Case Study: The Economics of Shopping: Time, Money, and Retirement

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Background

This case study explores how an individual, Carol, allocates her time between leisure, work, and shopping. The focus is on understanding why Carol might consume more after retirement despite a potential reduction in total income.

Scenario

Meet Carol, a woman in her late 50s who is contemplating how to allocate her time between work, leisure, and shopping as she approaches retirement. The time frame of the decision is one day (24 hours).

Setting Up the Problem

Given:

- c = Consumption goods.
- L = Leisure time measured in hours.
- s = Time spent on shopping measured in hours.
- l = Time spent on work measured in hours.
- T = Total available time in a day, such that $s + l + L = T$ (typically $T = 24$ hours).

- p = Price of consumption goods, determined by $p = f(s)$, such that (its derivative) $f'(s) < 0$ capturing the idea that as more time is spent shopping the better deals can the consumer obtain. We require that even if the consumer spends all the time shopping the price is positive ($f(T) > 0$).
- w = Wage rate per hour.
- R = Retirement income (converted to daily income).

Budget Constraint:

Still Working: Carol consumes c at prices $f(s)$, given labor income $w \times l$ before retirement.

Carol retires: Carol consumes c at prices $f(s)$, given retirement income R after retirement. The quantity of retirement she receives does not depends on savings so its fully non labor income.

Time Constraint:

Still working:

$$s + l + L = T.$$

Carol retires:

$$s + L = T.$$

Questions

1 Situation 1: Carol is Still Working

1. **Budget and Time Constraints**: Write down Carol's budget and time constraints when she is still working. Assume $R = 0$ in this case. Hint: use $f(s) = (s)^{-b}$ for $b > 0$. Explain in few sentences the meaning of the shopping technology $f(s)$ in terms of the parameter b and interpret its economic meaning.

2. **Optimization**: Use the Lagrangian method to solve for Carol's optimal choices of c, L, l and s when she is working Show all steps. Call the optimal value of this problem: c^1, L^1, l^1 and s^1. (Important: to

solve the problem pick a utility function, choose smart and on the basis of what you have learned. Use any utility you want but: Use the fact that consumption goods and leisure are neither perfect substitutes nor perfect complements. Also make sure your preferences are convex, hence the utility is a concave function. Finally, use a utility function that has only one parameter and call this preference parameter a).

(Hint: you can simplify the problem to 3 variables but there is no way to do only 2, when solving this Lagrangian notice that $\lambda > 0$, and think of ways to obtain the solution without having to compute the value of λ).

2 Situation 2: Carol Retires

1. **New Budget Constraint**: Write down Carol's new budget constraint after retirement. Assume $R > 0$.

2. **Optimization**: Use the Lagrangian method again to solve for Carol's optimal choices of c, L, and s after retirement ($l^2 = 0$). Show all steps. (You can reduce the problem here to 2 variables here) Call the optimal values of situation 2, c^2, L^2, s^2.

3. **Consumption Puzzle**: Find the range of R under which Carol ends up consuming more ($\frac{c^1}{c^2} \leq 1$) after retirement than when she was working, despite $0 < R \leq w \times l^1$, where l^1 is the solution to situation 1 (Hint: set the value of other other parameters distinct to R to numbers: $w = 1$, $a = \frac{1}{2}$, and $b = \frac{1}{2}$, $T = 24$).

3 Policy Implications and Real-world Connections

1. **Different Choices**: Discuss how Carol's choices in shopping and leisure might differ before and after retirement, even though her utility function remains the same.

2. **Real-world Relevance**: Link the findings of this model to the referenced paper on "Life-Cycle Prices and Expenditure." from Aguiar and

Hurst. Explain how this simplified model helps us understand real-world behaviors observed in the paper. This is the abstract of that paper: "Previous authors have documented a dramatic decline in food expenditures at the time of retirement. We show that this is matched by an equally dramatic rise in time spent shopping for and preparing meals. Using a novel data set that collects detailed food diaries for a large cross section of U.S. households, we show that neither the quality nor the quantity of food intake deteriorates with retirement status. We also show that unemployed households experience a decline in food expenditure and food consumption commensurate with the impact of job displacement on permanent income."

Your answer should be a well done one paragraph.

Answer to Case Study: The Economics of Shopping: Time, Money, and Retirement

1. Answer: Situation 1 Carol is Still Working

1.1 The budget is $f(s)c \leq wl$ and $s+l+L=T$, now we require $l=T-s-L$, and $f(s)c \leq w(T-s-L)$, and rearranging terms

$$f(s)c + w(s+L) \leq wT.$$

1.2. I pick the utility function $u(c,L) = alog(c) + (1-a)log(L)$, for $a \in (0,1)$, and $f(s) = (s+1)^{-b}$ for $b \in (0,1)$.

