

Atividade 01 - EDO - Exercícios 3

Thiêmala Eloisa Pimenta

④

$$a) \frac{dy}{dx} = (\cos x)^2 \cdot (\cos 2y)^2$$

$$\int \frac{dy}{(\cos 2y)^2} = \int (\cos x)^2 dx \quad ; \quad \int \frac{dy}{(\cos 2y)^2} = \int \sec^2 2y dy$$

$$\int \sec^2 2y dy = \int \frac{\sec^2 u du}{2} = \frac{\tan u}{2} + C_1 = \frac{\tan 2y}{2} + C_1$$

$$u = 2y \\ du = 2dy \Rightarrow dy = \frac{du}{2}$$

$$\int \cos^2 x dx = \int \left(\frac{\cos 2x}{2} + \frac{1}{2} \right) dx = \frac{\sin 2x}{4} + \frac{x}{2} + C_2$$

$$\int \frac{\cos 2x}{2} dx = \int \frac{\cos u du}{4} = \frac{1}{4} \sin u + C_2 = \frac{\sin 2x}{4} + C_2$$

$$u = 2x \\ du = 2dx \Rightarrow dx = \frac{du}{2}$$

$$\Rightarrow \frac{\tan 2y}{2} + C_1 = \frac{\sin 2x}{4} + \frac{x}{2} + C_2 \Rightarrow \frac{\tan 2y}{2} - \frac{\sin 2x}{4} - \frac{x}{2} + K = 0$$

$$\frac{\tan 2y}{2} - \frac{\sin 2x}{4} - \frac{x}{2} + K = 0$$

$$y(0) = 2 \Rightarrow \frac{\tan 4}{2} - \frac{\sin 0}{4} - \frac{0}{2} + K = 0 \Rightarrow K = -\frac{\tan 4}{2}$$

$$\boxed{\frac{\tan 2y}{2} - \frac{\sin 2x}{4} - \frac{x}{2} - \frac{\tan 4}{2} = 0}$$

b) $(xy^2 - x)dx + (2x^2y + 8y)dy = 0$

$$y(1) = 5$$

$$x.(y^2-1)dx + 2y(x^2+4)dy = 0$$

$$x(y^2-1)dx = -2y(x^2+4)dy$$

$$\int \frac{x dx}{x^2 + 4} = - \int \frac{2y dy}{y^2 - 1}$$

$$\frac{du}{u^2 + a^2} = \frac{1}{a} \arctan \frac{u}{a} + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right|$$

$$\int \frac{x dx}{x^2+4} = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ln u + C_1 = \frac{1}{2} \ln(x^2+4) + C_1$$

$$u = x^2 + 4 \rightarrow du = 2x dx \rightarrow \cancel{dx} du = \frac{du}{2}$$

$$\int \frac{2y dy}{y^2 - 1} = \int \frac{du}{u} = \ln|u| + C_2 = \ln|y^2 - 1| + C_2 //$$

$$u = y^2 - 1 \rightarrow du = 2y dy$$

$$\rightarrow \frac{1}{2} \ln|x^2+4| + C_1 = -\ln|y^2-1| + C_2$$

$$\ln|x^2+4| = -\ln|y^2-1| + C_3$$

$$\ln \sqrt{x^2+4} + \ln(y^2-1) = K$$

$$\ln |x^2 + 4| (y^2 - 1)^2 = K$$

1 para $y(1) = 5$

$$\ln|\sqrt{1^2+4} \cdot (5^2-1)^2| = K$$

$$K = \ln |5.24|$$

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$$K = \ln 24\sqrt{5}$$

$$\ln|x^2+4|(y^2-1)^2 = \ln 2415$$

$$d) \quad \frac{dy}{dx} = \frac{y \cos x}{1+2y^2}$$

$$\int \frac{(1+2y^2)}{y} dy = \int \cos x dx = \int \left(\frac{1}{y} + 2y\right) dy = \int \cos x dx$$

$$\ln y + \frac{2y^2}{2} - \sin x = K$$

~~$$\int (y + 2y^3) dy = \int \cos x dx$$~~

$$\ln y + y^2 - \sin x = K$$

~~$$\frac{y^2}{2} + 2y^4 =$$~~

$$y(3) = 2$$

$$K = \ln 2 + 4 - \sin 3$$

$$\boxed{\ln y + y^2 - \sin x = \ln 2 + 4 - \sin 3}$$

$$e) \quad \frac{dy}{dx} = \frac{x + e^{-x}}{y + e^y} \Rightarrow (y + e^y) dy = (x + e^{-x}) dx$$

