Psychological Review
1965, Vol. 72, No. 1, 3–26

A MODEL OF LOUDNESS SUMMATION[1]

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A psychophysical model is presented that explains why loudness summates across frequency as it does and that permits the precise calculation of loudness from the physical spectrum. Loudness is represented by geometrical patterns derived from the masking of pure tones by narrow bands of noise. The masking patterns are converted to loudness patterns by means of the critical-band function that relates tonalness in Barks to frequency in cycles per second and a power function that relates specific loudness, loudness per Bark, to sound pressure level (SPL). Plotted on the coordinates of specific loudness and tonalness, the geometrical patterns are integrated to yield a value in sones$_G$ for the overall loudness. Calculated values are compared to experimental values obtained from loudness balances with 3 types of sound.

Loudness summates across frequency. Thus two pure tones are usually louder than either alone. Their overall loudness is seldom, however, the sum of their individual loudnesses. The failure of loudness to summate perfectly is ascribed to mutual inhibition between the tones (cf. Stevens, 1956). Since the degree of inhibition depends upon both the intensity of the tones and their frequency separation, no simple rule can serve to predict their total loudness directly from their physical properties. Nevertheless, it is possible to represent each of the two tones, or indeed any number of tones including the infinite number contained in a continuous spectrum, by geometrical patterns that permit a precise analysis of loudness summation. These patterns are the essential feature of the model of loudness summation presented in this paper.

[1] Preparation of this paper was supported in part by United States Public Health Service Research Grant NB 04464 from the National Institute of Neurological Diseases and Blindness. The authors wish to thank S. S. Stevens for helpful discussions and suggestions concerning the paper.

Although often described below in broad physiological terms, the model is based entirely on psychophysical measures of masking, loudness, and the critical band, and is therefore wholly psychological. The scales of "specific loudness" and "tonalness," which replace the physical scales of intensity and frequency, provide the coordinates for the geometrical representation of loudness. The specific loudness scale, unlike the sone scale, is a derived scale and represents the loudness produced over each unit of tonalness. The scale of tonalness is based on the critical band as measured in experiments on the threshold for complex sounds, two-tone masking, phase sensitivity, and loudness summation itself (Feldtkeller, 1955; Scharf, 1961b).

The use of a geometrical representation of loudness follows from the place theory of hearing which assumes a local pattern of excitation produced within the auditory nervous system by a pure tone or by each pure tone component of a complex sound (a sound composed of two or more frequency components). In the present model, the shape and level of the local patterns are inferred from the masking of pure tones by narrow bands of noise. A complex sound is represented by a number of these local patterns whose integrated area yields a measure of the total loudness.

Some of the basic notions underlying the model such as the conversion of the intensity and frequency scales to psychological scales and the derivation of loudness patterns from masking patterns are not new (Fletcher & Munson, 1937; Harris, 1959; Howes, 1950; Munson & Gardner, 1950). New are the concept of the specific loudness and the use of the critical band and also the application of the model to a number of different types of sound.

Parts of the model have already been described elsewhere (Scharf, 1964; Zwicker, 1958, 1963). The whole model is here considered and tested against a variety of empirical measurements of loudness. As will be seen, the model, based on a few simple and general rules, accounts for most of the complex data on loudness summation.

Before introducing the model and testing it against experimental measures, we review briefly the basic facts of loudness summation.

BASIC FACTS OF LOUDNESS SUMMATION

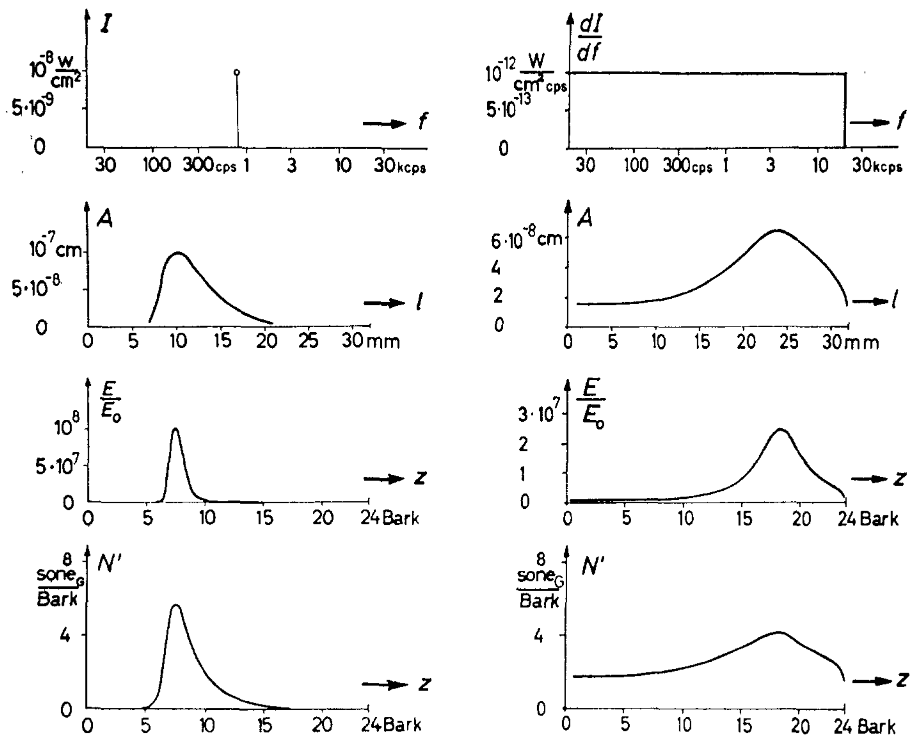
Perhaps the most striking fact of loudness summation, defined as the increase in loudness with the spread of energy over frequency, is that it does not begin until the energy is spread over more than a critical bandwidth (*Frequenzgruppe*) (Zwicker & Feldtkeller, 1955; Zwicker, Flottorp, & Stevens, 1957). Up to the critical band, the loudness of a complex sound is largely independent of the width of energy spread. Loudness first and rather abruptly begins to increase with bandwidth at the critical band. The value of a critical band centered on a given frequency is the same at all intensities, but beyond it the amount of the increase in loudness depends upon the intensity of the sound (Zwicker & Feldtkeller, 1955; Zwicker, Flottorp, & Stevens, 1957). Indeed, close to threshold the loudness decreases slightly rather than increases (Scharf, 1959a). At sensation levels above 10 decibels, however, some increase in loudness with bandwidth may be noted, with the greatest increases at moderate sensation levels between 40 and 60 decibels.

These rules hold regardless of where in the frequency spectrum the sound is located (Zwicker & Feldtkeller, 1955; Scharf, 1959a). They also hold whether the threshold is normal or is raised to an abnormally high level by the presence of a moderately intense uniform masking noise (Scharf, 1961a) or by a middle-ear impairment (Scharf, 1962b). The complex sound itself may be composed of any number of components from two up to the infinite number comprising a band of white noise (Scharf, 1959b). A complex sound at a given sensation level is loudest, however, when the components are all equally loud (Scharf, 1962a) and when they are separated by an equal number of critical bands (Zwicker, Flottorp, & Stevens, 1957).

The relation between loudness and bandwidth is the same even for sounds of short duration, although the loudness of a complex sound decreases as its duration is decreased below 70 milliseconds (Port, 1963a, 1963b).

DESCRIPTION AND DEVELOPMENT OF THE MODEL

Although the experimental measures of loudness summation yield a complex pattern of results, a single model



1a. First diagram is the physical spectrum of an 800-cps tone. (Intensity I plotted against frequency f. Next is plotted the Amplitude A of the displacement caused by the tone on the basilar membrane as a function of Distance l from the helicotrema. Third is shown the Excitation E at the Organ of Corti as a function of tonalness z. [The symbol E_0 is an arbitrary reference.] Last is shown the Specific Loudness N' as a function of Tonalness z. The area under the loudness pattern corresponds to the total loudness of the tone.)

1b. The spectrum of a white noise followed by the displacement, excitation, and loudness patterns produced by the noise (Zwicker, 1958, adapted with permission of *Acustica*).

FIG. 1. Sequential formation of a loudness pattern.

can encompass them all. First let us outline briefly the model as it is applied to a single tone and to white noise. Then we shall review the detailed development of the model.

