Final Exam: Microeconomics USFQ 2022

Victor H. Aguiar

December 16, 2022

Problem 1. Checking SARP.

Let $X = \mathbb{R}^L_+$, and consider the finite dataset of consumption bundles and prices $O^T = \{x^t, p^t\}_{t\in\{1,\cdots,T\}}$ where $x^t \in X$ and $p^t \in \mathbb{R}^L_{++}$ for all t.

WARP is satisfied if $p^t \cdot x^t \geq p^t \cdot x^s$ ($x^t \succeq^D x^s$) implies that we cannot have $p^s \cdot x^s > p^s \cdot x^t$ ($x^s \succ^D x^t$).

GARP is satisfied if $x^1 \succeq^D x^2 \cdots \succeq^D x^n$ implies that we cannot have $x^n \succ^D x^1$.

- Consider a data set O^3 with prices $p^1 = (4 \quad 1 \quad 5)'$, $p^2 = (5 \quad 4 \quad 1)'$, $p^3 = (1 \quad 5 \quad 4)'$, and bundles $x^1 = (4 \quad 1 \quad 1)'$, $x^2 = (1 \quad 4 \quad 1)'$, $x^3 = (1 \quad 1 \quad 4)'$.

1. Does O^3 satisfies (i) WARP, (ii) GARP. Show that O^3 cannot be rationalized by a locally-nonsatiated utility function $u : \mathbb{R}^3_+ \to \mathbb{R}$, (i.e., u rationalizes O^3 if $x^t \in argmax_y u(y)$ subject to $p^t \cdot y \leq p^t \cdot x^t$ for all t).

2. Consider a preference function $r : \mathbb{R}^3_+ \times \mathbb{R}^3_+ \to \mathbb{R}$ such that $r(x, y) \geq (>)0$ means that x is preferred to y. We assume that the preference function is skew-symmetric $r(x, y) = -r(y, x)$, and strictly increasing in the first entry (i.e., $r(\cdot, y)$ is strictly increasing for all $y \in \mathbb{R}^3_+$). We say $O^T = \{x^t, p^t\}_{t\in\{1,\cdots,T\}}$ is strictly rationalized by a preference function r if $r(x^t, y) > 0$ for all y such that $p^t \cdot y \leq p^t \cdot y^t$. Show that any O^T that is rationalized by a skew-symmetric and strictly increasing preference function must satisfy WARP. Consider O^3 above that cannot be rationalized by a utility function but that can be rationalized by a skew-symmetric and strictly increasing (in the first entry) preference function, can the rationalizing preference function be transitive (i.e., $r(x, y) \geq 0, r(y, z) \geq 0 \implies r(x, z) \geq 0$)?

3. Show that GARP is equivalent to WARP when there are two goods L=2.

Proof. 1)

$p^{1\prime} * x^1 = 22$,$p^{1\prime} * x^2 = 13$,$p^{1\prime} * x^3 = 25$,
$p^{2\prime} * x^1 = 25$,$p^{2\prime} * x^2 = 22$,$p^{2\prime} * x^3 = 13$,
$p^{3\prime} * x^1 = 13$,$p^{3\prime} * x^2 = 25$,$p^{3\prime} * x^3 = 22$,
it is easy to check that if $p^{j\prime} * x^i \leq p^{j\prime} * x^j$,then $p^{i\prime} * x^j > p^{i\prime} * x^i \forall i, j = 1, 2, 3$

Therefore this data set satisfies WARP.

From 1), we can also find that

$x^1 R^D x^2$, $x^2 R^D x^3$, $x^3 R^D x^1$,

so that $x^1 R x^3$, yet $x^3 R x^1$, which violates SARP.

It cannot be rationalized by a LNS utility function, the solution to this is in any textbook of grad level micro.

2) Assume towards contradiction that we observe a violation of WARP and that the dataset admits a rationalizing preference function that is skew-symmetric and strictly increasing. Then we have $p^t \cdot x^t \geq p^t \cdot x^s$ ($x^t \succeq^D x^s$) and $p^s \cdot x^s > p^s \cdot x^t$ ($x^s \succ^D x^t$). By rationalizability, $x^t \succeq^D x^s$ implies $r(x^t, x^s) \geq 0$, but we also have that $r(x^s, x^t) \geq 0$. There are two cases, either one of the inequalities are strict in which case skew-symmetry is violated, or both inequalities are in fact equalities. In the latter case, $r(x^t, x^s) = 0$ and $r(x^s, x^t) = 0$ cannot happen because $p^s \cdot x^s > p^s \cdot x^t$ means that there is x^k close enough to x^t such that $p^s \cdot x^s > p^s \cdot x^k$ and such that $r(x^s, x^k) < 0$ which by skew-symmetry means that $r(x^k, x^s) > 0$ contradicting the fact that x^s is a maximizer of r.

3)

If $L = 2$, we want to show that if O^T fails GARP then there must be a cycle of length 2. Suppose towards contradiction that we have an irreducible cycle of size 3:

$$x^1 \succeq^R x^2 \succeq^R x^3 \succeq^R x^1.$$

This implies that no observed bundle is larger than another. Indeed if $x^t \geq x^s$(in the vector order).

