# CIRCUITOS ELÉTRICOS III - LISTA EXERCÍCIOS DERIVADAS

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(1) $f(x) = 6x^3 - 5x^2 + x + 9$

$f'(x) = 3.6x^{3-1} - 5.2x^{2-1} + 1x^{1-1} + 0$

$f'(x) = 18x^2 - 10x + 1.$

(2) $f(x) = \text{sen}^2 x . \cos^3 x$ $\quad y' = u'v + v'u$

$f'(x) = \underbrace{2\,\text{sen}\,x . \cos x}. \cos^3 x + \text{sen}^2 x . 3\cos^2 x . (-\text{sen}\,x)$

$f'(x) = \text{sen}\,2x . \cos^3 x - 3\,\text{sen}^3 x . \cos^2 x$

(3) $g(x) = (x^3 - 7)(2x^2 + 3)$

$g'(x) = 3x^2.(2x^2+3) + 4x(x^3-7)$

$g'(x) = 6x^4 + 9x^2 + 4x^4 - 28x$

$g'(x) = 10x^4 + 9x^2 - 28x$

(4) $f(x) = \dfrac{1}{\sqrt{x^2+2x}} = \dfrac{1}{(x^2+2x)^{\frac{1}{2}}} = (x^2+2x)^{-\frac{1}{2}}$

$f'(x) = -\frac{1}{2}(x^2+2x)^{-\frac{3}{2}} . (2x+2)$

$f'(x) = -\dfrac{x+1}{(x^2+2x)^{\frac{3}{2}}} = -\dfrac{x+1}{\sqrt[2]{(x^2+2x)^3}}$

$f'(x) = -\dfrac{x+1}{\sqrt{x^2+2x} . (x^2+2x)}$

(5) $h(r) = r^2(3r^4 - 7r + 2)$

$h(r) = 3r^6 - 7r^3 + 2r^2$

$h'(r) = 18r^5 - 21r^2 + 4r$

(6) $f(x) = \ln(x^2+x+1)$

$f'(x) = \dfrac{1}{x^2+x+1} . 2x+1$

$f'(x) = \dfrac{2x+1}{x^2+x+1}$

$\frac{d}{dx}\,\text{cossec}\,x = -\text{cossec}\,x . \text{cotg}\,x$

(7) $f(x) = \sqrt{\text{cossec}(x)}$

$f(x) = (\text{cossec}(x))^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(\text{cossec}\,x)^{-\frac{1}{2}} . (-\text{cossec}\,x . \text{cotg}\,x)$

