Final Microeconomics 2021

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Abstract

This exam has three problems, the third one is modelling. You will be graded on the basis of your exam-structure over 90 points. Work group is not allowed, if I find out that several of you have essentially the same responses in any section, e.g., the same model, the same code, the same derivations, you will receive the total grade of this question divided by the number of students that share the same response. You can communicate and talk among you, I cannot prevent this anyway, but you have to write your own answers and you have to do your own derivations and your own code. If you share your code, answers explicitly and ideas for the model you are exposing yourself to have less than half the grade.

Problem 1. (Theory) Gravity model of trade.

We have I countries, with typical elements $i, j \in I$.

We assume that each country specializes in the production of only one good. The supply of each good is fixed (equivalently, each region is endowed with only positive quantity of one good, and there is no production).

If c_{ij} is the consumption by country j consumers of goods from region i, consumers in region j maximize

$$U(c) = \left(\sum_{i \in I} \beta_i^{(1-\sigma)/\sigma} c_{ij}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)},$$

subject to the budget constraint

$$\sum_i p_{ij} c_{ij} = y_j.$$

We have σ as the elasticity of substitution between all goods, β_i is a positive distribution parameter, y_j is a regional income of country j consumers, and p_{ij} is the price of region i good for j consumers.

Notice that p_{ij} differs per location j due to trade-costs.

Let p_i denote the exporter's supply price, net of trade costs, and $t_{ij} \geq 1$ is the trade cost factor between i, j (when $t_{ij} = 1$ then there is free-trade).

$$p_{ij} = p_i t_{ij}.$$

We assume that trade costs are absorbed by the exporter.

Formally, we assume that for each good shipped from i to j the exporter bears a costs equal to $t_{ij} - 1$ of country i goods. The exporter passes on these trade costs to the importer.

The nominal value of exports is:

$$x_{ij} = p_{ij} c_{ij}.$$

Market clearing implies here that in nominal terms endowments are equal to aggregate demand:

$$y_i = \sum_{j \in I} x_{ij}.$$

1) Show that the nominal demand for country i goods by country j consumer is:

$$x_{ij} = \left(\frac{\beta_i p_i t_{ij}}{P_j} \right)^{(1-\sigma)} y_j,$$

where P_j is the consumer price index of j, given by

$$P_j = \left[\sum_{i \in I} (\beta_i p_i t_{ij})^{1-\sigma} \right]^{1/(1-\sigma)}.$$

Important: I need step-by-step derivation.

2) Show, using the nominal market-clearing conditions and the results from 1) that: $\beta_i p_i = \frac{y_i^{1/(1-\sigma)}}{(\sum_{j \in I} (t_{ij}/P_j)^{1-\sigma} y_j)^{1/(1-\sigma)}}$.

Important: I need step-by-step derivation.

3) Let $y^W = \sum_{j \in I} y_j$ be the world income, and income share $\theta_j = y_j / y^W$. Show that, replacing $\beta_i p_i$ on the nominal demand, we can obtain:

$$x_{ij} = \frac{y_i y_j}{y^W} \left(\frac{t_{ij}}{\Pi_i P_j} \right)^{1-\sigma},$$

where

$$\Pi_i = \left(\sum_{j \in I} (t_{ij}/P_j)^{1-\sigma} \theta_j \right)^{1/(1-\sigma)}.$$

Important: I need step-by-step derivation.

4) Assume that $t_{ij} = t_{ji}$, where trade barriers are symmetric, show that:

$$\Pi_i = P_i.$$

Under this condition, we have an implicit solution to the price indexes P_i given by

$$P_j^{1-\sigma} = \sum_{i \in I} P_i^{\sigma-1} \theta_i t_{ij}^{1-\sigma} \forall j,$$

show that there is at least one solution to this system of equations.

For this part use the file \GitHub\Microeconomics1\finalexam\theory\fixedpoint_contraction_mapping_jacobian.p Theorem 1 and Theorem 2. In particular, show that the implicit solution to the price index P_i for all $i \in I$ for a contraction mapping. You can assume that P_i take values in a closed set for simplicity. Also note that, that you have to rewrite the price index equation in the form.

$$P_j = g_j(P_1, \cdots P_I).$$

For the gravity model g is continuously differentiable, so you can compute it's Jacobian. For the computing the norm of the Jacobian use whatever norm makes easier your computation. Euclidean or Max matrix norms are good candidates. I used the Euclidean matrix norm.

Important: Only for this item, assume that $I = \{1, 2\}$, $P_i \geq 1 \forall i$, $\sigma = \frac{1}{2}, t_{ij} = 1, \theta_i = \frac{1}{2}$. Remember you have to show that the the norm of the Jacobian of the mapping g is less than 1, for all values of P_i.

5) Show that, the gravity model implies that there are constants $\alpha_i \forall i \in I$, and $\rho = (1 - \sigma)$ such that:

$$z_{ij} = log x_{ij} - log(y_i) - log(y_j)$$

$$z_{ij} = -\alpha_i - \alpha_j + \rho log(t_{ij}).$$

Problem 2. (General Equilibrium) Consider a choice set X, and a consumer with preference defined over X, $\succeq \subseteq X \times X$, such that the preferences are a preorder (but not necessarily complete).

