Atividade 02 - Circuitos 3 - Thêndala Eloísa Pimenta

1) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 3y(x) = 4x^2 - 2x + 6$ $\quad y(0)=1$, $y'(0)=2$

$r^2 + 5r - 3 = 0$

$\Delta = 25 + 12 = 37$

$r = \frac{-5 \pm \sqrt{37}}{2} = \begin{cases} r_1 = \frac{-5+\sqrt{37}}{2} \\ r_2 = \frac{-5-\sqrt{37}}{2} \end{cases}$

$y_H(x) = C_1 . e^{\frac{-5+\sqrt{37}}{2}x} + C_2 e^{\frac{-5-\sqrt{37}}{2}x}$

$\begin{array}{r} 170 \\ +\ 54 \\ 24 \\ \hline 248 \end{array}$ $\quad \begin{array}{r} 34 \\ \times 5 \\ \hline 170 \end{array}$

$y_p(x) = Ax^2 + Bx + C$

$y_p' = 2Ax + B$

$y_p'' = 2A$

$y_p'' + 5y_p' - 3y_p = 4x^2 - 2x + 6$

$2A + 10Ax + 5B - 3Ax^2 - 3Bx - 3C = 4x^2 - 2x + 6$

$-3Ax^2 + 10Ax - 3Bx + 2A + 5B - 3C = 4x^2 - 2x + 6$

$\boxed{A = -\frac{4}{3}}$

$2A + 5B - 3C = 6$

$2.\left(-\frac{4}{3}\right) - 5.\frac{34}{9} - 3C = 6$

$3C = -\frac{8.3}{3.3} - \frac{170}{9} - \frac{6.9}{9} = \frac{-24-170-54}{9} = -\frac{248}{9} \rightarrow \boxed{C = -\frac{248}{27}}$

$10.\left(-\frac{4}{3}\right) - 3B = -2 \Rightarrow 3B = -\frac{40}{3} + \frac{2.3}{3}$

$3B = -\frac{34}{3} \rightarrow \boxed{B = -\frac{34}{9}}$

$\begin{array}{r} 248 \\ 27 \\ \hline 275 \end{array}$

$y(x) = C_1 . e^{\frac{-5+\sqrt{37}}{2}x} + C_2 e^{\frac{-5-\sqrt{37}}{2}x} - \frac{4}{3}x^2 - \frac{34}{9}x - \frac{248}{27}$

$y(0) = C_1 + C_2 - \frac{248}{27} = 1 \Rightarrow C_1 + C_2 = \frac{27}{27} + \frac{248}{27} \Rightarrow C_1 + C_2 = \frac{275}{27}$

$y'(x) = \left(\frac{-5+\sqrt{37}}{2}\right) . C_1 . e^{\frac{-5+\sqrt{37}}{2}x} - \frac{(5+\sqrt{37})}{2} C_2 . e^{\frac{-5-\sqrt{37}}{2}x} - \frac{8}{3}x - \frac{34}{9}$

$y'(0) = \frac{(-5+\sqrt{37})}{2}C_1 - \frac{(5+\sqrt{37})}{2}C_2 - \frac{34}{9} = 2$

$$\left(-\frac{5+\sqrt{37}}{2}\right)C_1 - \frac{(5+\sqrt{37})}{2}C_2 - \frac{34}{9} = \frac{2\cdot 9}{9}$$

$$\begin{array}{r} 34 \\ 18 \\ \hline 52 \end{array} \qquad \begin{array}{r} 52 \\ \times 2 \\ \hline 104 \end{array}$$

$$\frac{(-5+\sqrt{37})C_1 - (5+\sqrt{37})C_2}{2} = \frac{18}{9}+\frac{34}{9} = \frac{52}{9}$$

$$\begin{cases} (-5+\sqrt{37})C_1 - (5+\sqrt{37})C_2 = \dfrac{104}{9} \\ C_1 + C_2 = \dfrac{275}{27} \qquad \times(5+\sqrt{37}) \end{cases}$$

$$\begin{array}{r} 104 \\ \times 3 \\ \hline 312 \end{array} \qquad \begin{array}{r} 275 \\ \times 5 \\ \hline 1375 \end{array} \qquad \begin{array}{r} 1375 \\ +312 \\ \hline 1687 \end{array}$$

