 5104.156 \end{pmatrix},$$

$$\Phi_{3,1} = \begin{pmatrix} 0.55602070 & 0 & 0 & 0 \\ 0 & 0.475443 & 0 & 0 \\ 0 & 0 & 0.9531763 & 0 \\ 0 & 0 & 0 & 0.763194 \end{pmatrix}, \qquad \zeta_3 = \begin{pmatrix} 21292.61 \\ 2611.061 \\ 480.6702 \\ 4828.961 \end{pmatrix}.$$

Remark 9. Subsystem $k = 5$ has no inflow variable because it represents a transshipment node.

2.2.2 PVAR$_m$ initial conditions $\{a_0, \dots, a_{-\hat{p}+1}\}$

The initial inflow condition $\{a_0, \dots, a_{-\hat{p}+1}\}$ is necessary to initialize the stochastic process $\{a_t\}_{t=1}^T$ defined by the PVAR$_m$ model. Below, we recall the contributions of the initial inflow condition $\{a_0, \dots, a_{-\hat{p}+1}\}$ to the corresponding time-series equation from time $t = 1$ to time $t = \hat{p}$:

$$a_{t,k} = \left(\zeta_{t,k} + \sum_{\nu=1}^{\widehat{p}} \sum_{l=1}^{n_s} \phi_{t\nu,kl} a_{t-\nu,l} \right) \cdot \eta_{t,k}.$$

The initial inflow conditions $\{a_0, \dots, a_{-\hat{p}+1}\}$ is identified by the `PAST INFLOWS` keyword.

```
PAST INFLOWS:
 LAG    PLANT    1    PLANT    2    PLANT    3    PLANT    4
-----   ----------   ----------   ----------   ----------
   01    39717.564    6632.5141    15897.183    2525.2938
99999
```

In this particular example, we have the following initial inflow condition:

$$a_0 = \begin{pmatrix} 39717.564 \\ 6632.5141 \\ 15897.183 \\ 2525.2938 \end{pmatrix}.$$

Remark 10. Subsystem $k = 5$ has no inflow variable because it represents a transshipment node.

2.3 `scenarios.txt` file

2.3.1 PVAR$_m$ multiplicative error η_t

The multiplicative error scenario tree is implicitly described, on each stage, by a list of outcomes and marginal probabilities. This particular representation is possible due to the stagewise independence property of the multiplicative error η_t. Below, we provide an illustration of a general inflow tree (fig.1a) and a stagewise independent noise tree (fig.1b). For example, the outcomes of the inflow a_t from stage $t = 3$ are $\{a^5, a^6, a^7, a^8, a^9, a^{10}\}$. Because of the stagewise independence, the outcomes of the multiplicative noise η_t from stage $t = 3$ are just $\{\eta^5, \eta^6\}$. Also, the conditional probability $p_{2,5}$, for instance, from the inflow tree (fig.1a) is

$$p_{2,5} = \mathbb{P}[a_3 = a^5 \mid a_2 = a^2,\ a_1 = a^1].$$

On the other hand, the conditional probability $p_{2,5}$ from the multiplicative noise tree (fig.1b) can be further simplified

$$\begin{aligned} p_{2,5} &= \mathbb{P}[\eta_3 = \eta^5 \mid \eta_2 = \eta^2,\ \eta_1 = \eta^1] \\ &= \mathbb{P}[\eta_3 = \eta^5], \end{aligned}$$