$f'(x) = -\dfrac{\text{cossec}\,x . \text{cotg}\,x}{2\sqrt{\text{cossec}\,x}}$

(8) $f(x) = \ln^3 x = (\ln x)^3$

$f'(x) = 3\ln^2 x \cdot \frac{1}{x} = \dfrac{3\ln^2 x}{x}$

(9) $g(z) = \dfrac{8 - z + 3z^2}{2 - 9z}$ $\quad y' = \dfrac{u'v - v'u}{v^2}$

$g'(z) = \dfrac{(6z-1)(2-9z) - (-9)(8-z+3z^2)}{(2-9z)^2}$

$g'(z) = \dfrac{12z - 54z^2 - 2 + 9z + 72 - 9z + 27z^2}{(2-9z)^2}$

$g'(z) = \dfrac{-27z^2 + 12z + 70}{(2-9z)^2}$

(10) $f(x) = \ln(x^3) = 3\ln(x)$ $\quad \ln(m^n) = n . \ln(m)$

$f'(x) = 3 \cdot \frac{1}{x} = \dfrac{3}{x}$

(11) $f(t) = t^5 + \left(\dfrac{t+1}{t^2}\right)$

$f'(t) = 5t^4 + \dfrac{1(t^2) - 2t(t+1)}{t^4}$

$f'(t) = 5t^4 + \dfrac{t^2 - 2t^2 - 2t}{t^4}$

$f'(t) = 5t^4 - \dfrac{t^2 + 2t}{t^4} = \dfrac{5t^8 - t^2 + 2t}{t^4}$

$f'(t) = \dfrac{5t^7 . t + 2}{t^3}$

(12) $f(x) = \ln x \cdot \operatorname{sen} x$

$f'(x) = \frac{1}{x} \cdot \operatorname{sen} x + \cos x \cdot \ln x$

(13) $s(x) = 2x + \frac{1}{2x} = 2x + \frac{1}{2}x^{-1}$

$s'(x) = 2 - \frac{1}{2}x^{-2} = 2 - \frac{1}{2x^{-2}}$

(14) $f(x) = \ln(\operatorname{sen} x)$

$f'(x) = \frac{1}{\operatorname{sen} x} \cdot \cos x = \frac{\cos x}{\operatorname{sen} x} = \operatorname{cotg} x$

$y = \frac{1}{x^n} = x^{-n}$

$y = -n \cdot x^{-n-1}$

$y = -\frac{n}{x^{n+1}}$

(15) $p(x) = 1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}$

$p'(x) = -x^{-2} - 2x^{-3} - 3x^{-4} = -\frac{1}{x^2} - \frac{2}{x^3} - \frac{3}{x^4}$

(16) $f(x) = e^{3x^2} + x - 5$

$f'(x) = e^{3x^2} \cdot 6x + 1 = 6xe^{3x^2} + 1$

$\frac{d}{dx}(\sec x) = \frac{d}{dx}\frac{1}{\cos x} =$

$= \frac{0 \cdot \cos x - (-\operatorname{sen} x)}{\cos x^2} = \operatorname{tg} x \cdot \sec x$

(17) $f(x) = \sec^2 x$

$f'(x) = 2\sec x \cdot \operatorname{tg} x \cdot \sec x$

$f'(x) = 2\operatorname{tg} x \cdot \sec^2 x = \frac{2\operatorname{sen} x}{\cos^3 x}$

$\frac{d}{dx} a^x = a^x \ln a$

(18) $f(x) = x^3 \cdot 3^x$

$f'(x) = 3x^2 \cdot 3^x + x^3 \cdot 3^x \ln 3$

$= 3^x(3x^2 + x^3 \ln 3)$

(19) $f(x) = \frac{e^x + e^{-x}}{2}$

$f'(x) = \frac{(e^x - e^{-x}) \cdot \cancel{2} - 0 \cdot (e^x + e^{-x})}{\cancel{4}\,2}$

$f'(x) = \frac{e^x - e^{-x}}{2}$

(20) $f(x) = \operatorname{sen}^3(2x)$

$f'(x) = 3\operatorname{sen}^2(2x) \cdot 2\cos 2x$

$f'(x) = 6\operatorname{sen}^2 2x \cdot \cos 2x$

(7) $f(x) = \sqrt{\cos(\sec(x))} = (\cos(\sec x))^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(\cos(\sec x))^{-\frac{1}{2}} \cdot (-\operatorname{sen} x(\sec x)) \cdot \sec x \cdot \operatorname{tg} x$

$f'(x) = \frac{1}{2}(\cos(\sec x))^{-\frac{1}{2}} \cdot (-\operatorname{sen}(\sec x)) \cdot \frac{\operatorname{sen} x}{\cos^2 x}$

$f'(x) = -\frac{1}{2} \cdot \frac{\operatorname{sen}(\sec x) \cdot \operatorname{sen} x}{\sqrt{\cos(\sec x)} \cdot \cos^2 x}$

CIRCUITOS ELÉTRICOS III - LISTA EXERCÍCIOS EDO

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1-a) $\frac{dy}{dx} = \cos^2 x \cdot (\cos 2y)^2$ , $y(0) = 2$ (EDO LINEAR)

$\frac{dy}{(\cos 2y)^2} = \cos^2 x \, dx \rightarrow \int \frac{dy}{(\cos 2y)^2} = \int \cos^2 x \, dx$

$\int \cos^2 x \, dx = \frac{1}{2}x + \frac{\operatorname{sen} 2x}{4} + C$ //

FAZENDO $u = 2y$ $du = 2dy$

$\frac{1}{2}\int \frac{du}{\cos u^2} = \frac{1}{2}\int \sec^2 u \, du = \frac{1}{2} tg\, u + C = \frac{1}{2} tg\, 2y + C$