$$\begin{cases} (-\cancel{5}+\sqrt{37})C_1 - \cancel{(5+\sqrt{37})C_2} = \dfrac{104\cdot 3}{9\cdot 3} \\ (\cancel{5}+\sqrt{37})C_1 + \cancel{(5+\sqrt{37})}C_2 = \dfrac{275}{27}\times(5+\sqrt{37}) \end{cases}$$

$$2\sqrt{37}\,C_1 = \frac{312}{27} + \frac{275(5+\sqrt{37})}{27} = \frac{1687+275\sqrt{37}}{27}$$

$$\boxed{C_1 = \frac{1687+275\sqrt{37}}{54\sqrt{37}}}$$

$$C_2 = \frac{275}{27} - C_1 = \frac{275}{27}\cdot\frac{2\sqrt{37}}{2\sqrt{37}} - \frac{(1687+275\sqrt{37})}{54\sqrt{37}}$$

$$\boxed{C_2 = \frac{275\sqrt{37}-1687}{54\sqrt{37}}}$$

$$\boxed{y(x) = \frac{(1687+275\sqrt{37})\cdot e^{\frac{-5+\sqrt{37}}{2}x} + (275\sqrt{37}-1687)e^{-\frac{(5+\sqrt{37})}{2}x}}{54\sqrt{37}} - \frac{4}{3}x^2 - \frac{34}{9}x - \frac{248}{27}}$$

(2) $4\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y(x) = 4x+1$

$y(0)=3$

$y'(0)=1$

$4r^2 - 3r + 2 = 0$ $\quad r = \frac{3}{8} \pm \frac{\sqrt{23}}{8} i$

$\Delta = 9 - 32 = -23$

$r = a \pm bi$

$\begin{array}{r} 32 \\ -9 \\ \hline 23 \end{array}$

$y_H(x) = e^{ax} \cdot (C_1 \cos bx + C_2 \operatorname{sen} bx)$

$y_H(x) = e^{\frac{3}{8}x} \cdot \left(C_1 \cos \frac{\sqrt{23}}{8} x + C_2 \operatorname{sen} \frac{\sqrt{23}}{8} x\right)$

$y_p(x) = Ax + B$ $\quad 4.0 - 3.A + 2Ax + 2B = 4x + 1$

$y'_p(x) = A$ $\quad 2Ax - 3A + 2B = 4x + 1$

$y''_p(x) = 0$ $\quad \boxed{A=2}$ $\quad -6 + 2B = 1$

$\boxed{B = 7/2}$

$$\boxed{y(x) = e^{3/8 x} \cdot \left(C_1 \cos \frac{\sqrt{23}}{8} x + C_2 \operatorname{sen} \frac{\sqrt{23}}{8} x\right) + 2x + \frac{7}{2} B}$$

3

$$2\frac{d^2x}{dx^2} + 5\frac{dx}{dx} + 3y(x) = 3e^{5x} + 2e^{3x} \quad ; \quad y(0) = 5$$

$$y'(0) = 2$$

$$\boxed{y(x) = \frac{297}{130}e^{x} + \frac{133}{65}e^{-\frac{3}{2}x} + \frac{e^{5x}}{18} + \frac{e^{3x}}{26}}$$

$$2r^2 + 5r + 3 = 0$$

$$\Delta = 25 - 24 = 1$$

$$r = \frac{-5 \pm 1}{4} = \begin{cases} r_1 = 1 \\ r_2 = -\frac{3}{2} \end{cases}$$

$$y_H(x) = C_1 e^{x} + C_2 e^{-3/2 x}$$

$$y_p(x) = Ae^{5x} + Be^{3x}$$

$$y'_p(x) = 5Ae^{5x} + 3Be^{3x}$$

$$y''_p(x) = 25Ae^{5x} + 9Be^{3x}$$

$$2.(25Ae^{5x} + 9Be^{3x}) + 5.(5Ae^{5x} + 3Be^{3x}) + 3(Ae^{5x} + Be^{3x}) = 3e^{5x} + 2e^{3x}$$

$$50Ae^{5x} + 18Be^{3x} + 25Ae^{5x} + 15Be^{3x} + 3Ae^{5x} + 3Be^{3x} = 3e^{5x} + 2e^{3x}$$

$$78Ae^{5x} + 36Be^{3x} = 3e^{5x} + 2e^{3x}$$

$$78A = 3 \Rightarrow \boxed{A = \frac{3}{78}} \quad \boxed{B = \frac{1}{18}}$$

$$\boxed{A = \frac{1}{26}}$$

78 | 3, 18 | 26 ; 18 + 15 + 3 = 36 ; 172 − 39 = 133 ; 26 × 5 = 130 ; 172 × 5 = 860 ; 133 × 2 = 266 ; 13 × 5 = 65 ; 860 − 266 = 594 ; 594 | 2 = 297 ; 26 × 5 = 130 ; 52 − 13 = 39

