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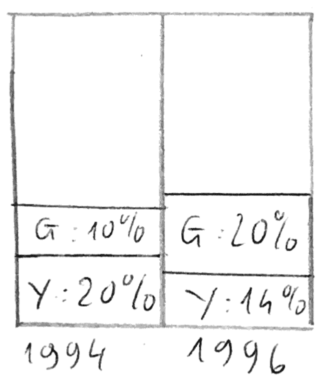
UML Artificial Intelligence Homework Quiz 03

Problem 3[15]. The blue M&M was introduced in 1995. Before then, the color mix in a bag of plain M&Ms was (30% Brown, 20% Yellow, 20% Red, 10% Green, 10% Orange, 10% Tan). Afterward it was (24% Blue, 20% Green, 16% Orange, 14% Yellow, 13% Red, 13% Brown).

A friend has two bags of M&Ms, and tells me that one is from 1994 and one from 1996. My friend won't tell me which is which, but gives me one M&M from each bag. One is yellow and one is green. What is the probability that the yellow M&M came from the 1994 bag?

Solve graphically *and* with Bayes'. Put your numeric answer in the box and show your work below.



Hypothese: A: Bag 1 from 1994, Bag 2 from 1996

B: Bag 2 from 1994, Bag 1 from 1996

$\Rightarrow P(A) = P(B) = \frac{1}{2}$.

E: Yellow from bag 1, Green from bag 2.

Bayes's theorem:

$$P(A|E) = \frac{P(E|A) \cdot P(A)}{P(E)}$$

$$= \frac{P(E|A) \cdot P(A)}{P(A) \cdot P(E|A) + P(B) \cdot P(E|B)} = \frac{(0.2)(0.2) \cdot (0.5)}{(0.5)(0.2)(0.2) + (0.5)(0.1)(0.14)}$$

$$= 0.74$$

Thanks Allen Downey for these two, who also points out that these are "urn problems."

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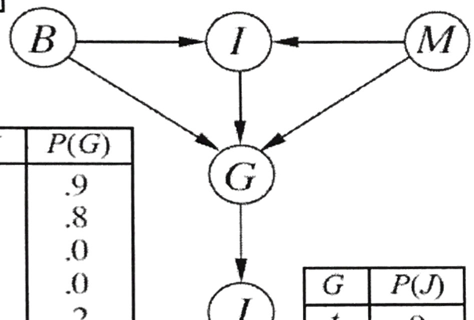
1. Consider the belief network shown below:

P(B)
.9

B	M	P(I)
t	t	.9
t	f	.5
f	t	.5
f	f	.1

P(M)
.1



B	I	M	P(G)
t	t	t	.9
t	t	f	.8
t	f	t	.0
t	f	f	.0
f	t	t	.2
f	t	f	.1
f	f	t	.0
f	f	f	.0

G	P(J)
t	.9
f	.0

Fig 1: A simple Bayes net with Boolean variables B =BrokeElectionLaw, I =Indicted M=PoliticallyMotivatedProsecutor , G=FoundGuilty , J =Jailed

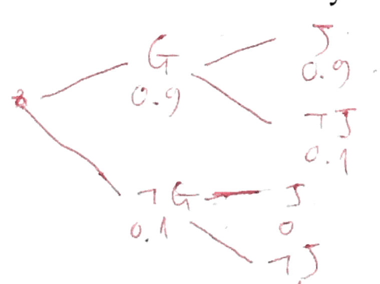
a) (4 points) Which of the following are asserted by the network structure?
   (i) P(B, I,M) = P(B)P(I)P(M).
   (ii) P(J |G) = P(J | G, I).
   (iii) P(M | G,B, I) = P(M | G,B, I, J).

b) (5 points) Calculate the value of P(b, i, ¬m, g, j).

P(b) P(i|B,¬m) P(¬m) P(g|B,I,¬m)
P(J|G)

c) (6 points) Calculate the probability that someone goes to jail given that they broke the law, have been indicted, and face a politically motivated prosecutor.

d) (5 points) Suppose we want to add the variable P =PresidentialPardon to the network; draw the new network and briefly explain any links you add.