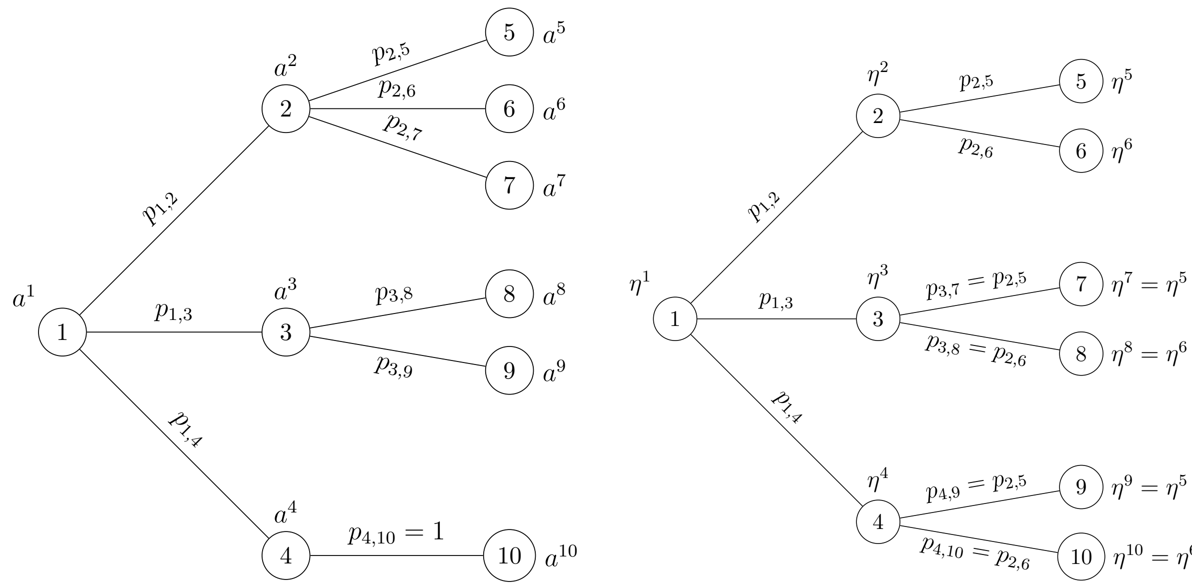
due to the stagewise independence property.

Below, we recall the contribution of the multiplicative error scenario tree to the Sample Average Approximation (SAA) of the cost-to-go function:

$$\mathcal{Q}_{t+1}([v_{t+1}, a_{[t+1-\widehat{p},t]}]) = \begin{cases} \rho_{t+1} \left[Q_{t+1}([v_{t+1}, a_{[t+1-\widehat{p},t]}], \eta_{t+1}) \right] & , t \in \{1, \dots, T-1\} \\ 0 & , t = T \end{cases},$$

where $\rho_t[Z_t] = (1-\lambda)\mathbb{E}[Z_t] + \lambda \mathrm{AV@R}_\alpha[Z_t]$, and the multiplicative error η_t contribution to the corresponding time-series equation:

$$a_{t,k} = \left(\zeta_{t,k} + \sum_{\nu=1}^{\widehat{p}} \sum_{l=1}^{n_s} \phi_{t\nu,kl} a_{t-\nu,l} \right) \cdot \eta_{t,k}.$$



(a) General inflow tree (b) Stagewise independent noise tree

Figure 1: Probability tree

The multiplicative error η_t and the corresponding marginal distribution block is identified by the `SCENARIOS` keyword.

```
SCENARIOS
 STAGE        PROB      NOISE    1    NOISE    2    NOISE    3    NOISE    4
-------   ----------   ----------   ----------   ----------   ----------
      1            1            1            1            1            1
      2         0.01     1.114361    0.9465053     1.129625     1.083902
      2         0.01      1.85045    0.9829512     2.512377     2.108396
      2         0.01    0.8134146       1.5349    0.5319363    0.6795749
      2         0.01     1.646692     1.574305     1.782563     1.774331
    ...          ...          ...          ...          ...          ...
    120         0.01     1.147625    0.5115445     1.301088     1.018563
    120         0.01      1.08623    0.8097364    0.9884022    0.9619588
    120         0.01    0.8509878     1.723977    0.6564894    0.6487821
    120         0.01     1.065119     1.893918    0.8602847    0.7493693
    120         0.01    0.7375268    0.8348935    0.5918711    0.8142026
9999999
```

2.4 scenarios_efs.txt

2.4.1 *Out-of-sample* multiplicative error scenario tree

After performing the optimization procedure, it is important to evaluate the corresponding policy approximation. This is done by sampling multiplicative error scenarios $\eta^i_{[T]} := (\eta^i_1, \eta^i_2, \ldots, \eta^i_T)$ and solving each stage problem (1) with the respective cost-to-go function approximation from stage 1 to T. In this way, we assess the costs and decisions distributions for those sampled scenarios and judge if this modeling approach is reasonable. The multiplicative error scenario samples are obtained from an implicit scenario tree, similar to the one used in the optimization procedure.

