Atividade 01 - Derivadas - ~~[illegible]~~ Circuitos 3

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(3) $g(x) = (x^3-7) \cdot (2x^2+3)$

$g'(x) = (x^3-7)' \cdot (2x^2+3) + (2x^2+3)' \cdot (x^3-7)$

$g'(x) = 3x^2 \cdot (2x^2+3) + (4x) \cdot (x^3-7)$

$g'(x) = 6x^4 + 9x^2 + 4x^4 - 28x$

$\boxed{g'(x) = 10x^4 + 9x^2 - 28x}$

(4) $f(x) = \dfrac{1}{\sqrt{x^2+2x}} = (x^2+2x)^{-1/2}$

$\boxed{f'(x) = -\frac{1}{2} \cdot (2x+2) \cdot (x^2+2x)^{-1/2+1} = -(x+1) \cdot \sqrt{x^2+2x}}$

(5) $h(r) = r^2(3r^4 - 7r + 2) = 3r^6 - 7r^3 + 2r^2$

$\boxed{h'(r) = 18r^5 - 21r^2 + 4r}$

(6) $f(x) = \ln(x^2+x+1)$

$\boxed{f'(x) = \dfrac{1}{x^2+x+1} \cdot (2x+1) = \dfrac{2x+1}{x^2+x+1}}$

(7) $f(x) = \sqrt{\operatorname{cossec}(x)} = (\cos^{-1}(x))^{1/2} = [\cos(x)]^{-1/2}$

$\boxed{f'(x) = -\frac{1}{2} \cdot (-\operatorname{sen} x) \cdot (\cos x)^{-1/2+1} = \dfrac{\operatorname{sen} x \cdot \sqrt{\cos x}}{2}}$

(9) $g(z) = \dfrac{(8 - z + 3z^2)}{2-9z}$ ~~1(8-z+3z²)(…~~

$$g'(z) = \frac{(8-z+3z^2)' \cdot (2-9z) - (2-9z)' \cdot (8-z+3z^2)}{(2-9z)^2}$$

$$g'(z) = \frac{(-1+6z) \cdot (2-9z) - (-9) \cdot (8-z+3z^2)}{(2-9z)^2}$$

$$g'(z) = \frac{(-2+\cancel{9z}+12z - 54z^2) + (72 - \cancel{9z} + 27z^2)}{(2-9z)^2}$$

$$\therefore \boxed{g'(z) = \frac{-27z^2 + 12z + 70}{(2-9z)^2}}$$

$$\begin{array}{r} 54 \\ -27 \\ \hline 27 \end{array}$$

(8) $f(x) = \ln^3 x$

$f'(x) = \dfrac{1}{x} \cdot 3 \cdot \ln^2 x \Rightarrow \boxed{f'(x) = \dfrac{3\ln^2 x}{x}}$

(10) $f(x) = \ln(x^3)$

$f'(x) = 3x^2 \cdot \dfrac{1}{x^3} \Rightarrow \boxed{f'(x) = \dfrac{3}{x}}$

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(11) $f(t) = t^5 + \left(\frac{t+1}{t^2}\right)$

$f'(t) = 5t^4 + \frac{(t+1)' \cdot t^2 - (t^2)' \cdot (t+1)}{t^4}$

$f'(t) = 5t^4 + \frac{t^2 - 2t(t+1)}{t^4}$

$f'(t) = 5t^4 + \frac{t - 2(t+1)}{t^3} = 5t^4 + \frac{t - 2t + 1}{t^3}$

$\boxed{f'(t) = \frac{5t^7 - t + 1}{t^3}}$

(12) $f(x) = (\ln x) \cdot (\operatorname{sen} x)$

$f'(x) = (\ln x)' \cdot \operatorname{sen} x + (\operatorname{sen} x)' \cdot \ln x$

$\boxed{f'(x) = \frac{\operatorname{sen} x}{x} + \ln x \cdot \cos x}$

(13) $s(x) = 2x + \frac{1}{2x}$ ~~[illegible]~~

$s(x) = 2x + (2x)^{-1}$ ~~[illegible]~~

~~[illegible]~~

$s'(x) = 2 - 2 \cdot (2x)^{-2}$

~~[illegible]~~

$\boxed{s'(x) = 2 - \frac{1}{2x^2}}$

(14) $f(x) = \ln(\operatorname{sen} x)$

$$f'(x) = \frac{1}{\operatorname{sen} x} \cdot \cos x \Rightarrow \boxed{f'(x) = \frac{1}{\operatorname{tg} x}}$$

