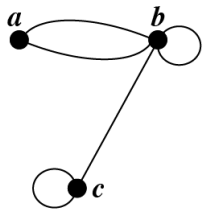
Section 10.4 Homework

1. Consider the graph G below. Let e_1 and e_2 be the two edges with endpoints a and b. Also, let $e_3 = \{b, b\}$, $e_4 = \{b, c\}$, and $e_5 = \{c, c\}$.



   (a) List all paths of length 2 from a to a.

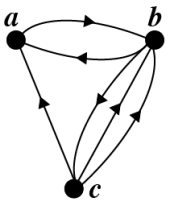
   (b) List all paths of length 3 from a to c.

   (c) Write down the adjacency matrix A for the graph G using the vertex order $V = \{a, b, c\}$.

   (d) Compute A^2, A^3.

   (e) Use the previous part to compute the number of paths of length 2 from a to b, and the number of paths of length 3 from a to a. Compare with your answers in parts (a) and (b).

2. Consider the directed graph G below. Label the edges as follows: Let $e_1 = (a, b)$, $e_2 = (b, a)$, $e_3 = (b, c)$, $e_4 = (c, a)$, and let e_5 and e_6 be the two edges from c to b.



   (a) List all paths of length 3 from b to c.

   (b) List all paths of length 3 from c to b.

   (c) Write down the adjacency matrix A for the graph G using the vertex order $V = \{a, b, c\}$.

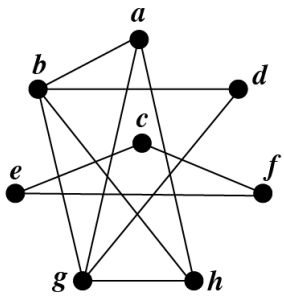
   (d) Compute A^2, A^3.

   (e) Use the previous part to compute the number of paths of length 3 from b to c, and the number of paths of length 3 from c to b. Compare with your answers in parts (a) and (b).

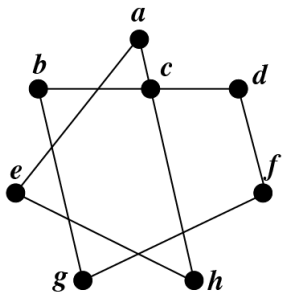
3. For each graph below, determine if the graph is connected. Then identify the connected components of the graph.

   A digraph is strongly connected if every vertex is reachable from every other following the directions of the arcs.
   A digraph is weakly connected if when considering it as an undirected graph it is connected.

   (a)

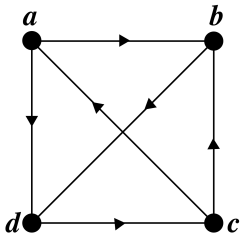


   (b)

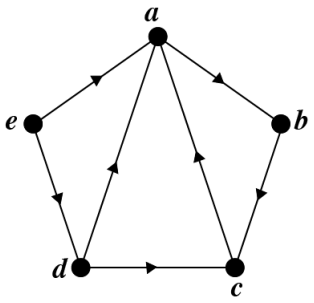


4. For each graph below, is the graph weakly connected? strongly connected?

(a)



(b)



5. (*Optional*) Consider K_{10} with vertices labeled with $a_1, a_2, \ldots, a_{10}$. How many different subgraphs of K_{10} are there which have exactly two connected components, each of which is isomorphic to the cycle graph C_5?

Answers:

1. (a) 4 paths:
   - e_1, e_1
   - e_1, e_2
   - e_2, e_1
   - e_2, e_2

   (b) 4 paths:
   - e_1, e_3, e_4
   - e_2, e_3, e_4
   - e_1, e_4, e_5
   - e_2, e_4, e_5

   (c) $\begin{bmatrix} 0 & 2 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

   (d) $A^2 = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 2 \end{bmatrix}$, $A^3 = \begin{bmatrix} 4 & 12 & 4 \\ 12 & 12 & 8 \\ 4 & 8 & 4 \end{bmatrix}$

2. (a) 3 paths
   - e_3, e_5, e_3
   - e_3, e_6, e_3
   - e_2, e_1, e_3

   (b) 6 paths:
   - e_5, e_2, e_1
   - e_6, e_2, e_1
   - e_5, e_3, e_5
   - e_5, e_3, e_6
   - e_6, e_3, e_5
   - e_6, e_3, e_6

   (c) $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$

   (d) $A^2 = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 3 & 0 \\ 2 & 1 & 2 \end{bmatrix}$, $A^3 = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 1 & 3 \\ 3 & 6 & 1 \end{bmatrix}$

3. (a) Not connected. There are two connected components: The subgraph induced by the vertices c, e, f, and the subgraph induced by the remaining vertices a, b, d, g, h.

   (b) Connected. There's only one connected component, namely the entire graph itself.

4. Both are weakly connected; (a) is strongly connected but (b) is not.

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