$\rightarrow \frac{1}{2} tg\, 2y = \frac{1}{2}x + \frac{\operatorname{sen} 2x}{4} + C$

$\rightarrow \frac{1}{2} \cdot 2y = tg^{-1}\left(\frac{1}{2}x + \frac{\operatorname{sen} 2x}{4} + C\right) \rightarrow y = tg^{-1}\left(\frac{1}{2}x + \frac{\operatorname{sen} 2x}{4} + C\right)$ //

Solução particular: $y(0) = tg^{-1}\left(\frac{1}{2}0 + \frac{\operatorname{sen} 2.0}{4} + C\right) = 2 \rightarrow C = tg\, 2$

$\therefore y = tg^{-1}\left(\frac{1}{2}x + \frac{\operatorname{sen} 2x}{4} + tg\, 2\right)$ //

b) $(xy^2 - x)\,dx + (2x^2y + 8y)\,dy = 0$ , $y(1) = 5$

$x \cdot (y^2 - 1)\,dx + y(2x^2 + 8)\,dy = 0$

$y(2x^2 + 8)\,dy = -x(y^2 - 1)\,dx$

$\int \frac{y\,dy}{y^2 - 1} = \int \frac{-x\,dx}{2x^2 + 8}$

FAZENDO $u = y^2 - 1$ $du = 2y\,dy$

$\int \frac{y\,dy}{y^2 - 1} = \frac{1}{2}\int \frac{u}{du} = \frac{1}{2} \ln u = \frac{\ln|y^2 - 1|}{2} + C$

FAZENDO $a = 2x^2 + 8$ $da = 4x\,dx$

$\int \frac{x\,dx}{2x^2 + 8} = \frac{1}{4}\int \frac{da}{a} = \frac{1}{4}\ln a = \frac{\ln|2x^2 + 8|}{4} + C$

Assim, $\frac{1}{2}\ln|y^2 - 1| = -\left(\frac{1}{4}\ln|2x^2 + 8| + C\right) \rightarrow \frac{1}{2}(y^2 - 1) = -\frac{1}{2}(x^2 + 4) + C$ //

$\rightarrow y^2 - 1 = -x^2 - 4 + C \rightarrow y^2 = -x^2 - 3 + C \rightarrow y = \sqrt{-x^2 - 3 + C}$

Solução particular: $y(1) = \sqrt{-1^2 - 3 + C} = 5 \rightarrow -4 + C = 25 \rightarrow C = 29$

$\therefore y = \sqrt{-x^2 - 3 + 29}$ //

d) $\frac{dy}{dx} = \frac{y \cos x}{1 + 2y^2}$ , $y(3) = 2$

$\rightarrow \int \frac{(1 + 2y^2)\,dy}{y} = \int \cos x \, dx \rightarrow \ln y + 2\frac{y^2}{2} = \operatorname{sen} x + C$ //

Solução particular $\ln 2 + 2^2 = \operatorname{sen} 3 + C$

$0{,}7 + 4 = 0{,}14 + C \rightarrow C = 4{,}56$

$\therefore \ln y + y^2 = \operatorname{sen} x + 4{,}56$

(e) $\dfrac{dy}{dx} = \dfrac{x + e^{-x}}{y + e^{+y}}$ , $y(0) = 3$

$$\int (y + e^{-y}) dy = \int (x + e^{-x}) dx \rightarrow \frac{y^2}{2} - e^{-y} = \frac{x^2}{2} - e^{-x} + C \rightarrow \frac{y^2}{2} + e^{+y} - \frac{x^2}{2} + e^{-x} - C = 0$$

Solução particular: $\dfrac{3^2}{2} + e^{+3} - \dfrac{0^2}{2} + e^{-0} - C = 0 \rightarrow C = \dfrac{9}{2} + 1 + e^{+3} = \dfrac{11}{2} + e^{+3}$

$$\therefore \frac{y^2}{2} - \frac{x^2}{2} + e^{+y} + e^{-x} - \frac{11}{2} + e^{-3}$$