(15) $p(x) = 1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}$

$$p(x) = 1 + x^{-1} + x^{-2} + x^{-3}$$

$$\boxed{p'(x) = -x^{-2} - 2x^{-3} - 3x^{-4}}$$

(16) $f(x) = e^{3x^2} + x - 5$

$$\boxed{f'(x) = 6x \cdot e^{3x^2} + 1}$$

(17) $f(x) = \sec^2 x = \frac{1}{\operatorname{sen}^2 x} = \operatorname{sen}^{-2} x$

$$f'(x) = -2 \cdot \operatorname{sen}^{-3} x \cdot \cos x$$

$$\boxed{f'(x) = -\frac{2\cos x}{\operatorname{sen}^3 x}}$$

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(18) $f(x) = x^3 \cdot 3^x$

$f'(x) = (x^3)' \cdot 3^x + (3^x)' \cdot x^3$

$f'(x) = 3x^2 \cdot 3^x + 3^x(\ln 3) \cdot x^3$

$f'(x) = 3x^2 \cdot 3^x + 3^x \cdot x^3 \cdot \ln 3$

$\boxed{f'(x) = (3 + x\ln 3) \cdot 3^x \cdot x^2}$

(19) $f(x) = \dfrac{e^x + e^{-x}}{2}$

$\boxed{f'(x) = \frac{1}{2}(e^x - e^{-x})}$

(20) $f(x) = \text{sen}^3 2x = (\text{sen}\, 2x)^3$

$\cancel{f'(x) = 3\,\text{sen}^2 2x}$

$f'(x) = 3 \cdot (\text{sen}\, 2x)^2 \cdot \cos 2x \cdot 2$

$\boxed{f'(x) = 6\,\text{sen}^2 2x \cos 2x}$

ou

$\boxed{f'(x) = 6\,\text{sen}\, 2x \cdot \text{sen}\, 4x}$

(1) $f(x) = 6x^3 - 5x^2 + x + 9$

$\dfrac{df(x)}{dx} = 18x^2 - 10x + 1$

(2) $f(x) = \text{sen}^2 x \cos^3 x$

$f'(x) = \cos^3 x (\text{sen}^2 x)' + \text{sen}^2 x (\cos^3 x)'$

$= 2\cos^3 x \cos x\, \text{sen}\, x + 3\,\text{sen}\, x \cos^2 x (-\text{sen}^2 x)$

$= 2\,\text{sen}\, x \cos^4 x - 3\,\text{sen}^3 \cdot \cos^2 x$

Atividade 01 - EDO - Circuitos 3

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(1)

a) $\frac{dy}{dx} = (\cos x)^2 \cdot (\cos 2y)^2$

$$\int \frac{dy}{(\cos 2y)^2} = \int (\cos x)^2 \cdot dx \quad ; \quad \int \frac{dy}{(\cos 2y)^2} = \int \sec^2 2y \, dy$$

$$\int \sec^2 2y \, dy = \int \frac{\sec^2 u \, du}{2} = \frac{tg\, u}{2} + C_1 = \frac{Tg\, 2y}{2} + C_1$$

$u = 2y$

$du = 2dy \Rightarrow dy = \frac{du}{2}$

$$\int \cos^2 x \, dx = \int \left(\frac{\cos 2x}{2} + \frac{1}{2} \right) dx = \frac{sen\, 2x}{4} + \frac{x}{2} + C_2$$

$$\int \frac{\cos 2x}{2} dx = \int \frac{\cos u \, du}{4} = \frac{1}{4} sen\, u + C_2 = \frac{sen\, 2x}{4} + C_2$$

