General strategy for solving problems:

- Find a procedure / algorithm which generates each possibility exactly once. Then use the rule of products.

- Use the sum rule (i.e. "case analysis") and "counting the complement."

1. An n-element DNA sequence is a string of n letters from the set $\{A, C, G, T\}$. (For example, $ATCTT$ and $GACCC$ are 5-element DNA sequences.) How many 5-element DNA sequences ...

   (a) ... start with A and end with T? A...T = 4*4*4 = 4^3 = 64

   (b) ... do not contain C? ..... = 3^5 = 243

2. A test consists of six true-false questions. In how many different ways can a student answer the questions on the test if ...   .

   (a) ... the student answers every question? 2^6 = 64

   (b) ... the student can leave answers blank?

   (c) ... the student leaves exactly one question blank and answers the remaining questions? 2^5 + 2^5 + 2^5 + 2^5 + 2^5 + 2^5
   = 32*6 = 192

3. A PIN is a string of four digits (e.g. 0118 or 9035). There are $10^4 = 10000$ PINs total. How many PINs ...

   (a) ... contain four different digits? [0...9][0...9-1][0...9-2][0...9-3] = 10*9*8*7 = 5040

   (b) ... contain at least one repeated digit? (e.g. 0103 or 7055) = Total - no digit repeated = 10,000 - 5,040 = 4,960

   (c) ... have exactly three digits which are 7's? (e.g. 7877) = 777x + 77x7 + 7x77 + x777 = 9 + 9 + 9 + 9 = 36

   (d) ... have exactly three digits that are the same? (e.g. 0333 or 5525) = (c) * [0...9] = 36*10 = 360

   (e) ... have at least three digits that are the same? 4 digits repeated (e.g: 1111, 2222) = 10
   (d) + 10 = 360 + 10 = 370

4. There are five students in a class. At the end of the semester, the professor will assign each student a grade of A, B, or C. In how many ways can she do this if ...

   (a) ... exactly one person gets a C, exactly one person gets a B, and the remaining students get As?

   (b) ... not everyone gets the same grade?

   (c) ... at least one student will receive an A?

   (d) ... at most one student gets an A?

5. A family consists of a mom, a dad, two sons, and a daughter. How many can the members of the family be lined up in a row for a family portrait if ...

   (a) ... the two sons must be next to each other?

   (b) ... the mom must be somewhere to the left of dad?

   4. (a) Pick 1 out of 5 students to assign 1C => C(5,1) = 5   ; Pick 1 out of 4 students to assign 1B => C(4,1) = 4
       Pick remaining 3 students to assign 3C's => C(3,3) = 1 => Ans. = 5*4*1 = 20 ways
      (b) Meaning 3 out of 5 students will be graded, remaining 2 will NOT be graded
          => C(5,3) =

Answers:

1. (a) 64
   (b) 243

2. (a) 64
   (b) 729
   (c) 192

3. (a) 5040
   (b) 4960
   (c) 36
   (d) 360
   (e) 370

4. (a) 20
   (b) 240
   (c) 211
   (d) 112

5. (a) 48
   (b) 60