CHAPTER 10

SYNCHRONOUS SEQUENTIAL CIRCUITS

10.1 Registers

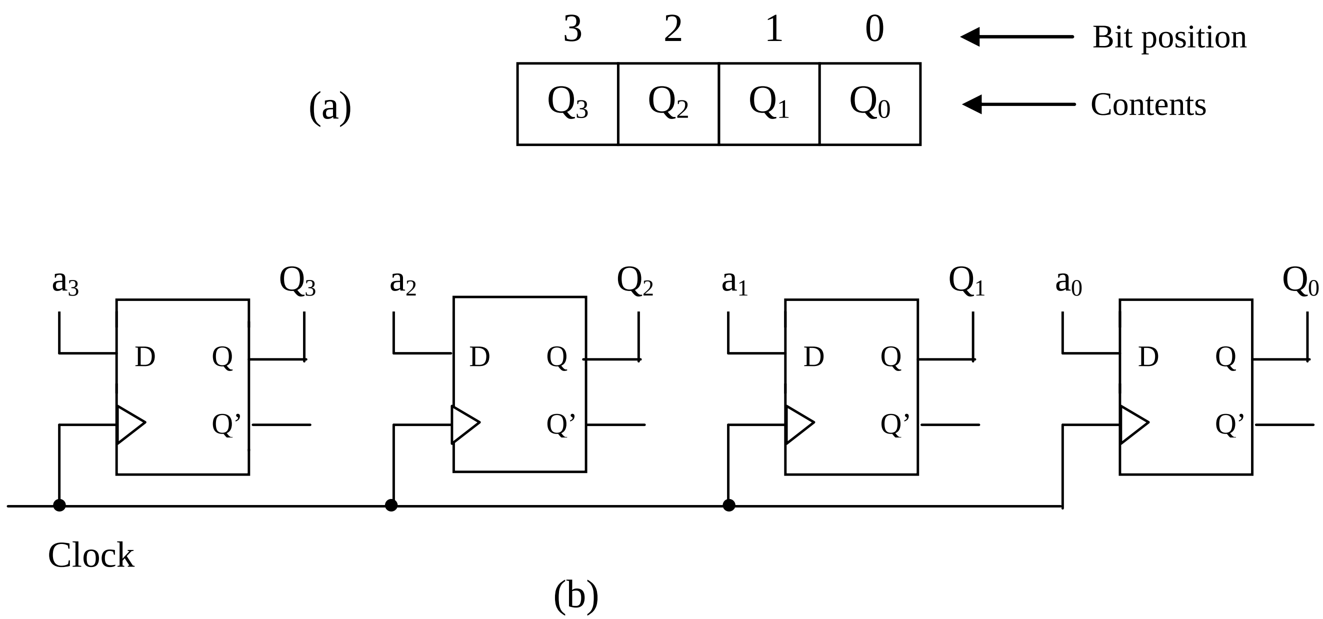
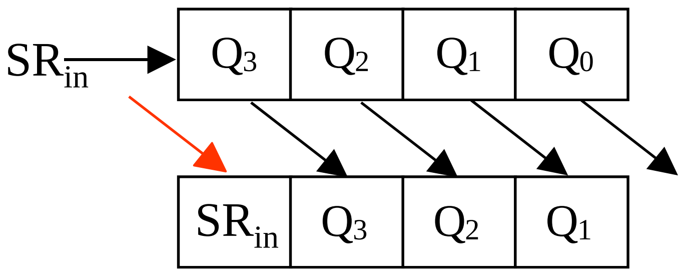
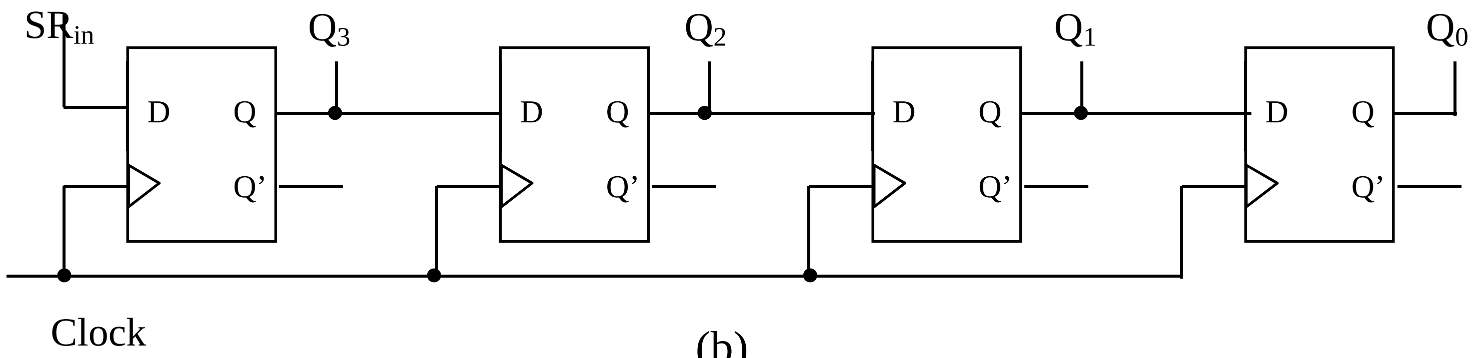


Figure 10.1 (a) Notation for a 4-bit register. (b) Circuit for a 4-bit register.

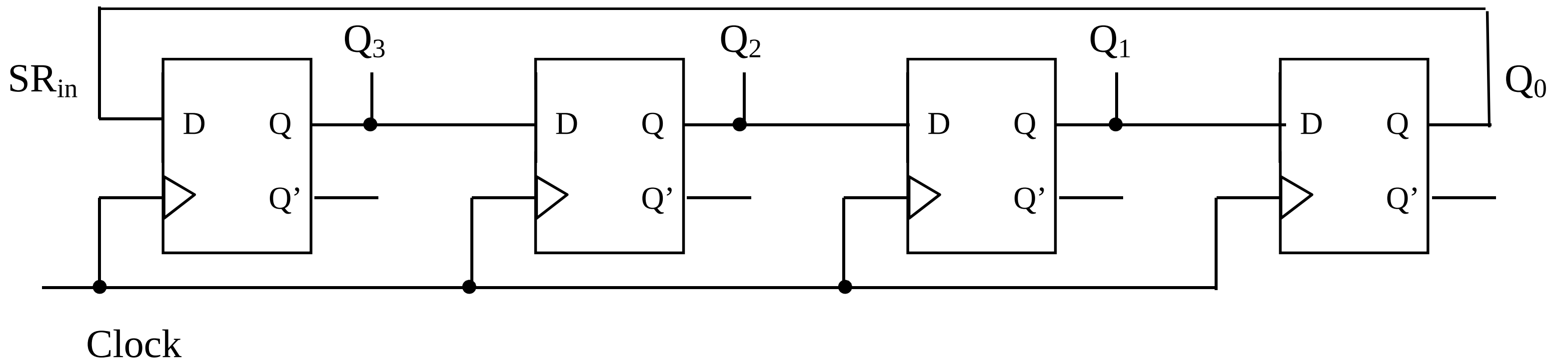
Parallel load

Figure 10.2 (a) Operation of a 4-bit shift-right register.
(b) Circuit of register.

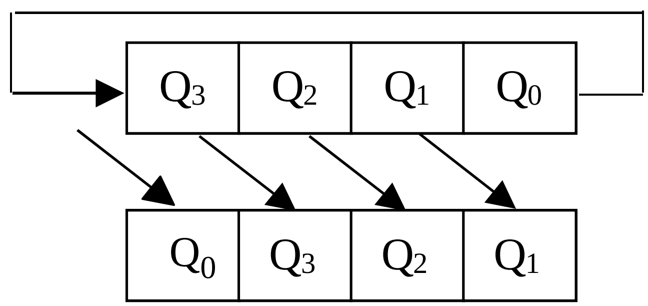


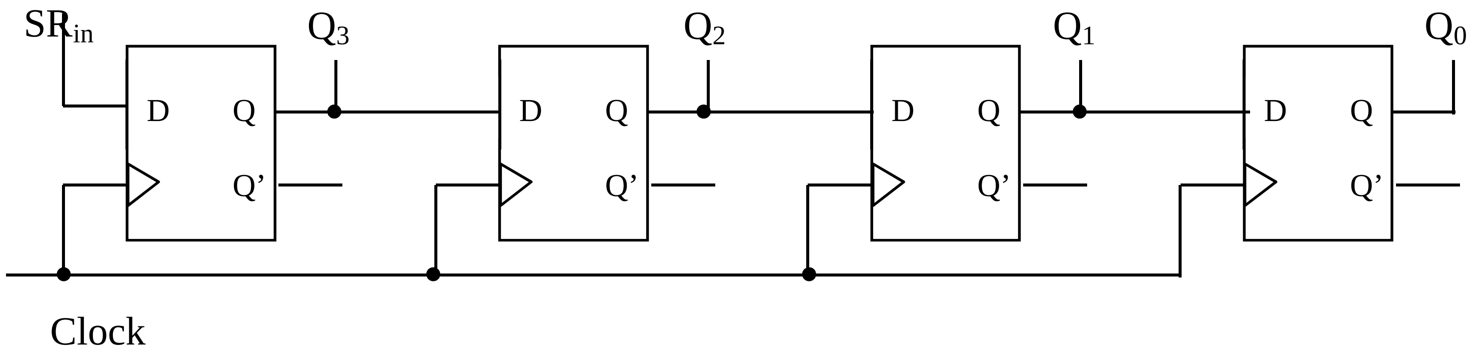


(b)



Rotate right




SR_in
Q3
Q2
Q1
Q0
D
Q
Q'
Clock

Serial load

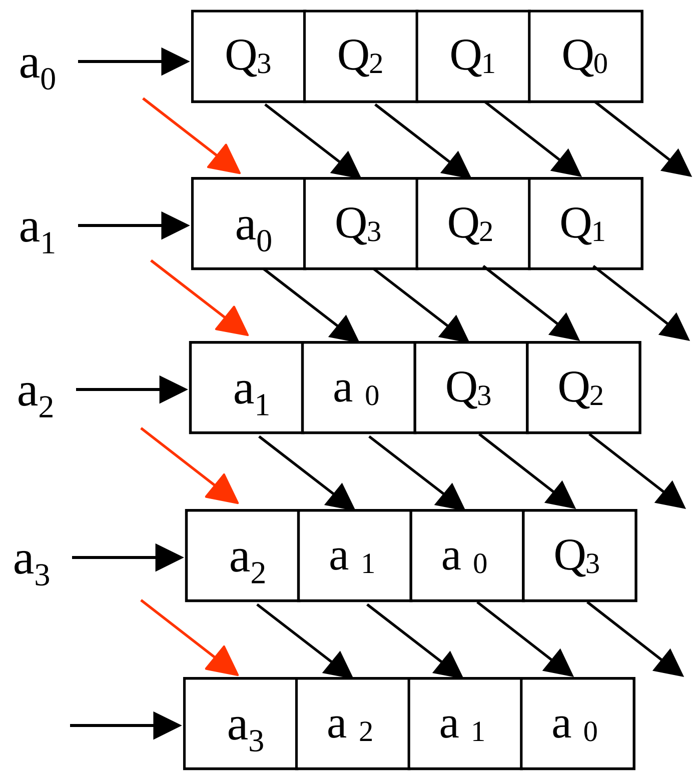

a0
a1
a2
a3
Q3 Q2 Q1 Q0
a0 Q3 Q2 Q1
a1 a0 Q3 Q2
a2 a1 a0 Q3
a3 a2 a1 a0

Table 10.1 Function table for a 4-bit universal shift register.