Figure 1 traces the transformations that are thought to occur when the ear is stimulated by a pure tone (Fig. 1a) and by white noise (Fig. 1b). The physical measurement of an ideal pure-tone stimulus at the ear drum yields a line spectrum that shows all the sound energy concentrated at a single frequency. Beyond the ear drum, the tonal intensity and frequency are transmitted by the ossicles of the middle ear to the oval window. At that point the inner-ear fluid and consequently the basilar membrane are set in motion. Békésy's measures of the vibration of the basilar membrane permit us to plot the maximum amplitude A of the displacement produced by the tone over the length of the basilar membrane, as shown in the second part of Figure 1a. This first, important step in the development of a model for loudness summation shows that energy confined to a single point on the physical frequency scale produces a displacement over a wide area of the basilar membrane. The displacement of the basilar membrane somehow activates the sensory cells whose neural response is then translated through several synaptic junctures to higher levels of the nervous system. The wide lateral displacement on the basilar membrane is apparently greatly reduced at the neural level, and the pattern of Excitation E looks like that in the third part of the figure. (The excitation pattern is based primarily upon masking data, as explained below.)

The value of the excitation relative to an arbitrary reference E_0 is plotted here against the scale of tonalness (z) which is related in an approximately linear fashion to distance on the basilar membrane. The excitation pattern is then transformed into a loudness pattern, depicted in the fourth part of Figure 1a, by means of a special equation relating Specific Loudness N' to Excitation E. The total loudness produced by the original tone is the integral of the area under the specific loudness pattern.

Thus the first part of the figure comes from a purely physical measure. The second part, the displacement over the basilar membrane, comes from physiological measures. The last two parts of the figure may, in the next years or decades, also be measured physiologically, but in the meantime they are derived from psychophysical measures to form the basis of our model of loudness and loudness summation.

Now let us look at the same sequence for white noise. The energy in white noise is distributed over frequency as shown in the first part of Figure 1b. The displacement produced along the basilar membrane is approximated from measurements on models of the basilar membrane (Oetinger & Haüser, 1961). The excitation and loudness patterns for the noise are quite different from the patterns produced by a pure tone.

Whereas the first two diagrams of Figure 1 are well-known physical and physiological measures, the representations of excitation and specific loudness are new and require explanation. The explanation may be broken down into five parts to which a sixth is added for sounds heard against a background of noise.

1. The source of the excitation measures
2. The relation between the tonalness scale and the physical frequency scale
3. The function relating the specific loudness to excitation
4. The dependence of the transmission characteristics of the middle ear on frequency
5. The generation of the total loudness
6. The effect of partial masking on loudness

1. It is assumed that both the excitation and the loudness patterns generated in the nervous system upon acoustic stimulation can be derived from the masking patterns, which can be empirically measured. This assumption is similar to that originally set forth by Fletcher and Munson (1937) and adopted by Steinberg and Gardner (1937) and also by Harris

(1959). The masking pattern for a given sound is obtained by using the sound to mask a pure tone. The plot of the masked threshold for the tone, as a function of its frequency, is the masking pattern.

In order to avoid the necessity of measuring the masked thresholds with each sound to which the model is applied, a set of standard masking patterns and their derived excitation patterns have been made available for narrow bands of noise (Zwicker, 1958, 1963). These narrow-band patterns represent the masking patterns of any sound narrower than or equal to a critical band, including pure tones, since the masking patterns are essentially the same for all bandwidths less than a critical band (Ehmer, 1959; Zwicker, 1956). Combinations of patterns centered at different frequencies can represent the masking patterns of sounds wider than a critical band. A set of masking patterns is shown in Figure 2 for the masking of pure tones by a narrow band of noise centered at 1,200 cps. The parameter on the curves is the sound pressure level (SPL) of the masking noise.

The first step in the transformation from masking patterns to excitation patterns is the conversion of the masking level at each frequency to the corresponding excitation level. But how do we determine from the measured masked threshold the excitation level within the nervous system produced by the masking sound? The answer comes from data on the self-masking of noise, i.e., from measures of the jnd for intensity. Since in such measures the excitation patterns of the masked noise, ΔI, and the masking noise, I, are almost identical, their ratio, $\Delta I/I$, may be taken to express the ratio between the excitation of the just masked stimulus and that of the masking stimulus.

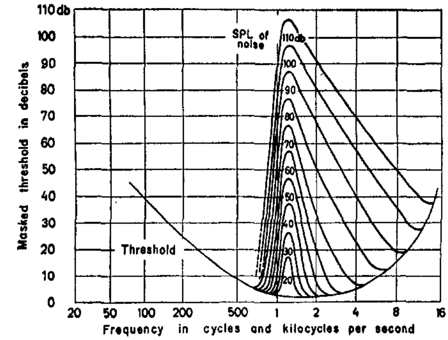


FIG. 2. Masking effect of a narrow band of noise with a center frequency of 1,200 cps. The parameter is the effective SPL of the noise. (On the ordinate the masked threshold for a pure tone is plotted for each level of noise as a function of frequency [Zwicker, 1958, adapted with permission of *Acustica*].)

Fortunately over a wide range of intensities the ratio, $\Delta I/I$, is fairly constant, being equal to $\frac{1}{2}$ for the lower and middle frequencies and to $\frac{1}{4}$ for the higher frequencies (Zwicker, 1956). The minimum excitation required then to mask completely at a given frequency is twice (3 decibels) or four times (6 decibels) the intensity at the masked threshold. The addition of 3 (or 6) decibels to the masked threshold gives the value for the excitation level. Excitation level will be expressed in decibels as $L_E = 10 \log E/E_0$ where E_0 is a reference value corresponding to $I_0 = 10^{-16}$ watt/centimeters^2.

2. The representation of the excitation pattern requires not only the conversion of the ordinate from masking level to excitation level, but also the conversion of the abscissa from the physical frequency scale to one that corresponds more closely to the way in which activity spreads in the auditory system. A good correspondence appears to be achieved by the tonalness or z scale. (The spread of activity represented by the z scale probably corresponds to a spatial spread of activity

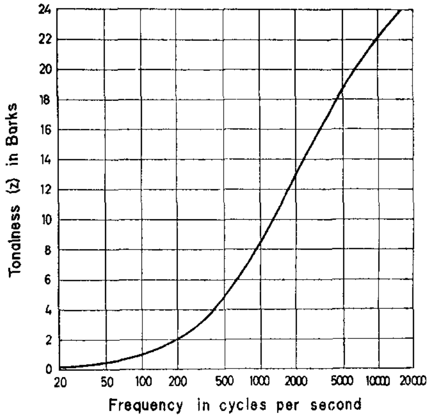


FIG. 3. Relation between tonalness in Barks and frequency (Zwicker, 1961, adapted with permission of *Journal of the Acoustical Society of America*).

on the basilar membrane, but at higher neural levels, the spread is not necessarily spatial.) The relation between tonalness and frequency is given by the function in Figure 3 (Zwicker, 1961). The unit, Bark (after the German acoustician Barkhausen), replaces the critical band in order to distinguish the Bark as a measure of sensory events from the critical band, a stimulus measure. The z function seems to be an appropriate measure of auditory activity because of its crucial role as the unit of integration in loudness summation, threshold measurements, and masking (Greenwood, 1961; Scharf, 1961b). Moreover, a similar function relates the jnd of pitch, the frequency representation on the basilar membrane, and the mel scale of pitch to frequency.

With the aid of the tonalness function, the masking patterns of Figure 2 have been converted to excitation patterns in Figure 4. (Excitation patterns for narrow bands of noise centered at lower and higher frequencies are similar to those in Figure 4; see Zwicker, 1958.) The excitation patterns are for a band of noise no wider than a critical band, centered at 1,200 cps and having the SPL shown as the parameter on the curve. These curves also represent the excitation produced by a 1,200-cps tone, since the masking patterns produced by pure tones and by narrow bands of noise are almost the same (except where beats and audible nonlinear distortions occur).