The budget is

$$(s+1)^{-b}c + w(s+L) \leq wT.$$

The Lagrangian is:

$$\mathcal{L}(c,L,s) = alog(c) + (1-a)log(L) + \lambda(wT - (s+1)^{-b}c - w(s+L))$$

$\frac{\partial \mathcal{L}}{\partial c} = \frac{a}{c} - \lambda[(s)^{-b}] = 0$
$\frac{\partial \mathcal{L}}{\partial L} = \frac{(1-a)}{L} - \lambda w = 0$
$\frac{\partial \mathcal{L}}{\partial s} = \lambda(bc(s)^{-1-b} - w) = 0$
$\frac{\partial \mathcal{L}}{\partial \lambda} = wT - (s+1)^{-b}c - w(s+L) = 0$
Using $\frac{\partial \mathcal{L}}{\partial c}$ and $\frac{\partial \mathcal{L}}{\partial L}$ we can form the MRS condition:

$$\frac{a}{1-a}\frac{L}{c} = \frac{s^{-b}}{w}$$

From this we obtain the first equation:

$$L = \frac{(1-a)}{a}\frac{cs^{-b}}{w}$$

Now from $\frac{\partial \mathcal{L}}{\partial s}$ we know that since $\lambda > 0$ this means that:

$$bc(s)^{-1-b} - w = 0$$

This means:

$$c = \frac{s^{1+b}w}{b}.$$

Now we obtain a second equation from L:

$$L = \frac{(1-a)}{a}\frac{s}{b}.$$

We replace the expressions above of c and L written in terms of s and parameters a, b in the budget, and we obtain the optimal shopping:

$$s^1 = \frac{ab}{(1+ab)}T$$

$$L^1 = \frac{(1-a)}{(1+ab)}T$$

$$c^1 = \frac{\left(\frac{ab}{(1+ab)}T\right)^{1+b} w}{b}$$

$$l^1 = \frac{a}{(1+ab)}T.$$

2. Answer: Situation 2 Carol is Retired

2.1. The new budget is $f(s)c \leq R$, $s + L = T$, in this case we will let $L = T - s$, and plug in the utility to form the Lagrangian.

2.2. The new lagrangian is the following:

$$\mathcal{L}_2(c,s) = aLog(c) + (1-a)Log(T-s) + \lambda(R - s^{-b}c)$$

The partial derivatives of lagrangians are:
$\frac{\partial \mathcal{L}_2}{\partial c} = \frac{a}{c} - \lambda s^{-b} = 0$

$\frac{\partial \mathcal{L}_2}{\partial s} = -\frac{(1-a)}{T-s} + \lambda bcs^{-1b} = 0$
$\frac{\partial \mathcal{L}_2}{\partial \lambda} = R - cs^{-b} = 0$
Forming the MRS condition from $\frac{\partial \mathcal{L}_2}{\partial c}$, $\frac{\partial \mathcal{L}_2}{\partial s}$ we obtain, the value of s:
$\frac{ab(T-s)}{(1-a)s} = 1$ meaning $s^2 = \frac{ab}{(1-a+ab)}T$.
Replacing this into the budget:
$c^2 = R\left(\frac{ab}{(1-a+ab)}T\right)^b$.
$L^2 = \frac{T}{1+ab}$.
2.3. Consumption puzzle:
Set $w = 1, a = \frac{1}{2}, b = \frac{1}{2}$, $T = 24$ compute

$$\frac{c^1}{c^2} = \frac{48\sqrt{\frac{3}{5}}}{5R}$$

This means that $\frac{c^1}{c^2} \leq 1 \iff \frac{48\sqrt{\frac{3}{5}}}{5R} \leq 1 \iff R \geq \frac{48\sqrt{\frac{3}{5}}}{5} = 7.43613$ and we verify that this number is below $wl^1 = \frac{48}{5} = 9.6$, so there is the range of R

$$\frac{48\sqrt{\frac{3}{5}}}{5} \leq R \leq \frac{48}{5}.$$

For this range of R even if the nonlabor retirement income of Carol is lower than its labor income in situation 1, Carol consumes more in situation 2.

3. Answer: Policy Implications and Real-world Connections.

3.1 Different Choices: Discuss how Carol's choices in shopping and leisure might differ before and after retirement, even though her utility function remains the same.

In the analysis above, we observe that Carol when facing a different budget and has no obligation to work but instead receives the nonlabor income R, even when it is smaller than the salary wl^1, may end up with Carol consuming more of c, of course, this happens because she now can spend more time shopping, this leads to smaller prices of consumption good, allowing her to buy more of c.

3.2. Real-world Relevance: Link the findings of this model to the referenced paper on "Life-Cycle Prices and Expenditure." from Aguiar and Hurst. Explain how this simplified model helps us understand real-world behaviors observed in the paper. This is the abstract of that paper: "Previous authors have documented a dramatic decline in food expenditures at the time of retirement. We show that this is matched by an equally dramatic rise in time spent shopping for and preparing meals. Using a novel data set that collects detailed food diaries for a large cross section of U.S. households, we show that neither the quality nor the quantity of food intake deteriorates with retirement status. We also show that unemployed households experience a decline in food expenditure and food consumption commensurate with the impact of job displacement on permanent income."

Our model shows that indeed, it is possible for Carol to consumer the same amount or more than before, while spending less, because she spends more time shopping when retired thus finding good deals and paying less. We did not model home production of goods, but we modeled the shopping technology that decreased as the shopping time increased.