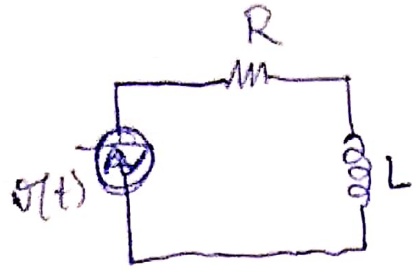
SC - S - C

# * Exemplo numérico 7

Circuito RL série: $v(t) = 100\, sen(wt+10)$

$R = 10\,\Omega$ e $L = 20mH$



LKT: $Ri + L\frac{di(t)}{dt} = v(t)$

$$\frac{di(t)}{dt} + \frac{R}{L}\, i(t) = \frac{100\, sen(wt+10)}{L}$$

$$\frac{di(t)}{dt} + \frac{10}{20.10^{-3}}\, i(t) = \frac{100\, sen(wt+10)}{20.10^{-3}}$$

$$\frac{di(t)}{dt} + \frac{10000}{20}\, i(t) = \frac{100000\, sen(wt+10)}{20}$$

Eq. caracteristica $\rightarrow$ $\frac{di(t)}{dt} + 500\, i(t) = 5000\, sen(wt+10)$

i) solução homogênea: $i_h(t) = Ke^{-500t}$

ii) Solução particular: $i_p(t) = K_1 \cos(wt+\theta) + K_2\, sen(wt+\theta)$

$$i_p(t) = K_1 \cos(wt+10) + K_2\, sen(wt+10)$$

$$\frac{di_p(t)}{dt} = -K_1 w\, sen(wt+10) + K_2 w \cos(wt+10)$$

$$-K_1 w\, sen(wt+10) + K_2 w \cos(wt+10) + 500K_1 \cos(wt+10) + 500K_2\, sen(wt+10) = 5000\, sen(wt+10)$$

$$[K_2 w + 500K_1]\cos(wt+10) + [500K_2 - K_1 w]\, sen(wt+10) = 5000\, sen(wt+10)$$

temos que: $\begin{cases} K_2 w + 500 K_1 = 0 \\ 500 K_2 - K_1 w = 5000 \end{cases}$ $\quad w = 2\pi f = 2 \cdot 3{,}14 \cdot 60$

$w = 376{,}8$

$$\begin{cases} w^2 K_2 + 500 w K_1 = 0 \\ 500^2 K_2 - 500 w K_1 = 5000 \cdot 500 \end{cases}$$

$$\begin{cases} w^2 K_2 + \cancel{500 w K_1} = 0 \\ 250000 K_2 - \cancel{500 w K_1} = 2500000 \end{cases} \quad (+)$$

$K_2 \cdot (w^2 + 250\,000) = 2.500.000 \Rightarrow K_2 = \dfrac{2{,}5 \cdot 10^6}{(376{,}8^2 + 2{,}5 \cdot 10^5)}$

$K_2 = \dfrac{2{,}5 \cdot 10^6}{391978{,}24} \Rightarrow \boxed{K_2 = 6{,}38}$

$K_1 = \dfrac{-w K_2}{500} = -\dfrac{376{,}8 \cdot 6{,}38}{500} \Rightarrow \boxed{K_1 = -4{,}81}$

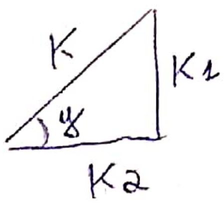
~~logo: $i_p(t) = 4{,}81 \cos(wt + 10) + K_2 \text{ sen}(w$~~

logo: $i_p(t) = -4{,}81 \cos(wt + 10) + 6{,}38 \text{ sen}(wt + 10)$

Na trigonometria temos que:

$\text{sen}(x+y) = \text{sen } x \underbrace{\cos y}_{\frac{K_2}{K}} + \underbrace{\text{sen } y}_{\frac{K_1}{K}} \cos x$

$\text{sen}(x+y) = \dfrac{K_2}{K} \text{sen } x + \dfrac{K_1}{K} \cos x \Rightarrow K \text{ sen}(x+y) = K_1 \text{sen } x + K_2 \cos x$




$\text{sen } y = \dfrac{K_1}{K} \quad \cos y = \dfrac{K_2}{K} \quad K = \sqrt{K_1^2 + K_2^2}$