3. In order to use these excitation patterns for the derivation of loudness, it is necessary to know the relation between loudness and excitation or more exactly, between Specific Loudness N' and Excitation E. A psychophysical equation expressing this relation has been formulated (Zwicker, 1958, 1963). It is based upon Stevens' power law, which says in one form that equal intensity ratios yield equal loudness ratios (Stevens, 1957); it is also based upon the assumption that the loudness of any sound is the integral of the specific loudness over the z scale. In this sense every sound involves the summation of loudness, for Figure 4 shows that the excitation aroused by even a pure tone spreads over a considerable portion of the z scale. The next few paragraphs trace the deriva-

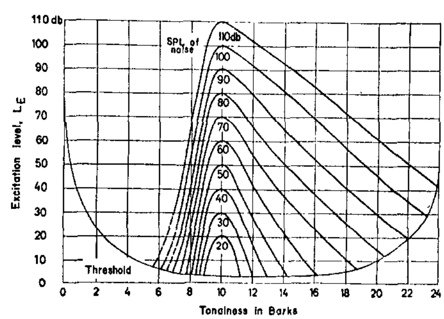


FIG. 4. Excitation patterns calculated from the masking produced by a narrow band of noise centered at 1,200 cps. (The level of the effective SPL of the band of noise is the parameter on the curves, Zwicker, 1958; adapted with permission of *Acustica*.)

tion of the basic formula that relates specific loudness to excitation.

Stevens' law may be expressed as:

$$\frac{\Delta N}{N} = k' \frac{\Delta I}{I} \qquad [1]$$

where I is the intensity of a tone and N is the loudness. In terms of excitation, we must assume that this law applies only to the excitation over a single Bark. It is not feasible to apply this law to the whole excitation produced by a pure tone since the excitation pattern changes considerably both with frequency and intensity. The critical band or Bark provides a constant unit over which the excitation can be converted to loudness by means of a single equation. The loudness due to the excitation over a single Bark is called the "specific loudness," represented by the symbol N'. With Intensity I replaced by Excitation E, Equation 1 becomes:

$$\frac{\Delta N'}{N'} = k \frac{\Delta E}{E}. \qquad [2]$$

Near threshold, where intensity discrimination is poor, the relationship breaks down unless a constant is added to the denominators of Equation 2. This constant, E_{gr}, may be thought to represent the excitation produced in the ear by an inaudible physiological background noise. This excitation can suppress a weak excitation produced by an external stimulus thereby setting a lower limit, the absolute threshold, for the ear's sensitivity. The corresponding inaudible specific "loudness" is N'_{gr}.

$$\frac{\Delta N'}{N' + N'_{gr}} = k \frac{\Delta E}{E + E_{gr}} \qquad [3]$$

Treating Equation 3 as a differential equation and integrating, we have

$$\log (N'_{gr} + N') = k \log (E_{gr} + E) + \log C \text{ or} \qquad [4]$$

$$N'_{gr} + N' = C\,(E_{gr} + E)^k. \qquad [4a]$$

Taking as our boundary conditions $N' = 0$ when $E = 0$, we obtain for the constant of Integration C:

$$C = \frac{N'_{gr}}{E_{gr}{}^k} \text{ or} \qquad [5]$$

$$N'_{gr} = C\,(E_{gr})^k. \qquad [5a]$$

The evaluation of the constant E_{gr} depends upon the same assumption used to convert from masking to excitation patterns, namely, that the masking excitation must be twice or four times the excitation produced by the just masked tone. It is assumed that the internal background excitation is twice the excitation E_t produced by an external tone at the absolute threshold (or four times if the tone is at a high frequency).

$$E_{gr} = 2E_t \qquad [6]$$

Using Equations 5 and 6 to substitute in Equation 4 we arrive at:

$$N' = N'_{gr}\left[\left(\frac{E}{2E_t} + 1\right)^k - 1\right]. \qquad [7]$$

In order to express the value N'_{gr} in relative values of Excitation E_t/E_o, we introduce the reference value N'_{gr_o}. Using Equation 5a with $2E_t$ replacing E_{gr}, we obtain:

$$\frac{N'_{gr}}{N'_{gr_o}} = \left(\frac{2E_t}{E_o}\right)^k \text{ or} \qquad [8]$$

$$N'_{gr} = N'_{gr_o}\left(\frac{2E_t}{E_o}\right)^k.$$

Equation 7 may now be written:

$$N' = N'_{gr_o}\left(\frac{2E_t}{E_o}\right)^k \times \left[\left(\frac{E}{2E_t} + 1\right)^k - 1\right]. \qquad [9]$$

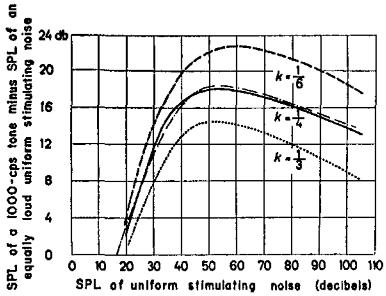


FIG. 5. Calculated and observed (dot-dash line) difference in SPL between a 1,000-cps tone and a uniform masking noise. (Calculations were made with three different values of k inserted in Equation 9 [Zwicker, 1958; adapted with permission of *Acustica*].)

N'_{gr_0} is an arbitrary constant that determines the units in which the specific loudness is measured; E_0 is the excitation corresponding to I_0 taken as 10^{-16} watt/centimeters^2; E is the excitation, at a given point on the z scale, evoked by the stimulating tone.

The exponent k remains to be determined. Its importance becomes clear at large values of E where Equation 9 approaches the simpler relation $N' \approx (E/E_t)^k$. The value of k can not, however, be determined from Equation 9 by the direct measurement of N', the loudness arising from excitation restricted to a single Bark. Such a restricted excitation pattern can not be produced in the ear. This difficulty has been bypassed, and the value of the exponent calculated from measurements of the loudness of two very different sounds (Zwicker, 1958). One sound was a 1,000-cps tone and the other a uniform stimulating noise, a noise that has the same overall intensity in each critical band. The two sounds were matched for loudness at various levels. The observed differences in loudness level between the noise and tone are plotted as the dot-dash line in Figure 5. On the same plot are shown the differences calculated from Equation 9 with 3 different values of k. In these calculations the Total Loudness N of the tone and also of the noise is the integral of the specific loudness over the z scale, that is

$$N = \int_{z=0}^{z=24} N'dz. \qquad [10]$$

The best fit between the observed and calculated differences is obtained with $k = 0.23$.

All the unknown constants of Equation 9 have now been determined except the arbitrary constant N'_{gr_0}. This constant is chosen so that the integral, $\int N'dz$, of the specific loudness of a 1,000-cps tone at 40-decibel SPL is equal to 1 sone. The unit assigned to the specific loudness is sone$_G$, where the subscript G distinguishes this calculated value from the sone obtained by direct loudness measurements.

After some smoothing at low levels, we arrive at the final equation for the specific loudness.

$$N' = 0.08\left(\frac{E_t}{E_o}\right)^{.23} \times \left[\left(1/2\,\frac{E}{E_t} + 1/2\right)^{.23} - 1\right] \quad \text{in sone}_G/\text{Bark} \qquad [11]$$

This equation is graphed in Figure 6 with both E_t and E expressed in decibels as L_t and L_E. Given the excitation level at threshold and given the excitation level due to the stimulating tone, the specific loudness, N' in sones$_G$, may be found on the graph at each Bark under the excitation pattern.

4. The ordinate of Figure 4 should be labeled not L_E, but $L_E - a$. The symbol a corrects the error introduced by the fact that the masked thresholds for pure tones vary not only because the excitation levels produced within the nervous system by a masking stimulus change with frequency, but also

because more energy is transmitted by the middle ear to the receptor cells at some frequencies than at others. The value of a is in a sense the amount of stimulus energy lost (or gained, depending on the reference value chosen) as a given tone is transmitted to the oval window. The masked threshold for the tone would then be greater by a decibels and would yield a value for the excitation level L_E of the masking stimulus that would be a decibels too high.

Direct measures of the transmission characteristics of the middle ear are apparently not available, but they may be approximated if we assume that the transmission characteristics govern the dependence of the absolute threshold on frequency (at least above 2,000 cps). Such an assumption implies that neural sensitivity is constant over most of the audible frequency scale (up to 9,000 or 10,000 cps) and that, for example, the average threshold is 18 decibels lower at 4,000 cps than at 8,000 cps (Robinson, 1957) because the energy transmitted to the oval window is 18 decibels less at 8,000 cps. The value of a would then be 18 decibels higher at 8,000 cps than at 4,000 cps.

Below 2,000 cps, the rise in the absolute threshold with decreasing frequency is ascribed to increased internal noise rather than to reduced middle-ear transmission. Since the internal noise has already been included in Equation 9 as part of the expression E_t/E_o, no other account need be taken of it here. Reference to a is omitted in the rest of this paper for the sake of simplicity.