Function	s_1 s_0	Contents: Bit position 3	2	1	0
Hold	0 0	Q_3	Q_2	Q_1	Q_0
Shift right	0 1	SR_{in}	Q_3	Q_2	Q_1
Shift left	1 0	Q_2	Q_1	Q_0	SL_{in}
Parallel load	1 1	a_3	a_2	a_1	a_0

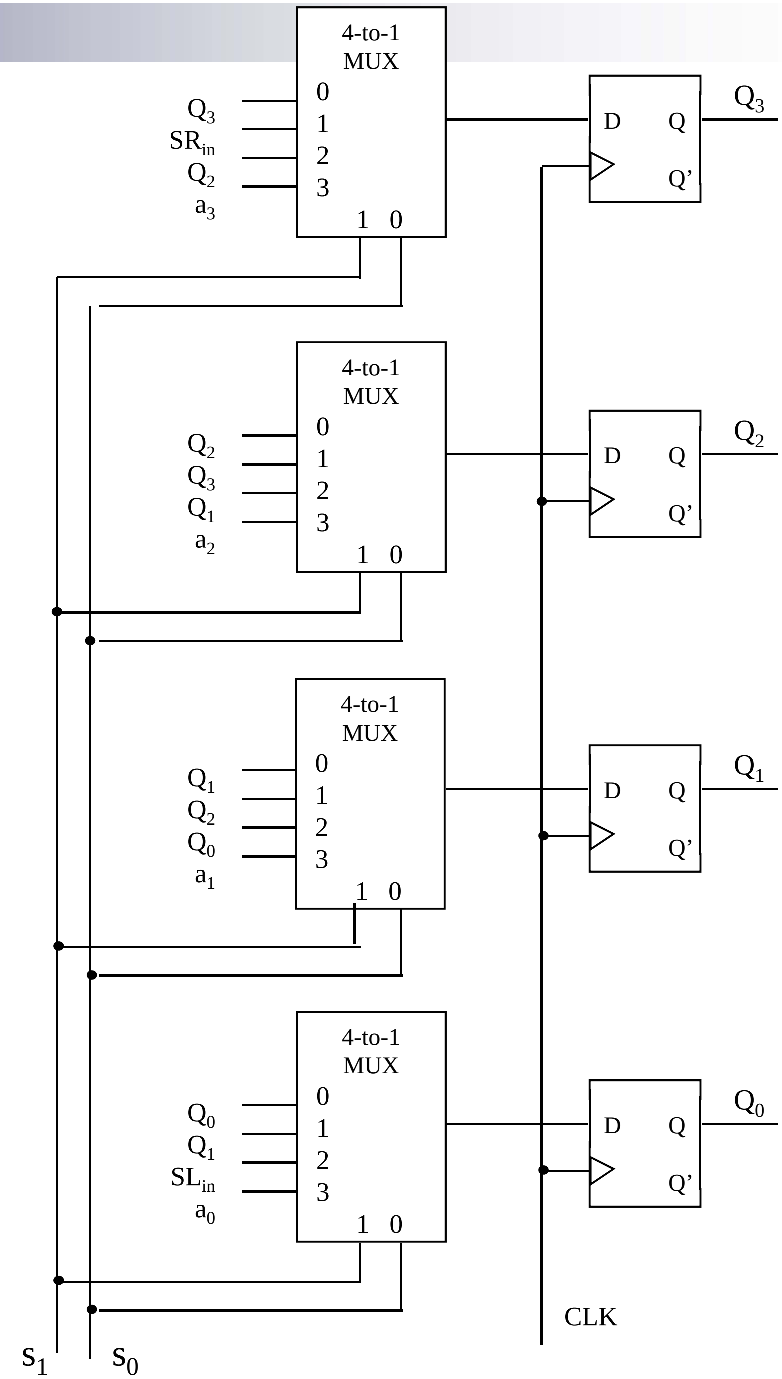


Figure 10.3 Design of a 4-bit universal shift register.

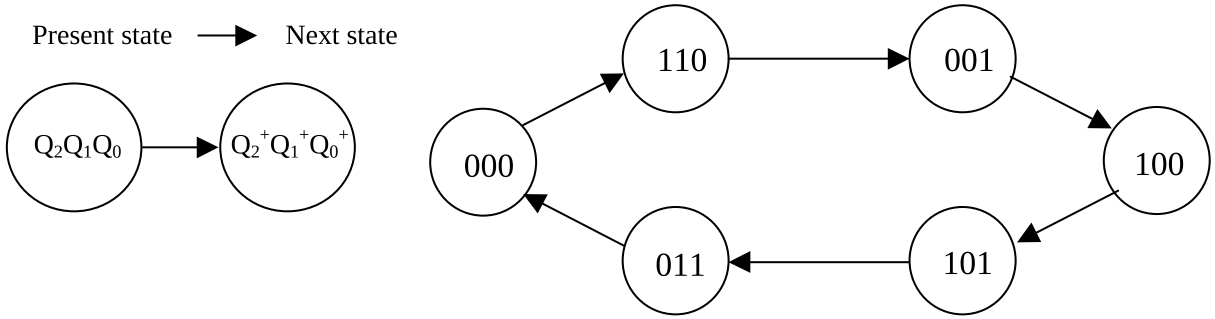


Figure 10.4 State diagram of a 6-state counter.

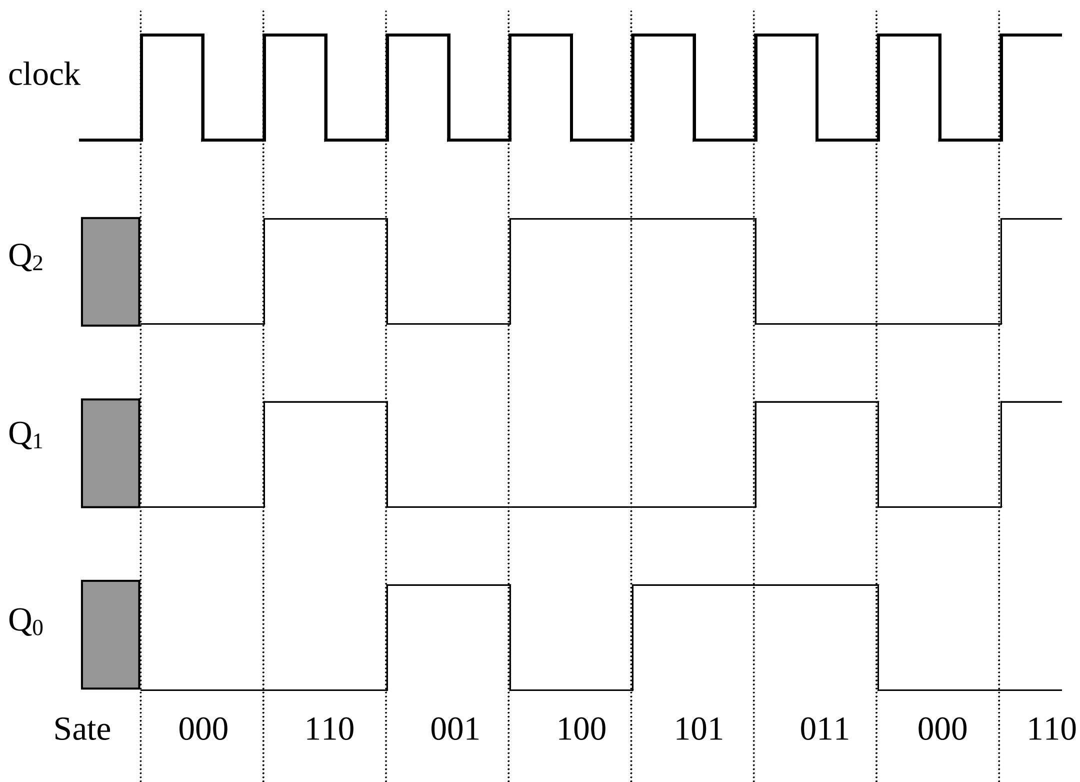


Figure 10.5 Timing diagram of the 6-state counter in Figure 10.4.

Ring Counter

Table 10.2 State assignment table for a 4-state ring counter

State	Q_0	Q_1	Q_2	Q_3
T_0	1	0	0	0
T_1	0	1	0	0
T_2	0	0	1	0
T_3	0	0	0	1

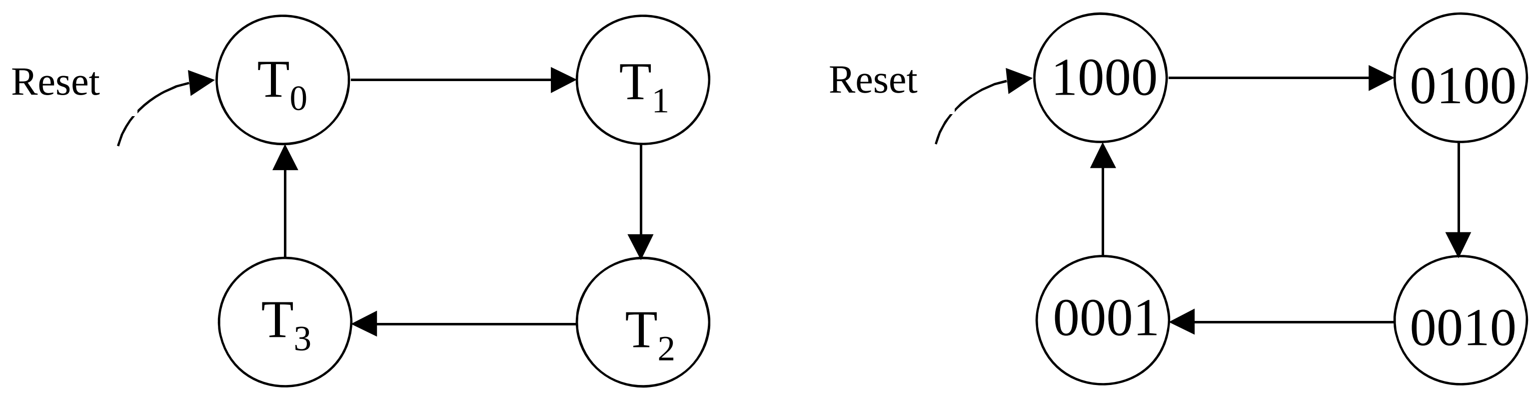


Figure 10.6 State diagram for a 4-state ring counter.

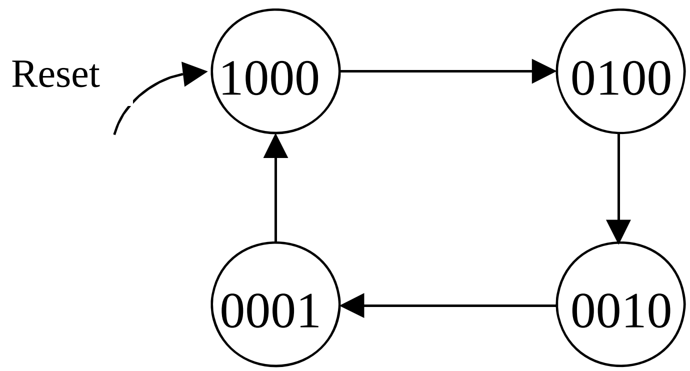


Figure 10.6 State diagram for a 4-state ring counter.

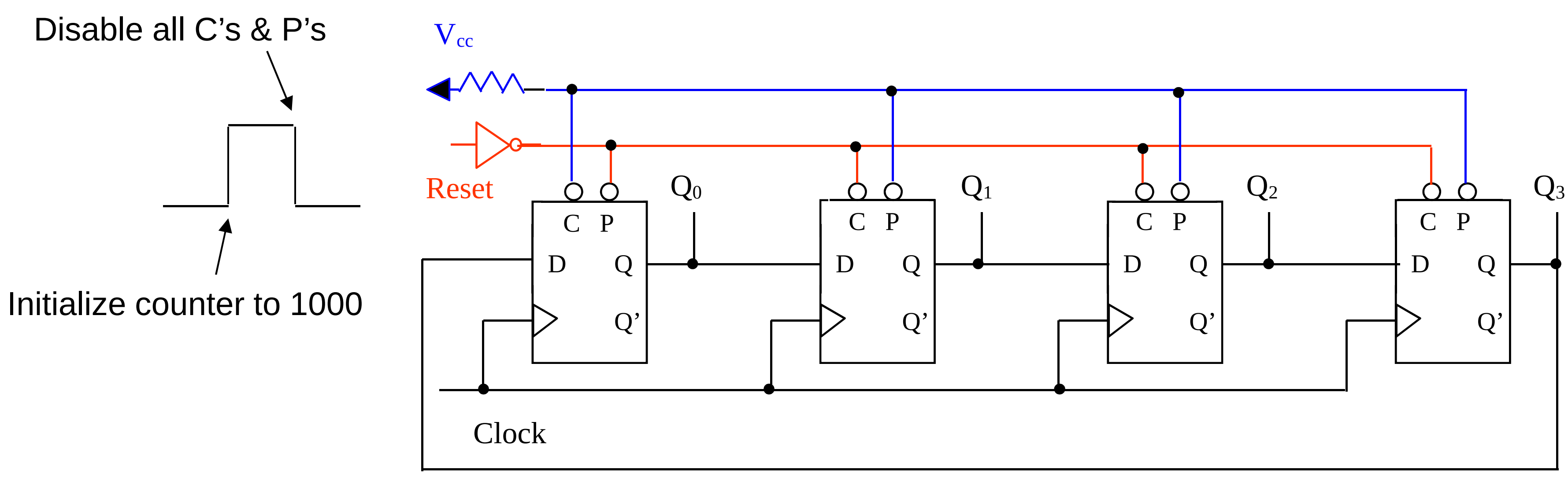


Figure 10.7 Circuit diagram for a 4-bit ring counter.

