Final Exam: Microeconomics USFQ 2022

Victor H. Aguiar

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Problem 1. Checking SARP.

Let $X = \mathbb{R}^L_+$, and consider the finite dataset of consumption bundles and prices $O^T = \{x^t, p^t\}_{t \in \{1, \cdots, T\}}$ where $x^t \in X$ and $p^t \in \mathbb{R}^L_{++}$ for all t.

WARP is satisfied if $p^t \cdot x^t \geq p^t \cdot x^s$ ($x^t \succeq^D x^s$) implies that we cannot have $p^s \cdot x^s > p^s \cdot x^t$ ($x^s \succ^D x^t$).

GARP is satisfied if $x^1 \succeq^D x^2 \cdots \succeq^D x^n$ implies that we cannot have $x^n \succ^D x^1$.

- Consider a data set O^3 with prices $p^1 = (4 \quad 1 \quad 5)'$, $p^2 = (5 \quad 4 \quad 1)'$, $p^3 = (1 \quad 5 \quad 4)'$, and bundles $x^1 = (4 \quad 1 \quad 1)'$, $x^2 = (1 \quad 4 \quad 1)'$, $x^3 = (1 \quad 1 \quad 4)'$.

1. Does O^3 satisfies (i) WARP, (ii) GARP. Show that O^3 cannot be rationalized by a locally-nonsatiated utility function $u : \mathbb{R}^3_+ \to \mathbb{R}$, (i.e., u rationalizes O^3 if $x^t \in argmax_y u(y)$ subject to $p^t \cdot y \leq p^t \cdot x^t$ for all t).

2. Consider a preference function $r : \mathbb{R}^3_+ \times \mathbb{R}^3_+ \to \mathbb{R}$ such that $r(x, y) \geq (>) 0$ means that x is preferred to y. We assume that the preference function is skew-symmetric $r(x, y) = -r(y, x)$, and strictly increasing in the first entry (i.e., $r(\cdot, y)$ is strictly increasing for all $y \in \mathbb{R}^3_+$). We say $O^T = \{x^t, p^t\}_{t \in \{1, \cdots, T\}}$ is strictly rationalized by a preference function r if $r(x^t, y) > 0$ for all y such that $p^t \cdot y \leq p^t \cdot y^t$. Show that any O^T that is rationalized by a skew-symmetric and strictly increasing preference function must satisfy WARP. Consider O^3 above that cannot be rationalized by a utility function but that can be rationalized by a skew-symmetric and strictly increasing (in the first entry) preference function, can the rationalizing preference function be transitive (i.e., $r(x, y) \geq 0, r(y, z) \geq 0 \implies r(x, z) \geq 0$)?

3. Show that GARP is equivalent to WARP when there are two goods L=2.

Problem 2. Price preferences. Consider a consumer that is characterized by the following utility function $V : X \times W \to \mathbb{R}$, where the consumer chooses x^t when prices are p_t, such that

$$x^t \in argmax_{x \in \mathbb{R}^L_+} V(x, -p^t \cdot x)$$

The utility $V(x, \cdot)$ is **strictly** increasing in the second argument. This consumer receives utility from consuming x, and experiences disutility when spending $p^t \cdot x$. The optimal bundle purchased by this consumer will balance the trade-offs between these two forces.

Also define the indirect utility of price by:

$$V(p^t) = max_x V(x, -p^t \cdot x).$$

Finally let's define the price-revealed preference

$$p^s \succeq_p p^t,$$

when

$$p^s x^t \leq p^t x^t.$$

Similarly $p^s \succ_p p^t$when $p^s x^t < p^t x^t$.

a) Show that a data set generated by the consumer above collected in $O^T = \{p^t, x^t\}$ satisfies the Generalized Axiom of Price Revealed Preference (GAPP).

(GAPP: If $p^t \succeq_p p^k \succeq_p \cdots \succeq_p p^s$ then it cannot be that $p^s \succ_p p^t$.)

b) Show that a data set generated by the special case of the model above with the following utility:

$$x^t \in argmax_{x \in X} u(x) - p^t x,$$

satisfies both the Generalized Axiom of Price Revealed Preference and the Generalized Axiom of Revealed Preference (GARP).

(If $p^t \cdot x^t \geq p^t \cdot x^s$ then we say $x^t \succeq^D x^s$ and if $p^s \cdot x^s > p^s \cdot x^t$ then we say $x^s \succ^D x^t$. GARP is satisfied if $x^1 \succeq^D x^2 \cdots \succeq^D x^n$ then it cannot be that $x^n \succ^D x^1$.)

c) Show that a data set generated by the special case of the model introduced in (b), satisfies the (uncompensated law of demand):

$$(p^t - p^s)'(x^t - x^s) \leq 0.$$