5. The model of loudness described so far for a single tone or narrow band of noise may also be applied to complex sounds such as wide bands of noise and multiple tones. Two alternative procedures are available.

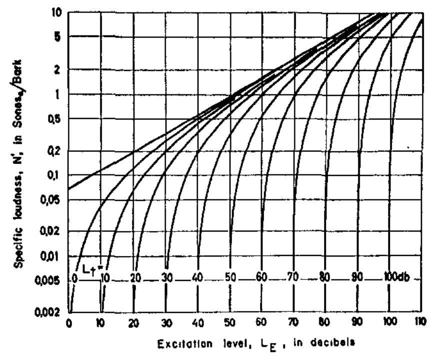


FIG. 6. Specific Loudness N' in sone$_G$/Bark as a function of the Excitation Level L_E. The exponent is $k = \frac{1}{4}$. (The parameter on the curves is the Excitation Level L_t of the test tone at threshold [Zwicker, 1958; adapted with permission of *Acustica*].)

The first procedure is exactly like that described above for obtaining the loudness value for a single tone. The masking pattern produced by the complex, as masking sound, is measured and transformed to the excitation pattern. The specific loudness can be read from Figure 6 as a function of tonalness to yield a loudness pattern whose integral over z is the total loudness. In the second procedure, the measurement of the masking pattern of the complex sound is replaced by the much simpler measurement of the SPL in each component critical band of the stimulus. Each band is treated as an independent stimulus that gives rise to an excitation pattern like those shown in Figure 4. The overlapping patterns thus obtained have a common upper envelope, which determines the excitation level; wherever two or more patterns overlap, only the highest excitation level is used. This procedure requires, of course, the prior measurement of the masking pattern for bands of noise at various SPLs and centered at various frequencies. Fortunately the shape of the masking patterns changes slowly enough with center fre-

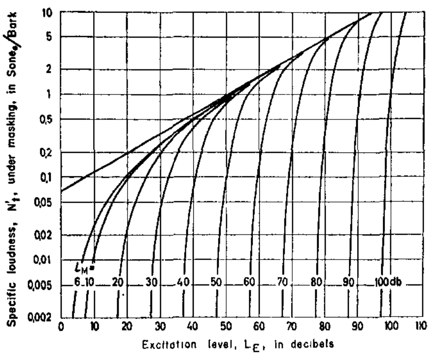


FIG. 7. Specific Loudness N'_t of partially masked pure tones as a function of the excitation level of the tone. (The parameter on the curves is the Excitation Level L_M of the masking sound [Zwicker, 1963; adapted with permission of *Acustica*].)

quency so that measurements of the masking patterns at only 6 or 7 scattered frequencies suffice to give good approximations of the patterns over the whole audible frequency spectrum.

Examples of the application of both procedures are given below.

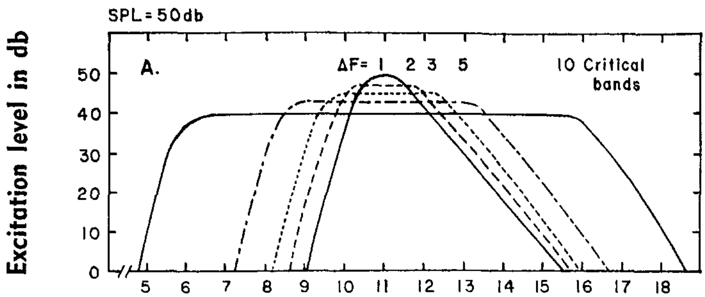
6. The loudness of partially masked sounds must be treated somewhat differently from the loudness of unmasked sounds. A sound is partially masked when its loudness is reduced by the presence of another sound. Partial masking occurs (and the measurement of the usual masked thresholds is possible) because the components of a complex sound may be distinguished from one another; their loudnesses do not always summate to yield an overall loudness. The question now posed is how our model can handle partial masking.

A series of measurements (Zwicker, 1963) showed that the present model underestimates the loudness of a 1,000-cps tone partially masked by either narrow-band or wide-band noise at various levels. This underestimation could be corrected, however, by an adjustment of the curves in Figure 6. Apparently when the excitation at threshold, L_t, is produced by an external masking noise, the loudness of a partially masked tone increases more rapidly with intensity than when the threshold excitation is produced by internal noise. Consequently, the curves of Figure 6 must be steeper if they are to be used with partially masked sounds. Adjustment of the curves in Figure 6 to yield loudness patterns for a partially masked tone whose integral better approximates the measured loudness leads to the curves of Figure 7. In place of the excitation level, L_t, at threshold, the excitation level, L_M, of the masking stimulus is the parameter on the curves.

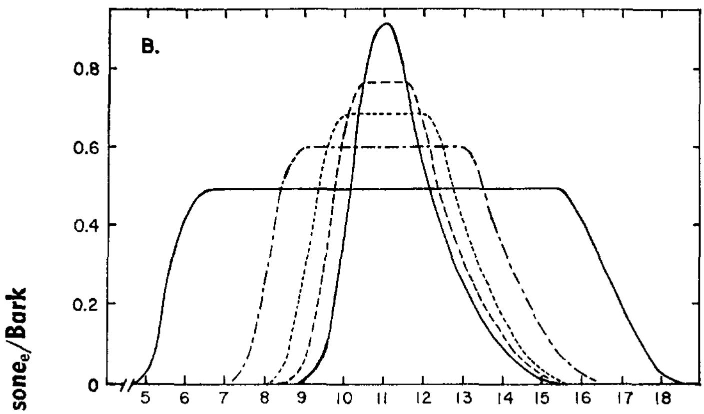
At low values of L_M and L_t, the curves in Figure 7 resemble those in Figure 6. Since, after the correction for the transmission characteristics of the middle ear, L_t is small for all but the low frequencies, perhaps the curves of Figure 7 ought to supercede those of Figure 6. Until it is known, however, which set of curves is more appropriate for the low frequencies, where internal masking is assumed to account for the large absolute threshold, it is expedient to retain the curves of Figure 6, which have already been in circulation for some time. Furthermore, the curves of Figure 7 are for pure tones masked by noise and probably do not accurately describe the growth of the specific loudness of tones masked by other tones or of narrow-band noise masked by tones or by other noises (Zwicker, 1963).

APPLICATION OF THE MODEL TO LOUDNESS SUMMATION AND ANALYSIS

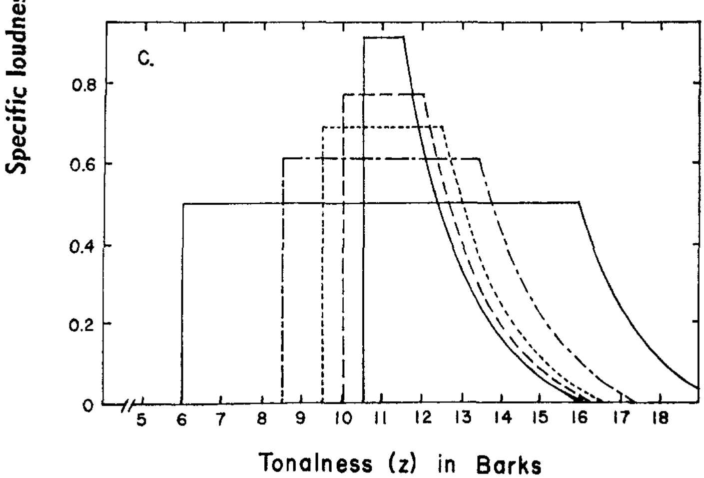
Our model of loudness serves two important and closely related functions. It permits an understanding of the changes in loudness under a variety of conditions involving both summation



8a. Excitation patterns for 5 bands of noise varying in width from 1 to 10 critical bands.



8b. Specific loudness patterns calculated by Equation 11 from A.



8c. Schematic version of the loudness patterns of B.

FIG. 8. Excitation and loudness patterns for bands of white noise at 50-decibel SPL.

and analysis. A convenient transformation of the input stimulus into excitation and loudness patterns provides a means for inferring the interactions within the auditory system. In terms of these interactions, most of the facts of loudness summation may be explained. At the same time the model permits the precise calculation of loudness so that manipulations of the model to predict (or postdict) the results of subjective measurements may be put to rigorous test. (The model is also presented elsewhere [Zwicker, 1959] in a more schematized version for third-octave band filters that simplifies the calculation of loudness where a numerical value is the only requirement as in audio engineering.)