10.3 Analysis of Synchronous Sequential Circuits

Moore model

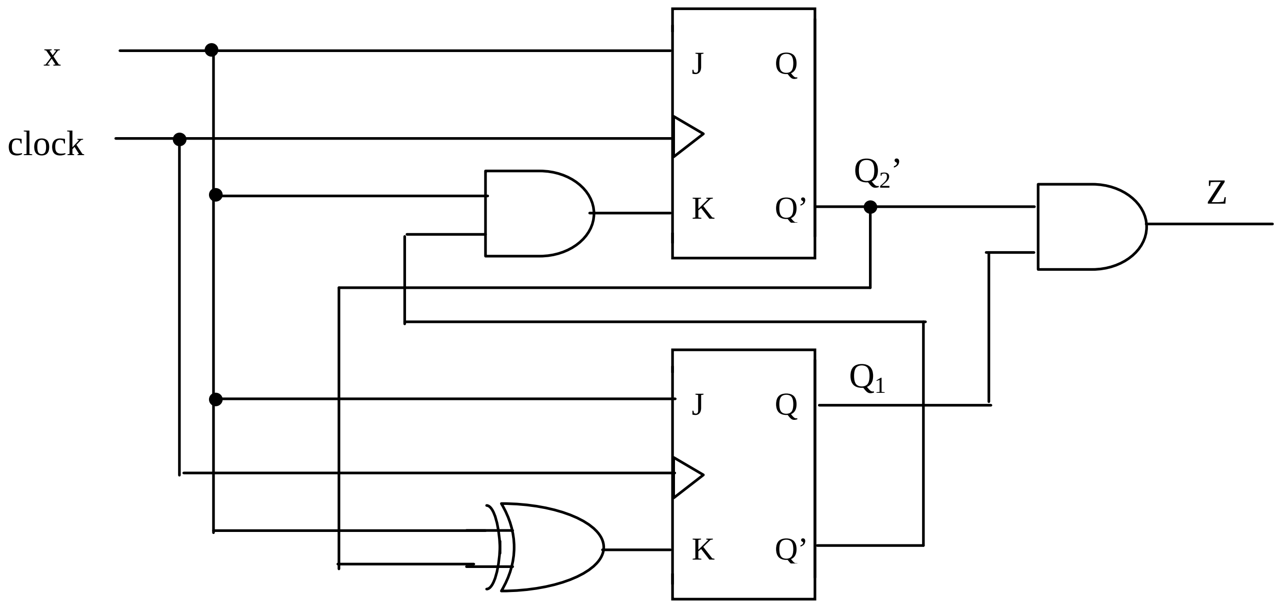


Figure 10.8 Synchronous sequential circuit of Moore model for analysis.

Step 1: Write the excitation and output functions.

$J_2 = x \qquad K_2 = x\,Q_1'$

$J_1 = x \qquad K_1 = x \oplus Q_2'$

$Z = Q_2'Q_1$

Step 1: Write the excitation and output functions.

$J_2 = x \qquad K_2 = x\,Q_1'$

$J_1 = x \qquad K_1 = x \oplus Q_2'$

$Z = Q_1$

Step 2: Substitute the excitation functions into the characteristic equations for the two flip-flops to get the next-state equations.

$$Q_2^+ = J_2Q_2' + K_2'Q_2 = xQ_2' + (x\,Q_1')'Q_2 = xQ_2' + x'Q_2 + Q_2Q_1$$

$$Q_1^+ = J_1Q_1' + K_1'Q_1 = xQ_1' + (x \oplus Q_2')'Q_1 = xQ_1' + x'Q_2Q_1 + xQ_2'Q_1$$

Step 3: Convert the next-state equations to next-state maps.

x \ Q_2Q_1	00	01	11	10
0	0	0	1	1
1	1	1	1	0

Q_2^+

x \ Q_2Q_1	00	01	11	10
0	0	0	1	0
1	1	1	0	1

Q_1^+

Figure 10.9 Next-state equations.

Step 3: Convert the next-state equations to next-state maps.

Q_2^+:

x \ Q_2Q_1	00	01	11	10
0	0	0	1	1
1	1	1	1	0

Q_1^+:

x \ Q_2Q_1	00	01	11	10
0	0	0	1	0
1	1	1	0	1

Figure 10.9 Next-state equations.

Step 4: Convert the next-state maps to a table. The table is called a transition table because it shows the transition from present states to next states. If the output is also included in the table, it is called a transition/output table.

Table 10.3 Transition/output table.

Q_2Q_1	$Q_2^+ Q_1^+$		Z
	x = 0	x = 1	
0 0	0 0	1 1	0
0 1	0 0	1 1	1
1 1	1 1	1 0	0
1 0	1 0	0 1	0

Step 4: Convert the next-state maps to a table. The table is called a transition table because it shows the transition from present states to next states. If the output is also included in the table, it is called a transition/output table.

Table 10.3 Transition/output table.

Q_2Q_1	$Q_2^+ Q_1^+$		Z
	x = 0	x = 1	
0 0	0 0	1 1	0
0 1	0 0	1 1	1
1 1	1 1	1 0	0
1 0	1 0	0 1	0

Step 5: Replace the states in the transition/output table using the state assignment in Table 10.4. The transition/output table is transformed into a state/output table.

Table 10.4 State assignment.

Q_2Q_1	State
0 0	A
0 1	B
1 1	C
1 0	D

Table 10.5 State/output table.

Present state	Next state		Z
	x = 0	x = 1	
A	A	C	0
B	A	C	1
C	C	D	0
D	D	B	0

Step 5: Replace the states in the transition/output table using the state assignment in Table 10.4. The transition/output table is transformed into a state/output table.

Table 10.4 State assignment.

Q_2Q_1	State
0 0	A
0 1	B
1 1	C
1 0	D

Table 10.5 State/output table.

Present state	Next state		Z
	x = 0	x = 1	
A	A	C	0
B	A	C	1
C	C	D	0
D	D	B	0

Step 6: Convert the state/output table to a state diagram.

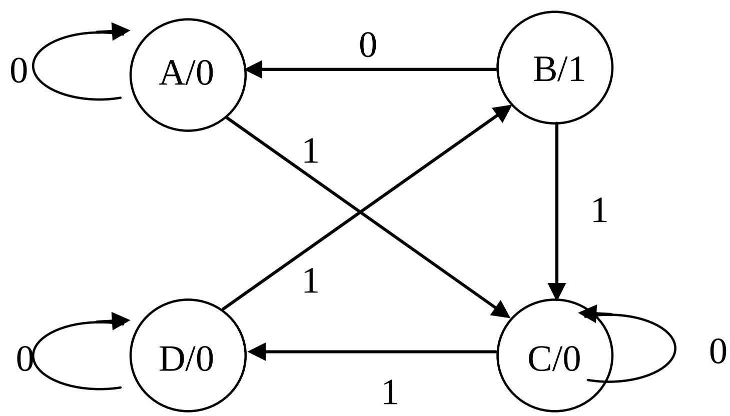


Figure 10.10 State diagram.

Write excitation functions & output functions

↓

Obtain next-state equations from excitation functions & characteristic equations

↓

Convert next-state equations to next-state maps

↓

Convert next-state maps & output functions to transition/output table

↓

State assignments

↓

Convert transition/output table to state/output table

↓

Convert state/output table to state diagram

Mealy model

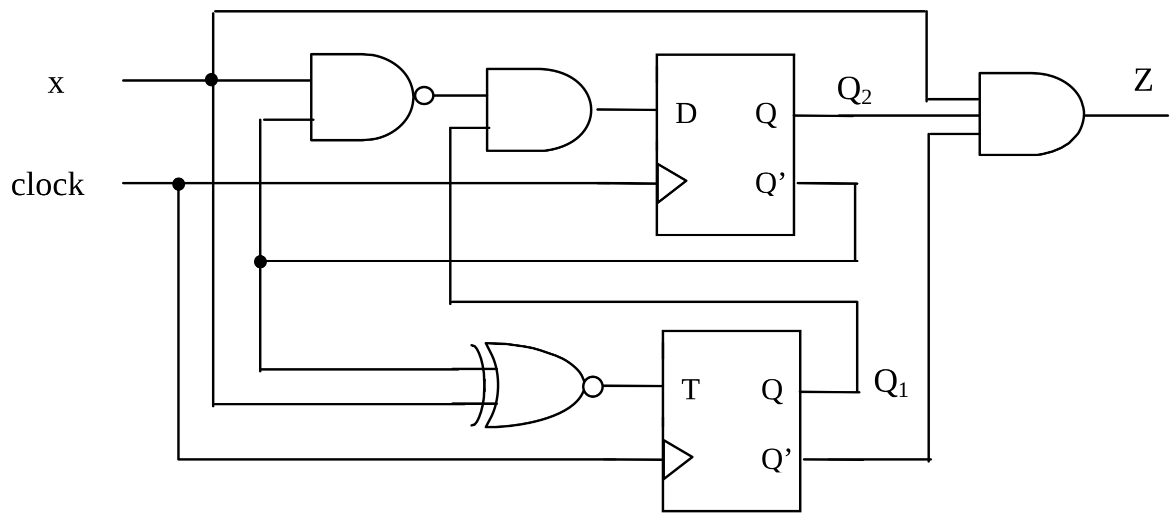


Figure 10.11 Synchronous sequential circuit of Moore model for analysis.

Step 1: Write the excitation and output functions.

$D_2 = (xQ_2')'Q_1$
$T_1 = x \odot Q_2'$
$Z = xQ_2Q_1'$

Step 2: Substitute the excitation functions into the characteristic equations to get the next-state equations.

$Q_2^+ = D_2 = (xQ_2')'Q_1 = x'Q_1 + Q_2Q_1$

$Q_1^+ = T_1 \oplus Q_1 = (x \odot Q_2') \oplus Q_1 = (x \odot Q_2)' \oplus Q_1 = x \oplus Q_2 \oplus Q_1$

Step 2: Substitute the excitation functions into the characteristic equations to get the next-state equations.

$$Q_2^+ = D_2 = (xQ_2')'Q_1 = x'Q_1 + Q_2Q_1$$
$$Q_1^+ = T_1 \oplus Q_1 = (x \odot Q_2') \oplus Q_1 = (x \odot Q_2)' \oplus Q_1 = x \oplus Q_2 \oplus Q_1$$

Step 3: Convert the next-state equations to next-state maps. Q_1^+ is the same as the function in Example 5.6.

x \ Q_2Q_1	00	01	11	10
0	0	1	1	0
1	0	0	1	0

Q_2^+

x \ Q_2Q_1	00	01	11	10
0	0	1	0	1
1	1	0	1	0

Q_1^+

Figure 10.12 Next-state equations.

Step 4: Convert the next-state maps to a transition/output table. Note that the values of Z do not have to be listed separately. They are placed next to the values of $Q_2^+Q_1^+$ because Z is also a function of Q2, Q_1, and x.

Table 10.6 Transition/output table.

Q_2Q_1	$Q_2^+ Q_1^+$, Z	
	x = 0	x = 1
0 0	0 0, 0	0 1, 0
0 1	1 1, 0	0 0, 0
1 1	1 0, 0	1 1, 0
1 0	0 1, 0	0 0, 1

Table 10.6 Transition/output table.

Q_2Q_1	$Q_2^+ Q_1^+, Z$	
	x = 0	x = 1
0 0	0 0, 0	0 1, 0
0 1	1 1, 0	0 0, 0
1 1	1 0, 0	1 1, 0
1 0	0 1, 0	0 0, 1

Step 5: Convert the transition/output table to a state/output table using the state assignment in Table 10.7.

Table 10.7 State assignment.

Q_2Q_1	State
0 0	A
0 1	B
1 1	C
1 0	D

Table 10.8 State/output table.