$u = 2x$

$du = 2dx \to dx = \frac{du}{2}$

$$\frac{Tg\, 2y}{2} + C_1 = \frac{sen\, 2x}{4} + \frac{x}{2} + C_2 \Rightarrow \cancel{\frac{Tg\, 2y}{2} - \frac{sen\, 2x}{4}}$$

$$\frac{Tg\, 2y}{2} - \frac{sen\, 2x}{4} - \frac{x}{2} + K = 0$$

$$y(0) = 2 \Rightarrow \frac{Tg\, 4}{2} - \frac{sen\, 0}{4} - \frac{0}{2} + K = 0 \Rightarrow K = -\frac{Tg\, 4}{2}$$

$$\boxed{\frac{Tg\, 2y}{2} - \frac{sen\, 2x}{4} - \frac{x}{2} - \frac{Tg\, 4}{2} = 0}$$

b) $(x y^2 - x)\,dx + (2x^2 y + 8y)\,dy = 0$

$y(1) = 5$

$x.(y^2-1)\,dx + 2y(x^2+4)\,dy = 0$

$x(y^2-1)\,dx = -2y(x^2+4)\,dy$

$$\int \frac{x\,dx}{x^2+4} = -\int \frac{2y\,dy}{y^2-1}$$

$$\cancel{\int \frac{du}{u^2+a^2} = \frac{1}{a} \operatorname{arctg} \frac{u}{a} + C}$$

$$\cancel{\int \frac{du}{u^2-a^2} = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right|}$$

$$\int \frac{x\,dx}{x^2+4} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2} \ln u + C_1 = \frac{1}{2} \ln|x^2+4| + C_1$$

$u = x^2+4 \rightarrow du = 2x\,dx \rightarrow x\,dx = \frac{du}{2}$

$$\int \frac{2y\,dy}{y^2-1} = \int \frac{du}{u} = \ln|u| + C_2 = \ln|y^2-1| + C_2$$

$u = y^2 - 1 \rightarrow du = 2y\,dy$

$\frac{1}{2} \ln|x^2+4| + C_1 = -\ln|y^2-1| + C_2$

$\ln|\sqrt{x^2+4}| = -\ln|y^2-1| + C_3$

$\ln\sqrt{x^2+4} + \ln(y^2-1) = K$

$\ln|\sqrt{x^2+4}\,(y^2-1)| = K$

$$\boxed{\ln|\sqrt{x^2+4}\,(y^2-1)^2| = \ln 24\sqrt{5}}$$

para $y(1) = 5$

$\ln|\sqrt{(1^2+4)}.(5^2-1)| = K$

$K = \ln|\sqrt{5}.24|$

$K = \ln 24\sqrt{5}$

d)
$$\frac{dy}{dx} = \frac{y\cos x}{1+2y^2}$$

$$\int \frac{(1+2y^2)}{y}\,dy = \int \cos x\,dx = \int\left(\frac{1}{y}+2y\right)dy = \int \cos x\,dx$$

$$\ln y + \frac{2y^2}{2} - \operatorname{sen} x = K$$

$$\cancel{\int\left(y+2y^3\right)dy = \int \cos x\,dx}$$

$$\ln y + y^2 - \operatorname{sen} x = K$$

$$\cancel{\frac{y^2}{2}+\frac{2y^4}{4}=}$$

$$y(3)=2$$

$$K = \ln 2 + 4 - \operatorname{sen} 3$$

$$\boxed{\ln y + y^2 - \operatorname{sen} x = \ln 2 + 4 - \operatorname{sen} 3}$$

e)
$$\frac{dy}{dx} = \frac{x+e^{-x}}{y+e^{y}} \Rightarrow (y+e^{y})\,dy = (x+e^{-x})\,dx$$

$$\int (y+e^{y})\,dy = \int (x+e^{-x})\,dx$$

$$\frac{y^2}{2} + e^{y} - \frac{x^2}{2} + e^{-x} = k \qquad y(0)=3$$

$$\frac{3^2}{2} + e^{3} + 1 = K = 5{,}5 + e^{3}$$

$$\boxed{\frac{y^2}{2} + e^{y} - \frac{x^2}{2} + e^{-x} = \frac{11}{2} + e^{3}}$$

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f) $\frac{dy}{dx} = \frac{x^2}{y(1+x^3)} \Rightarrow \int y\,dy = \int \frac{x^2}{1+x^3}\,dx$