In this section, the model is applied to three types of sound. In each case an important aspect of loudness summation or analysis is explained in terms of the model, and calculated values for loudness are compared to measured values. The first sound is a band of white noise. The second sound is a pure tone partially masked by a narrow band of noise. The third sound is a 4-tone complex partially masked by a wide-band noise.

Loudness of Bands of Noise

The loudness of a band of noise, of constant overall intensity, remains unchanged as its bandwidth is increased up to the critical bandwidth. Beyond the critical band the loudness of the noise usually increases, but at a faster rate for a noise at a moderate SPL than for one at either a high or low SPL. The results of Zwicker, Flottorp, and Stevens (1957) followed this typical pattern, and in the application of the model to their stimuli, the reasons for such results become clear.

Figure 8 presents the excitation patterns for five different bands of noise, all centered at a geometric mean of 1,480 cps and all with an overall SPL of 50 decibels. These excitation patterns are based upon masking patterns like those in Figure 4. For the noise one critical band wide, the excitation pattern corresponds to that produced by a pure tone or narrow band of noise. For a noise wider than a critical band, the patterns produced by contiguous critical bands have been combined. By means of Equation 11 as graphed in Figure 6, the values L_E are converted over tonalness to the specific loudness N'. These values are plotted as the loudness patterns in Figure 8b and also in Figure 8c. The more schematized versions of Figure 8c facilitate integration and show more clearly how the patterns change with level. The loudness levels computed from Figure 8c were almost identical to those from Figure 8b. Loudness values were also computed for bands of noise at 15 and 100 decibels from the loudness patterns presented in Figures 9 and 10.

It becomes clear from these patterns why loudness is constant within the critical band and why its increase beyond the critical band depends upon level. Consider first Figure 8c. The loudness of each of the 5 noises is proportional to the area under the loudness pattern

$$\left(N = \int_{z=0}^{z=24} N' dz\right).$$

Given the same overall intensity and center frequency, any stimulus no wider than a critical band, even a pure tone, produces a loudness pattern like that shown here for the noise one critical band wide (the highest pattern). Subcritical bands are represented by an invariant loudness pattern because they produce an invariant masking pattern (Zwicker, 1958). Once the critical

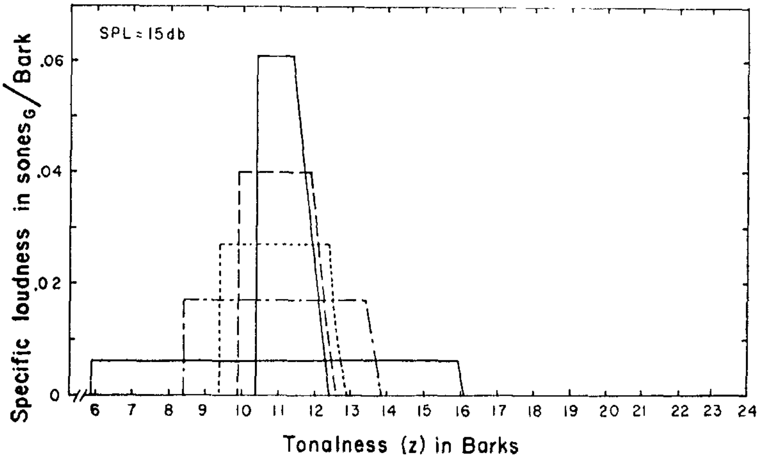


FIG. 9. Specific loudness patterns for 5 bands of noise, all at 15-decibel SPL.

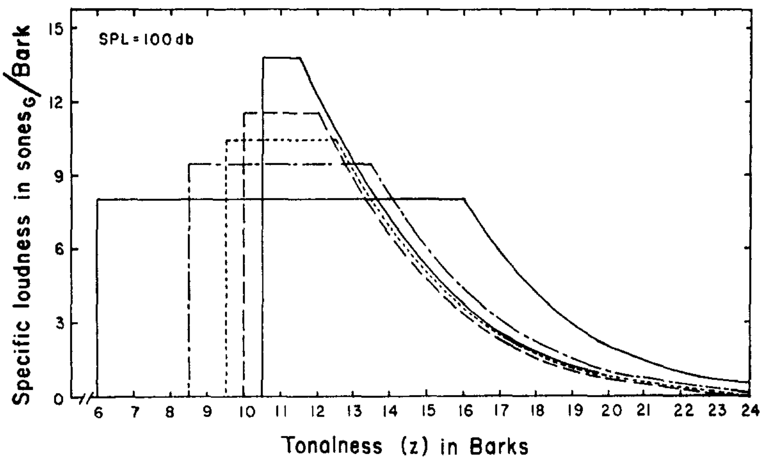


FIG. 10. Specific loudness patterns for 5 bands of noise, all at 100-decibel SPL.

band is exceeded, however, the shape of the loudness pattern begins to change. Its height decreases (owing to the decrease in the intensity of each critical band), and it extends over a greater portion of the z axis. At moderate levels, such as 50 decibels in Figure 8b, the vertical decrease of the curve is more than compensated by the horizontal increase, so that the total area and the total loudness increase. Thus although the excitation and with it the height of the central portions of the loudness pattern decrease as bandwidth increases, excitation at the lower and upper tonalness values increases sufficiently and enough new elements come into play to raise the overall loudness. Here we see why loudness increases with bandwidth at moderate levels. Why doesn't it increase as much at low and at high levels?

Figure 9 provides an answer for soft noises. Two changes from the 50-decibel noise are evident. First the height of the loudness patterns de-

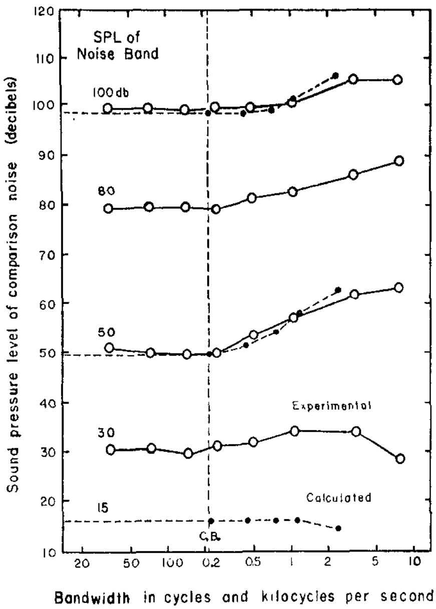


FIG. 11. The dependence of the loudness of a band of noise upon bandwidth. (The parameter on the curves is the overall SPL of the bands of noise. Open circles are values measured in an experiment by Zwicker, Flottorp, and Stevens, 1957. Filled circles are the values calculated from Equation 11. An excellent fit between measured and calculated values obtains.)

creases more rapidly as the stimulus energy is spread out, primarily because the specific loudness decreases with excitation level more rapidly at low than at moderate or high levels, as may be seen in Figure 6. (Close to threshold the loudness of a pure tone changes more rapidly as a function of intensity [Hellman & Zwislocki, 1961; Scharf & J. C. Stevens, 1961].) A second change from the moderate level is that the patterns are narrower and they increase in width somewhat more slowly as the stimulus energy is spread out. Owing primarily to the rapid decrease in the height of the patterns, loudness actually decreases as bandwidth increases beyond the critical band.

At the high levels, the situation is reversed as a comparison of Figure 10 with Figure 8c reveals. Here, the height of the loudness pattern changes with increasing bandwidth in approximately the same proportion as at the moderate level, but the loudness patterns are spread over considerably larger portions of the tonalness scale at the higher end. The spread is initially so extended that increasing the bandwidth beyond one critical band increases at first only the area at the lower tonalness values. At the same time, the specific loudness is reduced at the middle tonalness values, offsetting any gain from the greater spread. Consequently the loudness does not begin to increase until the stimulating bandwidth extends over 3 to 5 critical bands.

Integration of the areas under the loudness patterns of Figures 8c, 9, and 10 yields values for the loudness level of the bands of noise. These values are plotted as the filled circles in Figure 11. The open circles are the medians of 12 judgments by 12 subjects who adjusted the intensity of a band of noise 210 cps wide to match the loudness of each of the bandwidths shown on the abscissa (data from Zwicker, Flottorp, & Stevens, 1957). The overall SPL of the bands of noise was held constant at 30, 50, 80, or 100 decibels. The calculated and measured values lie close to one another, showing the same invariance of loudness summation within the critical band and the same dependence on level beyond the critical band.

