Section 10.8 Homework

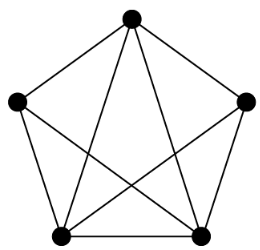
Note: To show that the chromatic number of a graph G equals k, you should both exhibit a valid coloring of the graph, and prove that k is the minimum number of colors that can be used. To do the second part, you can use the following facts:

- For any positive integer n, the chromatic number of K_n is n.
- For any integer $n \geq 3$, the chromatic number of C_n is 2 if n is even, and 3 if n is odd.
- Let G be a graph. If H is a subgraph of G, then $\chi(H) \leq \chi(G)$.

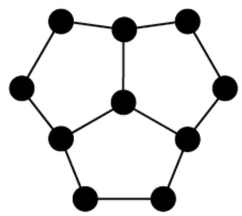
Chromatic Number: The smallest number of colors needed to color a graph G is called its chromatic number

1. Determine the chromatic number of each graph below.

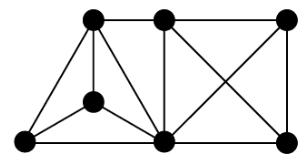
(a)



(b)



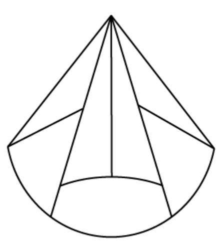
(c)



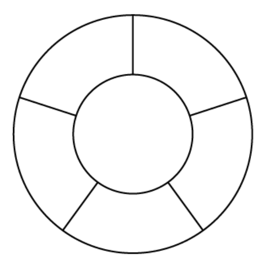
2. For each graph G in problem 1, is it possible to remove an edge to get a graph H which satisfies $\chi(H) < \chi(G)$? If so, which edge works and why? If not, explain your answer.

3. For each map below, draw the dual graph G. Then find the chromatic number of G. Use your coloring of the graph G to find a coloring of the map.

(a)



(b)



4. The math department must schedule final exams for courses labeled 1, 2, 3, 4, 5, 6, and 7. Suppose there are no students taking both courses 1 and 3, both courses 1 and 6, both courses 2 and 4, both courses 3 and 4, both courses 3 and 7, both courses 4 and 7, both courses 5 and 7, and both courses 6 and 7, but there are students in every other pair of courses. Using the appropriate graph model and graph coloring, determine the fewest number of time slots needed to schedule to final exams.

5. The mathematics department has six committees, each meeting once a month. The committees are

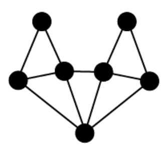
- $C_1 = \{Abel, Borel, Zorn\}$
- $C_2 = \{Borel, Laurent, Rosen\}$
- $C_3 = \{Abel, Smith, Zorn\}$
- $C_4 = \{Laurent, Rosen, Smith\}$
- $C_5 = \{Laurent, Smith\}$
- $C_6 = \{Abel, Rosen, Zorn\}$

Using the appropriate graph model and graph coloring, answer the following question: What's the minimum number of meeting times that must be used to ensure that no member is scheduled to attend two meetings at the same time?

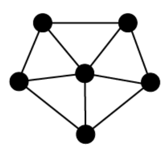
6. (*Optional*) Let G be a simple graph, and let $\Delta(G)$ be the maximum degree in G. (For example, if G has five vertices with degree $4, 3, 4, 3, 2$, then $\Delta(G) = 4$.) Prove that $\chi(G) \leq \Delta(G) + 1$.

Answers:

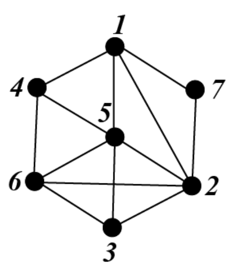
1. (a) 4
   (b) 3
   (c) 4
2. (a) yes
   (b) no
   (c) no
3. (a) $\chi(G) = 3$



   (b) $\chi(G) = 4$

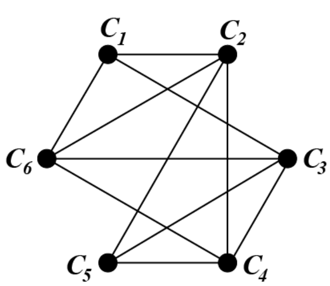


4.



$\chi(G) = 4$, so the minimum number of time slots is 4.

5.



$\chi(G) = 3$, so the minimum number of meeting times is 3.