(f) $\dfrac{dy}{dx} = \dfrac{x^2}{y(1+x^3)}$ , $y(1) = 2$ Fazendo $u = 1 + x^3$ $du = 3x^2\,dx$

$$\rightarrow \int y\,dy = \int \frac{x^2}{1+x^3} dx = \int \frac{\cancel{x^2}}{u} \cdot \frac{du}{3\cancel{x^2}} = \frac{1}{3}\int \frac{du}{u} = \frac{1}{3} \ln|u| + C = \frac{1}{3} \ln|1+x^3| + C$$

$$\rightarrow \frac{y^2}{2} = \frac{1}{3} \ln|1+x^3| + C \rightarrow y^2 = \frac{2}{3} \ln|1+x^3| + K \quad \text{, onde } K = 2C$$

Solução particular: $2^2 = \dfrac{2}{3} \ln|1+2^3| + K \rightarrow K = 4 - \dfrac{2}{3} \ln 7$

$$\therefore y^2 = \frac{2}{3} \ln|1+x^3| + 4 - \frac{2}{3} \ln 7 \rightarrow y = \sqrt{\frac{2}{3} \ln|1+x^3| + 4 - \frac{2}{3} \ln 7}$$

(g) $tg\,x \cdot \sec y\, dx - \tan y \cdot \sec x\, dy = 0$ $y(0) = 2$

$tg\,x \cdot \sec y\, dx = tg\,y \cdot \sec x\, dy$

$\dfrac{tg\,y}{\sec y} dy = \dfrac{tg\,x}{\sec x} dx \rightarrow \displaystyle\int sen\,y\, dy = \int sen\,x\, dx$

$$\frac{tg\,y}{\sec y} = \frac{\frac{sen\,y}{\cos y}}{\frac{1}{\cos y}} = \frac{sen\,y}{\cos y} \cdot \frac{\cos y}{1}$$

$$\frac{tg\,y}{\sec y} = sen\,y$$

$\rightarrow -\cos y = -\cos x + C \rightarrow -\cos y = \cos x - C \rightarrow y = \cos^{-1}(\cos x + C)$

Solução particular: $C = \cos x - \cos y = \cos 0 - \cos 2 \rightarrow C = 1 - \cos 2$

$$\therefore y = \cos^{-1}(\cos x + 1 - \cos 2)$$

(h) $2xy' - 10xy = 3x^2$ , $y(2) = 2$

$y' - 5y = \frac{3}{2}x$ ; FATOR INTEGRANTE: $y'\mu(x) - 5\mu(x).y = \frac{3}{2} x\, \mu(x)$ $\quad \frac{dv}{dx} = \frac{d\mu}{dx} = -5\mu(x)$

$\rightarrow \int \frac{d\mu}{\mu} = -\int 5\,dx \rightarrow \ln \mu = -5x \rightarrow \mu = e^{-5x}$

Assim,

$y'e^{-5x} - 5e^{-5x}y = \frac{3}{2}xe^{-5x}$ (Regra do Produto) $\rightarrow \frac{d(y.e^{-5x})}{dx} = \frac{3}{2}xe^{-5x}$

$\rightarrow \int d(y.e^{-5x}) = \frac{3}{2}\int xe^{-5x}dx$

$\int xe^{-5x}dx$ Por partes $\int u\,dv = uv - \int v\,du$

$u = x \quad dv = e^{-5x}$

$du = dx \quad v = -\frac{1}{5}e^{-5x}$

$\int xe^{-5x}dx = \frac{-xe^{-5x}}{5} - \int -\frac{1}{5}e^{-5x}dx$

$\rightarrow \int xe^{-5x}dx = \frac{-xe^{-5x}}{5} - \frac{e^{-5x}}{25} = \frac{-e^{-5x}(5x+1)}{25}$

$y = \frac{1}{e^{-5x}}\left[\frac{3}{2}\left(\frac{-e^{-5x}(5x+1)}{25}\right) + C\right]$