Loudness of Partially Masked Tones

Only with certain types of complex sounds, such as bands of white noise, do all the components fuse to yield an overall loudness. The sound of a plane passing overhead does not fuse with the sound of music coming from the nearby radio. Yet the presence of the one sound may distinctly affect the loudness of the other. Such a situation arises when a tone and a band of noise are presented together, and the model may be used to understand some of the interactions that take place within the auditory system and also to calculate the loudness of the tone.

Measurements are available of the loudness of pure tones partially masked by a narrow band of noise (Scharf,

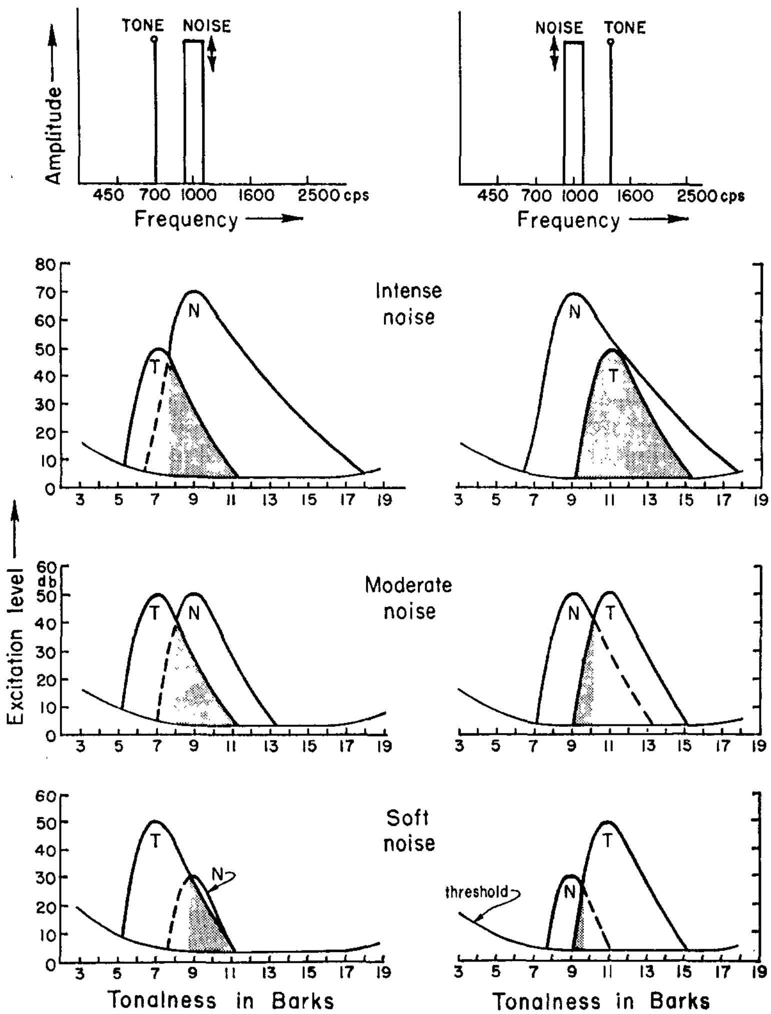


FIG. 12. Excitation patterns for a narrow-band masking noise and two tones. (The idealized spectra for the noise and tones are also shown. In the overlapping [shaded] areas the excitation produced by the tone is suppressed by the noise. Clearly as the noise intensity is reduced, interference with the higher frequency tone decreases more rapidly than that with the lower frequency tone. Thus although the higher frequency tone is completely masked at a noise level where the lower frequency tone is not, once its masked threshold is exceeded, the higher frequency tone grows in loudness more rapidly [Scharf, 1964; adapted with permission of *Acustica*].)

1964). The tones were located at frequencies above and below the frequency range of the noise, which was one critical band wide centered at 980 cps. Whereas complete masking was shown, as usual, to be more effective toward the higher frequencies, partial masking was found to be generally more effective toward the lower frequencies. This change appears paradoxical until the model is used to analyze the excitation patterns produced by the tone and band of noise.

Figure 12 shows the standard excitation patterns for two of the tones and for the band of noise used in the experiments. At the top of the figure are the idealized spectra of the three sounds. Analysis of these patterns is simplified if we assume that at those points where the excitation patterns of the tone and noise overlap, whichever pattern has the higher excitation level completely suppresses the other. (A difference of 3 decibels would actually be required for complete suppression.) In the figure an intense noise completely masks the higher frequency tone, but only partially masks the lower frequency tone. When the noise intensity is reduced, more of the excitation pattern of the lower frequency tone remains suppressed than of the higher frequency tone. The size of the suppressed area indicates the degree of loudness reduction, i.e., partial masking, so that it is clear that the moderate and soft noises partially mask the lower frequency more than the higher. This difference arises from the asymmetry of the excitation patterns toward the higher frequencies as examination of the patterns reveals.

The model can also be used to calculate the loudness of the partially masked tones. Figure 13 shows the excitation patterns for the masking noise at various levels and for one of the tones. These patterns differ somewhat from the standard excitation patterns; they were taken from the masked audiograms produced by the bands of noise used in the experiment. The same subjects made both the threshold and the loudness judgments. The loudness patterns for the 830-cps tone set at 65-decibel SPL and presented against five different levels of the narrow-band noise are shown in Figure 14. These loudness patterns were constructed by using the curves for the specific loud-

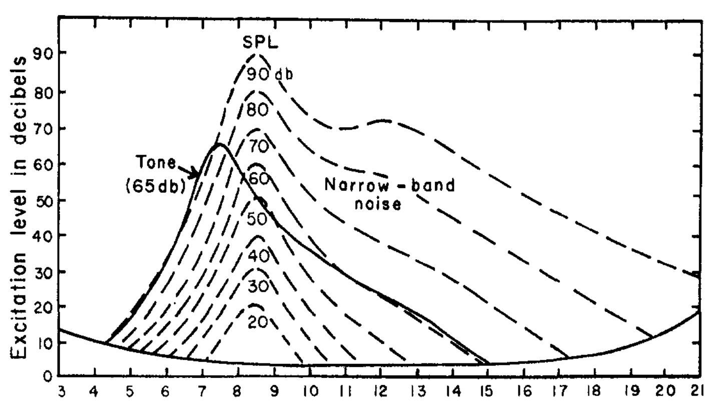


FIG. 13. Excitation patterns for an 830-cps tone (solid curve) and a narrow-band noise at various SPLs.

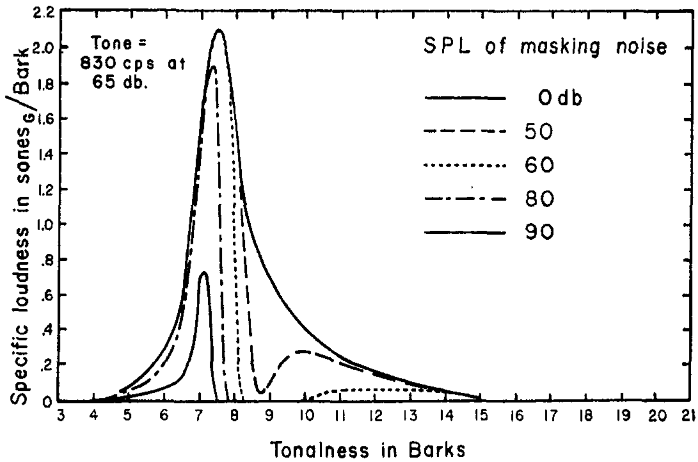


FIG. 14. Loudness patterns for an 830-cps tone (at 65-decibel SPL), partially masked by a narrow-band noise. (The noise was centered at 980 cps and set at the SPLs shown. The loudness patterns for the tone do not change until the noise SPL exceeds 35 decibels. When the noise SPL reaches 96 decibels, the tone is completely masked and is inaudible.)

ness under partial masking (Fig. 7) to convert the pure tone excitation levels of Figure 13 to the specific loudness in sones$_G$/Bark. The excitation level L_E of the noise pattern provided the values for L_M required in Figure 7. Loudness patterns (not reproduced here) were also constructed for the 830-cps tone at 45 decibels and also for tones at 980 and 1,355 cps at 45 and 65 decibels. Values calculated from the loudness patterns are plotted for the tones at 65 decibels in Figure 15 along with the measured loudness levels (Scharf, 1964) and the interquartile ranges. (The plot for the tones at 45 decibels is similar to that for tones at 65 decibels and is therefore not reproduced here.) The agreement of the observed with the calculated values is best for the 980-cps tone where the tone and noise were centered on the same frequency so that inaccuracies in the excitation patterns would tend to cancel out. The agreement is good for the 830-cps tone. The agreement is not as good for the 1,355-cps tone, although the general trend of the data is reproduced. Since this tone was at a higher frequency than the noise, the excitation levels at the higher tonalness values where the noise's excitation pattern had little effect on the tone's pattern were especially important in the loudness calculations. But it is precisely at the higher tonalness values on any given excitation pattern that the levels are most uncertain, for the masking patterns upon which they are based are most variable there both within and among subjects.

