Limited Consideration and Stochastic Choice: Theory and Evidence

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Motivation and Contribution

- The Random Utility Model (RUM) is the standard to describe population behavior: Heterogeneous preferences + Full Consideration.
- Decision makers (DMs) may not consider all alternatives. We extend RUM to a framework with **limited consideration**: Random Attention and Utility Model (RAUM).
- RAUM is a general and testable framework. Possible curse of dimensionality. One solution: restriction to attention-indexes.
- We run an experiment and do statistical testing for RAUM consistency with the attention-index restriction.
- **Preview of the Findings**: Reject RUM/ Fail to reject a RAUM that is an attention-index.

Our Model: Random Attention and Utility Model (RAUM)

- Dataset $P = (p(a, A))_{A \in \mathcal{A}}$ describing a population of DMs (vectorized p). $A \in \mathcal{A}$: $\emptyset \subset A \subseteq X$ with X finite.
- P has RAUM representation if:

$$p(a, A) = \sum_{(\succ, \phi) \in U \times \phi} \boldsymbol{\pi(\succ, \phi)} 1(a \succ b, \ \forall b \in \phi(A)),$$

for some distribution over preferences and consideration filters $\pi \in \Delta(U \times \Phi)$.

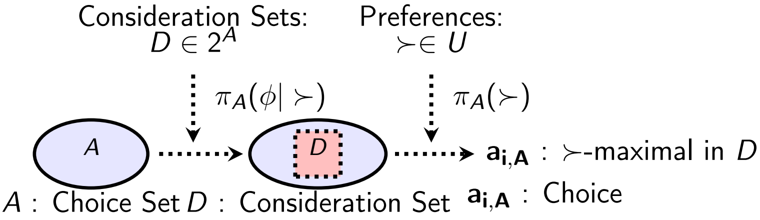
- U is the set of strict preferences. Φ is thes set of filters.
- For a fixed D, $\phi(A) = D$ when $D \subseteq A$ and $\phi(A) = \emptyset$ otherwise.

Lemma

Every P is a RAUM.

Our Model: Sources of Randomness

- Each DM is endowed with a preference $\succ$ and a random consideration rule $\pi_A(\phi|\succ)$.



- Sleeping agent: $X = \{a, b, c\}$, $b \succ^1 a \succ^1 c$, $a \succ^2 b \succ^2 c$, $\phi^1(X) = \{a, b, c\}$, $\phi^2(X) = \{a\}$. $\pi_X(\phi^1|\succ^1) = 1$, $\pi_X(\phi^2|\succ^2) = 1$. $\pi_X(\succ^1) = \frac{1}{2}$.
- An inattentive DM $p^*_\succ(a, A) = \sum_{\phi\in\Phi} \pi_A(\phi|\succ)1(a \succ b \forall b \in \phi(A))$. RAUM is a mixture of inattentive DMs.

Set-monotone and stable RAUM

- *Stability*: $\pi_A(\succ) = \pi_B(\succ)$ for all A, B and $\succ$, with $\pi_A(\succ) = \sum_{\phi \in \Phi} \pi(\succ, \phi)$.
- *Set-monotonicity*: For $A \subseteq B$, $\pi_A(\phi | \succ) \geq \pi_B(\phi | \succ)$ with $\phi(A) \neq \emptyset$.
- Not testable: Stable RAUM/ Set-monotone RAUM.
- Stable and testable RAUM is testable!
- Examples: Search and Satisfy, Rational Inattention, and *attention-index* models.

Theorem

TFAE:

- *P admits a set-monotone and stable RAUM representation.*
- *There exists a $\nu \in \mathbb{R}^{d_g}$, such that $g = G\nu$ (with a known matrix G of $1, 0, -1$, and $g = (p', 1_{dm}, 0_{dr})$, with $d_g = d_\rho + d_m + d_r$).*

- We generalize RUM ([McFadden and Richter, 1990]) and Random Attention Model (RAM, [Cattaneo et al., 2017]). Testing Kitamura and Stoye [2018].

