# Case Study: The Economics of Superstars

Victor Aguiar

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## Introduction

This case study, inspired by Krueger's 2015 research on the economics of the music industry, delves into the intriguing relationship between the sale of physical media (CDs)/song streaming services and live performance revenues (concerts). The focal point is Taylor S., a superstar artist, and how market forces and production constraints influence her pricing strategies for CDs/songs and concert tickets.

## Part 1: Consumer Behavior Analysis

**Context:**

- A consumer, a fan of Taylor S., is deciding how to spend their budget between music streaming services ($s$) and concert tickets ($c$) produced by Taylor S. For simplicity we will assume that the consumer can buy any fractional quantity of streaming services or concerts.
- The consumer faces a linear budget constraint with prices $p_s$ for CDs and $p_c$ for concerts, and a fixed income of $M = 1$.

**Task:**

- Question 1.1.: Write down the utility function of the consumer (Taylor S's fan) and their budget.
- Question 1.2. Obtain the optimal demand of CDs and concerts in terms of prices $p_s$,$p_c$ (call these optimal quantities $s^*(p_s, p_c)$ and $c^*(p_s, p_c)$).

## Part 2: Taylor S.'s Production Function

**Context:**

- Taylor S.'s production functions for concerts ($y_c$) and songs ($y_s$) are given by $y_c = f^c(l)$ and $y_s = f^s(l)$ with $l$ representing hours of work.

- The cost of working $l$ hours is $w$.
- Both production functions exhibit decreasing returns to scale.

**Task:**

- 2.1. Formulate Taylor S.'s production functions with decreasing returns to scale for the production of concerts and songs.
- 2.2. Obtain the associated cost function for the production of songs $cost^s(w, y_s)$, and separately the cost function for the production of concerts $cost^c(w, y_c)$. Notice that for simplicity we are assuming that this two production process are independent of each other and that they face the same labor costs.

# Part 3: Profit Maximization Strategy

**Context:**

- Taylor S. operates as a monopolist in the concert market (sets $p_c$), you can set the price of songs to $p_s = 1$. A monopolist can set up the price of concerts.

**Task:**

- 3.1 Write down explicitly Taylor S's revenue: $Revenue(p_s, p_c, \tau) = p_s \cdot \tau s^*(p_s, p_c) + p_c \cdot c^*(p_s, p_c)$, using the results from Part 1 ($c^*(p_s, p_c)$,$s^*(p_s, p_c)$). (Notice that in this market $y_c = c$ and $y_s = \tau s$ where $\tau \in (0, 1)$ is a fraction of the streaming services songs that belong to Taylor S., and is set by a third party (Spotify)). Set $p_s = 1$, and make sure your choices of utility specification and parameters are such that marginal revenue decreases with the price of concerts going up, if it doesn't reconsider the choices of the utility specification.
- 3.1. Solve Taylor S.'s profit maximization problem using 3.1 and Part 2 ($cost^c(w, y_c)$, $cost^s(w, y_s)$). Set the price of songs, $p_s = 1$, set $w = 1$. Solve for the equilibrium value of $p_c$ (it can be in implicit form):

  $$\pi(\tau, p_s, w) = \max_{p_c} \left( Revenue(p_s, p_c, \tau) - cost^s(w, \tau s^*(p_s, p_c)) - cost^c(w, c^*(p_s,p_c)) \right)$$

# Part 4: Price Dynamics and Production

**Context:**

- An observed trend in the market is the rising prices of concert tickets as $\tau$ decreases. A decrease of $\tau$ captures the idea that the revenue Taylor S earns from songs decreases due to the distribution systems such as Spotify.

**Task:**

- 4.1. In your model find parameters such that a decrease of $\tau$ (e.g., $\tau = 1$ as a baseline and then move to $\tau = 1/2$) leads to an increase in concert ticket prices. Here you are encouraged to plug-in numbers and you will need some help solving a polynomial equations (scientific calculator, Wolfram Mathematica, among others can do this).

- 4.2. Discuss, in one paragraph, the economic rationale behind this inverse relationship of $p_c$ and $\tau$, focusing on the production constraints and decreasing returns in both concert and song production but also discuss the role of market power yielded by Taylor S. in the concert market contrasting it with her lack of power on the song distribution side captured by $\tau$.