$\frac{y^2}{2} = \frac{1}{3}\ln|x^3+1| + K$

$\cancel{dy \cdot y(1+x^3)}$

$\frac{y^2}{2} - \frac{1}{3}\ln|x^3+1| = K$

$y(1) = 2 \Rightarrow \frac{4}{2} - \frac{1}{3}\ln 2 = K$

$$\boxed{\frac{y^2}{2} - \ln\sqrt[3]{x^3+1} = 2 - \frac{\ln 2}{3}}$$

$\boxed{S\ C\ -S\ -C}$

g) $tg\,x\,sec\,y\,dx - tg\,y\,sec\,x\,dy = 0$

$y(0) = 2$

$\frac{sen\,x}{cos\,x} \cdot \frac{1}{cos\,y}\,dx = \frac{sen\,y}{cos\,y} \cdot \frac{dy}{cos\,x}$

$\int sen\,x\,dx = \int sen\,y\,dy \Rightarrow cos\,x - cos\,y = K$

$K = cos\,0 - cos\,2 = 1 - cos\,2$

$$\boxed{cos\,x - cos\,y = 1 - cos\,2}$$

h) $\frac{dy}{dx} - 5y = \frac{3}{2}x$

$$\mu(x) = e^{-\int 5dx} = e^{-5x}$$

$$\int x.e^{-5x}dx = -\frac{x.e^{-5x}}{5} - \frac{e^{-5x}}{25}$$

$$\cancel{y(x) = -\frac{x}{5} - \frac{1}{25} + k}$$

$$y(x) = \frac{1}{e^{-5x}} \cdot \left[-\frac{x.e^{-5x}}{5} - \frac{e^{-5x}}{25} + K\right]$$

$$y(2) = 2$$

$$2 = \frac{1}{\cancel{e^{-10}}} \cdot \left[-\frac{2.\cancel{e^{-10}}}{5} - \frac{\cancel{e^{-10}}}{25} + \frac{K.\cancel{e^{-10}}}{e^{-10}}\right]$$

$$2 = -\frac{10}{25} - \frac{1}{25} + \frac{K}{e^{-10}}$$

$$\frac{25.2}{25} + \frac{11}{25} = \frac{K}{e^{-10}} \Rightarrow \boxed{K = \frac{61.e^{-10}}{25}}$$

$$\boxed{y(x) = -\frac{x}{5} - \frac{1}{25} + \frac{61.e^{-10+5x}}{25}}$$

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i) $\cancel{y}\cdot y' + 2y^{\cancel{2}} = 3\cancel{y}x^2 \qquad y(0)=0$

$$y' + 2y = 3x^2 \qquad A(x) = 2$$

$$B(x) = 3x^2$$

$$\mu(x) = e^{\int A(x)dx} = e^{\int 2dx} = e^{2x}$$

$$y(x) = \frac{1}{\mu(x)} \cdot \left[\int \mu(x)\cdot B(x)\,dx + C\right] = \frac{3}{2}x^2 - \frac{3x}{2} + \frac{3}{4} + \frac{C}{e^{2x}}$$

$$\int \mu(x)\cdot B(x)\,dx = \int e^{2x}\cdot 3\cdot x^2\,dx = H$$

$$3\cdot\int e^{2x}\cdot x^2\,dx = \frac{3x^2}{2}e^{2x} - \frac{3}{\cancel{2}}\int e^{2x}\cdot\cancel{2}x\,dx$$

$$u = x^2 \rightarrow du = 2x\,dx$$

$$dv = e^{2x}dx \rightarrow v = \frac{1}{2}e^{2x}$$

$$\int e^{2x}x\,dx = \frac{x\cdot e^{2x}}{2} - \int \frac{1}{2}e^{2x}dx = \frac{xe^{2x}}{2} - \frac{1}{4}e^{2x}$$

$$u = x \rightarrow du = dx$$

$$dv = e^{2x}dx \rightarrow v = \frac{1}{2}\cdot e^{2x}$$

$$H = \frac{3}{2}x^2\cdot e^{2x} - \frac{3xe^{2x}}{2} + \frac{3}{4}e^{2x}$$

$$y(0) = 0 \rightarrow C = -\frac{3}{4} \Rightarrow \boxed{y(x) = \frac{3}{2}x^2 - \frac{3}{2}x + \frac{3}{4} - \frac{3}{4e^{2x}}}$$

j) $\cancel{y}\cdot y' + 2y^{\cancel{2}} = 3\cancel{y}x \qquad y(0)=0$

$$y' + 2y = 3x \qquad A(x)=2$$
$$B(x)=3x$$

$$\mu(x) = e^{\int A(x)dx} = e^{\int 2dx} = e^{2x}$$

$$\int \mu(x)\cdot B(x)dx = \int e^{2x}\cdot 3x\cdot dx = 3\int e^{2x}\cdot x\cdot dx$$

$$\int e^{2x}\cdot x\cdot dx = \frac{x\cdot e^{2x}}{2} - \int \frac{1}{2}e^{2x}\cdot dx = \frac{xe^{2x}}{2} - \frac{e^{2x}}{4}$$