$$\int (y + e^y) dy = \int (x + e^{-x}) dx$$

$$\frac{y^2}{2} + e^y - \frac{x^2}{2} + e^{-x} = K$$

$$y(0) = 3$$

$$\frac{3^2}{2} + e^3 + 1 = K = 5,5 + e^3$$

$$\boxed{\frac{y^2}{2} + e^y - \frac{x^2}{2} + e^{-x} = \frac{11}{2} + e^3}$$

$$f) \frac{dy}{dx} = \frac{x^2}{y(1+x^3)} \Rightarrow \int y dy = \int \frac{x^2}{1+x^3} dx$$

$$\cancel{\frac{dy}{y(1+x^3)}} \quad \frac{y^2}{2} = \frac{1}{3} \ln|x^3+1| + K$$

$$\frac{y^2}{2} - \frac{1}{3} \ln|x^3+1| = K$$

$$y(1) = 2 \Rightarrow \frac{4}{2} - \frac{1}{3} \ln 2 = K$$

$$\boxed{\frac{y^2}{2} - \ln^3 \sqrt{x^3+1} = 2 - \frac{\ln 2}{3}}$$

SC-SC

$$g) \cancel{\text{VGAAX}} \quad \tan x \sec y dx - \tan y \sec x dy = 0$$

$$y(0) = 2$$

$$\cancel{\text{SC-SC}} \quad \frac{\tan x}{\cos x} \cdot \frac{1}{\cos y} dx = \frac{\tan y}{\cos y} \frac{dy}{\cos x}$$

$$\int \tan x dx = \int \tan y dy \Rightarrow \cos x - \cos y = K$$

$$K = \cos 0 - \cos 2 = 1 - \cos 2$$

$$\boxed{\cos x - \cos y = 1 - \cos 2}$$

$$h) \quad \frac{dy}{dx} - 5y = \frac{3}{2}x$$

$$u(x) = e^{-\int 5 dx} = e^{-5x}$$

$$\int x \cdot e^{-5x} dx = -\frac{x \cdot e^{-5x}}{5} - \frac{e^{-5x}}{25}$$

~~$$y(x) = -\frac{x}{5} - \frac{1}{25} + K$$~~

$$y(x) = \frac{1}{e^{-5x}} \cdot \left[-\frac{x \cdot e^{-5x}}{5} - \frac{e^{-5x}}{25} + K \right]$$

$$y(2) = 2$$

$$2 = \frac{1}{e^{-10}} \cdot \left[-\frac{2 \cdot e^{-10}}{5} - \frac{e^{-10}}{25} + \frac{K \cdot e^{-10}}{e^{-10}} \right]$$

$$2 = -\frac{10}{25} - \frac{1}{25} + \frac{K}{e^{-10}}$$

$$\frac{25 \cdot 2 + 11}{25} = \frac{K}{e^{-10}} \Rightarrow K = \frac{61 \cdot e^{-10}}{25}$$

$$y(x) = -\frac{x}{5} - \frac{1}{25} + \frac{61 \cdot e^{-10+5x}}{25}$$

$$i) \quad y \cdot y' + 2y^2 = 3yx^2 \quad y(0) = 0$$

$$y' + 2y = 3x^2$$

$$A(x) = 2$$

$$B(x) = 3x^2$$

$$u(x) = e^{\int A(x) dx} = e^{\int 2 dx} = e^{2x}$$

$$y(x) = \frac{1}{u(x)} \left[\int u(x) \cdot B(x) dx + C \right] = \frac{3}{2}x^2 - \frac{3x}{2} + \frac{3}{4} + \frac{C}{e^{2x}}$$

~~$$y(x) = \int u(x) \cdot B(x) dx = \int e^{2x} \cdot 3x^2 dx = H$$~~

$$3 \cdot \int e^{2x} \cdot x^2 dx = \frac{3x^2}{2} e^{2x} - \frac{3}{2} \int e^{2x} \cdot 2x dx$$

$$u = x^2 \rightarrow du = 2x dx$$

~~$$dV = e^{2x} dx \rightarrow \frac{dV}{dx} = e^{2x}$$~~

$$\rightarrow V = \frac{1}{2} e^{2x}$$

$$\int e^{2x} x dx = \frac{x \cdot e^{2x}}{2} - \int \frac{1}{2} e^{2x} dx = \frac{x e^{2x}}{2} - \frac{1}{4} e^{2x}$$

$$u = x \rightarrow du = dx$$

$$dV = e^{2x} dx \rightarrow V = \frac{1}{2} \cdot e^{2x}$$

$$H = \frac{3}{2} x^2 \cdot e^{2x} - \frac{3x e^{2x}}{2} + \frac{3}{4} e^{2x}$$

$$y(0) = 0 \rightarrow C = -\frac{3}{4} \Rightarrow y(x) = \frac{3}{2}x^2 - \frac{3}{2}x + \frac{3}{4} - \frac{3}{4e^{2x}}$$

$$j) \quad y \cdot y' + 2y^2 = 3y^2 x \quad y(0) = 0$$

$$y' + 2y = 3x$$

$$A(x) = 2$$

$$B(x) = 3x$$

$$\mu(x) = e^{\int A(x) dx} = e^{\int 2 dx} = e^{2x}$$

$$\int \mu(x) \cdot B(x) dx = \int e^{2x} \cdot 3x \cdot dx = 3 \int e^{2x} \cdot x \cdot dx$$