# Solution

## Part 1. Solution

1.1. Let the utility be $u(s, c) = s^\alpha + c^\alpha$ subject $p_s s + p_c c = 1$.

1.2. Choosing $\alpha = \frac{1}{2}$:

We pose the lagrangian of the problem:

$$L = s^{\frac{1}{2}} + c^{\frac{1}{2}} + \lambda(1 - p_s s - p_c c)$$

This lead to $s^*(p_s, p_c) = \frac{p_c}{p_s(p_c+p_s)}$, $c^*(p_s, p_c) = \frac{p_s}{p_c(p_c+p_s)}$.

## Part 2. Solution.

2.1. The following production functions are decreasing returns:

$y_c = l^{a_c}$, $y_s = l^{a_s}$, $a_s, a_c \in (0, 1)$.

2.2. Cost can be obtained by inverting the prodution function so that $cost^c(w, y_c) = w f^{c,-1}(y_c)$ and $cost^s(w, y_s) = w f^{s,-1}(y_s)$, for this particular specification:

$cost^c(w, y_c) = w(y_c)^{1/a_c}$, $cost^s(w, y_s) = w(y_s)^{1/a_s}$.

## Part 3. Solution.

3.1. The revenue function is:

$$Revenue(p_s, p_c, \tau) = p_s \cdot \tau s^*(p_s, p_c) + p_c \cdot c^*(p_s, p_c),$$

$$Revenue(p_s, p_c, \tau) = p_s \tau \frac{p_c}{p_s(p_c + p_s)} + p_c \frac{p_s}{p_c(p_c + p_s)}$$

$$Revenue(p_s, p_c, \tau) = \tau \frac{p_c}{p_c + p_s} + \frac{p_s}{p_c + p_s} = \frac{\tau p_c + p_s}{p_c + p_s}$$

Let $p_s = 1$, then:

$$Revenue(p_c, \tau) = \frac{\tau p_c + 1}{p_c + 1}$$

The marginal revenue is:

$$\frac{\partial Revenue(p_c, \tau)}{\partial p_c} = \frac{-1 + \tau}{(1 + p_c)^2} < 0.$$

3.2. Profit:

$$\pi(\tau, p_s, w) = \max_{p_c} \left( Revenue(p_s, p_c, \tau) - cost^s(w, \tau s) - cost^c(w, c^*(p_s, p_c)) \right)$$

$$\pi(\tau, p_s, w) = \max_{p_c} \left( \frac{\tau p_c + 1}{p_c + 1} - \left( \frac{\tau p_c}{(p_c + 1)} \right)^{1/a_s} - \left( \frac{1}{p_c(p_c + 1)} \right)^{1/a_c} \right)$$

Maximization happens when the first order conditions of profits are equal to zero or equivalently when marginal revenue is equal to marginal costs:

$$\frac{-1 + \tau}{(1 + p_c)^2} = \frac{a_c \left( \frac{p_c \tau}{p_c + 1} \right)^{\frac{1}{a_s}} - a_s(2p_c + 1) \left( \frac{1}{p_c(p_c + 1)} \right)^{\frac{1}{a_c}}}{a_c a_s p_c (p_c + 1)}$$

Now setting $a_c = a_s = \frac{1}{2}$:

$$\frac{-1 + \tau}{(1 + p_c)^2} = \frac{2 \left( p_c^4 \tau^2 - 2p_c - 1 \right)}{p_c^3 (p_c + 1)^3}$$

$$\tau - 1 = \frac{2 \left( p_c^4 \tau^2 - 2p_c - 1 \right)}{p_c^4 + p_c}$$

# Part 4. Analysis and Discussion

4.1. For $\tau = 1$, $p_c^* = 1.39$, for $\tau = 1/2$ , $p_c^* = 1.59$, the decrease of revenue due to the policies of Spotify, make the optimal price of concerts go up.

4.2. Taylor S's has monopoly power in concerts, that means she can set prices of concerts, however she faces decreasing marginal revenue in prices, optimality for Taylor S's profit maximization happen at concert price where marginal revenue is equal to marginal cost. For consumers an increase in price of concerts reduce their consumption of concerts and songs due to substitution and income effects respectively. But the gains in revenue of the price increase more than compensate this losses for Taylor S's and she increases the prices nevertheless.