Student: Phong Vo **Date:** 04/10/20	**Instructor:** Erica Yankowskas **Course:** Linear Algebra I (Spring 2020)	**Assignment:** Section 4.3 Homework

1. Determine whether the set $\left\{ \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 7 \end{bmatrix}, \begin{bmatrix} -5 \\ 2 \\ 11 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$. If the set is not a basis, determine whether the set is linearly independent and whether the set spans $\mathbb{R}^3$.

Which of the following describe the set? Select all that apply.

- [ ] **A.** The set is linearly independent.
- [ ] **B.** The set spans $\mathbb{R}^3$.
- [ ] **C.** The set is a basis for $\mathbb{R}^3$.
- [x] **D.** None of the above

2. Determine if the set of vectors shown to the right is a basis for $\mathbb{R}^3$. If the set of vectors is not a basis, determine whether it is linearly independent and whether the set spans $\mathbb{R}^3$.

$$\left\{ \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} -6 \\ 4 \\ 2 \end{bmatrix} \right\}$$

Which of the following describe the set? Select all that apply.

- [x] **A.** The set spans $\mathbb{R}^3$.
- [x] **B.** The set is linearly independent.
- [x] **C.** The set is a basis for $\mathbb{R}^3$.
- [ ] **D.** None of the above

3. Determine if the set of vectors shown to the right is a basis for $\mathbb{R}^3$. If the set of vectors is not a basis, determine whether it is linearly independent and whether the set spans $\mathbb{R}^3$.

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ -8 \end{bmatrix}, \begin{bmatrix} -5 \\ 4 \\ 16 \end{bmatrix} \right\}$$

Which of the following describe the set? Select all that apply.

- [ ] **A.** The set spans $\mathbb{R}^3$.
- [ ] **B.** The set is a basis for $\mathbb{R}^3$.
- [x] **C.** The set is linearly independent.
- [ ] **D.** None of the above

4. Find a basis for the null space of the matrix given below.

$$\begin{bmatrix} 1 & 1 & -4 & -1 & 5 \\ 0 & 1 & 0 & -3 & -1 \\ 0 & 0 & -8 & 0 & 8 \end{bmatrix}$$

A basis for the null space is $\left\{ \begin{bmatrix} -2 \\ 3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$.

(Use a comma to separate answers as needed.)

5. Find a basis for the set of vectors in $\mathbb{R}^3$ in the plane $x - 5y + 4z = 0$. [Hint: Think of the equation as a "system" of homogeneous equations.]

A basis for the set of vectors in $\mathbb{R}^3$ in the plane $x - 5y + 4z = 0$ is $\left\{ \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix} \right\}$.

6. Assume that, below, A is row equivalent to B. Find bases for Nul A and Col A.

$$A = \begin{bmatrix} 1 & 2 & 4 & -2 & 5 \\ 1 & 2 & 0 & 2 & 5 \\ 2 & 4 & -5 & 9 & 8 \\ 4 & 8 & 0 & 8 & 8 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 0 & 2 & 3 \\ 0 & 0 & 4 & -4 & 4 \\ 0 & 0 & 0 & 0 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

A basis for Col A is $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -5 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 8 \\ 8 \end{bmatrix} \right\}$.

(Use a comma to separate answers as needed.)

A basis for Nul A is $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$.

(Use a comma to separate answers as needed.)

YOU ANSWERED: $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ -\frac{3}{4} \\ 0 \\ 0 \end{bmatrix}$

7. Find a basis for the space spanned by the given vectors.

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ 0 \\ -6 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 19 \\ -12 \\ 12 \\ -10 \end{bmatrix}, \begin{bmatrix} 17 \\ -6 \\ 6 \\ -11 \end{bmatrix}$$

A basis for the space spanned by the given vectors is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ 0 \\ -6 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ 3 \\ -1 \end{bmatrix} \right\}$.

(Use a comma to separate answers as needed.)

8. Determine if each statement a. through e. below is true or false. Justify each answer.

a. A linearly independent set in a subspace H is a basis for H.