A special case: Attention-index/Link (L-RAUM)

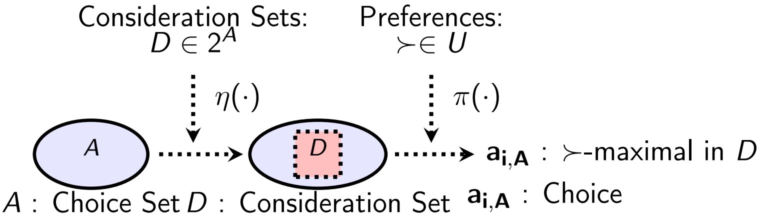
- P has an attention-index representation:

$$p(a, A) = \sum_{\succ \in U} \boldsymbol{\pi(\succ)} \sum_{D \subseteq A} \boldsymbol{m_A(D)} 1(a \succ b, \ \forall b \in D),$$

for some distribution over preferences $\pi \in \Delta(U)$ and some distribution over consideration sets $m_A \in \Delta(2^A)$.

- Attention index: $\eta : 2^X \to [0, 1]$. Analogous to utility but over menus.
- $m_A(D) = \psi(\eta(D), \sum_{C \in g(D,A)} \eta(C))$, where ψ is a link-function and g is an index. (Examples coming)
- This framework is stable and set-monotone (monotone ψ).
- Additional assumptions: (i) Independence: Preferences $\perp$ Consideration. (ii) (default) $o \in A$ such that $m_A(D) = 0$ if $o \notin D$.

Our Model: Sources of Randomness



- Two-stage system: First attention is determined, then DMs use preferences to decide.
- Simplify then choose.

Representative Agent: Optimal Consideration

- A representative DM with random utility with mean utility $u : X \to \mathbb{R}$, such that $u(x) + \xi_x$.
- McFadden's surplus: $\alpha(D) = \mathbb{E}[\max_{x \in D} u(x) + \xi_x]$.
- $m_A = argmax_{m \in \Delta(2^A)} \sum_{D \subseteq A} [m(D)\alpha(D) - K(m(D))]$.
- $K(t) = -t \log(t)/\theta$ then $m_A(D) = \frac{exp(\theta\alpha(D))}{\sum_{C \subseteq A} exp(\theta\alpha(C))}$.
- $\eta(D) = exp(\theta\alpha(D))$, and $\psi(t, s) = \frac{t}{s}$.
- Reduction of dimensionality from $\sum_{A \subseteq X} 2^{|A|} - 1$ to $2^{|X|} - 1$.

Attention index: Survival Race

- $\eta \in \Delta(2^X)$, attention index. $\eta(A)$ captures how attractive is A.
- LA-Logit Attention (Brady and Rehbeck 2016)

$$m_A^{LA}(D) = \frac{\eta(D)}{\sum_{C \subseteq A} \eta(C)}$$

- MM-Manzini-Mariotti (2014)/Independent attention

$$m_X^{MM}(D) = \eta(D) = \prod_{a \in D} \gamma(a) \prod_{a \in X \setminus D} (1 - \gamma(a))$$

- EBA Elimination-by-Aspects (Tversky, 1972)

$$m_A^{EBA}(D) = \sum_{C : C \cap A = D} \eta(C)$$

Underlying Full Consideration RUM

- $p_\pi(a, C) = \sum_{\succ \in U} \pi(\succ) 1(a \succ b \forall b \in C)$ (RUM/FC).
- We can re-write the L-RAUM as:

$$p(a, A) = \sum_{C \subseteq A} m_A(C) \boldsymbol{p_\pi(a, C)},$$

- Our main theoretical result is a decomposition of P into m_A, p_π.
- Note that:

$$p(o, A) = 1 - \sum_{C \subseteq A : C \neq \emptyset} m_A(C).$$

It does not depend on p_π: key assu. preferences and consideration are independent. We can identify $\eta^{\mathbf{L}}$ from $\{p(o, A)\}_{A \in \mathcal{A}}$.

Identification of the distribution of consideration

- If P admits a L-RAUM representation then: $m_A^L(C) = F^L(\{p(o, A \cup o)\}_{A \subseteq X})$.
- We recover m_A^L uniquely for $L \in \{LA, MM, EBA\}$.