$x^t \succeq^R x^s$ if $x^t \geq x^s$, then assume $x^2 \geq x^3$, then $p^1 x^1 \geq p^1 x^2 \implies p^1 x^1 \geq p^1 x^3$, so we can form the cycle

$$x^1 \succeq^R x^3 \succeq^R x^1,$$

but that is a contradiction.

Since $L = 2$, then the observed bundles can be ordered by the quantity of good 1 consumed in each bundle, this is a result from linear algebra.

□

Problem 2. Price preferences. Consider a consumer that is characterized by the following utility function $V : X \times W \to \mathbb{R}$, where the consumer chooses x^t when prices are p_t, such that

$$x^t \in argmax_{x \in \mathbb{R}^L_+} V(x, -p^t \cdot x)$$

The utility $V(x, \cdot)$ is **strictly** increasing in the second argument. This consumer receives utility from consuming x, and experiences disutility when spending $p^t \cdot x$. The optimal bundle purchased by this consumer will balance the trade-offs between these two forces.

Also define the indirect utility of price by:

$$V(p^t) = max_x V(x, -p^t \cdot x).$$

Finally let's define the price-revealed preference

$$p^s \succeq_p p^t,$$

when

$$p^s x^t \leq p^t x^t.$$

Similarly $p^s \succ_p p^t$when $p^s x^t < p^t x^t$.

a) Show that a data set generated by the consumer above collected in $O^T = \{p^t, x^t\}$ satisfies the Generalized Axiom of Price Revealed Preference (GAPP).

(GAPP: If $p^t \succeq_p p^k \succeq_p \cdots \succeq_p p^s$ then it cannot be that $p^s \succ_p p^t$.)

b) Show that a data set generated by the special case of the model above with the following utility:

$$x^t \in argmax_{x \in X} u(x) - p^t x,$$

satisfies both the Generalized Axiom of Price Revealed Preference and the Generalized Axiom of Revealed Preference (GARP).

(If $p^t \cdot x^t \geq p^t \cdot x^s$ then we say $x^t \succeq^D x^s$ and if $p^s \cdot x^s > p^s \cdot x^t$ then we say $x^s \succ^D x^t$. GARP is satisfied if $x^1 \succeq^D x^2 \cdots \succeq^D x^n$ then it cannot be that $x^n \succ^D x^1$.)

c) Show that a data set generated by the special case of the model introduced in (b), satisfies the (uncompensated law of demand):

$$(p^t - p^s)'(x^t - x^s) \leq 0.$$

Proof. If $p^s \succeq_p p^t$ then $p^s \cdot x^t \leq p^t \cdot x^t$, this implies that

$$V(x^t, -p^s \cdot x^t) \geq V(x^t, -p^t \cdot x^t)$$

by the fact that the function V is strictly increasing in the second argument.

Then notice that since $x^s \in argmax_x V(x, -p^s \cdot x)$ then

$$V(x^s, -p^s \cdot x^s) \geq V(x^t, -p^s \cdot x^t)$$

This implies:

$$V(p^s) = V(x^s, -p^s \cdot x^s) \geq V(x^t, -p^t \cdot x^t) = V(p^t).$$

In other words, if $p^s \succeq_p p^t$ then $V(p^s) \geq V(p^t)$.

Then, if $p^s \succeq_p p^t$ it cannot be that $p^t \succ_p p^s$ as this will imply that $V(p^t) > V(p^s)$ and $V(p^s) \geq V(p^t)$ at the same time. An analogous argument applies for indirect price preference revelation.

$\square$

b) Notice that I can write $x^t \in argmax_x u(x) - p^t x$ then
$u(x^t) - p^t x^t \geq u(y) - p^t y$ for all $y \in \mathbb{R}^L_+$
then
$u(x^t) - p^t x^t \geq u(x^s) - p^t x^s$
which implies
$u(x^t) - u(x^s) \geq p^t x^t - p^t x^s$
Then if
$p^t x^t - p^t x^s \geq 0 \iff x^t \succeq^D x^s$
this implies that $u(x^t) \geq u(x^s)$ with strict inequality if $p^t x^t > p^t x^s$.

Then if a violation of WARP cannot happen since $x^t \succeq^D x^s$ and $x^s \succ^D x^t$ would imply that

$$u(x^t) \geq u(x^s)$$

and

$$u(x^s) > u(x^t).$$

An analogous argument applies for indirect price preference revelation.

c)

Proof. $x^t \in argmax_x u(x) - p^t x$ then
$u(x^t) - p^t x^t \geq u(y) - p^t y$ for all $y \in \mathbb{R}^L_+$
(i) $u(x^t) - p^t x^t \geq u(x^s) - p^t x^s$
(ii) $u(x^s) - p^s x^s \geq u(x^t) - p^s x^t$
$\iff$
(i) $u(x^t) - u(x^s) \geq p^t x^t - p^t x^s$
(ii) $u(x^s) - u(x^t) \geq p^s x^s - p^s x^t$
Then: (i)+(ii)
$0 \geq [p^t x^t - p^t x^s] + [p^s x^s - p^s x^t]$
$[p^t - p^s]'[x^t - x^s] \leq 0.$

$\square$