

1. From Nicholas Quinn.

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① Slowest

← $\lg(2^{2^{\lg(n)}})$ $n^2 \lg \lg(n)$ $n^2 \lg(n)$ $(1/3)^n$ $(-3)^{2n}$ →

1(a) $\lg(2^{2^{\lg(n)}}) = \lg(n^3) = 3 \lg(n)$ ✓

$3 \lg(n) = O(n^2 \lg \lg(n))$

$0 \leq 3 \lg(n) \leq n^2 \lg \lg(n) \cdot C$, Let $C=1$

$0 \leq 3 \leq \frac{n^2 \lg \lg(n)}{\lg(n)}$ ✓

$n_0=3$

1(b) $n^2 \lg \lg(n) = O(n^2 \lg(n))$

$0 \leq n^2 \lg \lg(n) \leq n^2 \lg(n) \cdot C$, Let $C=1$

$0 \leq \lg \lg(n) \leq \lg(n)$

True when $C=1$ and $n_0=3$ ✓

1(c) $n^2 \lg \lg(n) = O((1/3)^n)$

$0 \leq n^2 \lg \lg(n) \leq 3^n C$, Let $C=1$

$0 \leq \lg \lg(n) \leq \frac{3^n}{n^2}$

$n_0=1$ ✓

1(d) $(1/3)^n = O((-3)^{2n})$

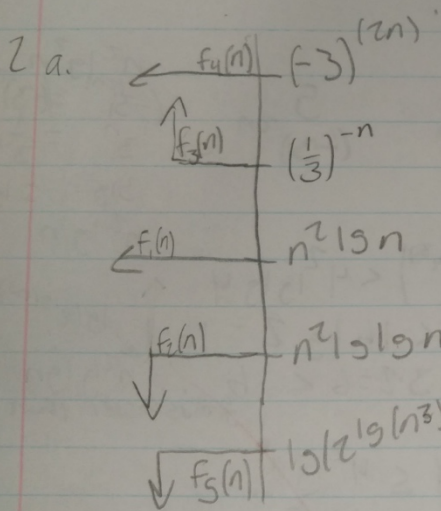
$0 \leq 3^n \leq 3^{2n}$, Let $C=1$

$n \leq 2n$

$1 \leq 2$

$n_0=1$

2. From Nicholas Bishop.



b. TRUE; $n^2 \lg \lg n \in O(n^2 \lg n) \therefore$

$$f_2(n) \in O(n^2 \lg n)$$

$$f_1(n) \in \Omega(n^2 \lg n) \therefore$$

$$f_2(n) \in O(f_1(n))$$

c. false; for $f_2(n) = \lg \lg n$ and $f_5(n) = \lg n$, $f_2(n)$ grows asymptotically slower than $f_5(n)$

d. false; for $f_4(n) = (-3)^{2n}$ and $f_3(n) = (-4)^{2n}$, $f_4(n)$ grows asymptotically slower than $f_3(n)$

e. false; for $f_4(n) = (-3)^{2n}$ and $f_1(n) = n^2 \lg n$, $f_4 \in \Omega(f_1(n))$, $f_4 \notin O(f_1(n)) \therefore f_4 \notin \Theta(f_1(n))$

3. From Ryan Duffy.

3. (e) $f(n) = O([f(n)]^2)$

Suppose $f(n) = \frac{1}{n} \Rightarrow [f(n)]^2 = \left[\frac{1}{n}\right]^2 = \frac{1}{n^2} = \frac{1}{n^2} = g(n)$

let $c = 1$

$f(n) \leq c \cdot g(n)$

$\frac{1}{n} \leq \frac{1}{n^2} \Rightarrow n \leq 1$

This implies if $n_0 > 1$ ~~then~~ and $c = 1$ then
 $f(n) > c g(n) \therefore f(n) \neq O([f(n)]^2)$

(g) $f(n) = \Theta(f(\frac{n}{2}))$

Suppose $f(n) = e^n \Rightarrow f(\frac{n}{2}) = e^{n/2} = g(n)$

let $c = 1$

$f(n) \leq c \cdot g(n)$

$e^n \leq e^{n/2} \Rightarrow 1 \leq \frac{e^{n/2}}{e^n} = e^{-n/2} = e^{n(k-1) - n/2} = e^{-n/2}$

$1 \leq e^{-n/2} \Rightarrow \ln(1) \leq -\frac{n}{2}$

$0 \leq -\frac{n}{2}$

$0 \leq -n$

$0 \geq n$

If $n_0 > 0$ and $c = 1$
 then $f(n) > c g(n)$

$\therefore f(n) \neq O(f(\frac{n}{2}))$

$\Rightarrow f(n) \neq \Theta(f(\frac{n}{2}))$

4. From Jacob Montpetit.

4. Analysis: (10 points) Your client is developing two new algorithms. $f_1(n)$ and $f_2(n)$ are the worst-case running time for these two algorithms: $f_1(n) = n \lg n$, and $f_2(n) = 128n$. As a consultant, which algorithm will you recommend to your client? Justify your answer. (Hint: Please consider the asymptotical growth of the functions and also consider the reality.)

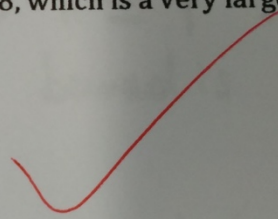
$n \lg(n) \leq 128n$ when n is less or equal to 2^{128} this inequality holds true. When n is greater than 2^{128} $128n$ gives better performance. I would recommend using the $n \lg(n)$ unless n is 2^{128} typically. 2^{128} is approximately $3.4028237e+38$, which is a very large number.

Work: $n \lg(n) \leq 128n$

$$n \lg(n) \leq 128n$$

$$\lg(n) \leq 2^{128}$$

$$n \leq 2^{128}$$



5. From Etienne Buhrle.

Since this input size is rather unusual, algorithm 1 with running time f_1 is probably still better in most applications.

5 Pseudocode Analysis

For the running time, we get

$$\begin{aligned} T_{\text{Mystery}}(n) &= c_1 + (n^2 + 1)c_2 + c_3 \sum_{i=1}^{n^2} (i + 1) + c_4 \sum_{i=1}^{n^2} i + c_5 \\ &= c_1 + c_5 + (n^2 + 1)c_2 + (c_3 + c_4) \sum_{i=1}^{n^2} i + c_3 \sum_{i=1}^{n^2} 1 \\ &= c_1 + c_5 + (n^2 + 1)c_2 + (c_3 + c_4) \frac{1}{2} n^2 (n^2 + 1) + c_3 n^2 \\ &= c_1 + c_5 + c_2 n^2 + c_2 + (c_3 + c_4) \frac{1}{2} (n^4 + n^2) + c_3 n^2 \\ &= \Theta(n^4) \end{aligned}$$