CMPSC 623 Problem Set 2
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Due: September 27, 2007, before class.

Problem 1. Solve the following recurrences:

1. $T(n) = T(n-2) + n$
2. $T(n) = T(n-1) + n$
3. $T(n) = 2T(n/2) + n^3$
4. $T(n) = 16T(n/4) + n^3$
5. $T(n) = 2T(n/16) + \sqrt{n}$

Problem 2. A sorting algorithm is thought of as stable if all equal elements are in the same relative order in the sorted sequence as in the original sequence. Which of insertion sort, quick sort, and merge sort are stable, and which are unstable? Give a simple fix to make the unstable sorts stable.

Problem 3. Use the version of the master theorem stated in Exercise 4.4-2 (page 84) to solve the recurrence $T(n) = 2T(n/2) + n \lg n$. (You need not prove Exercise 4.4-2.)

Problem 4. Suppose we run QuickSort (not randomized version) on an array of size n. Assume that each recursive partition function call divides its input array of size k into two sub-arrays, one with size $\lfloor \frac{k}{20} \rfloor$ and the other one with size $\lceil \frac{19k}{20} \rceil - 1$. What is the running time (as a function of n) of this QuickSort? Why?

Problem 5. Problem 28.2-6 on page 741.

Problem 6. (Divide-and-Conquer) Suppose that you can multiply two 4×4 matrices using k multiplications, then do the following:

1. Design a divide-and-conquer algorithm to multiply two $n \times n$ matrices.
2. In addition, suppose that you want to beat Strassen's algorithm by showing that your algorithm's running time $T(n)$ is $o(n^{lg7})$, then what is the largest k you can choose? Recall that, as we discussed in class, the larger k is, the longer time your algorithm will take.

Problem 7. (Probabilistic Analysis) Use indicator random variables to compute the expected value of the sum of n dice.

Problem 8. (Probabilistic Analysis) Page 118, Problem 5-2. Only do part (a), (b), (c).

Problem 9. Give a $\Theta(n)$-time, randomized procedure that takes as input an array $A[1...n]$ and performs a random permutation on the array elements. You need to write pseudo code.

Problem 10. Page 160, Problem 7-2. Only do part (a), (b), (c), and (e). Note, you need to use substitution method for (e), and you can use the result in part (d) without proof.