Present state	Next state	
	x = 0	x = 1
A	A, 0	B, 0
B	C, 0	A, 0
C	D, 0	C, 0
D	B, 0	A, 1

Step 5: Convert the transition/output table to a state/output table using the state assignment in Table 10.7.

Table 10.7 State assignment.

Q_2Q_1	State
0 0	A
0 1	B
1 1	C
1 0	D

Table 10.8 State/output table.

Present state	Next state	
	x = 0	x = 1
A	A, 0	B, 0
B	C, 0	A, 0
C	D, 0	C, 0
D	B, 0	A, 1

Step 6: Convert the state/output table to a state diagram. Because Z is a function of the present state and the input x, its values are placed after x and separated by a slash.

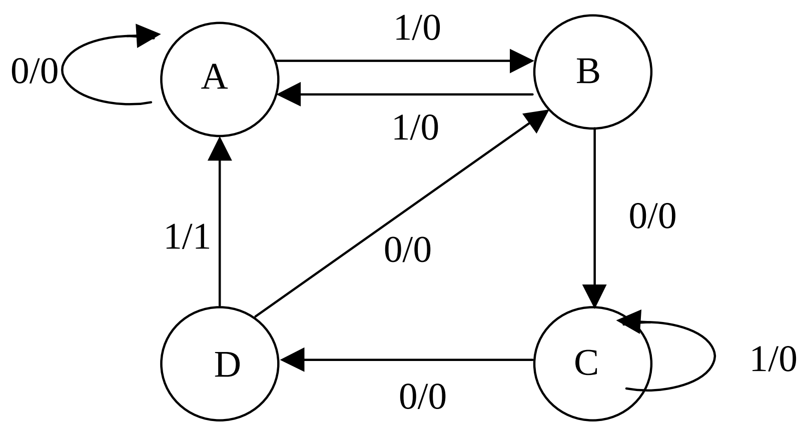


Figure 10.13 State diagram.

10.4 Synthesis of Synchronous Sequential Circuits

10.4.1 Counter Design

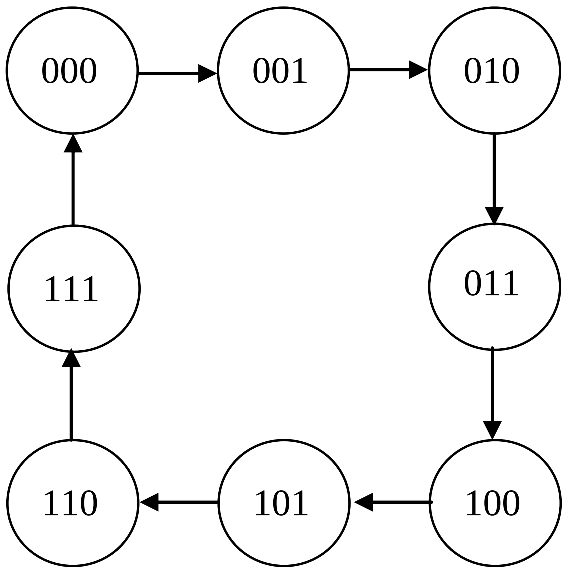


Figure 10.14 State diagram for a modulo-8 counter.

Table 10.9 Transition table for a modulo-8 counter.

Present state $Q_2Q_1Q_0$	Next state $Q_2^+Q_1^+Q_0^+$
0 0 0	0 0 1
0 0 1	0 1 0
0 1 0	0 1 1
0 1 1	1 0 0
1 0 0	1 0 1
1 0 1	1 1 0
1 1 0	1 1 1
1 1 1	0 0 0

Q_2^+

Q_0 \ Q_2Q_1	00	01	11	10
0	0	0	1	1
1	0	1	0	1

Q_1^+

Q_0 \ Q_2Q_1	00	01	11	10
0	0	1	1	0
1	1	0	0	1

Q_0^+

Q_0 \ Q_2Q_1	00	01	11	10
0	1	1	1	1
1	0	0	0	0

Figure 10.15 Next-state maps for a modulo-8 counter.

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	0	0	1	1
1	0	1	0	1

Q_2^+

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	0	1	1	0
1	1	0	0	1

Q_1^+

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	1	1	1	1
1	0	0	0	0

Q_0^+

Figure 10.15 Next-state maps for a modulo-8 counter.

$$Q_2^+ = Q_2'Q_1Q_0 + Q_2Q_0' + Q_2Q_1' = Q_2'Q_1Q_0 + (Q_0' + Q_1')\,Q_2$$
$$= (Q_1Q_0)\,Q_2' + (Q_1Q_0)'\,Q_2 = (Q_1Q_0) \oplus Q_2$$

$$Q_1^+ = Q_1Q_0' + Q_1'Q_0 = Q_1 \oplus Q_0$$

$$Q_0^+ = Q_0'$$

Design with D flip-flops

$$D_2 = Q_2^+ = (Q_1 Q_0) \oplus Q_2$$
$$D_1 = Q_1^+ = Q_1 \oplus Q_0$$
$$D_0 = Q_0^+ = Q_0'$$

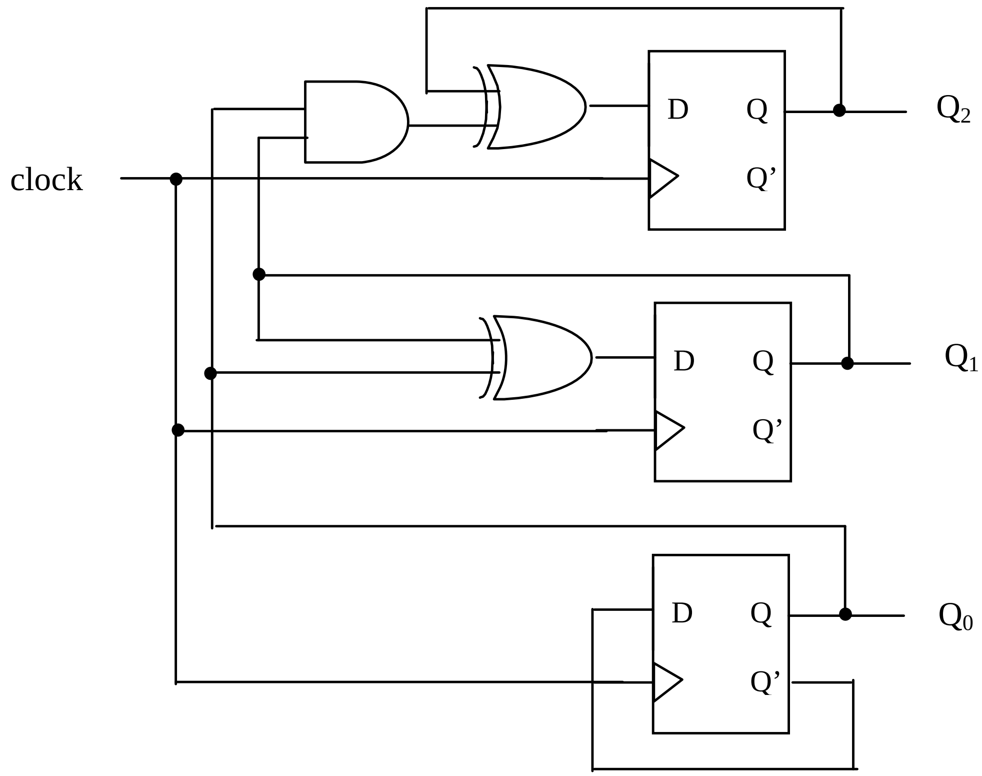


Figure 10.16 Circuit diagram for modulo-8 counter.

Design with JK flip-flops

Table 10.10 Excitation table for JK flip-flops.

Q	Q^+	J	K	Function
0	0	0	d	No change (JK = 00) or reset (JK = 01)
0	1	1	d	Set (JK = 10) or toggled (JK = 11)
1	0	d	1	Reset (JK = 01) or toggle (JK = 11)
1	1	d	0	No change (JK = 00) or set (JK = 10)

Table 10.11 J and K excitations for a modulo-8 counter.

Present state $Q_2Q_1Q_0$	Next state $Q_2^+Q_1^+Q_0^+$	Excitations					
		J_2	K_2	J_1	K_1	J_0	K_0
0 0 0	0 0 1	0	d	0	d	1	d
0 0 1	0 1 0	0	d	1	d	d	1
0 1 0	0 1 1	0	d	d	0	1	d
0 1 1	1 0 0	1	d	d	1	d	1
1 0 0	1 0 1	d	0	0	d	1	d
1 0 1	1 1 0	d	0	1	d	d	1
1 1 0	1 1 1	d	0	d	0	1	d
1 1 1	0 0 0	d	1	d	1	d	1

Table 10.11 J and K excitations for a modulo-8 counter.

Present state $Q_2Q_1Q_0$	Next state $Q_2^+Q_1^+Q_0^+$	Excitations		
		J_2 K_2	J_1 K_1	J_0 K_0
0 0 0	0 0 1	0 d	0 d	1 d
0 0 1	0 1 0	0 d	1 d	d 1
0 1 0	0 1 1	0 d	d 0	1 d
0 1 1	1 0 0	1 d	d 1	d 1
1 0 0	1 0 1	d 0	0 d	1 d
1 0 1	1 1 0	d 0	1 d	d 1
1 1 0	1 1 1	d 0	d 0	1 d
1 1 1	0 0 0	d 1	d 1	d 1

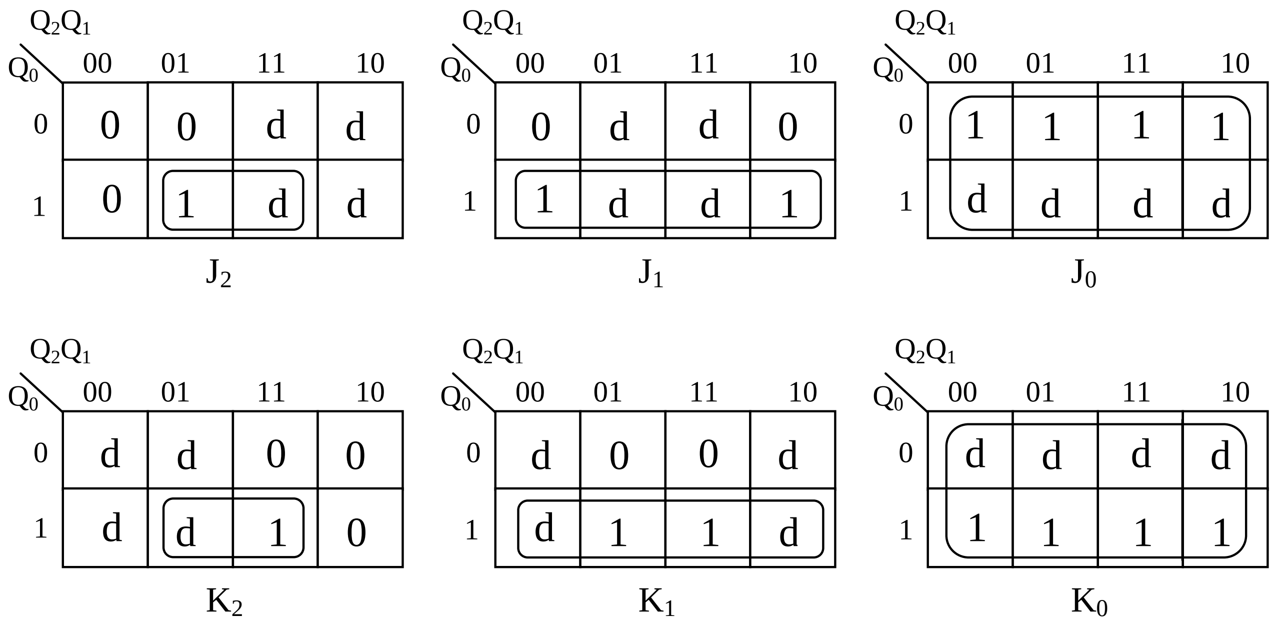


Figure 10.17 K-maps for excitation functions.