$$y(x) = C_1 e^{x} + C_2 e^{-3/2 x} + \frac{e^{5x}}{18} + \frac{e^{3x}}{26}$$

$$y(0) = C_1 + C_2 + \frac{1}{18} + \frac{1}{26} = 5 \Rightarrow C_1 + C_2 = \frac{130}{26} - \frac{2}{26} - \frac{1}{26} = \frac{127}{26}$$

$$C_1 + C_2 = \frac{172}{26}$$

$$y'(x) = C_1 e^{x} - \frac{3}{2}C_2 e^{-3/2 x} + 5\frac{e^{5x}}{18} + 3\frac{e^{3x}}{26}$$

$$y'(0) = C_1 - \frac{3}{2}C_2 + \frac{5.2}{2.18} + \frac{3}{26} = \frac{2.26}{26}$$

$$C_1 - \frac{3}{2}C_2 = \frac{52 - 3 - 10}{26}$$

$$\begin{cases} -\cancel{C_1} + \frac{3}{2}C_2 = \frac{-39}{26} \\ \cancel{C_1} + \frac{2C_2}{2} = \frac{172}{26} \end{cases}$$

$$\frac{5}{2}C_2 = \frac{133}{26} \Rightarrow \boxed{C_2 = \frac{133}{65}}$$

$$C_1 = \frac{172.5}{26.5} - \frac{133.2}{65.2} = \frac{594}{2.26.5} = \frac{297}{26.5} \quad \boxed{C_1 = \frac{297}{130}}$$

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(4) $\frac{d^2}{dx^2} + \frac{2dy}{dx} - 5y(x) = \operatorname{sen}4x + 2\cos 4x$

$r^2 + 2r - 5 = 0$

$\Delta = 4 + 20 = 24$

$r = \frac{-2 \pm 2\sqrt{6}}{2} = -1 \pm \sqrt{6} = \begin{cases} r_1 = -1+\sqrt{6} \\ r_2 = -1-\sqrt{6} \end{cases}$

$\begin{array}{r|l} 24 & 2 \\ 12 & 2 \\ 6 & 2 \\ 3 & 3 \\ 1 & \end{array}$

$y_H(x) = C_1 e^{(-1+\sqrt{6})x} + C_2 e^{(-1-\sqrt{6})x}$

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$y_p(x) = A\operatorname{sen}4x + B\cos 4x$

$y'_p(x) = 4A\cos 4x + 4B(-\operatorname{sen}4x) = 4A\cos 4x - 4B\operatorname{sen}4x$

$y''_p(x) = 16A(-\operatorname{sen}4x) - 16B\cos 4x = -16A\operatorname{sen}4x - 16B\cos 4x$

$y''_p + 2y'_p - 5y_p = \operatorname{sen}4x + 2\cos 4x$

$21 \times 21 = 441$, $21 \times 8 = 168$

$-16A\operatorname{sen}4x - 16B\cos 4x + 8A\cos 4x - 8B\operatorname{sen}4x$
$-5A\operatorname{sen}4x - 5B\cos 4x = \operatorname{sen}4x + 2\cos 4x$

$-(21A + 8B)\operatorname{sen}4x + (8A - 21B)\cos 4x = \operatorname{sen}4x + 2\cos 4x$

$\begin{cases} 21A + 8B = -1 & \times 8 \\ 8A - 21B = 2 & \times 21 \end{cases} \Rightarrow \begin{cases} 168A + 64B = -8 \\ -168A + 441B = -42 \end{cases}$

$441 + 64 = 505$; $210 - 202 = 8$

$505B = -50 \Rightarrow \boxed{B = -\frac{10}{101}}$

$8A = 2 + 21B = 2 - \frac{210}{101} = \frac{202-210}{101} = \frac{-8}{101} \Rightarrow \boxed{A = -\frac{1}{101}}$

$\boxed{y(x) = C_1 e^{(-1+\sqrt{6})x} + C_2 e^{(-1-\sqrt{6})x} - \frac{\operatorname{sen}4x}{101} - \frac{10\cos 4x}{101}}$

