# Midterm Exam Microeconomics: 2021

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**Problem 1.** (30 points) Let $X$ be a nonempty and finite set of alternatives. Consider the following bounded rational consumer. Given a menu $A \subseteq X$, the consumer searches sequentially through the menu according to a fixed search order on $X$. At each step in the search process, the consumer compares the utility of the item that she sees to a fixed threshold. If the utility of the item is above the threshold then the consumer stops the search and picks the item. Formally, let $S \subseteq X \times X$ be a linear order on $X$, let $u : X \to \mathbb{R}$ be a utility function, and $u^* \in \mathbb{R}$ be a threshold. The choice correspondence of this bounded rational consumer is given by

$$c_S(A) = \{b \in A | u(b) \geq u^*; \text{there exists no} \quad aSb; u(a) \geq u^*\}$$

for all $A \subseteq X$, where the subscript $S$ denotes the dependence of the choice correspondence of the linear order that models the search process.

(a) Show that the choice correspondence $c_S$ defined above is in fact a choice function.

(b) Show that the choice correspondence $c_S$ satisfies WARP (define WARP using the notes and textbook).

(c) Assume you observe a dataset $(2^X \setminus \emptyset, c)$ (such that $c : 2^X \setminus \emptyset \to 2^X \setminus \emptyset$ is an observed choice function -possibly different from $c_{BR}$-) that satisfies WARP. Show that there is a bounded rational consumer characterized by a triple $(S, u, u^*)$ a linear order on $X$, a utility function and a thresholds as in (a) such that $c(A) = c_{BR}(A)$ for all $A \in 2^X \setminus \emptyset$.

d) Assume now that there is a distribution over search orders. Denote the collection of all search orders/linear orders on $X \times X$ by $\mathcal{S}$. The distribution over search orders is $\pi \in \Delta(\mathcal{S})$. The probability of choice of item $a$in menu $A$ such is given by $\rho_A(a)$ where $a \in A$ and $\rho_A \in \Delta(A)$. Let the stochastic version of the bounded rational model above generate $\rho_A$ such that:

$$\rho_A(a) = \sum_{S \in \mathcal{S}} \pi(S) 1(a = c_S(A)).$$

Show that if you have two menus $A \subseteq B$ and $a \in A \cap B$, it has to be that $\rho_B(a) \leq \rho_A(a)$ (i.e., regularity).

**Solutions:**

a)

*Proof.* Assume that $a, b \in c_{BR}(A)$ for some $A \subseteq X$. Then $u(a) \geq u^*$ and $u(b) \geq u^*$. Given that $a \in c_{BR}(A)$ then there it must be that either (i) $u(b) \leq u^*$ (which cannot happen), or (ii) $aSb$. If the latter then $b \notin c_{BR}(A)$ this is a contradiction.

□

b)

*Proof.* Define the revealed preference relation $aR^Db$ iff $a = c_{BR}(A)$ for some $A$ and $b \in A$. If $a^1 R^D a^2$ then either $u(a^1) \geq u^* > u(a^2)$ or (exclusive) $u(a^1) \geq u^*$, $u(a^2) \geq u^*$and $a^1 S a^2$.

Say that $a^2 P^D a^1$, or equivalently given the fact that $c_{BR}$ is a function, $a^2 R^D a^1$ then (i) $u(a^2) \geq u^* > u(a^1)$ or exclusive $u(a^1) \geq u^*$, $u(a^2) \geq u^*$and $a^2 S a^1$. This is a contradiction.

□

c)

*Proof.* We can use the same proof of rationalizability by a preference relation (preorder). Then we do the following, $c(A) = c^*(A, \succeq)$ as in the notes, then we identify the preferences with the transitive closure of the direct preference relation $R^D$. Since the choice $c$ is a function $R^D = P^D$ the revealed preference relation is strict. Hence, $\succeq$ is a linear order. Now, we let the search order be $S = \succeq$, we build $u : X \to \mathbb{R}$ in a way such that $min_{b \in X} u(b) = 1$, then we let $u^* = 0$. This means that all items are satisficing, i.e., for all $x \in X$ , $u(x) \geq u^*$. This shows that if $a = c(A)$ then $a = c_{BR}(A) = \{b \in A | u(b) \geq u^*; \text{there exists no} \quad aSb; u(a) \geq u^*\}$.

□

d)

*Proof.* $1(a = c_S(A)) \leq 1(a = c_S(B))$ where $A \supseteq B$, because WARP holds for $c_S(A)$. Then regularity is implied.

□

**Problem 2.** (30 points) Let $X$ be a nonempty and finite set of alternatives. Consider the following stochastic bounded rational consumer called "stochastic limited consideration (SLC)".