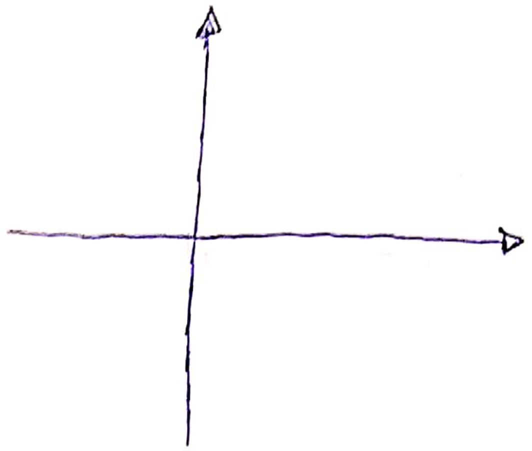
$y = \text{arctg}\left(\dfrac{K_1}{K_2}\right)$

$$i_p(t) = -4{,}81 \cdot \cos(wt+10) + 6{,}38\ \text{sen}(wt+10)$$

$$K = \sqrt{K_1^2 + K_2^2} = \sqrt{(-4{,}81)^2 + 6{,}38^2}$$

$$K = 7{,}99 \qquad \cancel{\text{arctg}\, y} \qquad y = \text{arctg}\left(\frac{K_1}{K_2}\right) = -37^\circ$$

$$\cancel{i_p(t) = 7{,}99\ \text{sen}(wt - 3}$$



$$\text{sen}\, y = \frac{K_1}{K} = \frac{-4{,}81}{7{,}99} \cong -0{,}6$$

$$\cos y = \frac{K_2}{K} = \frac{6{,}38}{7{,}99} \cong 0{,}8$$

$$i_p(t) = 7{,}99\ \text{sen}(wt + 10^\circ - 37^\circ)$$

$$\boxed{i_p(t) = 7{,}99\ \text{sen}(wt - 27^\circ)}$$

Lei de Ohm: $V_{rp}(t) = R\, i_p(t) = 79{,}9\ \text{sen}(wt - 27^\circ)$

$$\boxed{V_{rp}(t) = 79{,}9\ \text{sen}(wt - 27^\circ)}$$

Indutor: $V_{Lp}(t) = L \dfrac{d i_p(t)}{dt} = 20 \cdot 10^{-3} \cdot 7{,}99 \cdot w \cdot \cos(wt - 27^\circ)$

$$\boxed{V_{Lp}(t) = 60{,}2\ \cos(wt - 27^\circ)}$$

# * Exemplo numérico 8

$dq = C\,dV$

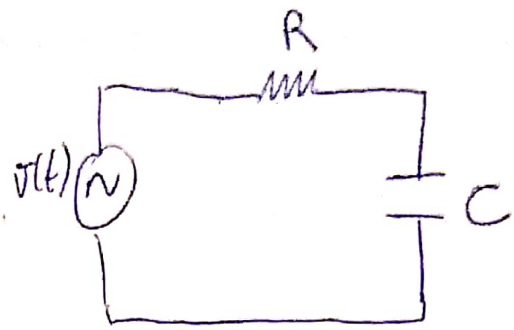
$i = \frac{dq}{dt}$

$i\,dt = C\,dV$

$V = \frac{1}{C}\int i\,dt$

$v(t) = 200\,\text{sen}(\omega t + 30^\circ)$

$R = 10\,\Omega \qquad C = 5\,mF$




$$R\,i(t) + \frac{1}{C}\int i(t)\,dt = v(t)$$

$$R\frac{di(t)}{dt} + \frac{1}{C}\cdot i(t) = \frac{dv(t)}{dt} \quad (\div R)$$

$$\frac{di(t)}{dt} + \frac{1}{RC}\,i(t) = \frac{1}{R}\cdot\frac{dv(t)}{dt} \Rightarrow \frac{di(t)}{dt} + \frac{1}{5\cdot 10^{-2}}\,i(t) = \frac{1}{10}\cdot 200\,\frac{d}{dt}\left[\text{sen}(\omega t + 30^\circ)\right]$$

$$\frac{di(t)}{dt} + 20\,i(t) = 20\cdot\omega\cdot\cos(\omega t + 30^\circ)$$

$$\frac{di(t)}{dt} + 20\,i(t) = 7540\cos(\omega t + 30^\circ)$$

$$i_p(t) = K_1\cos(\omega t + 30^\circ) + K_2\,\text{sen}(\omega t + 30^\circ)$$

$$\frac{di_p(t)}{dt} = -K_1\omega\cdot\text{sen}(\omega t + 30^\circ) + K_2\omega\cdot\cos(\omega t + 30^\circ)$$

$$\rightarrow -K_1\omega\,\text{sen}(\omega t + 30^\circ) + K_2\omega\cos(\omega t + 30^\circ) + 20K_1\cos(\omega t + 30^\circ) + 20K_2\,\text{sen}(\omega t + 30^\circ) = 7540\cos(\omega t + 30^\circ)$$

$$(20K_2 - K_1\omega)\,\text{sen}(\omega t + 30^\circ) + (K_2\omega + 20K_1)\cos(\omega t + 30^\circ) = 7540\cos(\omega t + 30^\circ)$$