$J_2 = Q_1Q_0$ $\quad$ $J_1 = Q_0$ $\quad$ $J_0 = 1$

$K_2 = Q_1Q_0$ $\quad$ $K_1 = Q_0$ $\quad$ $K_0 = 1$

Design with JK Flip-Flops by Partition

Shannon's expansion theorem

$$Q_i^+(Q_{n-1}, Q_{n-2}, ..., Q_i, ... , Q_1, Q_0, x_{m-1}, x_{m-2}, ..., x_1, x_0)$$

$$= Q_i' \bullet Q_i^+(Q_{n-1}, Q_{n-2}, ..., Q_i = 0, ... , Q_1, Q_0, x_{m-1}, x_{m-2}, ..., x_1, x_0)$$

$$+ Q_i \bullet Q_i^+(Q_{n-1}, Q_{n-2}, ..., Q_i = 1, ... , Q_1, Q_0, x_{m-1}, x_{m-2}, ..., x_1, x_0)$$

$$Q_i^+ = J_i Q_i' + K_i' Q_i$$

$$J_i = Q_i^+(Q_{n-1}, Q_{n-2}, ..., Q_i = 0, ... , Q_1, Q_0, x_{m-1}, x_{m-2}, ..., x_1, x_0) = (Q_i^+)_{Qi = 0}$$

$$K_i' = Q_i^+(Q_{n-1}, Q_{n-2}, ..., Q_i = 1, ... , Q_1, Q_0, x_{m-1}, x_{m-2}, ..., x_1, x_0) = (Q_i^+)_{Qi = 1}$$

Binary Tree

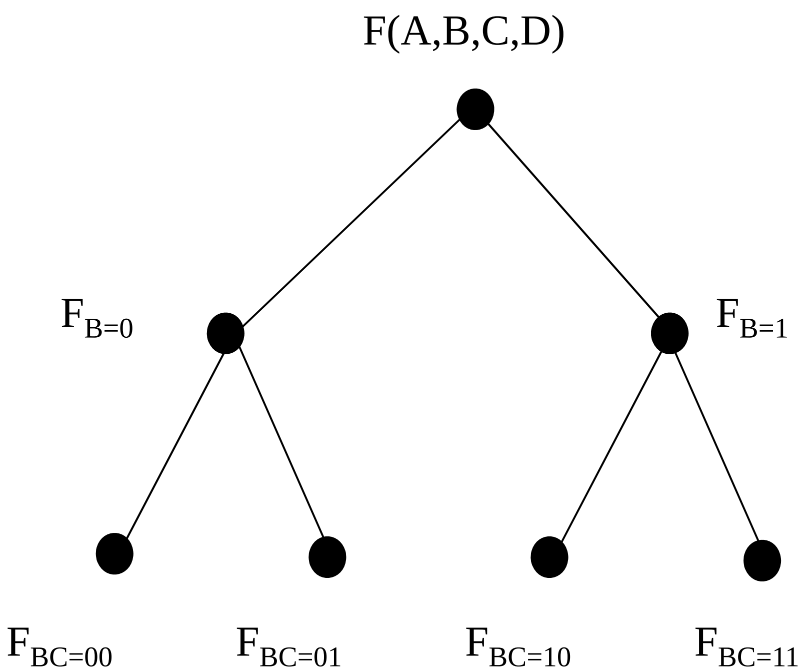
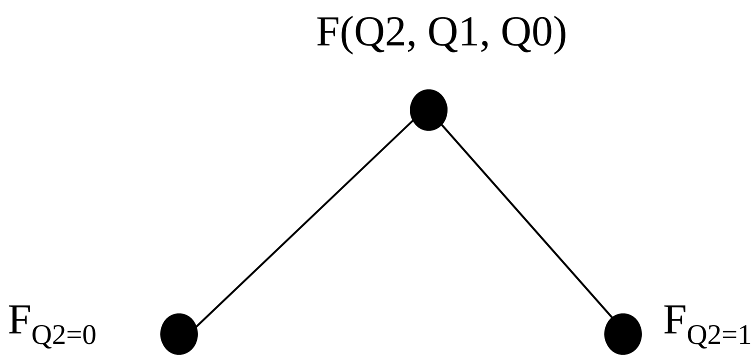


Figure 4.3 A binary tree for the expansion of a Boolean function.

Binary Tree



$$F = Q_2' F_{Q2=0} + Q_2 F_{Q2=1}$$

$$F = J2\ Q_2' + K_2' Q_2$$

$$J_2 = F_{Q2=0} \qquad K_2 = (F_{Q2=1})'$$

Design with JK Flip-Flops by Partition

$$J_i = Q_i^+(Q_{n-1}, Q_{n-2}, \ldots, Q_i = 0, \ldots, Q_1, Q_0, x_{m-1}, x_{m-2}, \ldots, x_1, x_0) = (Q_i^+)_{Qi = 0}$$

$$K_i' = Q_i^+(Q_{n-1}, Q_{n-2}, \ldots, Q_i = 1, \ldots, Q_1, Q_0, x_{m-1}, x_{m-2}, \ldots, x_1, x_0) = (Q_i^+)_{Qi = 1}$$

From the next state equations of the modulo-8 counter, which are

$$Q_2^+ = (Q_1Q_0) \oplus Q_2$$

$$Q_1^+ = Q_1 \oplus Q_0$$

$$Q_0^+ = Q_0'$$

the excitation functions can be obtained as follows:

$$J_2 = (Q_2^+)_{Q2 = 0} = (Q_1Q_0) \oplus 0 = Q_1Q_0$$

$$K_2 = [(Q_2^+)_{Q2 = 1}]' = [(Q_1Q_0) \oplus 1]' = Q_1Q_0$$

$$J_1 = (Q_1^+)_{Q1 = 0} = 0 \oplus Q_0 = Q_0$$

$$K_1 = [(Q_1^+)_{Q1 = 1}]' = (1 \oplus Q_0)' = Q_0$$

$$J_0 = (Q_0^+)_{Q0 = 0} = 0' = 1$$

$$K_0 = [(Q_0^+)_{Q0 = 1}]' = (1')' = 1$$

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	0	0	1	1
1	0	1	0	1

Q_2^+

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	0	1	1	0
1	1	0	0	1

Q_1^+

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	1	1	1	1
1	0	0	0	0

Q_0^+

Figure 10.15 Next-state maps for a modulo-8 counter.

$Q_0 \backslash Q_1$	0	1
0	0	0
1	0	1

$J_2 = (Q_2^+)_{Q2 = 0}$

$Q_0 \backslash Q_2$	0	1
0	0	0
1	1	1

$J_1 = (Q_1^+)_{Q1 = 0}$

Q_2Q_1	00	01	11	10
	1	1	1	1

$J_0 = (Q_0^+)_{Q0 = 0}$

$Q_0 \backslash Q_1$	1	0
0	1	1
1	0	1

$K_2' = (Q_2^+)_{Q2 = 1}$

$Q_0 \backslash Q_2$	0	1
0	1	1
1	0	0

$K_1' = (Q_1^+)_{Q1 = 1}$

Q_2Q_1	00	01	11	10
	0	0	0	1

$K_0' = (Q_0^+)_{Q0 = 1}$

$Q_0 \backslash Q_1$	1	0
0	0	0
1	1	0

K_2

$Q_0 \backslash Q_2$	0	1
0	0	0
1	1	1

K_1

Q_2Q_1	00	01	11	10
	1	1	1	1

K_0

$$J_2 = K_2 = Q_1Q_0$$
$$J_1 = K_1 = Q_0$$
$$J_0 = K_0 = 1$$

Design with T flip-flops

Table 10.12 Excitation table for T flip-flops.

Q	Q^+	T	Function
0	0	0	No change
0	1	1	Toggled
1	0	1	Toggled
1	1	0	No change

$$T = Q \oplus Q^+$$

$$T_2 = Q_2 \oplus Q_2^+ = Q_2 \oplus (Q_1 Q_0) \oplus Q_2 = Q_1Q_0$$
$$T_1 = Q_1 \oplus Q_1^+ = Q_1 \oplus Q_1 \oplus Q_0 = Q_0$$
$$T_0 = Q_0 \oplus Q_0^+ = Q_0 \oplus Q_0' = 1$$

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	0	0	1	1
1	0	1	0	1

Q_2^+

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	0	1	1	0
1	1	0	0	1

Q_1^+

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	1	1	1	1
1	0	0	0	0

Q_0^+

Figure 10.15 Next-state maps for a modulo-8 counter.

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	0	0	0	0
1	0	1	1	0

T_2

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	0	0	0	0
1	1	1	1	1

T_1

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	1	1	1	1
1	1	1	1	1

T_0

Figure 10.19 K-maps for the excitations of T flip-flops.

$$T_2 = Q_2 \oplus Q_2^+ = Q_2 \oplus (Q_1\, Q_0) \oplus Q_2 = Q_1Q_0$$

$$T_1 = Q_1 \oplus Q_1^+ = Q_1 \oplus Q_1 \oplus Q_0 = Q_0$$

$$T_0 = Q_0 \oplus Q_0^+ = Q_0 \oplus Q_0' = 1$$

10.4.2 Self-Correcting Counter

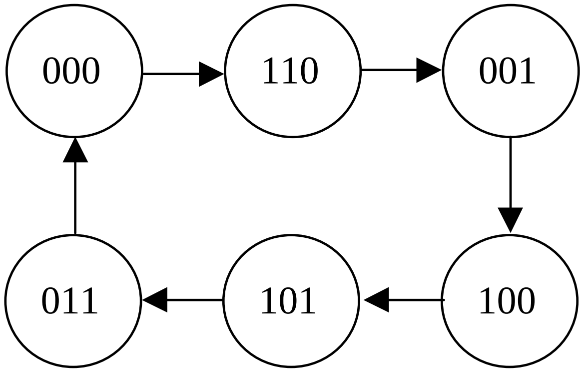


Figure 10.21 State diagram for a 6-state self-correcting counter.

Table 10.13 Transition table for a 6-state self-correcting counter.

Present state $Q_2Q_1Q_0$	Next state $Q_2^+Q_1^+Q_0^+$
0 0 0	1 1 0
0 0 1	1 0 0
0 1 0	d d d
0 1 1	0 0 0
1 0 0	1 0 1
1 0 1	0 1 1
1 1 0	0 0 1
1 1 1	d d d

Q_2^+

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	1	d	0	1
1	1	0	d	0

Q_1^+

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	1	d	0	0
1	0	0	d	1

Q_0^+

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	0	d	1	1
1	0	0	d	1

Figure 10.22 Next-state maps for the 6-state self-correcting counter.

$$D_2 = Q_2^+ = Q_2'Q_1' + Q_1'Q_0'$$
$$D_1 = Q_1^+ = Q_2'Q_0' + Q_2Q_0 = (Q_2 \oplus Q_0)'$$
$$D_0 = Q_0^+ = Q_2$$

Q_2^+

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	1	d	0	1
1	1	0	d	0

Q_1^+

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	1	d	0	0
1	0	0	d	1

Q_0^+

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	0	d	1	1
1	0	0	d	1

Figure 10.22 Next-state maps for the 6-state self-correcting counter.

$D_2 = Q_2^+ = Q_2'Q_1' + Q_1'Q_0'$

$D_1 = Q_1^+ = Q_2'Q_0' + Q_2Q_0 = (Q_2 \oplus Q_0)'$

$D_0 = Q_0^+ = Q_2$

Q_2^+

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	1	d	0	1
1	1	0	d	0

Q_1^+

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	1	d	0	0
1	0	0	d	1

Q_0^+

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	0	d	1	1
1	0	0	d	1

Figure 10.23 Re-design of the 6-state self-correcting counter.

$D_1 = Q_1^+ = Q_2'Q_1'Q_0' + Q_2Q_1'Q_0 = Q_1'(Q_2 \oplus Q_0)'$

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	1	d	0	1
1	1	0	d	0

Q_2^+

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	1	d	0	0
1	0	0	d	1

Q_1^+

$Q_0 \backslash Q_2Q_1$	00	01	11	10
0	0	d	1	1
1	0	0	d	1

Q_0^+

Figure 10.22 Next-state maps for the 6-state self-correcting counter.