- **A.** The statement is false because the set must be linearly dependent.
- **B.** The statement is true by the Spanning Set Theorem.
- **C.** The statement is true by the definition of a basis.
- **D.** The statement is false because the subspace spanned by the set must also coincide with H.

b. If a finite set S of nonzero vectors spans a vector space V, then some subset of S is a basis for V.

- **A.** The statement is false because the subset must be independent.
- **B.** The statement is true by the Spanning Set Theorem.
- **C.** The statement is true by the definition of a basis.
- **D.** The statement is false because the subspace spanned by the set must also coincide with V.

c. A basis is a linearly independent set that is as large as possible.

- **A.** The statement is false because a basis is a linearly dependent set.
- **B.** The statement is false because a basis is the smallest independent set that spans the subspace.
- **C.** The statement is true by the Spanning Set Theorem.
- **D.** The statement is true by the definition of a basis.

d. The standard method for producing a spanning set for Nul A sometimes fails to produce a basis for Nul A.

- **A.** The statement is true because the set produced may not be independent.
- **B.** The statement is false because a spanning set for Nul A also spans A.
- **C.** The statement is true because the only set produced may be the trivial solution.
- **D.** The statement is false because the method always produces an independent set.

e. If B is an echelon form of a matrix A, then the pivot columns of B form a basis for Col A.

- **A.** The statement is true by the Invertible Matrix Theorem.
- **B.** The statement is false because the columns of an echelon form B of A are not necessarily in the column space of A.
- **C.** The statement is false because the pivot columns of A form a basis for Col B.
- **D.** The statement is true by the definition of a basis.

9. In the vector space of all real-valued functions, find a basis for the subspace spanned by {**sin** t, **sin** 2t, **sin** t **cos** t}.

A basis for this subspace is {**sin** t, **sin** 2t}.

10. Suppose that $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is a subset of V and T is a one-to-one linear transformation, so that an equation $T(\mathbf{u}) = T(\mathbf{v})$ always implies $\mathbf{u} = \mathbf{v}$. Show that if the set of images $\{T(\mathbf{v}_1), \ldots, T(\mathbf{v}_p)\}$ is linearly dependent, then $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is linearly dependent.

If the set $\{T(\mathbf{v}_1), \ldots, T(\mathbf{v}_p)\}$ is linearly dependent, which statement below is true?

- **A.** For scalars $c_1, \ldots, c_p$, the vector equation $c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p = \mathbf{0}$ has only the trivial solution, $c_1 = 0, \ldots, c_p = 0$.
- **B.** There exist scalars $c_1, \ldots, c_p$, not all zero, such that $c_1 T(\mathbf{v}_1) + \cdots + c_p T(\mathbf{v}_p) = \mathbf{0}$.
- **C.** For scalars $c_1, \ldots, c_p$, the vector equation $c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p = \mathbf{0}$ has no solution.
- **D.** There exist scalars $c_1, \ldots, c_p$, not all zero, such that the vector equation $c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p = \mathbf{0}$.

Since T is linear, which statement below is true?

- **A.** $c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p = (c_1 + \cdots + c_p)(\mathbf{v}_1 + \cdots + \mathbf{v}_p)$
- **B.** $c_1 T(\mathbf{v}_1) + \cdots + c_p T(\mathbf{v}_p) = T(c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p)$
- **C.** $c_1 T(\mathbf{v}_1) + \cdots + c_p T(\mathbf{v}_p) = c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p$
- **D.** $c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p = T(c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p)$

Which statement below follows from the true statements found previously?

- **A.** $T(c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p) = \mathbf{0}$
- **B.** $c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p = T(\mathbf{0})$
- **C.** $c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p = \mathbf{v}_1 + \cdots + \mathbf{v}_p$
- **D.** $T(c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p) = T(c_1 + \cdots + c_p) + T(\mathbf{v}_1 + \cdots + \mathbf{v}_p)$

Complete the statement below so it follows from the true statements found previously and leads to the conclusion.

Since T is linear, $T(\mathbf{0}) = \mathbf{0}$. Since T is one-to-one, $c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p = \mathbf{0}$, where $c_1, \ldots, c_p$ are not all zero. Therefore, $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is linearly dependent.