Final Exam: Case Study on Determinants of Bilateral Trade

Victor H. Aguiar

Introduction

The Law of Gravity of Trade is a fundamental principle in international economics, positing that trade between two countries is directly proportional to their economic size, often measured by GDP, and inversely proportional to the geographic distance between them. This model, akin to Newton's law of gravity, suggests that larger economies have a greater gravitational pull in trade relationships, while distance acts as a trade barrier, diminishing trade interactions. This case study aims to apply this principle in a simplified scenario involving two fictional countries, Arabasta and Baltigo, to illustrate the microfoundations of the law of gravity of trade.

Part 1: Consumption Problem for Arabasta

We want to model the consumption of domestic and foreign products for the country of Arabasta.

Model Setup:

- There are 2 countries, Arabasta (a) and Baltigo (b). Goods are differentiated by their country of origin.
- $c_{b,a}$ denotes the consumption in Arabasta of the good produced in Baltigo, and $c_{a,a}$ represents local consumption in Arabasta of the good produced in Arabasta.
- Arabasta total revenue for selling his good both to itself and to Baltigo is y_a.
- The prices faced by Arabasta for the good originating in Baltigo is $p_{b,a}$, and the price for the good originating in Arabasta is $p_{a,a}$.

Task for the Student

1.1. Write down a utility function for the representative consumer of Arabasta, $u_a(c_{a,a}, c_{b,a})$, and make sure this utility has one parameter σ. Also write down the budget constraint faced by Arabasta.

1.2. Write down the Lagrangian associated with the constrainted maximization of the utility and budget you wrote down before and obtain: the consumption levels at Arabasta, $c^*_{b,a}$ and $c^*_{a,a}$. (Notice that by carefully relabeling this optimal consumptions to $c^*_{a,b}$ and $c^*_{b,b}$ you have obtained as well the Baltigo's optimal consumption with the exact same utility parameter σ.).

Part 2: Market Clearing Conditions

Equilibrium Conditions:

- For market equilibrium: the total self-exports ($p_{a,a}$ times consumption in Arabasta of Arabasta good), and the export to Baltigo ($p_{a,b}$ times consumption in Baltigo of Arabasta good) must equal Arabasta's revenue, y_a.
- The analogous case holds for Baltigo revenue y_b.

Price Formulation:

- Prices $p_{a,j}$ depend on base prices at origin, p_a, and trade costs, $t_{a,j}$ for $j \in \{a, b\}$: $p_{a,b} = p_a t_{a,b}$ and $p_{a,a} = p_a t_{a,a}$. Note that $t_{a,b}$ reflects the trade cost due to distance and other barriers. We restrict that $t_{a,a} = 1$, so we can simplify it in the analysis, we also assume $t_{a,b} > 1$ and $t_{a,b} = t_{b,a}$. In addition, we will assume that the only trade barrier is distance between a, b, $d_{a,b}$.

Trade Costs and Consumption:

- The cost of importing goods from Baltigo to Arabasta increases with distance, affecting the prices and, consequently, the consumption patterns.

Task for the Student

2.1. Write down the equilibrium condition that equates Arabasta's revenue y_a to its exports as described above. Taking as given $t_{a,a}, t_{a,b}$, and using the optimal demands of Part 1 $c^*_{a,a}$ and $c^*_{a,b}$ find the equilibrium value of the good in Arabasta, p^*_a, using the market equilibrium conditions. Make sure that this price is a function of y_a and $t_{a,b}$ as well as the parameter of preferences σ. (Hint: You have to normalize p^*_b, e.g., $p^*_b = 1$. Here we need an explicit solution and, if you must, choose a number for your preference parameter. Note that there is only one solution to this problem, if you are obtaining more than one solution one of the possible solutions may not satisfy that $p_a > 0$.).

2.2. Write down a production function, with a single parameter α, with non increasing returns for transporting goods and services a certain distance $d_{a,b} = f(k)$, using k units of capital (ships). The rental rate (price of k) is $r = 1$. For simplicity, we assume here that neither Arabasta or Baltigo own this transport company so the profits of this firm do not enter their budgets. Find a cost function $c(d_{a,b})$ associated with the transportation production function. Write down an equation for bilateral trade barriers $t_{a,b} = c(d_{a,b}) + 1$. Make sure to verify that the larger the distance the larger the trade barrier, make sure that when the distance is zero then $t_{a,b} = f(0) = 1$. Verify, using derivation, that as distance grows so does the trade barrier.

Part 3: Comparative Statics Analysis in Equilibrium

- As the distance between Arabasta and Baltigo increases, the trade cost t_{ab} rises, leading to a decrease imports.
- Bilateral trade is more substantial when both countries are economically larger, even if the distance remains constant.