$u = x \rightarrow du = dx$

$dv = e^{2x}dx \rightarrow v = \frac{1}{2}e^{2x}$

$$\rightarrow \int \mu(x)\cdot B(x)dx = \frac{x\cdot e^{2x}}{2} - \frac{e^{2x}}{4}$$

$$y(x) = \frac{1}{\mu(x)}\cdot\left[\int \mu(x)\cdot B(x)\,dx + C\right]$$

$$y(x) = \frac{1}{e^{2x}}\cdot\left[\frac{3x\cdot e^{2x}}{2} - \frac{3e^{2x}}{4} + C\right]$$

$$y(x) = \frac{3x}{2} - \frac{3}{4} + \frac{C}{e^{2x}}$$

$$y(0)=0 \Rightarrow 0 = 0 - \frac{3}{4} + \frac{C}{1} \Rightarrow C = \frac{3}{4}$$

$$\boxed{y(x) = \frac{3x}{2} - \frac{3}{4} + \frac{3}{4e^{2x}}}$$

k)

$$3dy + 7xy\,dx = 10x^2dx \qquad y(0)=10$$

$$3\frac{dy}{dx} + 7xy\frac{dx}{dx} = 10x^2\frac{dx}{dx}$$

$$3\frac{dy}{dx} + 7xy = 10x^2 \quad (\div 3)$$

$$\frac{dy}{dx} + \frac{7}{3}xy = \frac{10}{3}x^2 \qquad A(x) = \frac{7}{3}x \qquad B(x) = \frac{10}{3}x^2$$

$$\mu(x) = e^{\int \frac{7}{3}x\,dx} \Rightarrow \mu(x) = e^{\frac{7}{6}x^2}$$

$$\int \mu(x)\cdot B(x)\cdot dx = \int e^{\frac{7x^2}{6}}\cdot\frac{10}{3}x^2dx = \frac{10}{3}\cdot\int e^{\frac{7x^2}{6}}\cdot x^2dx$$

$$\int e^{\frac{7x^2}{6}}\cdot x^2\cdot dx = \frac{6}{7}x^3\cdot e^{\frac{7x^2}{6}} - \frac{12}{7}\int x^2\cdot e^{\frac{7x^2}{6}}dx$$

$$u = x^2 \rightarrow du = 2x\,dx$$

$$dv = e^{\frac{7x^2}{6}}dx \rightarrow v = \frac{6}{7}x\,e^{\frac{7x^2}{6}}$$

$$\rightarrow \int e^{\frac{7x^2}{6}}\cdot x^2dx + \frac{12}{7}\int x^2\cdot e^{\frac{7x^2}{6}}dx = \frac{6}{7}\cdot x^3\cdot e^{\frac{7x^2}{6}}$$

$$\frac{19}{7}\int e^{\frac{7x^2}{6}}\cdot x^2dx = \frac{6}{7}x^3\cdot e^{\frac{7x^2}{6}}$$

$$\int e^{\frac{7x^2}{6}}\cdot x^2dx = \frac{6}{19}x^3\cdot e^{\frac{7x^2}{6}}$$

$$y(x) = \frac{1}{u(x)} \cdot \left[\int u(x) \cdot B(x) \cdot dx + C \right]$$

$$y(x) = \frac{1}{e^{7/6 x^2}} \cdot \left[\frac{10}{3} \cdot \frac{6}{19} \cdot x^3 \cdot e^{\frac{7x^2}{6}} + C \right]$$

$$y(x) = \frac{20}{19} \cdot x^3 + \frac{C}{e^{7/6 x^2}}$$

$$y(0) = 10 \rightarrow 10 = \frac{C}{1} \Rightarrow C = 10$$

$$\boxed{y(x) = \frac{20}{19} x^3 + \frac{10}{e^{\frac{7x^2}{6}}}}$$

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