Assume there is a default alternative $o \notin X$, that is always present in any given menu. Let $X^* = X \cup \{o\}$. Also let the collection of menus that contain $o$ be defined as $\mathcal{A}$, such that $A \in \mathcal{A}$ if $A \subseteq X^*$ and $o \in A$. Given a menu $A \in \mathcal{A}$, the probability of this consumer of choosing an alternative $a$ in menu $A$, denoted by $\rho_A(a)$, where $\rho_A \in \Delta(A)$, is given by the following decision algorithm: First the consumer is endowed with a strict preference relation $\succ$ defined over $X^*$ with the restriction that for any $x \in X$, it must be that $x \succ o$ (i.e., the default is the worst item). You can assume that $\succ$ is complete and transitive and there is no indifference among any two items. The consumer is also endowed with a distribution over mental categories. Formally, mental categories, $\mathcal{D}$, is the collection all subsets of $X^*$. At each decision trial, the consumer draws a mental category from $\mathcal{D}$ with probability $m \in \Delta(\mathcal{D})$ with the restriction that $m(D) = 0$ if $o \notin D$ (i.e., the probability of drawing a category that does not have the default is zero). Then the consumer forms a "consideration set" by taking the intersection of menu $A$ and $D$ such that the consideration set is equal to $D \cap A$. Then the consumer maximizes her preferences $\succ$ on the consideration set $D \cap A$. Finally, she chooses the item that maximizes her preferences on $D \cap A$. The resulting probability of this decision algorithm is:

$$\rho_A(a) = \sum_{D \in \mathcal{D}} m(D) 1(a \succ b \forall b \in (D \cap A), b \neq a),$$

where $1(\cdot) = 1$ when the argument is true and zero otherwise.

- A complete stochastic dataset, $\rho$, is a collection of $\rho_A$ for all menus $A \in \mathcal{A}$.
- We say that $a$ is stochastically revealed preferred to $b$ (i.e., $aRb$) when $p(b, A \cup \{a\}) < p(b, A)$.
- We say $R$ is a acyclic if there is no integer $n \geq 2$ and $a_1, \cdots, a_n \in X$ such that $a_i R a_{i+1}$ for $i = 1, \cdots, n-1$ and $a_n P a_1$, where $P$ is the strict part of $R$.
- We say that a complete stochastic dataset, $\rho$, is regular if for any pair of menus $A, B \in \mathcal{A}$ such that $A \subseteq B$ it must be that $\rho_B(a) \leq p_A(a)$ for any $a \in A \cap B$.

a) Show that a complete stochastic dataset $\rho$ that is generated by a SLC, as described above, implies that the stochastic revealed preference relation $R$ is acyclic.

b) Show that a complete stochastic dataset $\rho$ that is generated by a SLC, implies that $\rho$ is regular.

c) Let $\rho$ be generated by a SLC with the additional restriction that $m(X^*) = 1$ and zero otherwise. This means that this consumer always considers all alternatives.

Show that the choice correspondence defined by:

$$c(A) = \{a \in A : \rho_A(a) = 1\},$$

is a choice function (i.e., single-valued), and that $c(A)$ satisfies SARP (first define SARP as seen in class and in the problem sets).

d) Show that a complete stochastic dataset $\rho$ that is generated by a SLC, satisfies the ASRP (Axiom of Stochastic Revealed Preference) as defined in the notes. (Hint: use the equivalence theorem between the ASRP and random utility and define ASRP using the notes.).

a)

*Proof.* A cycle implies the following:

$a_i R a_{i+1}$ means that $\rho(a_{i+1}, A \cup \{a_i\}) < \rho(a_{i+1}, A)$ for $i = 1, \cdots, n-1$, and $a_n P a_1$ means that $\rho(a_1, A \cup \{a_n\}) < \rho(a_1, A)$ and not $\rho(a_n, B \cup \{a_1\}) < \rho(a_n, B)$.

The first part means that

$$\sum_{D \in \mathcal{D}} m(D) 1(a_{i+1} \succ b \forall b \in (D \cap A \cup \{a_i\}), b \neq a_{i+1}) < \sum_{D \in \mathcal{D}} m(D) 1(a_{i+1} \succ b \forall b \in (D \cap A), b \neq a_{i+1}),$$

this means that there is some $D \cap (A \cup \{a_i\})$ for which $a_i \succ a_{i+1}$, for all $i = 1, \cdots, n-1$.

This means that by transivity and completeness that $a_1 \succ a_n$, however, the presence of a cycle $\rho(a_1, A \cup \{a_n\}) < \rho(a_1, A)$ implies that

$$a_n \succ a_1,$$

which is a contradiction.

