Advanced Microeconomics: 2022

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This version: 2022

Problem 1. Consider the following collective household model with exponential discounting. Consider a household that consists of two individuals labeled A and B. The consumption vector $c_t \in \mathbb{R}^L_+ \backslash \{0\}$ is partitioned into $c_t = (c'_{t,I}, c'_{t,H})'$ where $c_{t,I}$ is the privately consumed consumption vector and $c_{t,H}$ is the publicly consumed consumption vector. Similarly prices $p_t \in \mathbb{R}^L_{++}$ can be partitioned into $p_t = (p'_{t,I}, p'_{t,H})'$, where $p_{t,I}$ is the price of the private consumption goods and $p_{t,H}$ is the price of the public consumption goods . The collective household problem with exponential discounting corresponds to:

$$V_\tau(c) = \omega_A u_A(c_{\tau,A}, c_{\tau,H}) + \omega_B u_B(c_{\tau,B}, c_{\tau,H}) +$$

$$\sum_{j=1}^{T-\tau} [d^j_A \omega_A u_A(c_{\tau+j,A}, c_{\tau+j,H}) + d^j_B \omega_B u_B(c_{\tau+j,B}, c_{\tau+j,H}),$$

where ω_A,ω_B are Pareto weights that remain constant across time and represent the bargaining power of each household member. Individual utilities u_A, u_B are assumed to be continuous, locally nonsatiated and concave. The individual discount factors are denoted by d_A, $d_B \in (0, 1]$.

The quantities $c_{t,A}, c_{t,B}$ are assumed to be unobserved, we only observe $c_{t,I} = c_{t,A} + c_{t,B}$.

The constraint is:

$$p'_{\tau,I} c_{\tau,I} + p'_{\tau,H} c_{\tau,H} + s_\tau - y_\tau - (1 + r_\tau)s_{\tau-1} = 0.$$

Under the assumption of full efficiency this joint optimization problem can be decentralized using Lindhal prices. In particular if the consumers are able to purchase the public good such as to eliminate all externalities then there exists prices $p_{t,A}$ and $p_{t,B}$ for the public good such that $p_{t,A} + p_{t,B} = p_{t,H}$, such that: $c_{t,A}, c_H$ maximizes:

$$V_\tau(c) = u_A(c_{\tau,A}, c_{\tau,H}) +$$

$$\sum_{j=1}^{T-\tau} d^j_A u_A(c_{\tau+j,A}, c_{\tau+j,H}),$$

subject to:

$$p'_{\tau,I} c_{\tau,A,I} + p'_{\tau,A} c_{\tau,H} + s_\tau - y_\tau - (1 + r_\tau)s_{\tau-1} = 0.$$

and $c_{t,B}, c_H$ maximizes the analogous problem:

$$V_\tau(c) = u_B(c_{\tau,B}, c_{\tau,H}) +$$

$$\sum_{j=1}^{T-\tau} d^j_B u_B(c_{\tau+j,B}, c_{\tau+j,H}),$$

subject to:

$$p'_{\tau,I} c_{\tau,B,I} + p'_{\tau,B} c_{\tau,H} + s_\tau - y_\tau - (1 + r_\tau)s_{\tau-1} = 0.$$

Note that $c_{t,A} + c_{t,B} = c_{t,I}$ and $p_{t,A} + p_{t,B} = p_{t,H}$ and $c_t = (c'_{t,I}, c'_{t,H})'$.

(a) Show the following Theorem. Let $\rho_{t,h} = p_{t,h} / \prod_{j=1}^{t}(1 + r_j)$ for $h \in \{I, H, A, B\}$

Theorem 1. *A dataset $(\rho_t, c_t)_{t=0}^{T}$ can be generated by a collective household exponential discounting model with full efficiency if and only if there exists $d_A, d_B \in (0, 1]$ strictly positive vectors $(v_{t,A})_{t=0}^{T}$, $(v_{t,B})_{t=0}^{T}$ individual private consumption quantities $(c_{t,A}, c_{t,B})_{t=0}^{T}$ with $c_{t,A} + c_{t,B} = c_{t,I}$, and personalized Lindhal prices $(p_{t,A}, p_{t,B})_{t=0}^{T}$ with $(p_{t,A} + p_{t,B} = p_{t,H})$ such that for all $s, t \in \{0, 1 \cdots, T\}$*

$$v_{t,A} - v_{s,A} \geq d_A^{-t}[\rho'_{t,I}[c_{t,A} - c_{s,A}] + \rho'_{t,A}[c_{t,H} - c_{s,H}]]$$

$$v_{t,B} - v_{s,B} \geq d_B^{-t}[\rho'_{t,I}[c_{t,B} - c_{s,B}] + \rho'_{t,B}[c_{t,H} - c_{s,H}]].$$

Problem 2. Use the results in Problem 1 to test the collective exponential discounting model in the dataset of Aguiar Kashaev 2029, for **couples**. This is the couples household dataset extracted from the Spanish Household dataset.

- Write the code in Julia, using JuMP and Gurobi.
- Write a report of your findings (pass rate). Presentation matters and also try to write this the best you can to practice writing, and clarity in expressing ideas.

Problem 3. Challenge question: Using the **singles** dataset extracted from the Spanish Household dataset. Implement the ELVIS procedure in Aguiar and Kashaev 2020 for the simple case of the law of demand. Namely, with the moments:

$$g_{s,t}(x, e) = 1(0 \geq (\rho_t - \rho_s)'(c_t - w_t - c_s + w_s)) - 1 \forall s, t \in \mathcal{T}$$

$$g_M(x, e) = \rho'_t w_t \forall t \in \mathcal{T},$$

with $x = (\rho, c)$ and $e = w$.

If you are able to run the replication code in Aguiar Kashaev 2020, this is also a complete answer to this question, but you can implement yourself in Julia, since this is a substantially simplify problem.