Given the difficulty of the loudness judgments and the many steps in the loudness calculation in this complex situation, the model fares well not only in a qualitative analysis (Fig. 12), but also in the quantitative calculations. Perhaps, better calculations could be obtained with further refinements of the model. Nevertheless, using the model as it stands, let us turn to a still more complex stimulus arrangement.

Loudness of a Partially Masked Complex Sound

In the same way that the loudness of a pure tone can be distinguished

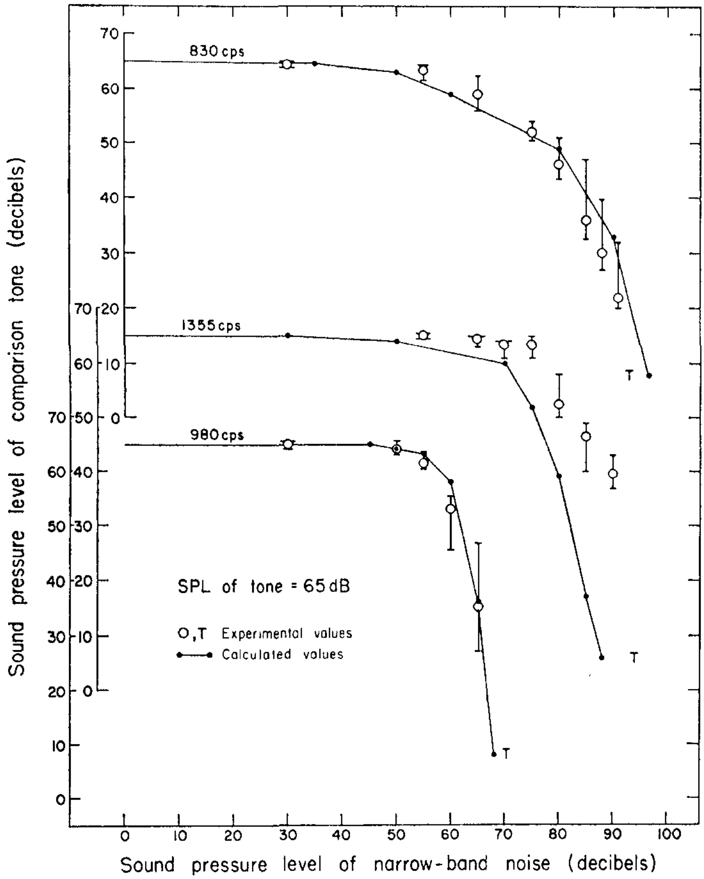


FIG. 15. Dependence of the loudness of a pure tone at 65 decibels on the SPL of a partially masking narrow-band noise. (The ordinate gives the SPL of the equally loud comparison tone which had the same frequency as the masked tone, but was presented in the quiet. The frequency of the tone is the parameter on the curves. Each open circle is the median of 8 to 10 loudness matches by four to five subjects. Interquartile ranges are also shown. The symbol T indicates the measured noise SPL required to mask the tone completely. The filled circles were calculated by means of the curves in Figure 7 from excitation patterns like those in Figure 14.)

from a neighboring band of noise, a multitone complex can be distinguished from a background noise. The loudness of a 4-tone complex has been measured against various levels of a uniform masking noise (Scharf, 1961a). Results showed loudness remains invariant within the critical band. Beyond the critical band, the changes in loudness depend primarily on the sensation level (number of decibels above threshold) of the complex. Even a moderately intense complex, which in the quiet shows the greatest amount of loudness summation, decreased in loudness as bandwidth increased whenever an intense masking noise was simultaneously presented. How does our model account for this finding?

The excitation patterns for the complex are assumed to remain unchanged in the presence of the noise. Figure

16 shows the excitation pattern for some of the complexes used in the experiment and for a uniform masking noise at an overall level of 55 decibels, which corresponds to an SPL of about 40 decibels in each critical band. These patterns are combined from the standard excitation patterns measured on narrow bands of noise. The individual excitation patterns evoked by the four tones that comprise the complex stand out rather clearly when the tones are spread out as in the 2,200-cps wide complex.

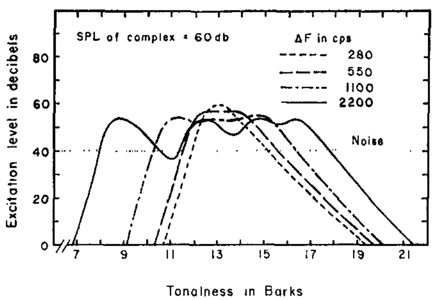


FIG. 16. Excitation patterns for a 4-tone complex at an overall SPL of 60 decibels and centered at 2,000 cps. (Patterns for four different values of ΔF are shown. The dotted line is the excitation pattern for the uniform masking noise when the SPL in each critical band was 40 decibels.)

Whereas the excitation patterns are unaffected by the noise, the loudness patterns may be greatly affected as shown by Figures 17, 18, and 19. Each figure presents the loudness patterns for the complexes in a different level of noise. In Figure 17, where the noise was inaudible at 15 decibels, the loudness patterns grow larger as the complex becomes wider. Noise at 35 or 45 decibels did not produce a measurable change in the loudness patterns for this 60-decibel complex, but with the overall noise level raised to 55 decibels, the patterns do not increase as much with ΔF; they increase still less

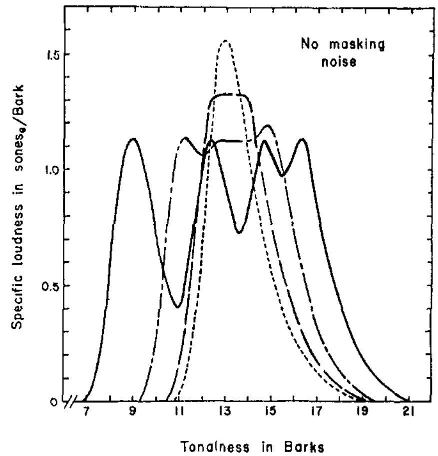


FIG. 17. Loudness patterns for four different 4-tone complexes at 60-decibel SPL. (No background noise.)

when the noise level was 65 decibels (Fig. 18). With the noise at 70 decibels and the complex almost completely masked (Fig. 19), the size of the loudness patterns, and therefore the loudness, actually decreases as ΔF increases.

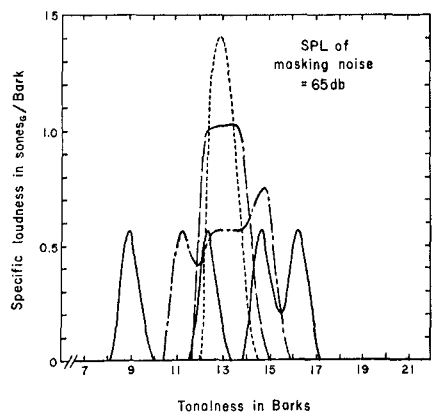


FIG. 18. Loudness patterns for the four complexes presented against a uniform masking noise with an SPL of 50 decibels in each critical band (overall SPL approximately 65 decibels). (Both the height and spread of the patterns are somewhat reduced by the addition of the noise background.)