$\square$

b)

*Proof.* Let $B \subseteq A$, assume a violation of regularity $\rho_B(a) < \rho_A(a)$ for some $a \in A \cap B$:

$$\rho_A(a) = \sum_{D \in \mathcal{D}} m(D) 1(a \succ b \forall b \in (D \cap A), b \neq a) >$$

$$\rho_B(a) = \sum_{D \in \mathcal{D}} m(D) 1(a \succ b \forall b \in (D \cap B), b \neq a).$$

This means that

$$1(a \succ b \forall b \in (D \cap A), b \neq a) > 1(a \succ b \forall b \in (D \cap B)$$

for at least one $D$, but this is impossible because:

$$D \cap B \subseteq D \cap A,$$

which means that

$$1(a \succ b \forall b \in (D \cap A), b \neq a) \leq 1(a \succ b \forall b \in (D \cap B),$$

due to completeness and transivity of $\succeq$ that imply WARP.

$\square$

c)

*Proof.* This follows from the notes in the class after realizing that $c(A) = \{a \in A : \rho_A(a) = 1\} = \{a \in A : a \succ b \forall b \in A\}$.

$\square$

d)

*Proof.* This follows from the notes after realizing that SLC is the weighted average of deterministic rational behavior with weights $m$, since the average of random utility models is random utility (according to the notes) then SLC admits a Random Utility representation. Then ASRP must hold. For each category $D$, you can create an order or preference relation $\succ_D$, that essentially ranks the original preference relation with the restriction to the $D$ category. $\succ |D = \succ_D$ and then $b \notin D$, $b \succ_D a$ for any $a \in D$. You could rewrite the model

$$m(D) = \pi(\succ_D),$$

because RUM is equivalent to ASRP then you were done.

□

**Problem 3.** (Modeling question). (30 points) Modeling the "Endowment Effect". I am going to present to you a summary of an experiment showing a behavioral bias called the Endowment Effect. The task is to write a decision model algorithm to explain the Endowment Effect.

**The Endowment Effect Experiment.**

Kahneman, Knetch and Thaler [1990], did an experiment with the following features:

- 44 subjects
- 22 subjects given mugs
- The other 22 subjects given nothing
- Subjects who owned mugs asked to announce the price at which they would be prepared to sell mug
- Subjects who did not own mug announced price at which they are prepared to buy mug
- Experimenter figured out prices at which supply of mugs equals demand.

If our subjects are rational consumers then the prediction in this market is:

- Prediction for rationality: As mugs are distributed randomly, we should expect half the mugs (11) to get traded.
- Explanation: Consider the group of mug lovers (i.e. those that have valuation above the median), of which there are 22.
  - Half of these should have mugs, and half should not.
  - The 11 mug haters that have mugs should trade with the 11 mug lovers that do not.

The **Experiment Outcome** however differs from the **Prediction**, and we call this the **Endowment Effect.**

- In 4 sessions, the number of trades was 4,1,2 and 2 (respectively per session).
- Median seller valued mug at \$5.25
- Median buyer valued mug at \$2.75
- Willingness to pay/willingness to accept gap
- **Subject's preferences seem to be affected by whether or not their reference point was owning the mug.**

1. Explain why the Endowment Effect is not consistent with a traditional model of rationality. Hint: The answer has less to do with the idea of revealed preferences, but more with the idea of stability of preferences. Write down the model of trade formally. You can assume a parametric distribution of preferences, make as simple as possible (uniform distribution).
2. Propose a decision algorithm that explains the endowment effect given the trading scheme of the experiment.
3. Calibrate your model parameters to reproduce the experiment outcome.

## Partial Solution

Consider now two time periods $t = 0, 1$, $t = 0$ means that this is before the experiments happens, and $t = 1$ is after the treatment is assigned.

The valuation of a mug for subject $i$ at $t$ is $v_{i,t}$.

- Assumption: Under Rationality, assume that $v_{i,t}$ is i.i.d. from some uniform distribution on $[0, 1]$, and pretty much $v_{i0} = v_{i1}$ (i.e., this means the subject is not changing her preferences).

Under the assumption of rationality, if we select $N$ individuals, at random.

With probability $\frac{1}{2}$, I assign an individual to the Mug Treatment group.

This implies that that the share of individual that like the mugs (e.g., $v_i \geq \frac{1}{2}$) in the treatment group is $\frac{1}{2}$ of the size of Mug lovers and the other $\frac{1}{2}$ of the Mug lovers is in the control group.

If preferences are rational, i.e., “stable”, the number of transaction in expectation has to be $N/2$.