Problem Set 9

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Problem 1. (30 points) Expected Completion time: 20 minutes) Consider an economy with two goods, two consumers

$u_1(x_{11}, x_{21}) = x_{11} - \frac{1}{8}(x_{21})^{-8}$ and $u_2(x_{12}, x_{22}) = -\frac{1}{8}(x_{12})^{-8} + x_{22}$ with $\omega_1 = (2, r)$ and $\omega_2 = (r, 2)$, with $r = 2^{8/9} - 2^{1/9}$.

(a) Compute the excess demand for any of the 2 goods. (You can assume nonnegativity of the prices $p >> 0$). And obtain all equilibria analytically.

(b) Is the economy regular? Describe formally what is a regular economy and then find out whether this economy is regular.

(c) Can you intuitively describe what means that the economy is regular? What would happen if the economy is not regular if I perturb the endowments by a bit. In particular, what would happen with the multiple equilibria?

(d) Prove the following statement:

Any regular (normalized) equilibrium price vector is locally isolated (or locally unique). That is, there is an $\epsilon > 0$ such that if $p' \neq p$, $p'_2 = p_L = 1$ (where p_L, p'_L are the prices of the last good L in vector p, p' respectively), and $||p' - p|| < \epsilon$, then $z(p') \neq 0$. Moreover, if the economy is regular, then the number of normalized equilibrium price vectors is finite.

e) Does the aggregate excess demand function satisfy the gross substitutes property?

Problem 2. (30 points) Computational Robinson Crusoe.

Robinson Crusoe is trapped in an island and he spends his days eating coconuts (c) or napping on the beach -getting Vitamin D-(d). His utility funciton is

$$U(c, d) = \alpha ln(c) + (1 - \alpha) ln(d) \tag{1}$$

with $\alpha \in [0,1]$ a parameter. Coconuts c are measured in kg in a week so $c \in \mathbb{R}^L_+$, and $d \in [0, \overline{L}]$ is measured as the total of hours in a week Robinson spends in the beach, where $\overline{L}$ is the total amount of hours in a week.

Coconuts need to be harvested from the Island's trees. The time Robinson spends collecting coconuts from trees is measured in ours $l \in [0, \overline{L}]$. Since the island is very isolated we assume that

$$l + d = \overline{L}. \tag{2}$$

Hence, Robinson can only work or nap in the beach and the time he spends consuming the coconuts is negligible.

The productivity of Robinson is given by

$$f(l) = Al^{\beta}, \tag{3}$$

where $A > 0$ and $\beta \in (0,1)$ are fixed parameters. We assume, that Robinson cannot consume coconuts he has not collected himself so:

$$c = f(l). \tag{4}$$

Since Robinson wants to be happy his consumption and working decision are the solution to the following problem:

Centralized problem:

$$\max_{c,l} \alpha ln(c) + (1-\alpha) ln(\overline{L} - l)$$

$$s.t.$$

$$c = Al^{\beta}.$$

1) Let $\overline{L} = 100$. Show this centralized problem has a solution, and also show that this solution is unique.

2) Compute the close-form solution of the consumption and labor decisions of Robinson Crusoe's centralized problem.

3) Write down the equivalent descentralized general equilibrium problem associated to the centralized problem above. In other words, break down the problem of Robinson into a consumer problem and a firm problem. Show that both problems produce the same solution. Show that the problem has a solution using a fixed-point argument.

Check the notes for this. In the descentralized problem you need to find the allocations c^*, l^*, assume the price of coconuts is set to 1 ($p = 1$) and the wage for labor l is w. Profits of the firm are denoted by π.

4) Calibration. Imagine an observer (Friday) has collected the following dataset that corresponds to the behavior of the descentralized economy in 3).

variable	
c^*	60.056
l^*	6.250
w	0.961
p	1
π^*	54.01
A	50

Use this dataset to compute the implied values α and β using 3).

5) Implement the descentralized general equilibrium problem in Julia, with the calibrated α, β from 4) and solve it numerically. Verify the solutions of your sytem of equations reproduce the dataset Friday collected.