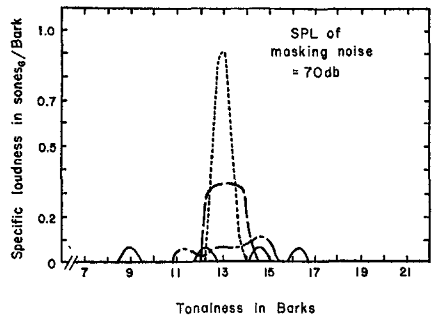


FIG. 19. Loudness patterns for the four complexes presented against a uniform masking noise with an SPL of 55 decibels per critical band (overall SPL approximately 70 decibels).

Loudness diminishes with increased ΔF in the presence of intense noise because near the masked threshold, even more so than near the absolute threshold, the specific loudness changes very rapidly with the excitation level. Consequently, as the excitation spreads beyond a single Bark, even a small decrease in the level of excitation within each Bark produces a large drop in the loudness pattern. A glance at Figure 7, which is used to transform the excitation levels L_E to the specific loudness values N'_t, shows that near the masked threshold (approximately where the curves cross the abscissa) a reduction of only a few decibels in the excitation level over a single Bark causes a large reduction in specific loudness. Loudness summation appears then to be poorer in the presence of a noise because the noise steepens the loudness functions for the component critical bands of the complex, and

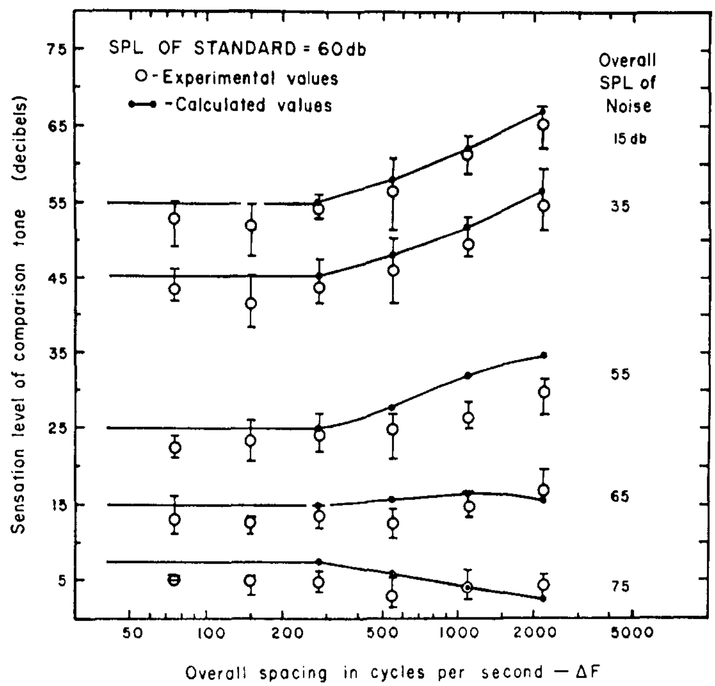


FIG. 20. Dependence of the loudness of a 4-tone complex on ΔF. (The SPL of the complex was held constant at 60 decibels and the SPL of a noise background was varied from 15 to 75 decibels. Open circles are the medians of 12 loudness matches by six subjects; interquartile ranges are also shown. Filled circles are the values calculated from the loudness patterns shown in Figures 17 to 19.)

the more intense the noise relative to the tone, the steeper these functions become.

Now the question remains of how well the calculated loudness values agree with the experimentally measured values. A direct comparison can not be made because the loudness levels of the complexes were not measured in the original experiment. Results were given in terms of the sensation level of an equally loud pure tone heard against the same noise background as the complex. Conversion of the calculated measures to the sensation level of the comparison tone in noise can be made with the help of data recently published on the loudness relations between a pure tone in noise and the same tone in the quiet (Zwicker, 1963). Figure 20 shows the sensation level of the comparison tone that sounded as loud as the complex whose overall spacing is given on the abscissa. The open circles are the medians of 12 judgments by six subjects; the solid lines are fitted to the calculated loudness values. In terms of the absolute values, the agreement is not too good; the calculated values are generally a little too high. However, both the calculated and measured values show the same progressive decrease in the amount of loudness summation as the masking noise becomes more intense, until close to the masked threshold, both sets of values show loudness decreasing as ΔF increases.

Summary of Calculations

With respect to the three types of sounds examined, the agreement between the observed and the calculated loudness values must be considered remarkably good. For each type of sound, the shape of the calculated functions reproduces quite closely that of the measured functions. Although the absolute values may occasionally differ by many decibels, they are usually in good agreement, especially if we consider that the loudness judgments themselves were highly variable with the interquartile ranges for the various experiments averaging over 4 decibels. It seems unlikely, therefore, that any major modification of the model could eliminate the present discrepancies. As it now stands, the model provides a valid representation of the kinds of physiological processes which can account for most of the psychophysical data on loudness and loudness summation and which also agree with current theories of hearing (cf. Békésy, 1963).

SOME FINAL CONSIDERATIONS

Our model does not try to deal with attentional, attitudinal, or other central factors that certainly affect the experience of loudness. It is intended to apply to normal, unidirectional hearing under conditions of applied attention. Whether, under conditions of attention (such as sleep) that greatly alter the subjective experience of loudness, the model remains a valid representation of neural events occurring at some early stage in the nervous system, we do not know. The whole problem of the role of efferent innervation in the auditory mechanism is under intensive study (Davis, 1962); perhaps some clarification will soon be available. However, even if conditions in the cochlea turn out to be relatively constant, we still do not know whether the summations and interactions pictured in our model are complete at the cochlear level. Indeed our ignorance may be even greater. Ades (1959) writes that, "for the moment, we can only exercise caution in theorizing about the (neural) mediation of loudness . . . [p. 609]." Nevertheless "subjective loudness" is "usually attributed to the number of nerve impulses per second traversing the auditory nerve [Davis, 1959, p. 583]." While our model does not depend on this particular conceptualization (or any physiological theories or data for that matter), the specific loudness is perhaps most easily interpreted on a physiological level as dependent upon the number of neural impulses. Such a view served in the formulation of the model (Zwicker, 1958).

Another underlying assumption that facilitates interpretation of the model is that loudness summation is primarily peripheral. This assumption receives compelling support from Niese's (1960) demonstration that loudness summation takes place monaurally. Niese showed that the loudness of two narrow bands of noise increases as their frequency separation increases beyond the critical bandwidth only if both bands are presented to the same ear. If the two noises are presented separately to each ear, no increase in loudness with increasing frequency separation can be detected. Perhaps more direct evidence for the peripheral nature of loudness summation can be obtained from studies of humans and animals with lesions at various levels within the auditory nervous system. Additionally, studies of interaural effects in multidirectional hearing may provide interesting data.

Physiologically the critical band is more of an enigma than loudness, for we do not know where it is formed or how.[2] That it is formed upon stimulation and is not already there as some fixed structural unit, something like a variable band-pass filter (cf. De Boer, 1962), is clear. The critical band is a continuous function of frequency, not a step function—it may be measured at all the audible frequencies. Thus the critical band seems to form around the stimulating frequency as stimulation occurs. The formation of the critical band during stimulation suggests the necessity for a period of time during which the locus of stimulation is made known to the auditory system. Just such a temporal lag has been demonstrated by Scholl (1962) in measurements of the masked threshold for short pulses. The usual critical band does not appear until the duration of the pulse exceeds 10 milliseconds. Below 10 milliseconds the critical band appears to become wider and below 5 milliseconds matches the width of almost the whole auditory spectrum. Perhaps the most likely process by which the critical band is formed is of an inhibitory nature, similar to that postulated by Békésy (1960) for the neural unit on the skin and in the eye. If the critical band is a measure of inhibitory processes, we may wonder why the masking pattern measured for a pure tone shows no trace of a discontinuity at the critical bandwidth. Masking (and supposedly excitation) spread out continuously toward the higher frequencies.

The problems raised here are primarily physiological. Perhaps our model can serve to guide research in this area as well as in the psychoacoustical area. Whatever its advantages as a generator of new research, the model provides a comprehensive summary of

[2] Indeed, detection experiments using the method of forced choice have led some investigators (Swets, 1963) to infer a critical band whose width depends upon the experimental instructions and the subject's attitude. Such an inference is open to question, and in any case no one has reported that the average critical band in loudness summation experiments varies in any consistent fashion.

available psychoacoustical data on loudness and permits manipulations and calculations that lead to fruitful new insights into the nature of the auditory processes.

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(Received February 4, 1964)