Due Date: Oct. 12, 2018 (F), BEFORE the lecture starts

This assignment covers textbook Chapter 4 & Chapter1~3.

In problem 1 to 3, solve the following recurrence with three different methods that we learned in class.

$$T(n) = \begin{cases} \Theta(1) & n \le k \\ T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + \Theta(1) & n > k \end{cases}$$

where n = 2^j for a positive integer j, and k is a small positive integer. That is, find a function g(n) such that T(n) ∈Θ(g(n)). The Θ(1) terminating condition is intended to represent some small constant.

1. Use the **Master Theorem** to solve this recurrence. (10 points)
2. Use the **recursion tree** to solve the recurrence. (15 points)
3. Use **substitution** method to prove the correctness of your answer to problem 1 and 2 above. Be sure to justify both the O and Ω parts of the Θ notation. (20 points)

4. Recurrence (15 points)

An algorithm solves a problem of size *n* by dividing them into four sub-problems of size n/3, recursively solving each sub-problem, and then combining the solutions in $2^{\lg n}$ time. You may assume that n = 3^k for some positive integer k.

Derive a recurrence for the running time of above algorithm. Use an appropriate method (just pick one method) to solve the recurrence by finding a tight upper and lower bound solution for the recurrence. You must show the procedure of calculation.

5. **Recurrence** (10 points)

For each of the following recurrences, give an expression for the runtime T(n) if the recurrence can be solved with the Master Theorem. Otherwise, explain why the Master Theorem does not apply. Justify your answer.

(1) $T(n) = 4T\left(\frac{n}{4}\right) + n$

(2) $T(n) = 3T\left(\frac{n}{2}\right) + \sqrt{10}n^2$

(3) $T(n) = T(n-1) + 10$

(4) $T(n) = 3T\left(\frac{3n}{2}\right) + n$

(5) $T(n) = 2^n T\left(\frac{n}{3}\right) + n$

6. **Design an Algorithm** (30 points)

You are given an array A, which stores n distinct numbers. The sequence of numbers in the array is unimodal. In other words, there is an index *i* such that the sequence A[1 . . . i] is increasing (A[j] < A[j + 1] for 1 ≤ j < i), and the sequence

A[i . . . n] is decreasing. The index *i* is called the mode of A. For example, <1, 3, 5, 7, 12, 13, 14, 10, 9, 6, 2> is a unimodal array and the mode of A is 7 (the index of element 14).

Design an *efficient* **divide-and-conquer** algorithm that only accepts a unimodal array *A* and *n* as inputs and returns the mode of A, i.e., the **index *i*** as the output.

(1) (10 points) Pseudocode *(please use the textbook conventions)*

(2) (10 points) Analysis: Derive a recurrence for the running time of your algorithm. Justify your answer by listing the cost for executing each line of code. Solve the recurrence and show your work.

(3) (10 points) Prove your answer using the **substitution method**

Algorithms -- COMP.4040 Honor Statement
(Courtesy of Prof. Tom Costello and Karen Daniels with modifications)

Must be attached to each submission, otherwise, your homework will not be graded.

Academic achievement is ordinarily evaluated on the basis of work that a student produces independently. Infringement of this Code of Honor entails penalties ranging from reprimand to suspension, dismissal or expulsion from the University.

Your name on any exercise is regarded as assurance and certification that what you are submitting for that exercise is the result of your own thoughts and study. Where collaboration is authorized, you should state very clearly which parts of any assignment were performed with collaboration and name your collaborators.

In writing examinations and quizzes, you are expected and required to respond entirely on the basis of your own memory and capacity, without any assistance whatsoever except such as what is specifically authorized by the instructor.

I certify that the work submitted with this assignment is mine and was generated in a manner consistent with this document, the course academic policy on the course website on Blackboard, and the UMass Lowell academic code.

Date: ______________________

Name (please print): ______________________

Signature: ______________________