The multiplicative error η_t and the corresponding marginal distribution block is identified by the `SCENARIOS` keyword.

```
SCENARIOS
 STAGE       PROB     NOISE   1   NOISE   2   NOISE   3   NOISE   4
-------  ----------  ----------  ----------  ----------  ----------
      1           1           1           1           1           1
      2        0.01    1.114361   0.9465053    1.129625    1.083902
      2        0.01     1.85045   0.9829512    2.512377    2.108396
      2        0.01   0.8134146      1.5349   0.5319363   0.6795749
      2        0.01    1.646692    1.574305    1.782563    1.774331
    ...         ...         ...         ...         ...         ...
    120        0.01    1.147625   0.5115445    1.301088    1.018563
    120        0.01     1.08623   0.8097364   0.9884022   0.9619588
    120        0.01   0.8509878    1.723977   0.6564894   0.6487821
    120        0.01    1.065119    1.893918   0.8602847   0.7493693
    120        0.01   0.7375268   0.8348935   0.5918711   0.8142026
9999999
```

3 Output description

The first stage linear programming problem for the initial iteration od the SDDP algorithm (that is, before any cut is generated) is given below.

```
\Problem name:

Minimize
Stage_1:       0.00100 SpillVol_1 +       0.00100 SpillVol_2 +
               0.00100 SpillVol_3 +       0.00100 SpillVol_4 +
            21.49000 ThermalGen_1 +   18.96000 ThermalGen_13 +
                 937 ThermalGen_7 +  194.79000 ThermalGen_88 +
          222.22000 ThermalGen_89 + 140.58000 ThermalGen_104 +
            6.27000 ThermalGen_12 + 505.92000 ThermalGen_153 +
            0.01000 ThermalGen_65 + 112.46000 ThermalGen_183 +
         159.97000 ThermalGen_187 + 250.87000 ThermalGen_189 +
         550.66000 ThermalGen_155 +  188.89000 ThermalGen_63 +
           645.30000 ThermalGen_2 +        150 ThermalGen_54 +
          145.68000 ThermalGen_15 + 274.54000 ThermalGen_178 +
         253.83000 ThermalGen_179 +  37.80000 ThermalGen_171 +
          51.93000 ThermalGen_172 +  90.69000 ThermalGen_173 +
         131.68000 ThermalGen_174 +  317.98000 ThermalGen_72 +
         152.80000 ThermalGen_105 +  470.34000 ThermalGen_50 +
          317.98000 ThermalGen_66 +   523.35000 ThermalGen_9 +
           730.54000 ThermalGen_3 +   310.41000 ThermalGen_4 +
          730.54000 ThermalGen_87 + 101.33000 ThermalGen_197 +
         140.34000 ThermalGen_198 +  292.49000 ThermalGen_51 +
         610.33000 ThermalGen_193 + 487.56000 ThermalGen_194 +
          122.65000 ThermalGen_75 +  214.48000 ThermalGen_77 +
         1047.38000 ThermalGen_19 +   0.01000 ThermalGen_196 +
          329.57000 ThermalGen_49 +  197.85000 ThermalGen_34 +
         733.54000 ThermalGen_108 +  564.57000 ThermalGen_31 +
               219 ThermalGen_199 +        219 ThermalGen_48 +
          50.47000 ThermalGen_156 +  541.93000 ThermalGen_64 +
```

```
        154.10000 ThermalGen_29 + 180.51000 ThermalGen_169 +
        218.77000 ThermalGen_28 +  189.54000 ThermalGen_26 +
        143.04000 ThermalGen_27 +  142.86000 ThermalGen_25 +
        116.90000 ThermalGen_24 +        780 ThermalGen_30 +
        115.90000 ThermalGen_22 +  115.90000 ThermalGen_23 +
        248.31000 ThermalGen_32 +  141.18000 ThermalGen_35 +
       464.64000 ThermalGen_109 + 464.64000 ThermalGen_111 +
       455.13000 ThermalGen_112 + 464.64000 ThermalGen_113 +
        834.35000 ThermalGen_93 + 509.86000 ThermalGen_164 +
       509.86000 ThermalGen_166 + 464.64000 ThermalGen_117 +
       464.64000 ThermalGen_119 + 185.09000 ThermalGen_175 +
       492.29000 ThermalGen_177 + 464.64000 ThermalGen_121 +
       464.64000 ThermalGen_125 +  188.15000 ThermalGen_74 +
         82.34000 ThermalGen_42 +  329.37000 ThermalGen_53 +
        329.37000 ThermalGen_55 + 464.64000 ThermalGen_127 +