$$\begin{cases} -377K_1 + 20K_2 = 0 & (\times 20) \\ 377K_2 + 20K_1 = 7540 & (\times 377) \end{cases}$$

$$\begin{cases} \cancel{-377.20K_1} + 400K_2 = 0 \\ \cancel{20.377K_1} + 377^2K_2 = 7540.377 \end{cases} \; (+)$$

$$K_2(400 + 377^2) = 377.7540$$

$$\boxed{K_2 = 19,94} \rightarrow K_1 = \frac{20K_2}{377} \rightarrow \boxed{K_1 = 1,058}$$

$$K = \sqrt{K_1^2 + K_2^2} = \sqrt{19,94^2 + 1,058^2}$$

$$\boxed{K = 19,96} \qquad \alpha = \operatorname{arctg}\left(\frac{1,058}{19,94}\right) =$$

$$\boxed{i_p(t) = 19,96\,\text{sen}(\omega t + 33,03^\circ)}$$

$$v_{rp}(t) = R.i_p(t) \Rightarrow \boxed{v_{rp}(t) = 199,6\,\text{sen}(\omega t + 33,03^\circ)}$$

$$v_{cp}(t) = \frac{1}{C}\int i_p(t)\,dt = \frac{19,96}{5.10^{-3}}\int \text{sen}(\omega t + 33,03^\circ)\,dt$$

$$\boxed{v_{cp}(t) = -10,59\cos(\omega t + 33,03^\circ)}$$

\# Outra maneira de se determinar a tensão do capacitor:

$$v_{Rp} = R\,i(t) = RC\frac{dv_{cp}}{dt}$$

$$v_{cp} + v_{Rp} = v(t)$$

$$v_{cp} = \frac{1}{C}\int i_p(t)\,dt$$

$$i_p(t) = C\frac{dv_{cp}}{dt}$$

Temos então:

$$v_{cp} + RC\frac{dv_{cp}}{dt} = v(t)$$

$$\frac{dv_{cp}}{dt} + \frac{v_{cp}}{RC} = \frac{v(t)}{RC}$$

$$\frac{dv_{cp}}{dt} + \frac{1}{10.5.10^{-3}} \cdot v_{cp} = \frac{\overset{40}{\cancel{200}}[\text{sen}(wt+30°)]}{5.10^{-3}}$$

$$\frac{dv_{cp}}{dt} + 20v_{cp} = 4000\,\text{sen}(wt+30°)$$

$$v_{cp}(t) = K_1\cos(wt+30°) + K_2\,\text{sen}(wt+30°)$$

$$\cancel{\frac{dv_{cp}(t)}{dt} = -K_1w(}$$

$$\frac{dv_{cp}(t)}{dt} = -K_1w\,\text{sen}(wt+30°) + K_2w\cos(wt+30°)$$

$$\to -K_1w\,\text{sen}(wt+30°) + K_2w\cos(wt+30°) + 20K_1\cos(wt+30°) + 20K_2\,\text{sen}(wt+30°) = 4000\,\text{sen}(wt+30°)$$

$$(20K_2 - K_1w)\,\text{sen}(wt+30°) + (K_2w + 20K_1)\cos(wt+30°) = 4000\,\text{sen}(wt+30°)$$

$$\begin{cases} 20K_2 - K_1w = 4000 \\ K_2w + 20K_1 = 0 \end{cases} \Rightarrow \begin{cases} 20K_2 - 377K_1 = 4000 & (\times 20) \\ 377K_2 + 20K_1 = 0 & (\times 377) \end{cases}$$

$$\Rightarrow \begin{cases} 400K_2 - \cancel{7540K_1} = 80000 \\ 142129K_2 + \cancel{7540K_1} = 0 \end{cases} \; (+)$$

$$142529K_2 = 80000 \Rightarrow \boxed{K_2 = 0{,}56} \to \boxed{K_1 = -10{,}55}$$

Então : $K = \sqrt{0{,}56^2 + (10{,}55)^2}$

$$\boxed{K = 10{,}56}$$

$$\alpha = \text{arctg}\left(\frac{-10{,}55}{0{,}56}\right)$$

$$\boxed{\alpha = -87^\circ}$$

$$v_{cp}(t) = 10{,}56\ \text{sen}(wt - 57^\circ)$$

$$v_{cp}(t) = 10{,}56 \cos(wt - 147^\circ)$$

$$\begin{array}{r} 180 \\ -147 \\ \hline 33 \end{array}$$

$$\boxed{v_{cp}(t) = -10{,}56 \cos(wt + 33^\circ)}$$

↳ Conforme resultado anterior.