Name:

Linear Algebra I: Exam 3 (Spring 2020)

Show ALL work, as unjustified answers may receive no credit. Calculators are not allowed on any quiz or test paper. *Make sure to exhibit skills discussed in class*. Box all answers and simplify answers as much as possible.

Good Luck! ☺

1. The Dimension of a Vector Space (4.5) & Rank (4.6)

Suppose that $T: \mathbb{R}^4 \to \mathbb{R}^6$ is a Linear Transformation.

(a) [2pts] If the $dim[Nul(T)] = 2$, then find $dim[range(T)]$.

(b) [2pts] If the $dim[range(T)] = 3$, then find $dim[Nul(T)]$.

2. ***Vector Spaces & Subspaces (4.1)***

[9pts] Determine which of the following sets is a Vector Space. Construct geometric figures that illustrate why each set *is* or *is not* a Vector Space. For the two sets that are not Vector Spaces, find a specific example to show that they are *not* a Vector Space.

$$\boldsymbol{E \text{ is the line } y = x \text{ in the } xy-\text{plane}}\text{: } E = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : y = x \right\}$$

$$\boldsymbol{R \text{ is the union of the } 1^{st} \text{ \& } 2^{nd} \text{ quadrants in the } xy-\text{plane}}\text{: } R = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : y \geq 0 \right\}$$

$$\boldsymbol{Y \text{ is the line } y = x + 1 \text{ in the } xy-\text{plane}}\text{: } Y = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : y = x + 1 \right\}$$

3. Linearly Independent Sets; Bases (4.3)

[5pts] Determine whether the set of polynomials is linearly independent or linearly dependent:

$$\overrightarrow{p_1}(t) = 1\,, \qquad \overrightarrow{p_2}(t) = -2 + 4t^2\,, \qquad \overrightarrow{p_3}(t) = 2t\,, \qquad \overrightarrow{p_4}(t) = -12t + 8t^3$$

4. Change of Basis (4.7)

Consider the Vector Space $V = \mathbb{P}_2$, with Bases:

$$\mathcal{B} = \{\, t-1\,, t+1\,, t^2-1\,\} \;\;\&\;\; \mathcal{C} = \{\, 1\,, t+1\,, t^2+t\,\}$$

(a) [6pts] Find the Change of Coordinates Matrix from $\mathcal{B}$ to $\mathcal{C}$.

(b) [6pts] Express the vector $\vec{p}(t) = t^2 - t + 5$ relative to Basis $\mathcal{B}$.

5. Null Space & Column Space (4.2) & Rank (4.6)

Define the Linear Transformation $T: \mathbb{R}^4 \rightarrow \mathbb{R}^3$ by:

$$T(\,x_1\,,x_2\,,x_3\,,x_4\,) = (\,x_1 + x_2 - x_3 + x_4\;,\; 2x_1 + x_2 + 4x_3 + x_4\;,\; 3x_1 + x_2 + 9x_3\,)$$

(a) [6pts] Find the Basis for the Column Space of T.

(b) [6pts] Find the Basis for the Row Space of T.

(c) [6pts] Find the Basis for the Null Space of T.

(d) [2pts] List the $rank(T)$ and $dim[Nul(T)]$.