Task for the Student

3.1 Once you have obtained the prices p_a^* and $p_b^* = 1$ from the good of Arabasta and Baltigo: Obtain the consumptions in Arabasta of Baligo's good in equilibrium, $c_{b,a}^{**}$ (Hint: you can do this by pluggin in p_a^*, p_b^*=1 from Part 2 into the expression you obtained in Part 1 for $c_{a,b}^*$). Now, do the same above, to obtain the nominal imports of Baltigo's good in Arabasta $x_{b,a}^{**} = p_{b,a}^* c_{b,a}^{**}$ (make sure to replace p_{ab}^* by the appropiate terms given the previous analysis using p_a^*). Notice that equilibrium quantities and equilibrium imports should not longer be a function fo prices but only of $y_a, y_b, t_{a,b}(d_{a,b})$.

3.2. Demostrate the law of gravity of trade in your model. The law of gravity of trade works at the equilibrium nominal imports $x_{b,a}^{**}$. First, show that if we duplicate both y_a and y_b (you may assume specific values for GDP, (y_a,y_a) of Arabasta and Baltigo, and distance) then $x_{b,a}^{**}$ strictly increases as well. Second, by pluggin in your function for bilateral trade barrier in terms of distance, show that if distance is larger, keeping the sizes of Arabasta and Baltigo fixed, then $x_{b,a}^{**}$ strictly decreases (you can also plug in numerical values here).

Part 4: Microeconomic Foundation of the Law of Gravity of Trade

4.1. Provide a one paragraph explanation of why the Law of Gravity of Trade explains bilateral trade flows between two countries using the concepts we learned in our class and the mode you have developed. Hint: Chapter 1 to 4 and Chapter 8 are the relevant parts here.

Solutions

Solution Part 1:

1.1. The utility is $u_a(c_{a,a}, c_{b,a}) = c^{\sigma}_{a,a} + c^{\sigma}_{b,a}$ for σ the only parameter. The budget is $p_{aa}c_{aa} + p_{ba}c_{ba} = y_a$, total imports (self imports + imports from Baltigo) equal to total revenue.

1.2. The first step is to pose the Lagrangian

$L(c_{a,a}, c_{b,a}, \lambda) = c^{\sigma}_{a,a} + c^{\sigma}_{b,a} + \lambda(y_a - p_{aa}c_{aa} + p_{ba}c_{ba})$

(I) $\frac{\partial L}{\partial c_{a,a}} = \sigma c^{\sigma-1}_{a,a} - \lambda p_{a,a} = 0$

(II) $\frac{\partial L}{\partial c_{b,a}} = \sigma c^{\sigma-1}_{b,a} - \lambda p_{b,a} = 0$

(III) $\frac{\partial L}{\partial \lambda} = y_a - p_{aa}c_{aa} + p_{ba}c_{ba} = 0$

From (I) and (II) we obtain the MRS equal to the ratio of prices conditions:

$$\frac{c^{\sigma-1}_{a,a}}{c^{\sigma-1}_{b,a}} = \frac{p_{a,a}}{p_{b,a}}$$

From this condition we isolate x_1:

$$c_{a,a} = c_{b,a}\left(\frac{p_{a,a}}{p_{b,a}}\right)^{\frac{1}{\sigma-1}}$$

Then we plug-in this into (III)/budget:

$$p_{a,a}\left(c_{b,a}\left(\frac{p_{a,a}}{p_{b,a}}\right)^{\frac{1}{\sigma-1}}\right) + p_{b,a}c_{b,a} = y_a.$$

Now solve for $c_{b,a}$ in terms of $p_{a,a}, p_{b,a}, y_a$:

$$c^*_{b,a} = \frac{p^{\frac{1}{\sigma-1}}_{b,a} y_a}{p^{\frac{\sigma}{\sigma-1}}_{a,a} + p^{\frac{\sigma}{\sigma-1}}_{b,a}}$$

$$c^*_{a,a} = \frac{p^{\frac{1}{\sigma-1}}_{a,a} y_a}{p^{\frac{\sigma}{\sigma-1}}_{a,a} + p^{\frac{\sigma}{\sigma-1}}_{b,a}}.$$

Baltigo Solution:

$$c^*_{a,b} = \frac{p_{a,b}^{\frac{1}{\sigma-1}} y_b}{p_{b,b}^{\frac{\sigma}{\sigma-1}} + p_{a,b}^{\frac{\sigma}{\sigma-1}}}$$

$$c^*_{b,b} = \frac{p_{b,b}^{\frac{1}{\sigma-1}} y_b}{p_{b,b}^{\frac{\sigma}{\sigma-1}} + p_{a,b}^{\frac{\sigma}{\sigma-1}}}.$$

Cobb-Douglas*: $c_{b,a} = \frac{(1-\sigma)y_a}{p_{b,a}}$, $c_{a,a} = \frac{\sigma y_a}{p_{a,a}}$. This would get full points if no errors as well.