1) Consider the following economy:

Definition 1. Consider a triple of consumers, firms and endowments $(\{X_i, \succeq_i\}_{i=1}^I, \{Y_j\}_{j=1}^J, \omega)$, where $X_i = X \subseteq \mathbb{R}_+^L$, where $L \geq 2$ is the number of goods, $\succeq_i$ is a (possibly incomplete preorder) **that is locally non-satiated** for all consumers $i \in \{1, \cdots, I\}$, $Y_j \subseteq \mathbb{R}^L$ is the production set, and $\omega \in \mathbb{R}_+^{L \times I}$ is a vector of endowments.

Prove that if an allocation (x^*, y^*) in the above defined economy, an a price vector $p = (p_1, \cdots, p_L)$ constitute a price equilibrium with transfers, then the allocation (x^*, y^*) is Pareto optimal. (Hint: Write down the definition of a price equilibrium with transfers using the (possibly incomplete) preorder $\succeq_i$. Note that when preferences are incomplete you have to modify the notion of preference maximization.)

2) Consider the economy in 1). What conditions on $\succeq_i$ for all $i \in \{1, \cdots, I\}$ and on Y_j for all $j \in \{1, \cdots, J\}$ would suffice to guarantee that every Pareto efficient allocation (x^*, y^*) can be decentralized as a Price **quasi-equilibrium** with transfers? Be formal about the conditions you impose (define them), and show that they imply that the second welfare theorem holds in your setup with incomplete preferences.

Problem 3. (Modeling) Modeling the "(Dynamic) Endowment Effect". I am going summarize an in class experiment that took place on December 2020 regarding Top Trading Cycles (TTC) activity, that exhibits a behavioral bias called the Endowment Effect with dynamic traits.

The task is to write a decision model algorithm to explain the stylized facts our our TTC activity: namely, the number of trades realized with respect to total trades is smaller in the treatment group than in the control. More important, the expected value of trade in the treatment group is smaller than in the control group. I remind you that the treatment group was involved in a dynamic implementation of the TTC algorithm, participating in two rounds of trade. The control group instead stated their preferences as a ranking of all exams, and the TTC algorithm was implemented using that information and the randomly drawn endowments to obtain their assignments.

Due to randomization, under the rationality hypothesis of behavior, both the treatment and control group should have the same expected value of trade, and the same probability of trade.

Since the results of our randomization reject the rationality hypothesis, you have to come up with a model that explains the two stylized facts that we have described.

The TTC randomization.

- 15 subjects
- 10 subjects were randomly assigned to treatment, and 5 to control.
- A set of 15 exam-structures where randomly assigned as initial endowments to the 15 students. Exam structures differ in the points assigned to 3 different parts of the exam. The students faced a similar structure in the midterm exam, but otherwise did not had access to the final exam. An exam structure is a positive vector (a_s, b_s, c_s) where $a_s + b_s + c_S = 100$, and a_s gives the percentage points assigned to question 1 for student s, b_s the percentage points assigned to question 2 for student s, and c_s the percentage points assigned to question 3 for student s. Each exam structure assigned to student s then gives a different weight to each of the three question. Since students may have heterogeneous skills and preferences that affect which exam structure they like.
- The treatment subjects, had to do live/dynamic trade using a Qualtrics survey. They submitted their most preferred alternative and they knew their endowment. Trade can only happen inside their group.
- The control subjects, had to to submit a complete ranking of all exams in their group.
- Trade happen live for the treatment group, with the first round taking 7 hours, and the second round taking 12 hours (with 3 waking hours). There was no need for more rounds.
- The preferences of the control group students was used in a TTC algorithm given their endowments to produce the new assignments. The algorithm terminated in two rounds.
- The assignments were announced publicly to all 15 students, and they were asked to fill a post-match survey. The treatment group had to submit all their ranked preferences with the clear understanding that it would be used only for analysis purposes. Both the treatment and control submitted their subjective and expected gains from trades as percentage points.

If our subjects are rational consumers then the prediction in this market is:

- Prediction for rationality: As exam-structures are distributed randomly, and the treatment and control membership was uniformly chosen, we should expect that the probability of trade in both the treatment and control is the same.

- As exam-structures are distributed randomly, and the treatment and control membership was uniformly chosen, we expect that the subjective expected gains from trade are the same among treatment and control.

The **Experiment Outcome** however differs from the **Prediction**, and we call this the **(Dynamic) Endowment Effect.**

- In 2 rounds, the total number of trades was: Treatment had 4 trades (4 and 0 trades per round) over 10 possible. Control had 4 trades (2 and 2 trade per round) over 5 possible.
- Mean expected gain from trade for the **control** group was 7 percentage points.
- Mean expected gain from trade for the **treatment** group was 3 percentage points.
- Dynamic vs static trade expected gains from trade gap.
- **Subject's preferences seem to be affected by the trading protocol.**

1. Propose a decision algorithm that explains the stylized facts above and explain how it generalizes or innovates with respect to the rational TTC model we cover in class.
2. Is the TTC algorithm Pareto efficient under your new decision algorithm. Is the TTC matching still in the core?

Grading Criterion

1. Work group is not allowed, if I find out that several of you have essentially the same model, you will receive the total grade of this question divided by the number of students that share the same response.
2. The model will be graded on the following 5 items: Logical consistency of the decision algorithm. Clarity of the explanation of the decision algorithm. Degree of success in explaining the endowment effect qualitatively. Calibration to the actual numbers of the experiment. Creativity and degree to which you have included topics that we covered in class (limited attention, reference dependence, among others).

Hint: write down the TTC algorithm using utilities instead of rankings, since you have to explain the gap in gains from trade.