$$\boxed{y = \frac{-15x-3}{50} + \frac{C}{e^{-5x}}}$$

$y = \frac{-15x-3}{50} + K$

Solução particular $y = \frac{-30-3}{50} + \frac{C}{e^{-10}} = 2$

$\frac{+33}{50} + 2$

$\frac{33+100}{50} = \frac{133}{50}$

$C = \frac{133}{50}e^{-10}$

$$\boxed{y = \frac{-15x-3+133e^{-10}}{50}}$$

(i) $y.y' + 2y^2 = 3yx^2$ , $y(0) = 0$ $\quad A(x) = 2 \quad B(x) = 3x^2$

$y' + 2y = 3x^2$ FATOR INTERANTE $\mu(x) = e^{\int 2\,dx} = e^{2x}$

Assim, $y = \frac{1}{e^{2x}}\left[3\int x^2 \cdot e^{2x}dx + C\right]$

Integração por partes $\int u.dv = u.v - \int v.du$

FAZENDO $u = x^2 \quad dv = e^{2x}$

$du = 2x\,dx \quad v = \frac{1}{2}e^{2x}$

$\int x^2.e^{2x} = x^2.\frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x}2x\,dx$

FAZENDO $z = x \quad dw = e^{2x}$

$dz = 1dx \quad w = \frac{e^{2x}}{2}dx$

$\int x^2.e^{2x} = x^2.\frac{1}{2}e^{2x} - \left(\frac{xe^{2x}}{2} - \int \frac{e^{2x}}{2}dx\right)$

$= \frac{x^2e^{2x} - xe^{2x}}{2} + \frac{e^{2x}}{4} + K$

$y = \frac{1}{e^{2x}}\left[3\left(\frac{x^2e^{2x} - xe^{2x}}{2} + \frac{e^{2x}}{4}\right) + C\right]$

$y = \frac{1}{e^{2x}}\left[3\left(\frac{e^{2x}(2x^2-2x+1)}{4}\right) + C\right]$

$$\boxed{y = \frac{3}{4}(2x^2-2x+1) + \frac{C}{e^{2x}}}$$

Solução Particular

$0 = \frac{3}{4}(0-0+1) + \frac{C}{1} \rightarrow C = -3/4$

$$\therefore \boxed{y = \frac{3}{4}\left[2x^2 - 2x + 1 - e^{-2x}\right]}$$

(j) $y \cdot y' + 2y^2 = 3yx \quad , \quad y(0) = 0$

$y' + 2y = 3x$ FATOR INTEGRANTE $\mu(x) = e^{\int 2\,dx} = e^{2x}$

Assim, $y = \frac{1}{e^{2x}} \cdot \left[\int 3x\, e^{2x}\, dx + C\right]$

$\int x e^{2x}\, dx$ Fazendo $x = u$, $dx = du$; $dv = e^{2x}$, $v = \frac{e^{2x}}{2}$,

$\int x e^{2x} dx = x\frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} dx$

$= x\frac{e^{2x}}{2} - \frac{e^{2x}}{4}$

$y = \frac{1}{e^{2x}} \cdot \left[3\left(\frac{x e^{2x}}{2} - \frac{e^{2x}}{4}\right) + C\right]$

$y = \frac{1}{e^{2x}} \left[\frac{3e^{2x}(2x-1)}{4} + C\right]$

Solução particular: $0 = \frac{3}{4}(0-1) + \frac{C}{e^0} \to C = \frac{3}{4}$

$\boxed{y = \frac{3}{4}(2x-1) + \frac{C}{e^{2x}}}$ $\therefore$ $\boxed{y = \frac{3}{4}\left(2x - 1 + \frac{1}{e^{2x}}\right)}$

(K) $3\,dy + 7xy\,dx = 10x^2\,dx, \quad y(0) = 10$

$\frac{\cancel{3}\,dy}{\cancel{3}dx} + \frac{7xy\,\cancel{dx}}{3\cancel{dx}} = \frac{10x^2\,\cancel{dx}}{3\cancel{dx}} \to \frac{dy}{dx} + \frac{7}{3}xy = \frac{10}{3}x^2 \to y' + \frac{7}{3}xy = \frac{10}{3}x^2$

FATOR DE INTEGRAÇÃO $\mu(x) = e^{\int \frac{7}{3}x\,dx} \to \mu(x) = e^{\frac{7}{6}x^2}$

Assim, $y = \frac{1}{e^{\frac{7}{6}x^2}}\left[\int 10x^2 \cdot e^{\frac{7}{6}x^2} dx + C\right]$

$e^{\frac{7}{6}x^2} \to$ $\nexists$ tem anti-derivada elementar.