⑥ $8\frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 3y(x) = 5x + 6$

$\begin{array}{r} 12 \\ \times 8 \\ \hline 96 \end{array}$

$8r^2 + 10r + 3 = 0$

$\Delta = 100 - 96 = 4$

$r = -\frac{10 \pm 2}{2 \cdot 8} = \frac{5 \pm 1}{8} = \begin{cases} r_1 = 6/8 = 3/4 \\ r_2 = 4/8 = 1/2 \end{cases}$

$y_H(x) = C_1 e^{3/4 x} + C_2 e^{1/2 x}$

$y_p(x) = Ax + B \rightarrow y'_p(x) = A \rightarrow y''_p(x) = 0$

$\begin{array}{r} 4 \\ \times 8 \\ \hline 32 \end{array}$

$8 \cdot 0 + 10 \cdot A + 3Ax + 3B = 5x + 6$

$3Ax + 10A + 3B = 5x + 6 \Rightarrow \boxed{A = \frac{5}{3}}$

$10A + 3B = 6 \Rightarrow \frac{50}{3} + 3B = \frac{6 \cdot 3}{3} \Rightarrow 3B = \frac{18}{3} - \frac{50}{3}$

$\boxed{B = -\frac{32}{9}}$

$\boxed{y(x) = C_1 e^{3/4 x} + C_2 e^{1/2 x} + \frac{5}{3}x - \frac{32}{9}}$

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(8) $\frac{d^2y}{dx^2} + 10y(x) = 10e^x$

~~$Ae^x + B$~~

~~Ae^x~~

~~Axe^x~~

$r^2 + 10r = 0$

$\Delta = 10^2$

$r = \frac{-10 \pm 10}{2} = -5 \pm 5 = \begin{cases} r_1 = 0 \\ r_2 = -10 \end{cases}$

$y_H(x) = C_1 + C_2 e^{-10x}$

$y_p(x) = Ae^x \rightarrow y_p'(x) = Ae^x \rightarrow y_p''(x) = Ae^x$

$Ae^x + 10Ae^x = 10e^x \Rightarrow 11Ae^x = 10e^x$

~~$A + 10 = 10 \Rightarrow A = 0$ impossível.~~ $\hookrightarrow \boxed{A = \frac{10}{11}}$

~~$y_p(x) = Axe^x \rightarrow y_p'(x) = Ae^x + Axe^x \rightarrow y_p''(x) = 2Ae^x + Axe^x$~~

~~$2Ae^x + Axe^x + 10Ae^x + 10Axe^x = 10e^x$~~

$$\boxed{y(x) = C_1 + C_2 e^{-10x} + \frac{10}{11}e^x}$$

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9

$$\frac{d^2y}{dx^2} = 3x - 5 \rightarrow$$

~~ANS~~

$$\frac{3x^2}{2} - 5x$$

$$\frac{\cancel{3}x^3}{2\cdot\cancel{3}} - \frac{5x^2}{2}$$

$$\boxed{\frac{x^3}{2} - \frac{5x^2}{2} + K}$$

(10) $\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} = 3y(x) + 2x^2 + 3$

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 3y(x) = 2x^2 + 3$$

$r^2 + 5r - 3 = 0 \qquad r = \dfrac{-5 \pm 4}{2} \Rightarrow \begin{cases} r_1 = -1/2 \\ r_2 = -9/2 \end{cases}$

$\Delta = 25 - 9 = 16 \qquad y_H(x) = C_1 e^{-1/2 x} + C_2 e^{-9/2 x}$

$y_p(x) = Ax^2 + Bx + C \qquad \cancel{2A + 10Ax + 5B - 3Ax^2 + B = 2x^2 + 3}$

$y'_p(x) = 2Ax + B \qquad \cancel{2A + B = 3}$

$\cancel{10A = 0}$

$y''_p(x) = 2A \qquad \cancel{-3A = 2}$

$$2A + 10Ax + 5B - 3Ax^2 - 3Bx - C = 2x^2 + 3$$

$\boxed{A = -\dfrac{3}{2}} \qquad 10A = 3B \quad \boxed{B = -\dfrac{1}{20}} \qquad 2A + 5B - C = 3$

$-\dfrac{12}{4} - \dfrac{1}{4} - C = \dfrac{12}{4} \Rightarrow \boxed{C = -\dfrac{25}{4}}$

$$\boxed{y(x) = C_1 e^{-1/2 x} + C_2 e^{-9/2 x} - \frac{3}{2}x^2 - \frac{x}{20} - \frac{25}{4}}$$