$Q_0 \backslash Q_1$	0	1
0	1	d
1	1	0

$J_2 = (Q_2^+)_{Q2 = 0}$

$Q_0 \backslash Q_2$	0	1
0	1	0
1	0	1

$J_1 = (Q_1^+)_{Q1 = 0}$

Q_2Q_1: 00	01	11	10
0	d	1	1

$J_0 = (Q_0^+)_{Q0 = 0}$

$Q_0 \backslash Q_1$	1	0
0	0	1
1	d	0

$K_2' = (Q_2^+)_{Q2 = 1}$

$Q_0 \backslash Q_2$	0	1
0	d	0
1	0	d

$K_1' = (Q_1^+)_{Q1 = 1}$

Q_2Q_1: 00	01	11	10
0	0	d	1

$K_0' = (Q_0^+)_{Q0 = 1}$

$Q_0 \backslash Q_1$	1	0
0	1	0
1	d	1

K_2

$Q_0 \backslash Q_2$	0	1
0	d	1
1	1	d

K_1

Q_2Q_1: 00	01	11	10
1	1	d	0

K_0

Figure 10.24 K-maps for JK excitations.

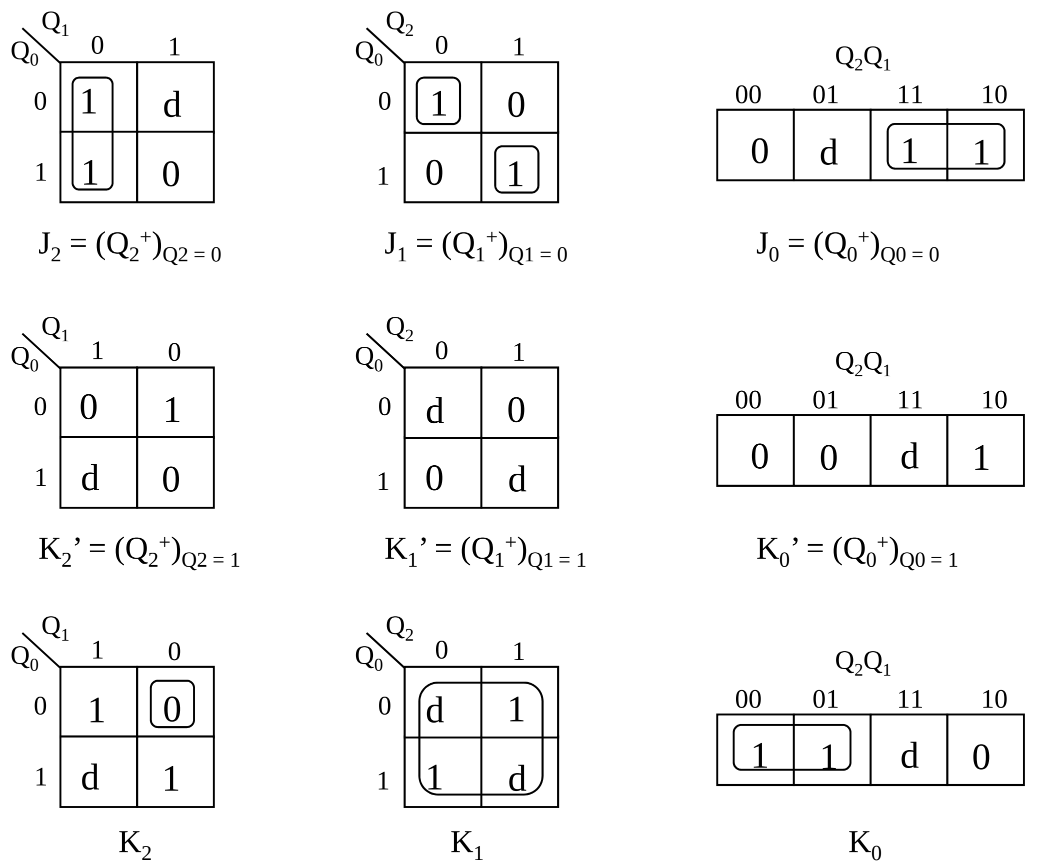


Figure 10.24 K-maps for JK excitations.

$$Q_2^+ = J_2Q_2' + K_2'Q_2 = Q_1'Q_2' + (Q_1 + Q_0)'Q_2 = Q_2'Q_1' + Q_2Q_1'Q_0'$$

$$Q_1^+ = J_1Q_1' + K_1'Q_1 = (Q_2 \oplus Q_0)'Q_1' + (1)'Q_1 = Q_2'Q_1'Q_0' + Q_2Q_1'Q_0$$

$$Q_0^+ = J_0Q_0' + K_0'Q_0 = Q_2 Q_0' + Q_2Q_0 = Q_2$$

Up-Down Counter

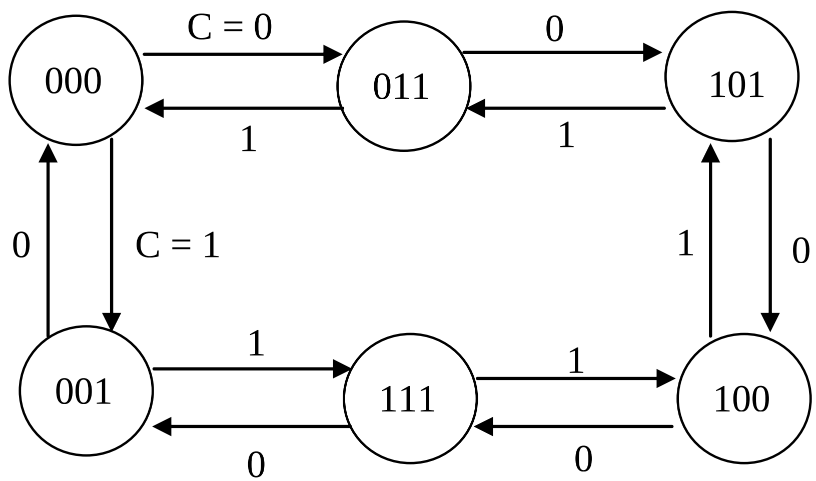


Figure 10.25 State diagram for a 6-state up-down counter.

Table 10.14 Transition table for a 6-state up-down counter.

Present state $Q_2Q_1Q_0$	Next state $Q_2^+Q_1^+Q_0^+$ C = 0	Next state $Q_2^+Q_1^+Q_0^+$ C = 1
0 0 0	0 1 1	0 0 1
0 0 1	0 0 0	1 1 1
0 1 0	d d d	d d d
0 1 1	1 0 1	0 0 0
1 0 0	1 1 1	1 0 1
1 0 1	1 0 0	0 1 1
1 1 0	d d d	d d d
1 1 1	0 0 1	1 0 0

Table 10.14 Transition table for a 6-state up-down counter.

Present state $Q_2Q_1Q_0$	Next state $Q_2^+Q_1^+Q_0^+$	
	C = 0	C = 1
0 0 0	0 1 1	0 0 1
0 0 1	0 0 0	1 1 1
0 1 0	d d d	d d d
0 1 1	1 0 1	0 0 0
1 0 0	1 1 1	1 0 1
1 0 1	1 0 0	0 1 1
1 1 0	d d d	d d d
1 1 1	0 0 1	1 0 0

CQ_0 \ Q_2Q_1	00	01	11	10
00	0	d	d	1
01	0	1	0	1
11	1	0	1	0
10	0	d	d	1

Q_2^+

CQ_0 \ Q_2Q_1	00	01	11	10
00	1	d	d	1
01	0	0	0	0
11	1	0	0	1
10	0	d	d	0

Q_1^+

CQ_0 \ Q_2Q_1	00	01	11	10
00	1	d	d	1
01	0	1	1	0
11	1	0	0	1
10	1	d	d	1

Q_0^+

Figure 10.26 Next-state maps for a 6-state up-down counter.

CQ0 \ Q2Q1	00	01	11	10
00	0	d	d	1
01	0	1	0	1
11	1	0	1	0
10	0	d	d	1

Q_2^+

CQ0 \ Q2Q1	00	01	11	10
00	1	d	d	1
01	0	0	0	0
11	1	0	0	1
10	0	d	d	0

Q_1^+

CQ0 \ Q2Q1	00	01	11	10
00	1	d	d	1
01	0	1	1	0
11	1	0	0	1
10	1	d	d	1

Q_0^+

Figure 10.26 Next-state maps for a 6-state up-down counter.

CQ0 \ Q2Q1	00	01	11	10
00	0	d	d	0
01	0	1	1	0
11	1	0	0	1
10	0	d	d	0

T_2

CQ0 \ Q2Q1	00	01	11	10
00	1	d	d	1
01	0	1	1	0
11	1	1	1	1
10	0	d	d	0

T_1

CQ0 \ Q2Q1	00	01	11	10
00	1	d	d	1
01	1	0	0	1
11	0	1	1	0
10	1	d	d	1

T_0

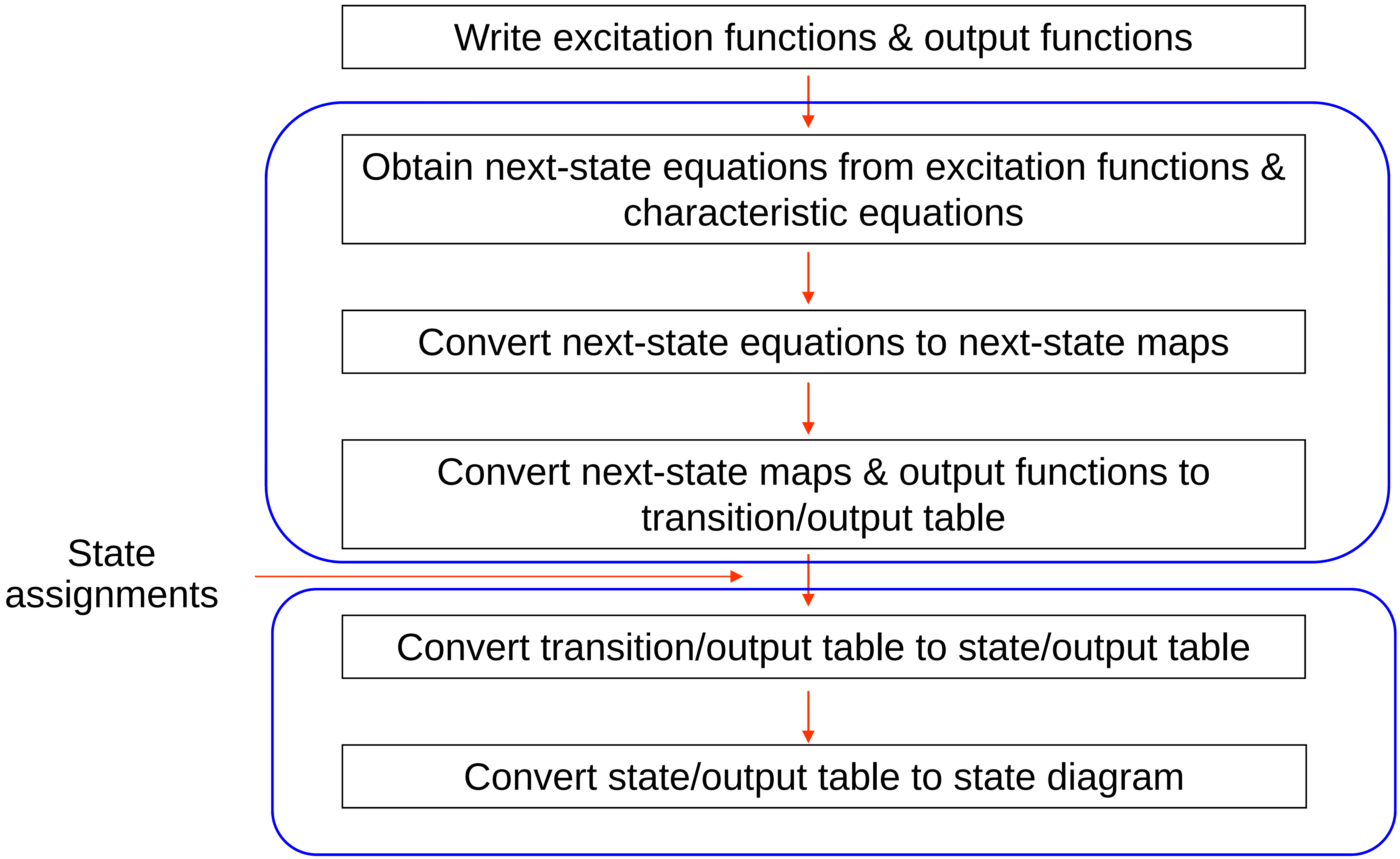
Figure 10.27 T excitations for the 6-state up-down counter in Figure 10.25.