~~Do it~~

$$\int e^{2x} \cdot x \cdot dx = \frac{x \cdot e^{2x}}{2} - \int \frac{1}{2} e^{2x} \cdot dx = \frac{x e^{2x}}{2} - \frac{e^{2x}}{4}$$

$$u = x \rightarrow du = dx$$

$$dv = e^{2x} dx \rightarrow v = \frac{1}{2} e^{2x}$$

$$\rightarrow \int \mu(x) \cdot B(x) dx = \frac{x e^{2x}}{2} - \frac{e^{2x}}{4}$$

$$y(x) = \frac{1}{\mu(x)} \cdot \left[\int \mu(x) \cdot B(x) dx + C \right]$$

$$y(x) = \frac{1}{e^{2x}} \cdot \left[\frac{3x \cdot e^{2x}}{2} - \frac{e^{2x}}{4} + C \right]$$

$$y(x) = \frac{3x}{2} - \frac{1}{4} + \frac{C}{e^{2x}}$$

$$y(0) = 0 \Rightarrow 0 = 0 - \frac{1}{4} + \frac{C}{1} \Rightarrow C = \frac{1}{4}$$

$$\boxed{y(x) = \frac{3x}{2} - \frac{1}{4} + \frac{1}{4e^{2x}}}$$

k)

$$3dy + 7xydx = 10x^2dx$$

$$y(0) = 10$$

$$3\frac{dy}{dx} + 7xy\frac{dx}{dx} = 10x^2\frac{dx}{dx}$$

$$3\frac{dy}{dx} + 7xy = 10x^2 \quad (\div 3)$$

$$\frac{dy}{dx} + \frac{7}{3}xy = \frac{10}{3}x^2$$

$$A(x) = \frac{7}{3}x$$

$$B(x) = \frac{10}{3}x^2$$

$$u(x) = e^{\int \frac{7}{3}x dx} \Rightarrow u(x) = e^{\frac{7}{6}x^2}$$

$$\int u(x) \cdot B(x) \cdot dx = \int e^{\frac{7}{6}x^2} \cdot \frac{10}{3}x^2 dx = \frac{10}{3} \cdot \int e^{\frac{7}{6}x^2} \cdot x^2 dx$$

$$\int e^{\frac{7}{6}x^2} \cdot x^2 dx = \frac{6}{7}x^3 \cdot e^{\frac{7}{6}x^2} - \frac{12}{7} \int x^2 \cdot e^{\frac{7}{6}x^2} dx$$

$$u = x^2 \rightarrow du = 2x dx$$

$$dv = e^{\frac{7}{6}x^2} dx \rightarrow v = \frac{6}{7}x e^{\frac{7}{6}x^2}$$

$$\rightarrow \int e^{\frac{7}{6}x^2} \cdot x^2 dx + \frac{12}{7} \int x^2 \cdot e^{\frac{7}{6}x^2} dx = \frac{6}{7} \cdot x^3 \cdot e^{\frac{7}{6}x^2}$$

$$\frac{19}{7} \int e^{\frac{7}{6}x^2} \cdot x^2 dx = \frac{6}{7} x^3 \cdot e^{\frac{7}{6}x^2}$$

$$\int e^{\frac{7}{6}x^2} \cdot x^2 dx = \frac{6}{19} x^3 \cdot e^{\frac{7}{6}x^2}$$

$$y(x) = \frac{1}{u(x)} \cdot \left[\int u(x) \cdot B(x) \cdot dx + C \right]$$

$$y(x) = \frac{1}{e^{\frac{7}{6}x^2}} \cdot \left[\frac{10}{3} \cdot \frac{6}{19} \cdot x^3 \cdot e^{\frac{7}{6}x^2} + C \right]$$

$$y(x) = \frac{20}{19} x^3 + \frac{C}{e^{\frac{7}{6}x^2}}$$

$$y(0) = 10 \rightarrow 10 = \frac{C}{1} \Rightarrow C = 10$$

$$y(x) = \frac{20}{19} x^3 + \frac{10}{e^{\frac{7}{6}x^2}}$$