       464.64000 ThermalGen_131 + 464.64000 ThermalGen_133 +
        317.19000 ThermalGen_57 + 464.64000 ThermalGen_135 +
       464.64000 ThermalGen_138 + 678.03000 ThermalGen_160 +
       559.39000 ThermalGen_144 + 611.57000 ThermalGen_151 +
       611.56000 ThermalGen_161 +  204.43000 ThermalGen_43 +
       325.67000 ThermalGen_152 + 678.03000 ThermalGen_159 +
        329.20000 ThermalGen_67 +   70.16000 ThermalGen_96 +
        287.83000 ThermalGen_83 +  329.56000 ThermalGen_70 +
        329.56000 ThermalGen_73 +   1142.80000 Deficit_1_1 +
         2465.40000 Deficit_1_2 +   5152.46000 Deficit_1_3 +
         5845.54000 Deficit_1_4 +   1142.80000 Deficit_2_1 +
         2465.40000 Deficit_2_2 +   5152.46000 Deficit_2_3 +
         5845.54000 Deficit_2_4 +   1142.80000 Deficit_3_1 +
         2465.40000 Deficit_3_2 +   5152.46000 Deficit_3_3 +
         5845.54000 Deficit_3_4 +   1142.80000 Deficit_4_1 +
         2465.40000 Deficit_4_2 +   5152.46000 Deficit_4_3 +
         5845.54000 Deficit_4_4 +   1142.80000 Deficit_5_1 +
         2465.40000 Deficit_5_2 +   5152.46000 Deficit_5_3 +
         5845.54000 Deficit_5_4 +     0.00100 Exchange_1_2 +
           0.00100 Exchange_1_3 +     0.00050 Exchange_1_5 +
           0.00050 Exchange_3_5 +     0.00050 Exchange_4_5 +
           0.00100 Exchange_2_1 +     0.00100 Exchange_3_1 +
           0.00050 Exchange_5_1 +     0.00050 Exchange_5_3 +
           0.00050 Exchange_5_4 +             0.99060 alfa
Subject To
D_Supply_1:             TurbVol_1 +             ThermalGen_1 +
                    ThermalGen_13 +             ThermalGen_7 +
                    ThermalGen_88 +            ThermalGen_89 +
                   ThermalGen_104 +            ThermalGen_12 +
                   ThermalGen_153 +            ThermalGen_65 +
                   ThermalGen_183 +           ThermalGen_187 +
                   ThermalGen_189 +           ThermalGen_155 +
                    ThermalGen_63 +             ThermalGen_2 +
                    ThermalGen_54 +            ThermalGen_15 +
                   ThermalGen_178 +           ThermalGen_179 +
                   ThermalGen_171 +           ThermalGen_172 +
                   ThermalGen_173 +           ThermalGen_174 +
                    ThermalGen_72 +           ThermalGen_105 +
                    ThermalGen_50 +            ThermalGen_66 +
                     ThermalGen_9 +             ThermalGen_3 +
                     ThermalGen_4 +            ThermalGen_87 +
                   ThermalGen_197 +           ThermalGen_198 +
                    ThermalGen_51 +           ThermalGen_193 +
                   ThermalGen_194 +            ThermalGen_75 +
```

```
                     ThermalGen_77 +            ThermalGen_19 +
                    ThermalGen_196 +            ThermalGen_49 +
                     ThermalGen_34 +           ThermalGen_108 +
                       Deficit_1_1 +              Deficit_1_2 +
                       Deficit_1_3 +              Deficit_1_4 -
                      Exchange_1_2 -             Exchange_1_3 -
                      Exchange_1_5 +             Exchange_2_1 +
                      Exchange_3_1 +             Exchange_5_1 = 45515
D_Supply_2:              TurbVol_2 +            ThermalGen_31 +
                    ThermalGen_199 +            ThermalGen_48 +
                    ThermalGen_156 +            ThermalGen_64 +
                     ThermalGen_29 +           ThermalGen_169 +
                     ThermalGen_28 +            ThermalGen_26 +
                     ThermalGen_27 +            ThermalGen_25 +
                     ThermalGen_24 +            ThermalGen_30 +
                     ThermalGen_22 +            ThermalGen_23 +
                     ThermalGen_32 +            ThermalGen_35 +