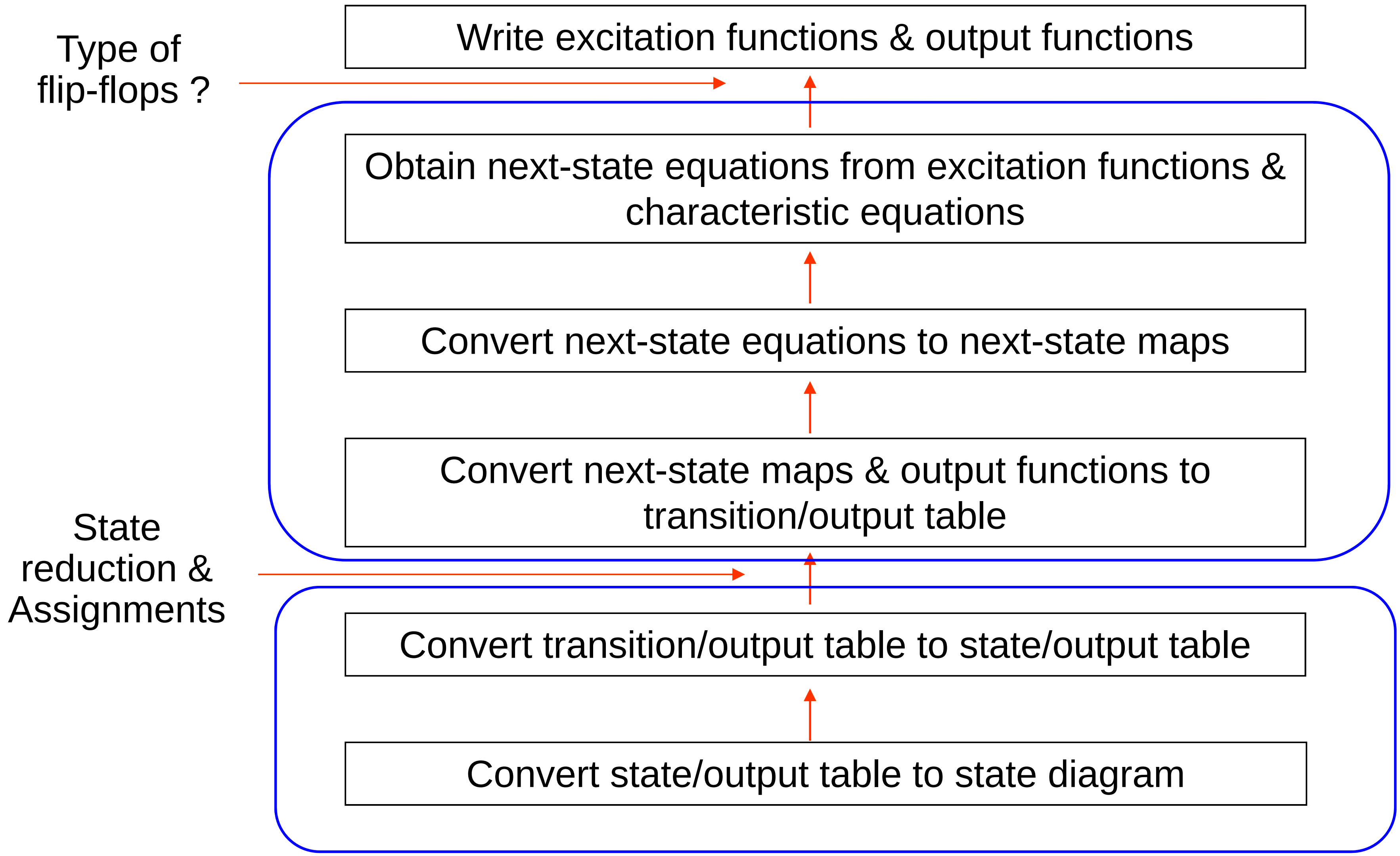
$T_2 = C'Q_1Q_0 + CQ_1'Q_0 = Q_0(C \oplus Q_1)$

$T_1 = (C + Q_1 + Q_0')(C' + Q_1 + Q_0) = Q_1 + (C \oplus Q_0)'$

$Q_2Q_1Q_0 = 010 \qquad Q_2^+Q_1^+Q_0^+ = 001$

$Q_2Q_1Q_0 = 110 \qquad Q_2^+Q_1^+Q_0^+ = 101$


Write excitation functions & output functions
Obtain next-state equations from excitation functions & characteristic equations
Convert next-state equations to next-state maps
Convert next-state maps & output functions to transition/output table
State assignments
Convert transition/output table to state/output table
Convert state/output table to state diagram



Type of flip-flops ?
Write excitation functions & output functions
Obtain next-state equations from excitation functions & characteristic equations
Convert next-state equations to next-state maps
Convert next-state maps & output functions to transition/output table
State reduction & Assignments
Convert transition/output table to state/output table
Convert state/output table to state diagram

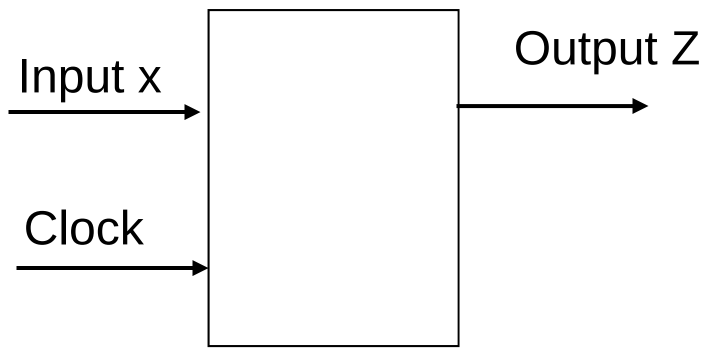
Bit sequence detector /recognizer to detect a 3-bit sequence 101.
Output Z = 1 when sequence is detected.

Moore model - Non-overlapping 00101010100101

Mealy model - Non-overlapping

Moore model – Overlapping 00101010100101

Mealy model - Overlapping



10.4.3 Design of Bit-Sequence Detector

Moore model

Table 10.15 Sample sequences of input and output for a Moore bit-sequence detector.

Clock cycle	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Input x	1	1	0	0	1	0	1	0	1	0	1	1	0	1	….
Output Z	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1

Condition A: Nothing has been detected, not even the first bit of the sequence.
Condition B: The first bit, 1, has been detected.
Condition C: The first two bits, 10, have been detected.
Condition D: All three bits, 101, have been detected.

Condition A: Nothing has been detected, not even the first bit of the sequence.
Condition B: The first bit, 1, has been detected.
Condition C: The first two bits, 10, have been detected.
Condition D: All three bits, 101, have been detected.

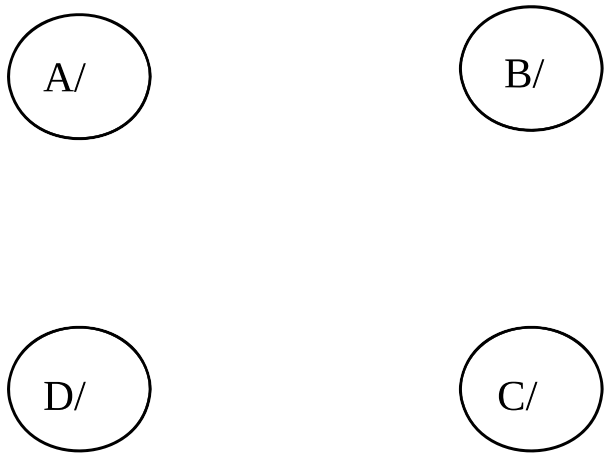


Figure 10.28 State diagram for a circuit of Moore model to detect 101.

Condition A: Nothing has been detected, not even the first bit of the sequence.
Condition B: The first bit, 1, has been detected.
Condition C: The first two bits, 10, have been detected.
Condition D: All three bits, 101, have been detected.

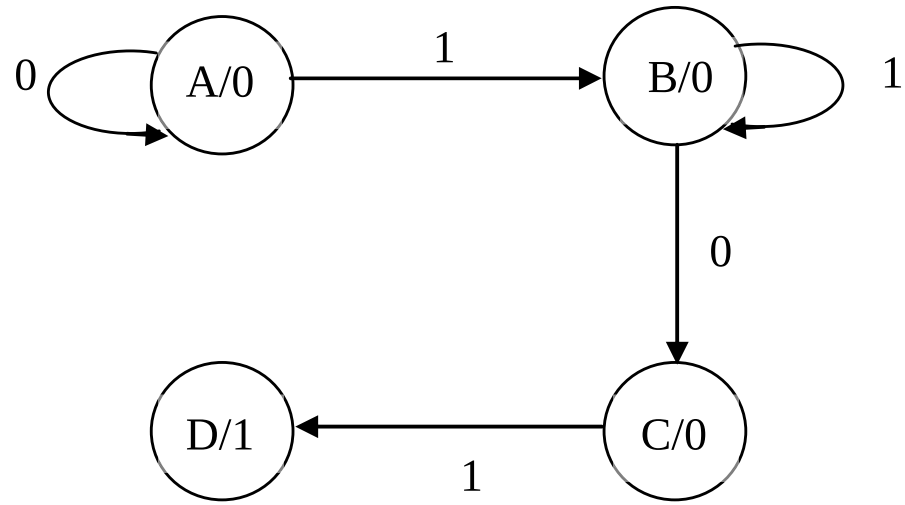


Figure 10.28 State diagram for a circuit of Moore model to detect 101.

Condition A: Nothing has been detected, not even the first bit of the sequence.
Condition B: The first bit, 1, has been detected.
Condition C: The first two bits, 10, have been detected.
Condition D: All three bits, 101, have been detected.

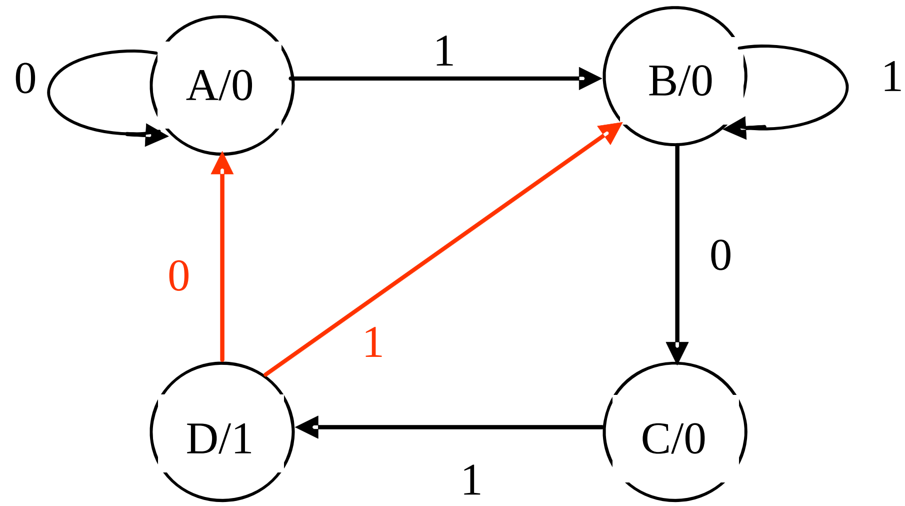


Figure 10.28 State diagram for a circuit of Moore model to detect 101.

Condition A: Nothing has been detected, not even the first bit of the sequence.
Condition B: The first bit, 1, has been detected.
Condition C: The first two bits, 10, have been detected.
Condition D: All three bits, 101, have been detected.

1 0 0 ?

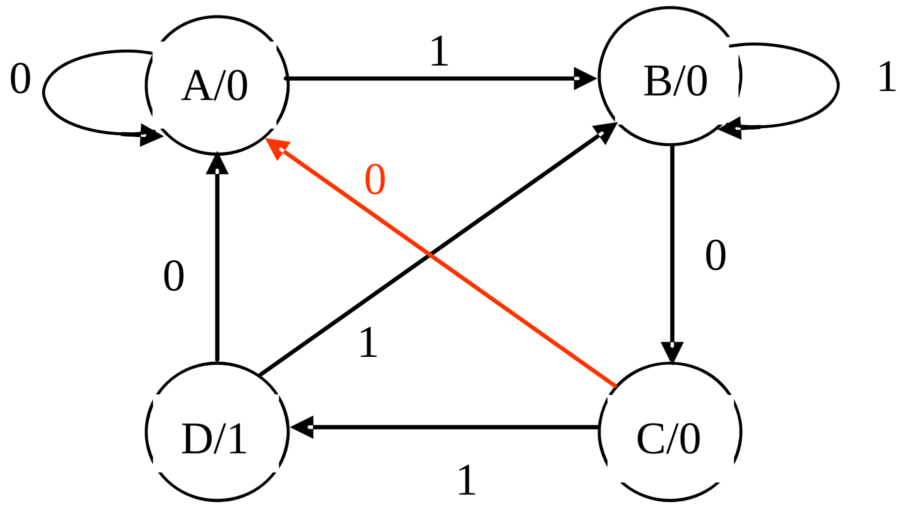


Figure 10.28 State diagram for a circuit of Moore model to detect 101.

