MICROECONOMICS 1
PROBLEM SET 10

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1. Contingent Commodities.

Exercise 1. Demand for state contingent commodities. An expected utility maximizer faces uncertainty at date 0 as to which state of the world will prevail at date 1. At date 1 there are two possible states s_1 and s_2. His subjective probability assessment is that state $s_i, \quad i = 1,2$ will happen with probability $\pi_i > 0$. He knows that he will receive 10000 unites of income at date 1, regardless of the state. His ex-post preferences over money in each state s_i are represented by the utility function $u(x_i) = x_i^{\alpha}$ for $0 < \alpha$. Suppose that relative price of the two state contingent commodities at date 0 is 1.

(a) What is this consumer's demand for state contingent commodities if $\alpha = 1$?

(b) How will your answer change to (a) if $\alpha < 1$? What if $\alpha > 1$? Explain your answers in therms of agent's attitudes towards risk.

Exercise 2. [An exchange economy with idiosyncratic risk] There are two risk averse expected utility maximizing agents, $i = 1,2$, each with some initial endowment of wealth, $W_i > 0$, facing the uncertain outcomes c_1 and c_2. Agent i suffers a loss of wealth L, $0 < L < W_i$, if c_i occurs while the other agent's wealth is unaffected. (Note that there is no aggregate risk in this economy since the sum of agent's wealth is a constant $W_1 + W_2 - L$, no matter which outcome occurs). Define a risk sharing trade agreement as an agreement between these agents reached prior to the resolution of uncertainty that Agent 2 pays Agent 1 a certain sum if outcome c_1 occurs, in exchange for Agent 1 paying Agent 2 a certain sum if c_2 occurs, these need not be the same sums. Suppose agent I believes that p_i is the probability the outcome c_1 will occurs. Characterize the Pareto efficient gambles in this situation.

In particular show that if the agents have strictly concave, differentiable Bernoulli utility functions , then pareto efficiency exchange between them agents leaves neither bearing any risk if an only if $p_1 = p_2$.

2. Overlapping Generations.

In this problem we study an overlapping generations model under two market interpretation and isolate the conditions for Pareto optimality.

All individuals live for only two periods. Generation t (denoted by the superscript) has utility function:

$$u(c_t^t) + \beta u(c_{t+1}^t)$$

where $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly concave and increasing in a single consumption good. Individuals are endowed with one unit of labor in the first period of their lives and supply it inelastically. Capital is owned by individuals and rented to firms. Competitive firms rent capital and labor services at prices r_t and w_t (in terms of time t consumptions goods). At time zero all initial capital is held by the old (i.e. generation $t = -1$).

The resource constraint is

$$k_{t+1} + c_t^t + c_t^{t-1} \leq F(k_t, 1) + (1-\delta)k_t$$

where F is a constant returns to scale CRS production function and $\delta \in (0, 1]$.

Exercise 3. [Sequential trade.] Consider the sequential competitive market arrangement where individuals in generation t face the budget constraints:

$$c_t^t + k_{t+1} = w_t$$

$$c_{t+1}^t = R_{t+1}k_{t+1}$$

with $R_{t+1} = 1 - \delta + r_t$.

Given k_o, define a competitive equilibrium for this market arrangement for c_t^{-1}, $\{(c_t^t, c_{t+1}^t), k_{t+1}\}_{t=0}^{\infty}$ and prices $\{r_t, w_t\}_{t=0}^{\infty}$.

Exercise 4. [Time-zero trade] Now consider the complete market arrangement where we imagine all generations (the born and the yet unborn) and firms meeting at time zero and competitively trading in claims for future consumption, labor and capital. We generalize the notation and specialize it to interpret our overlapping generations model.

Generation t faces the budget constraint:

$$\sum_{s=0}^{\infty} q_s^o (c_s^t + k_{s+1}^t - R_s k_s^t) \leq \sum_{s=0}^{\infty} q_s^0 w_s \overline{n}_s^t + R_o \overline{k}_o^t$$

where we normalize $q_o^o = 1$.

Notation:

$\overline{n}_s^t$ and $\overline{k}_o^t$ represent endowment of labor in period s and initial capital owned by generation t. Thus in OLG: $\overline{n}_s^t = 1$ for $s = t$ and $\overline{n}_s^t = 0$ for $s \neq t$; $\overline{k}_o^{-1} = k_o$ and $\overline{k}_o^s = 0$ for $s \neq -1$.

Think of each generation t as having a utility function U^t defined over the entire consumption stream $\{c_s^t\}_{s=0}^t$. In this OLG model: $U^t(\{c_t^t\}_{s=0}^{\infty}) = u(c_t^t) + \beta u(c_{t+1}^t)$.

In A-D notation:

$Max_{c_t^t, k_{t+1}} U(\{c_s^t\}_{s=0}^{\infty})$

subject to

$\sum_s q_s^o z_s^t \leq 0$

$c_s^t + k_{s+1} \leq w_s \overline{n}_s^t + R_s k_s + R_o \overline{k}_o^t + z_s^t$ for $\forall s$

Define an equilibrium for $c_t^{-1}, \{(c_t^t, c_{t+1}^t), k_{t+1}\}_{t=0}^{\infty}$ and prices $\{q_t^o\}_{t=o}^{\infty}$ and $\{r_t, w_t\}_{t=0}^{\infty}$. Show that the equilibria must satisfy the arbitrage condition $\frac{q_t^0}{q_{t+1}^0} = R_{t+1}$. Argue that the sequential equilibrium of the previous point is a time-zero equilibrium for appropriate chosen prices $\{q_t^o\}$.

Exercise 5. Consider the case of log utility: $u(c) = log(c)$ and Cobb-Douglas production function $F(n_t, k_t) = Ak_t^{\alpha} n_t^{1-\alpha}$. Characterize the entire equilibrium allocation $c_t^{-1}, \{(c_t^t, c_{t+1}^t), k_{t+1}\}_{t=o}^{\infty}$ and prices $\{r_t, w_t\}_{t=0}^{\infty}$. Solve for the steady state level of capital k_{ss}. Show that the equilibrium is not Pareto efficient if steady state capital is higher than the golden rule $k_g = argmax_k\{F(k, 1) - \delta k\}$. Show that there are parameters for which $k_g < k_{gg}$.

Exercise 6. If terms of the time-zero, complete market arrangement: whay does the First Welfare Theorem fail to apply? Argue that the condition

$$\sum_{s=o}^{\infty}\sum_{t=o}^{\infty} q_t^o w_t \overline{n}_s^t < \infty$$

is necessary for the proof of the first welfare theorem and that is not satisfied here. Show that the welfare theorem does apply if $k_{ss} \leq k_g$.