Section 8.1 Homework

Note: For each problem, justify why your recurrence relation works by giving a verbal description of what each term of your recurrence relation represents.

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1. For the following problem, tiles can be of the following types:

   - 1×1 tiles that are red, blue, or green.
   - 1×2 tiles that are green or blue (which can be rotated to be vertical or horizontal)

   For each sequence a_n below, find a linear recurrence relation for a_n. Also, compute a_1, a_2, a_3, and a_4.

   (a) a_n is the number of tilings of a $1 \times n$ rectangle.

   (b) a_n is the number of tilings of a $1 \times n$ rectangle where only blue or green tiles are used.

   (c) a_n is the number of tilings of a $1 \times n$ rectangle so that only 1×1 tiles are used, and red tiles occur consecutively.

   (d) a_n is the number of tilings of a $1 \times n$ rectangle using only red and green tiles and so that red tiles do not occur consecutively.

   (e) a_n is the number of tilings of a $2 \times n$ rectangle using only blue or green 1×2 tiles which can be placed horizontally or vertically.

   (f) a_n is the number of tilings of a $2 \times n$ rectangle using only blue or green 1×2 tiles, where only the green tiles can be placed horizontally. (*Note:* For the $2 \times n$ rectangle, the vertical dimension is 2 and the horizontal dimension is n.)

2. Consider strings of digits consisting of the numbers 1, 2, 4. For each sequence a_n below, find a linear recurrence relation for a_n. Also, compute a_1, a_2, a_3, a_4, a_5.

   (a) a_n is the number of such strings that sum to n.

   (b) a_n is the number of such strings that sum to n and don't contain two consecutive 1s.

3. A *ternary* string is a string of numbers from the set $\{0, 1, 2\}$. For each sequence a_n below, find a linear recurrence relation for a_n. Also, compute a_1, a_2, a_3, a_4, and a_5.

   (a) a_n is the number of ternary strings of length n that don't contain 00 consecutively.

   (b) a_n is the number of ternary strings of length n that contain 000 consecutively.

Answers:

1. (a) $a_n = 3a_{n-1} + 2a_{n-2}$
   $a_0 = 1, \quad a_1 = 3, a_2 = 11, \quad a_3 = 39, \quad a_4 = 139$
   (b) $a_n = 2a_{n-1} + 2a_{n-2}$
   $a_0 = 1, \quad a_1 = 2, a_2 = 6, \quad a_3 = 16, \quad a_4 = 44$
   (c) $a_n = 2a_{n-1} + 2a_{n-2} + 3^{n-2}$
   $a_0 = 0, \quad a_1 = 0, a_2 = 1, \quad a_3 = 5, \quad a_4 = 21$
   (d) $a_n = a_{n-1} + 2a_{n-2} + a_{n-3}$
   $a_0 = 1, \quad a_1 = 2, a_2 = 4, \quad a_3 = 9, \quad a_4 = 19$
   (e) $a_n = 2a_{n-1} + 4a_{n-2}$
   $a_0 = 1, \quad a_1 = 2, a_2 = 8, \quad a_3 = 24, \quad a_4 = 80$
   (f) $a_n = 2a_{n-1} + a_{n-2}$
   $a_0 = 1, \quad a_1 = 2, a_2 = 5, \quad a_3 = 12, \quad a_4 = 29$
2. (a) $a_n = a_{n-1} + a_{n-2} + a_{n-4}$
   $a_0 = 1, \quad a_1 = 1, a_2 = 2, \quad a_3 = 3, \quad a_4 = 6, \quad a_5 = 10$
   (b) $a_n = a_{n-2} + a_{n-3} + a_{n-4} + a_{n-5}$
   $a_0 = 1, \quad a_1 = 1, \quad a_2 = 1, \quad a_3 = 2, \quad a_4 = 3, \quad a_5 = 5$
3. (a) $a_n = 2a_{n-1} + 2a_{n-2}$
   $a_0 = 1, \quad a_1 = 3, a_2 = 8, \quad a_3 = 22, \quad a_4 = 60, \quad a_5 = 164$
   (b) $a_n = 2a_{n-1} + 2a_{n-2} + 2a_{n-3} + 3^{n-3}$
   $a_0 = 0, \quad a_1 = 0, a_2 = 0, \quad a_3 = 1, \quad a_4 = 5, \quad a_5 = 21$