L	m_A^L	η^L
$\mathcal{M}^{LA}$	$m_A^{LA}(D) = \frac{\eta(D)}{\sum_{C \subseteq A} \eta(C)} > 0$	$\eta^{LA}(D) = \sum_{B \subseteq D} (-1)^{\|D \setminus B\|} \frac{p(o,X)}{p(o,B)}$
$\mathcal{M}^{MM}$	$\eta(\cdot) = \prod_{a \in X \setminus \cdot} (1 - \gamma(a)) \prod_{b \in \cdot} \gamma(b)$	$\gamma^{MM}(a) = 1 - \frac{p(o,A)}{p(o,A \setminus \{a\})}$
$\mathcal{M}^{EBA}$	$m_A(D) = \sum_{C : C \cap A = D} \eta(C)$	$\eta^{EBA}(D) = \sum_{A \subseteq D : D \in \mathcal{A}} (-1)^{\|D \setminus A\|} p(o, X \setminus A)$

Identification of the distribution of preferences

- Once we have identified m_A^L, we can identify p_π^L uniquely:
- $p_\pi^L(a, A) = \frac{p(a,A) - \sum_{C \subset A} m_A^L(C) p_\pi^L(a,C)}{\mathbf{m_A^L(A)}}$ ($P_\pi^L = \{p_\pi^L(a, A)\}_{a \in A, A \in \mathcal{A}}$).

Theorem

Suppose that for given $L \in \{LA, MM, EBA\}$ and P, (i) m^L is well-defined, (ii) $m_A^L(A) > 0$ for all $A \in \mathcal{A}$. Then the following are equivalent.

1. *P is a L-RAUM-rule;*
2. *P_π^L is a FC-RAUM-rule (i.e., RUM).*

- We have reduced our problem to testing RUM! (Kitamura and Stoye 2018).
- m_A, p_π are identified uniquely!

Testing Procedure (I)

Theorem (Testing)

The following are equivalent.

1. P^L *is RUM and* m^L *is well-defined;*
2. $\inf_{v \in \mathbb{R}_+^d} \|g^L - Gv\| = 0$*, where* $g^L = (P^{L'}, m^{L'})'$.

$$G = \begin{bmatrix} B & 0_{d_p \times d_m} \\ 0_{d_m \times \|X\|!} & I_{d_m} \end{bmatrix}, \qquad B_{k,l} = 1(a \in A)1(a \succ_l c, : \forall : c \in A)$$

We can apply [Kitamura and Stoye, 2018].

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Testing Procedure (II)

Given testing theorem, a natural test statistic is

$$T_n = n \min_{[v - \tau_n \iota / d] \in \mathbb{R}^d_+} (\hat{g}^L - Gv)'(\hat{g}^L - Gv)$$

where $\hat{g}^L = (\hat{P}^{L'}, \hat{m}^{L'})'$; τ_n is a tuning parameter; and ι is a vector of ones of dimension d.

Testing Procedure (III)

Let $\hat{g}_l^{L,*}$, $l = 1, \ldots, L$ be bootstrap replications of $\hat{g}^L$. To compute CV of T_n we follow the bootstrap procedure proposed in [Kitamura and Stoye, 2018]:

1. Compute $\hat{\eta}_{\tau_n} = Gv_{\tau_n}$, where v_{τ_n} solves

$$n \min_{[v - \tau_n \iota / d] \in \mathbb{R}_+^d} (\hat{g}^L - Gv)'(\hat{g}^L - Gv);$$

2. Compute

$$\hat{g}_l^{L,*} = \hat{g}_l^{L,*} - \hat{g}^T + \hat{\eta}_{\tau_n},$$

3. Compute the bootstrap test statistics

$$T_{n,l}^* = n \min_{[v - \tau_n \iota / d] \in \mathbb{R}_+^d} (\hat{g}_l^{L,*} - Gv)'(\hat{g}_l^{L,*} - Gv), \quad l = 1, \ldots, L;$$

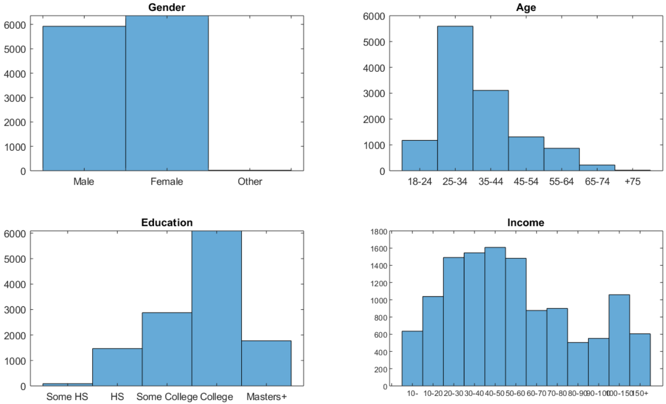
4. Use the empirical distribution of the bootstrap statistic to compute critical values of T_n. Reject if $T_n > \hat{c_{1-\alpha}}$, $\alpha \in (0, 1/2)$ significance level.