Solution Part 2:

2.1 Equilibrium equation says:

$$y_a = p_{a,a}c_{a,a} + p_{a,b}c_{a,b}.$$

$$y_a = p_a c_{a,a} + p_a t_{a,b} c_{a,b}$$

Now, we plug in $c^*_{a,a}, c^*_{a,b}$

$$y_a = p_a \left(\frac{p_{a,a}^{\frac{1}{\sigma-1}} y_a}{p_{a,a}^{\frac{\sigma}{\sigma-1}} + p_{b,a}^{\frac{\sigma}{\sigma-1}}} \right) + p_a t_{a,b} \left(\frac{p_{a,b}^{\frac{1}{\sigma-1}} y_b}{p_{b,b}^{\frac{\sigma}{\sigma-1}} + p_{a,b}^{\frac{\sigma}{\sigma-1}}} \right)$$

Replace the price expressions

$$y_a = p_a^{\frac{\sigma}{\sigma-1}} \left(\frac{y_a}{p_a^{\frac{\sigma}{\sigma-1}} + t_{a,b}^{\frac{\sigma}{\sigma-1}}} + \frac{t_{a,b}^{\frac{\sigma}{\sigma-1}} y_b}{1 + p_a^{\frac{\sigma}{\sigma-1}} t_{a,b}^{\frac{\sigma}{\sigma-1}}} \right)$$

Letting $\sigma = \frac{1}{2}$ then, we can solve explicitly the implicit solution above. In this case there are two numerical solutions, but only one economic solution:

$$p^*_a = \frac{y_b - y_a + \sqrt{y_a^2 - 2y_a y_b + 4t_{a,b}^2 y_a y_b + y_b^2}}{2t_{a,b} y_a}$$

You can verify this is the unique nonnegative solution.

Cobb-Douglas*: your equilibrium conditions do not contain p_a, that means that any price can be an equilibrium price, if you did not notice that indeterminacy then you will get 2 points (raw).

2.2. The cost function with $d_{a,b} = \frac{1}{\alpha}k$, this is constant returns, for $\alpha > 0$, then by inversion of the production function $c(d_{a,b}) = \alpha d_{a,b}$ with $r = 1$. Then $t_{a,b} = \alpha d_{a,b} + 1$, $\frac{\partial t_{a,b}}{\partial d_{a,b}} = \alpha > 0$. Other correct solutions involved decreasing returns.

Common error: Many of you wrote this in terms of k, so this will mean loss of all points in some cases.

Another error was to confuse f^{-1} the inverse of f with the multiplicative inverse, so you divided in a fraction this is incorrect.

Part 3 Solution

3.1 We obtain $c_{b,a}^{**}$ by replacing

$$c_{b,a}^{*}=\frac{p_{b,a}^{\frac{1}{\sigma-1}}y_a}{p_{a,a}^{\frac{\sigma}{\sigma-1}}+p_{b,a}^{\frac{\sigma}{\sigma-1}}},$$

with the appropiate terms:

$$c_{b,a}^{*}=\frac{t_{a,b}^{\frac{1}{\sigma-1}}y_a}{p_a^{\frac{\sigma}{\sigma-1}}t_{a,a}^{\frac{\sigma}{\sigma-1}}+t_{a,b}^{\frac{\sigma}{\sigma-1}}},$$

now replacing p_a^* for $\sigma=\frac{1}{2}$, we obtain:

$$c_{b,a}^{**}=\frac{2y_a^2}{(-1+2t_{a,b}^2)y_a+y_b+\sqrt{y_a^2-2y_ay_b+4t_{a,b}y_ay_b+y_b^2}},$$

by multiplying by $p_a^*t_{a,b}$ we obtain $x_{a,b}^{**}$

$$x_{a,b}^{**}=\frac{y_a+y_b-\sqrt{y_a^2-2y_ay_b+4t_{a,b}^2y_ay_b+y_b^2}}{2-2t_{a,b}^2}$$

3.2. $y_a=1, y_b=1, d_{a,b}=1$, with $\alpha=1$

$$x_{ab}^{**}=\frac{1}{3}$$

$$y_a^*=2, y_b^*=2, d_{a,b}=1$$

$$x_{ab}^{**}=\frac{2}{3}$$

$$y_a=1, y_b=1, d_{a,b}=3$$

$$x_{ab}^{**}=\frac{1}{5}.$$

Cobb Douglas: if you did a numerical analysis with Cobb douglas without errors you got most likely 5 points raw because you did not get the effect of the trading patters as in the CES.

Part 4 Solution

The effect of the size of countries is driven by the income effect since both goods are normal as the income increases in both countries demand will go up. The effect of distance is driven by the substitution effect as distance increases prices of the imported good hence this will drive demand down, but these findings are only possible because imports is equal to export revenues, which is reminicisent of the Edgeworth effect, due to that increases in prices will have a secondary income effect, so the total effect is ambiguous but we have shown using numerical examples that the gravity of bilateral trade works out in our model as it does in real life.

Note: I graded all the exam carefully, it's very unlikely I change my mind in regrades, so be mindful of this.