1 a) (ii) and (iii).

b). $P(b, i, \neg m, g, j)$:

$= P(b) . P(\neg m) . P(i \mid b, \neg m) . P(g \mid b, i, \neg m) P(j \mid g)$

$= 0.9 \times 0.9 \times 0.5 \times 0.8 \times 0.9 = 0.2916$

c). $P(J \mid b, i, m)$

$= \alpha \sum_{g} P(J, g) = \alpha [P(J, g) + P(J, \neg g)]$

① $= \alpha [\langle P(j, g), P(\neg j, g)\rangle + \langle P(j, \neg g), P(\neg j, \neg g)\rangle]$

- $P(j, g) = P(J \mid g) . P(g) = 0.9 \times 0.9 = 0.81$

$P(\neg J, g) = P(\neg J \mid g) . P(g) = 0.1 \times 0.9 = 0.09.$

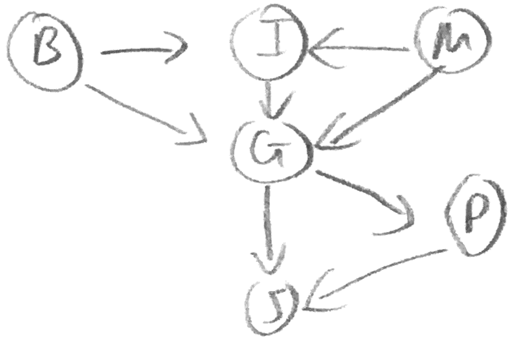
$P(J, \neg g) = P(J \mid \neg g) . P(\neg g) = 0 \times 0.1 = 0$

$P(\neg J, \neg g) = P(\neg J \mid \neg g) . P(\neg g) = 1 \times 0.1 = 0.1$

$\Rightarrow$ ① $= \alpha [\langle 0.81, 0.09\rangle + \langle 0, 0.1\rangle] = \langle 0.81, 0.09\rangle$

$\Rightarrow$ the probability of going to jail is 0.81

d)



A pardon is not necessary if the person is not indicted or not found guilty $\Rightarrow$ I and G are parent of P. P is the parent of J since Pardon means get out of jail.

3. Consider the Bayesian network below

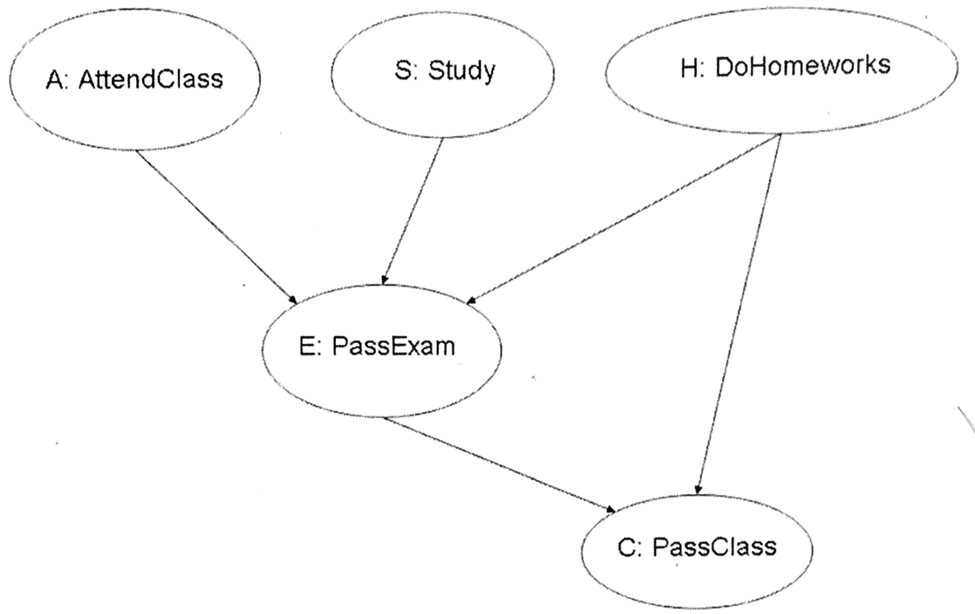


Fig 3: Bayes net for studying and passing

a) (5points) Write down the joint distribution as it factorizes according to the graph below.