The Experiment

- We design and implement an (mTurk) experiment to collect a **standard stochastic choice dataset** with a default from a **large** sample of individuals.
- We require full choice set variation, but no requirement to have repeated individual choices. We vary exogenously the cost of consideration and the choice set.
- Our experiment: 12297 choices from 2135 individuals (3 levels of complexity/ 3 frames).

Sample

- Run between Aug 25th and Sept 17th 2018 on Amazon MTurk
- 2135 individuals with 12,297 independent decisions
- Avg. payment \$1.09 (\$ 0.25 participation fee)
- Avg duration 251.68 secs - Hourly rate $\sim$ \$15



Design - Consideration Cost H/M/L

Select (only) one of the following alternatives. Click over the selected option and hit next [-->].

Get 12 tokens for sure.

Prize	Probability
(3-8+22+4 +9) tokens	50%
(5-7 + 41 +10 -39) tokens	50%

Prize	Probability
(27-10 -2 +11 +24) tokens	25%
(25+6-1+3+15) tokens	20%
(1-18 +3 +17 +11) tokens	15%
(9-6 -7 -4 + 8) tokens	40%

Design - Consideration Cost H/M/L

Select (only) one of the following alternatives. Click over the selected option and hit next [->].

Get 12 tokens for sure.

Prize	Probability
(18+2+10) tokens	50%
(4+5+1) tokens	50%

Prize	Probability
(66+5-21) tokens	25%
(28-24+44) tokens	20%
(10-3+7) tokens	15%
(2-5+3) tokens	40%

Design - Consideration Cost H/M/L

Select (only) one of the following alternatives. Click over the selected option and hit next [-->].

Get 12 tokens for sure.

Prize	Probability
50 tokens	25%
48 tokens	20%
14 tokens	15%
0 tokens	40%

Prize	Probability
30 tokens	50%
10 tokens	50%

Evidence against RUM

Under the null of RUM-consistency there should be no effect of the Costly Consideration Treatments on behavior.

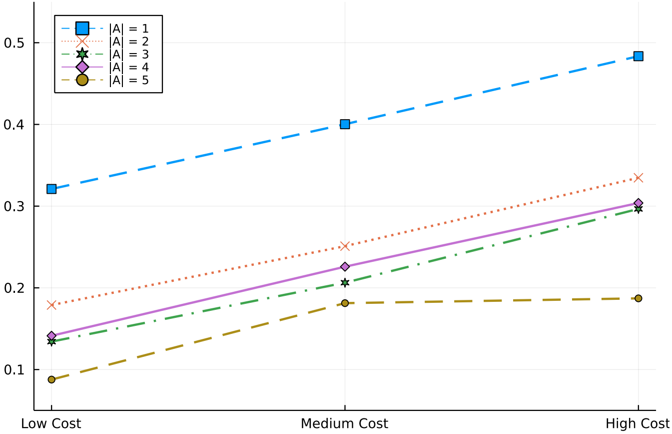


Figure: Estimated frequency of the Outside Option: Average by menu size.

RUM vs LA:RAUM

- We test RUM, LA-RAUM
- Rich menu variation increases the statistical power.
- We have 5 lotteries and one certain outcome.
- The certain outcome is always available and dominated. It is also pre-selected. (Default).
- The number of possible rankings over the choice set is 120 under LA-RAUM and 720 under RUM.
- We reject RUM. We cannot reject LA-RAUM.

Stability of Preferences across frames/cost treatments

- Unique design allows to test RUM (stability of preferences).
- RUM is description independent (implicit assumptions: *consequentialism*+ full consideration.
- *Consequentialism* means preferences do not change with descriptions/frames.
- We cannot reject LA under consequentialism. Different attention-index per frame, but same stable preference.

Model	T_n	p-value
RUM	3231.59	<0.001
LA	24959.06	0.524
EBA	24840.23	0.001

Notes: Number of bootstrap replications=1000.

Conclusion

- We extend existing econometrics tools for RUM, to allow for consideration-mediated choice: testing and identification results
- We have designed a novel experiment that allows us to discern among competing models for population behavior.
- We find that:
  1. RUM fails
  2. LA with preference heterogeneity describes behavior across all costs.
  3. Preferences are stable across costs for LA-RAUM
- [Abaluck and Adams, 2017] have used our findings to choose a semiparametric specification in their setup for insurance buying.
- Take-home message: we need to model inattention.

Abaluck, J. and Adams, A. (2017).
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Technical report, National Bureau of Economic Research.

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