                       Deficit_2_1 +              Deficit_2_2 +
                       Deficit_2_3 +              Deficit_2_4 +
                      Exchange_1_2 -             Exchange_2_1 = 11692
D_Supply_3:              TurbVol_3 +           ThermalGen_109 +
                    ThermalGen_111 +           ThermalGen_112 +
                    ThermalGen_113 +            ThermalGen_93 +
                    ThermalGen_164 +           ThermalGen_166 +
                    ThermalGen_117 +           ThermalGen_119 +
                    ThermalGen_175 +           ThermalGen_177 +
                    ThermalGen_121 +           ThermalGen_125 +
                     ThermalGen_74 +            ThermalGen_42 +
                     ThermalGen_53 +            ThermalGen_55 +
                    ThermalGen_127 +           ThermalGen_131 +
                    ThermalGen_133 +            ThermalGen_57 +
                    ThermalGen_135 +           ThermalGen_138 +
                    ThermalGen_160 +           ThermalGen_144 +
                    ThermalGen_151 +           ThermalGen_161 +
                     ThermalGen_43 +           ThermalGen_152 +
                    ThermalGen_159 +            ThermalGen_67 +
                     ThermalGen_96 +            ThermalGen_83 +
                       Deficit_3_1 +              Deficit_3_2 +
                       Deficit_3_3 +              Deficit_3_4 +
                      Exchange_1_3 -             Exchange_3_5 -
                      Exchange_3_1 +             Exchange_5_3 = 10811
D_Supply_4:              TurbVol_4 +            ThermalGen_70 +
                     ThermalGen_73 +              Deficit_4_1 +
                       Deficit_4_2 +              Deficit_4_3 +
                       Deficit_4_4 -             Exchange_4_5 +
                      Exchange_5_4 = 6507
D_Supply_5:            Deficit_5_1 +              Deficit_5_2 +
                       Deficit_5_3 +              Deficit_5_4 +
                      Exchange_1_5 +             Exchange_3_5 +
                      Exchange_4_5 -             Exchange_5_1 -
                      Exchange_5_3 -             Exchange_5_4 = 0
H_Balance_1:             TurbVol_1 +                SpillVol_1 +
                          StoVol_1 = 112861.99944
H_Balance_2:             TurbVol_2 +                SpillVol_2 +
                          StoVol_2 = 11904.78797
H_Balance_3:             TurbVol_3 +                SpillVol_3 +
                          StoVol_3 = 31014.06993
H_Balance_4:             TurbVol_4 +                SpillVol_4 +
                          StoVol_4 = 10785.90696
```

```
Bounds
 0          <= TurbVol_1      <= 45414.30000
 0          <= TurbVol_2      <= 13081.50000
 0          <= TurbVol_3      <= 9900.90000
 0          <= TurbVol_4      <= 7629.90000
 0          <= StoVol_1       <= 200717.60000
 0          <= StoVol_2       <= 19617.20000
 0          <= StoVol_3       <= 51806.10000
 0          <= StoVol_4       <= 12744.90000
 520        <= ThermalGen_1   <= 657
 1080       <= ThermalGen_13  <= 1350
 0          <= ThermalGen_7   <= 36
 59.30000   <= ThermalGen_88  <= 250
 27.10000   <= ThermalGen_89  <= 250
 0          <= ThermalGen_104 <= 28
 0          <= ThermalGen_12  <= 529
 0          <= ThermalGen_153 <= 44
 219.78000  <= ThermalGen_65  <= 255
 199.99000  <= ThermalGen_183 <= 235
 0          <= ThermalGen_187 <= 386
 0          <= ThermalGen_189 <= 386
 0          <= ThermalGen_155 <= 145
 0          <= ThermalGen_63  <= 226
 0          <= ThermalGen_2   <= 131
 0          <= ThermalGen_54  <= 87
 0          <= ThermalGen_15  <= 204
 0          <= ThermalGen_178 <= 923
 0          <= ThermalGen_179 <= 923
 399.99000  <= ThermalGen_171 <= 400
 0          <= ThermalGen_172 <= 100
 0          <= ThermalGen_173 <= 200
