

14.7 CHANGE OF VARIABLES (Circular & Hyperbolic) 26-7

Evaluate $\iint_R (x^4 - y^4) dA$ $R: \begin{cases} 16 \leq x^2 + y^2 \leq 25 \\ 4 \leq x^2 - y^2 \leq 16 \end{cases}$

$$= \iint_R (x^2 + y^2)(x^2 - y^2) dA$$

TRY $u = x^2 + y^2$
 $v = x^2 - y^2$

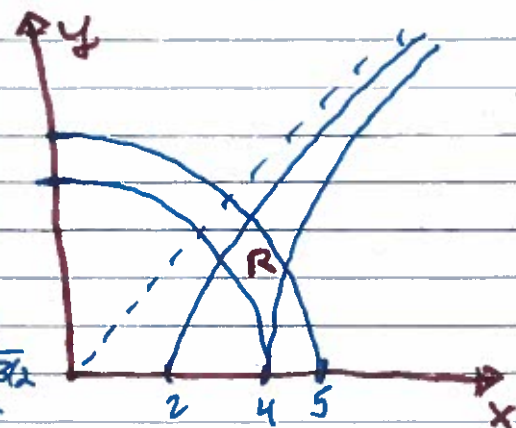
$$\Rightarrow \begin{cases} x = (u+v)^{1/2} / \sqrt{2} \\ y = (u-v)^{1/2} / \sqrt{2} \end{cases}$$

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} (u+v)^{-1/2} & (u+v)^{-1/2} \\ (u-v)^{-1/2} & -(u-v)^{-1/2} \end{vmatrix} \cdot \frac{1}{2^{3/2}} \cdot \frac{1}{2^{3/2}}$$

$$= \frac{1}{8} \left| \frac{-1}{u^2 - v^2} - \frac{1}{u^2 - v^2} \right| = \frac{1}{4} (u^2 - v^2)^{-1/2}$$

Note:

$$\begin{aligned} 16 &\leq u \leq 25 \\ 4 &\leq v \leq 16 \end{aligned}$$



14.7 CHANGE OF VARIABLES (Circular & Hyperbolic) 26-8

$$\iint_R (x^2 + y^2)(x^2 - y^2) dA = \int_4^{16} \int_{16}^{25} uv \frac{1}{4} (u^2 - v^2)^{-1/2} du dv$$

$$= \frac{1}{4} \int_4^{16} v (u^2 - v^2)^{1/2} dv = \frac{1}{4} \int_4^{16} v \left((625 - v^2)^{1/2} - (256 - v^2)^{1/2} \right) dv$$

$$= -\frac{1}{12} \left[(625 - v^2)^{3/2} - (256 - v^2)^{3/2} \right] \Big|_4^{16}$$

$$= -\frac{1}{12} \left[(625 - 256)^{3/2} - 0 - \left[(625 - 16)^{3/2} - (256 - 16)^{3/2} \right] \right]$$

$$= \frac{1}{12} (609^{3/2} - 369^{3/2} - 240^{3/2}) \approx 351.8778...$$