Calculus III Quiz #2 Spring 2018 NAME: SOLUTION

1. Given $\vec{u} = \langle 1, -4, 2 \rangle$ and $\vec{v} = \langle -3, 2, 1 \rangle$ compute the following:

1.a. (4 pts) $\vec{u} \cdot \vec{v}$ 1.b. (4 pts) $proj_{\vec{v}}\vec{u}$ 1.c. (4 pts) $\vec{u} \times \vec{v}$ or $\vec{v} \times \vec{u}$

You may use the reverse side but you MUST clearly label your answers and use correct notation!

1.a. $\vec{u} \cdot \vec{v} = \langle 1, -4, 2 \rangle \cdot \langle -3, 2, 1 \rangle = -3 - 8 + 2 = -9$

1.b. $proj_{\vec{v}}\vec{u} = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2}\vec{v} = \frac{-9}{14}\langle -3, 2, 1 \rangle$

1.c. $$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -4 & 2 \\ -3 & 2 & 1 \end{vmatrix} = \langle -4-4, -6-1, 2-12 \rangle$$

$$= \langle -8, -7, -10 \rangle$$

$$= -\langle 8, 7, 10 \rangle$$

$$\vec{v} \times \vec{u} = -\vec{u} \times \vec{v} = \langle 8, 7, 10 \rangle$$

Extra: what if I had asked for the angle between $\vec{u}$ and $\vec{v}$?

$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos(\theta) = -9$

$|\vec{u}| = \sqrt{21}$ $|\vec{v}| = \sqrt{14}$ $|\vec{u}||\vec{v}| = 7\sqrt{6}$

Therefore $\cos(\theta) = -9/7\sqrt{6}$

$\theta = \cos^{-1}(-9/7\sqrt{6})$

You would leave answer in this form.

$\approx \cos^{-1}(-0.52489)$

≈ 2.123383 rad

≈ 121.661 deg

What if I had asked for the area of the parallelogram formed by $\vec{u}$ and $\vec{v}$?

$$\text{Area} = |\vec{u} \times \vec{v}| = \sqrt{64 + 49 + 100} = \sqrt{213}$$

Area of triangle formed by $\vec{u}$ and $\vec{v}$?

$$\text{Area of triangle} = \frac{|\vec{u} \times \vec{v}|}{2} = \frac{\sqrt{213}}{2}$$