 0          <= ThermalGen_174 <= 169
 0          <= ThermalGen_72  <= 386
 0          <= ThermalGen_105 <= 28
 0          <= ThermalGen_50  <= 200
 0          <= ThermalGen_66  <= 272
 0          <= ThermalGen_9   <= 30
 0          <= ThermalGen_3   <= 168
 0          <= ThermalGen_4   <= 440
 0          <= ThermalGen_87  <= 400
 0          <= ThermalGen_197 <= 258
 0          <= ThermalGen_198 <= 258
 0          <= ThermalGen_51  <= 258
 0          <= ThermalGen_193 <= 64
 0          <= ThermalGen_194 <= 340
 71.70000   <= ThermalGen_75  <= 1058
 28.80000   <= ThermalGen_77  <= 1058
 0          <= ThermalGen_19  <= 10
 132.98000  <= ThermalGen_196 <= 197
 0          <= ThermalGen_49  <= 175
 0          <= ThermalGen_34  <= 206
 0          <= ThermalGen_108 <= 54
 0          <= ThermalGen_31  <= 66
 0          <= ThermalGen_199 <= 485
 0          <= ThermalGen_48  <= 485
 210        <= ThermalGen_156 <= 350
 0          <= ThermalGen_64  <= 161
 27         <= ThermalGen_29  <= 72
 0          <= ThermalGen_169 <= 4
```

```
9.56000   <= ThermalGen_28  <= 20
25        <= ThermalGen_26  <= 100
79.46000  <= ThermalGen_27  <= 132
147.54000 <= ThermalGen_25  <= 262
228.02000 <= ThermalGen_24  <= 363
0         <= ThermalGen_30  <= 24
49.66000  <= ThermalGen_22  <= 126
105       <= ThermalGen_23  <= 320
5         <= ThermalGen_32  <= 20
0         <= ThermalGen_35  <= 640
0         <= ThermalGen_109 <= 13
0         <= ThermalGen_111 <= 11
0         <= ThermalGen_112 <= 32
0         <= ThermalGen_113 <= 11
0.70000   <= ThermalGen_93  <= 347
0         <= ThermalGen_164 <= 152
0         <= ThermalGen_166 <= 150
0         <= ThermalGen_117 <= 13
0         <= ThermalGen_119 <= 15
0         <= ThermalGen_175 <= 220
0         <= ThermalGen_177 <= 220
0         <= ThermalGen_121 <= 13
0         <= ThermalGen_125 <= 15
0         <= ThermalGen_74  <= 138
223       <= ThermalGen_42  <= 347
0         <= ThermalGen_53  <= 149
0         <= ThermalGen_55  <= 149
0         <= ThermalGen_127 <= 15
0         <= ThermalGen_131 <= 102
0         <= ThermalGen_133 <= 15
0         <= ThermalGen_57  <= 168
0         <= ThermalGen_135 <= 13
0         <= ThermalGen_138 <= 13
0         <= ThermalGen_160 <= 103
0         <= ThermalGen_144 <= 136
0         <= ThermalGen_151 <= 53
0         <= ThermalGen_161 <= 66
0         <= ThermalGen_43  <= 186
0         <= ThermalGen_152 <= 50
0         <= ThermalGen_159 <= 156
0         <= ThermalGen_67  <= 171
348.80000 <= ThermalGen_96  <= 533
0         <= ThermalGen_83  <= 323
0         <= ThermalGen_70  <= 166
0         <= ThermalGen_73  <= 166
0         <= Deficit_1_1    <= 2275.75000
0         <= Deficit_1_2    <= 2275.75000
0         <= Deficit_1_3    <= 4551.50000
0         <= Deficit_1_4    <= 36412
0         <= Deficit_2_1    <= 584.60000
0         <= Deficit_2_2    <= 584.60000
0         <= Deficit_2_3    <= 1169.20000
0         <= Deficit_2_4    <= 9353.60000
0         <= Deficit_3_1    <= 540.55000
0         <= Deficit_3_2    <= 540.55000
0         <= Deficit_3_3    <= 1081.10000
0         <= Deficit_3_4    <= 8648.80000
0         <= Deficit_4_1    <= 325.35000
0         <= Deficit_4_2    <= 325.35000
```

```
 0          <= Deficit_4_3    <= 650.70000
 0          <= Deficit_4_4    <= 5205.60000
 0          <= Deficit_5_1    <= 0
 0          <= Deficit_5_2    <= 0
 0          <= Deficit_5_3    <= 0
 0          <= Deficit_5_4    <= 0
 0          <= Exchange_1_2   <= 7379
 0          <= Exchange_1_3   <= 1000
 0          <= Exchange_1_5   <= 4000
 0          <= Exchange_3_5   <= 2236
 0          <= Exchange_4_5   <= 99999999
 0          <= Exchange_2_1   <= 5625
 0          <= Exchange_3_1   <= 600
 0          <= Exchange_5_1   <= 3154
 0          <= Exchange_5_3   <= 3951
 0          <= Exchange_5_4   <= 3053
End
```