b) (5 points) Use variable elimination and your result from (a) above to write down the expression for the probability of passing the class, given that you attend class and study, but don't do the home works.

c) (10 points) Use the following CPTs for the graph of question 1 to compute $P(A \mid C, H)$.

$$P(A) = 0.5,\ P(S) = 0.7,\ P(H) = 0.9$$

A	S	H	$P(E \mid A, S,$
0	0	0	0.2
0	0	1	0.5
0	1	0	0.4
0	1	1	0.8
1	0	0	0.3
1	0	1	0.7
1	1	0	0.6
1	1	1	0.9

E	H	$P(C \mid E, H)$
0	0	0.1
0	1	0.4
1	0	0.3
1	1	0.9

3a) $P(A,S,H,E,C) = P(A) . P(S) . P(H) . P(E|A,S,H) . P(C|E,H)$

b) $$P(C|A,S,\neg H) = \frac{P(C,A,S,\neg H)}{P(A,S,\neg H)}$$

$$= \frac{\sum_e P(A,S,\neg H, E=e, C)}{\sum_e \sum_c P(A,S,\neg H, E=e, C=c)}$$

$$= \frac{\sum_e P(A).P(S).P(\neg H).P(E=e|A,S,\neg H).P(C|E=e,\neg H)}{\sum_e \sum_c P(A).P(S).P(\neg H).P(E=e|A,S,\neg H)P(C=c|E=e,\neg H)}$$

$$= \frac{P(A).P(S).P(\neg H).\sum_e P(E=e|A,S,\neg H)\ P(C|E=e,\neg H)}{P(A).P(S).P(\neg H).\sum_e P(E=e|A,S,\neg H).\sum_c P(C=c|E=e,\neg H)}$$

$$= \frac{P(A)\ P(S).P(\neg H) \sum_e P(E=e|A,S,\neg H).P(C|E=e,\neg H)}{P(A).P(S).P(\neg H)}$$

$$= \sum_e P(E=e|A,S,\neg H).P(C|E=e,\neg H)$$

c) $$P(A|C,H) = \frac{P(A,C,H)}{P(C,H)}$$

$$= \frac{\sum_e \sum_s P(A, S=s, H, E=e, C)}{\sum_a \sum_e \sum_s P(A=a, S=s, H, E=e, C)}$$

$$= \frac{\sum_e \sum_s P(A).P(S=s).P(H).P(E=e|A,S=s,H).P(C|E=e,H)}{\sum_a \sum_e \sum_s P(A=a)\ P(S=s).P(H).P(E=e|A=a,S=s,H).P(C|E=e,H)}$$

$$= \frac{P(A).P(H).\sum_s P(S=s).\sum_e P(E=e|A,S=s,H)\ P(C|E=e,H)}{P(H).\sum_a P(A=a).\sum_s P(S=s).\sum_e P(E=e|A=a,S=s,H)\ P(C|E=e,H)}$$

$$= \frac{P(A).\sum_s P(S=s)\ \sum_e P(E=e|A,S=s,H)\ P(C|E=e,H)}{\sum_a P(A=a) \sum_s P(S=s) \sum_e P(E=e|A=a,S=s,H)\ P(C|E=e,H)}$$

$$= \frac{0.5[0.7(0.9\times0.9+0.1\times0.4)+0.3(0.7\times0.9+0.3\times0.4)]}{0.5[0.7(0.9\times0.9+0.1\times0.4)+0.3(0.7\times0.9+0.3\times0.4)]+0.5[0.7(0.8\times0.9+0.2\times0.4)+0.3(0.5\times0.9+0.5\times0.4}$$

$$= \frac{0.41}{0.7875}$$