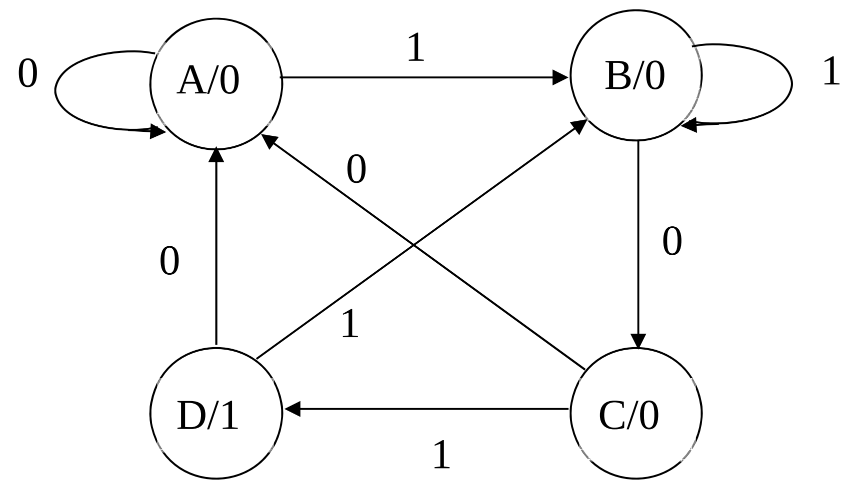


Table 10.16 State/output table for bit sequence detector.

Present state	Next state		Z
	x = 0	x = 1	
A	A	B	0
B	C	B	0
C	A	D	0
D	A	B	1

Table 10.16 State/output table for bit sequence detector.

Present state	Next state x = 0	Next state x = 1	Z
A	A	B	0
B	C	B	0
C	A	D	0
D	A	B	1

Table 10.17 Transition/output table for bit sequence detector.

	Q_2Q_1	$Q_2^+Q_1^+$ x = 0	$Q_2^+Q_1^+$ x = 1	Z
A	0 0	0 0	0 1	0
B	0 1	1 1	0 1	0
C	1 1	0 0	1 0	0
D	1 0	0 0	0 1	1

Q_2^+

x \ Q_2Q_1	00	01	11	10
0	0	1	0	0
1	0	0	1	0

Q_1^+

x \ Q_2Q_1	00	01	11	10
0	0	1	0	0
1	1	1	0	1

Figure 10.29 Next-state maps and the partition for excitations.

$x \backslash Q_2Q_1$	00	01	11	10
0	0	1	0	0
1	0	0	1	0

Q_2^+

$x \backslash Q_2Q_1$	00	01	11	10
0	0	1	0	0
1	1	1	0	1

Q_1^+

Figure 10.29 Next-state maps and the partition for excitations.

$Q_0 \backslash Q_1$	0	1
0	0	1
1	0	0

$J_2 = (Q_2^+)_{Q2 = 0}$

$Q_0 \backslash Q_2$	0	1
0	0	0
1	1	1

$J_1 = (Q_1^+)_{Q1 = 0}$

$Q_0 \backslash Q_1$	1	0
0	1	1
1	0	1

$K_2 = [(Q_2^+)_{Q2 = 1}]'$

$Q_0 \backslash Q_2$	0	1
0	0	1
1	0	1

$K_1 = [(Q_1^+)_{Q1 = 1}]'$

$J_2 = x'Q_1$ $K_2 = (xQ_1)'$

$J_1 = x$ $K_1 = Q_2$

$Z = Q_2Q_1'$

Figure 10.30 K-maps for the excitations of the bit sequence detector.

$J_2 = x'Q_1$ $K_2 = (xQ_1)'$ $J_1 = x$ $K_1 = Q_2$ $Z = Q_2Q_1'$

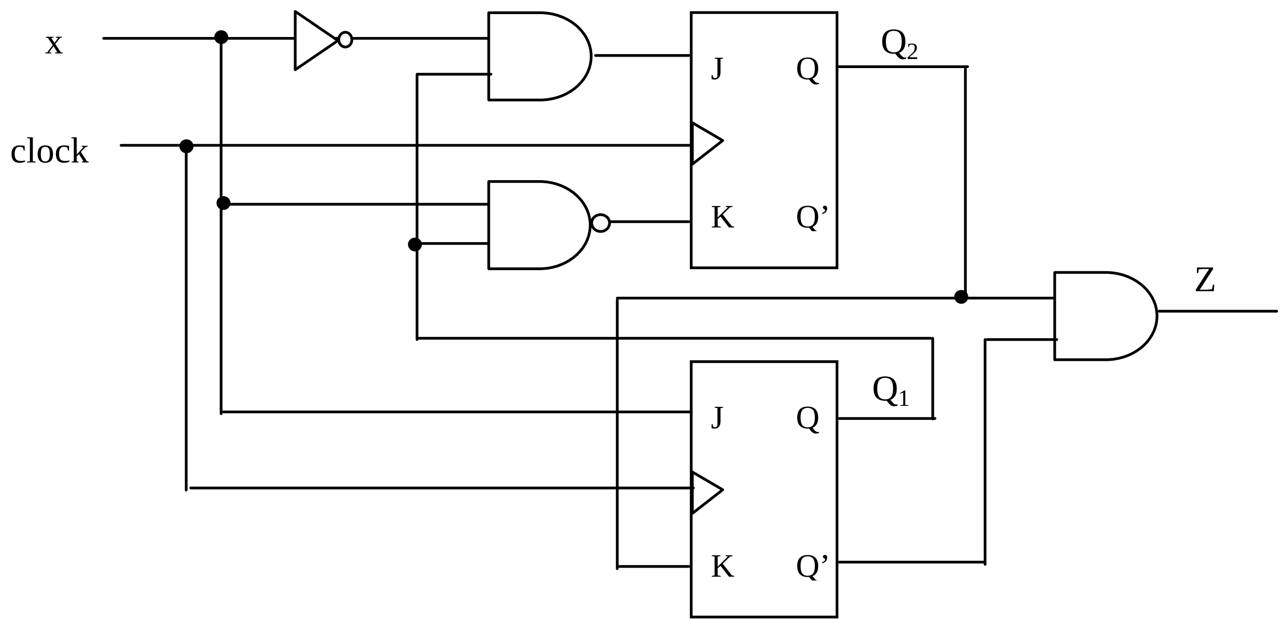


Figure 10.31 Sequential circuit of Moore model to detect a sequence of 101.

Mealy model

Table 10.18 Sample sequences of input and output for a Mealy bit-sequence detector.

Clock cycle	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Input x	1	1	0	0	1	0	1	0	1	0	1	1	0	1	….
Output Z	0	0	0	0	0	0	1	0	0	0	1	0	0	1	….

Condition A: Nothing has been detected, not even the first bit of the sequence.
Condition B: The first bit, 1, has been detected.
Condition C: The first two bits, 10, have been detected. If present input x = 0, Z = 0. If x = 1, Z = 1.

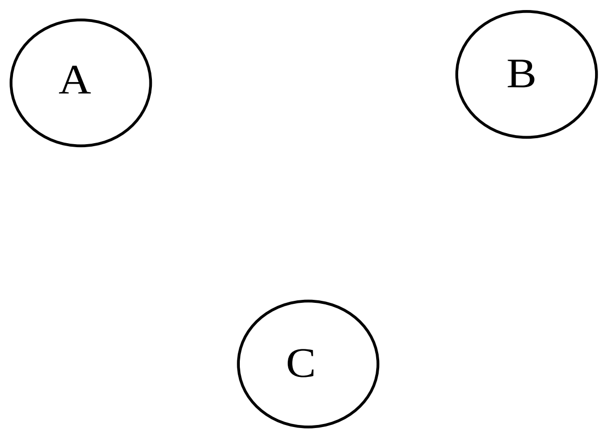


Figure 10.32 State diagram for a circuit of Mealy model to detect 101.

Mealy model

Table 10.18 Sample sequences of input and output for a Mealy bit-sequence detector.

Clock cycle	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Input x	1	1	0	0	1	0	1	0	1	0	1	1	0	1	….
Output Z	0	0	0	0	0	0	1	0	0	0	1	0	0	1	….

Condition A: Nothing has been detected, not even the first bit of the sequence.
Condition B: The first bit, 1, has been detected.
Condition C: The first two bits, 10, have been detected. If present input x = 0, Z = 0. If x = 1, Z = 1.

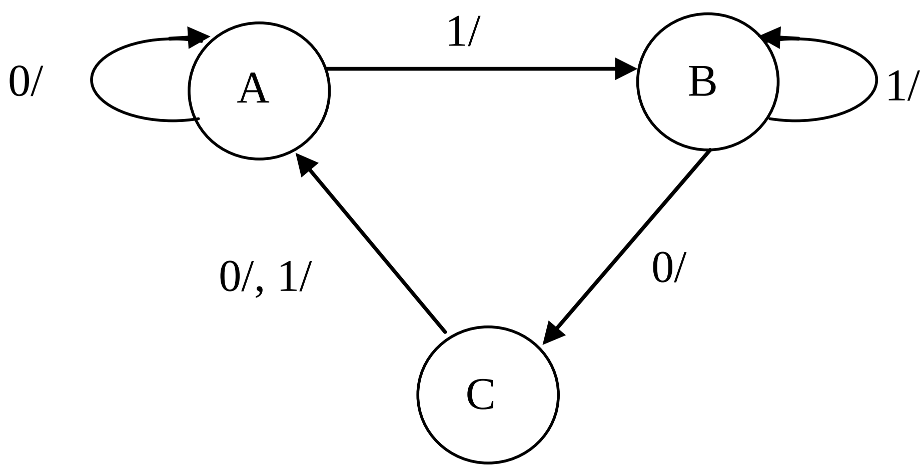


Figure 10.32 State diagram for a circuit of Mealy model to detect 101.

Mealy model

Table 10.18   Sample sequences of input and output for a Mealy bit-sequence detector.

Clock cycle	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Input x	1	1	0	0	1	0	1	0	1	0	1	1	0	1	….
Output Z	0	0	0	0	0	0	1	0	0	0	1	0	0	1	….

Condition A: Nothing has been detected, not even the first bit of the sequence.
Condition B: The first bit, 1, has been detected.
Condition C: The first two bits, 10, have been detected. If present input x = 0, Z = 0. If x = 1, Z = 1.

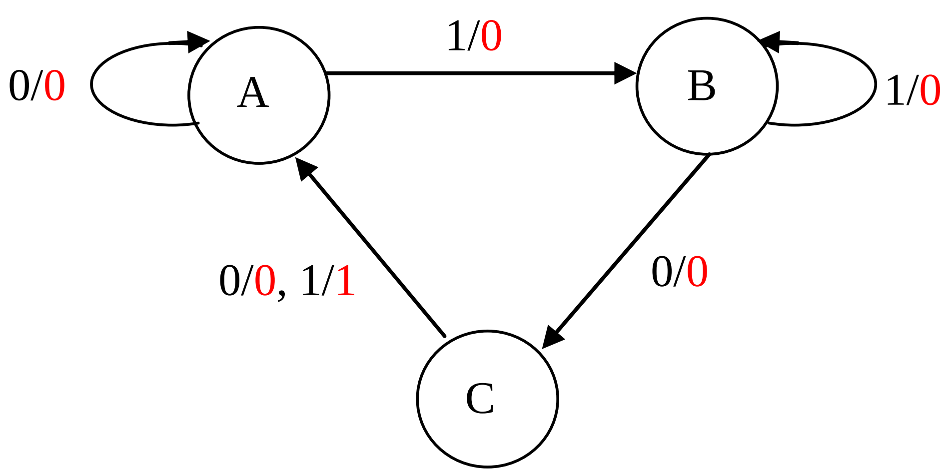


Figure 10.32 State diagram for a circuit of Mealy model to detect 101.

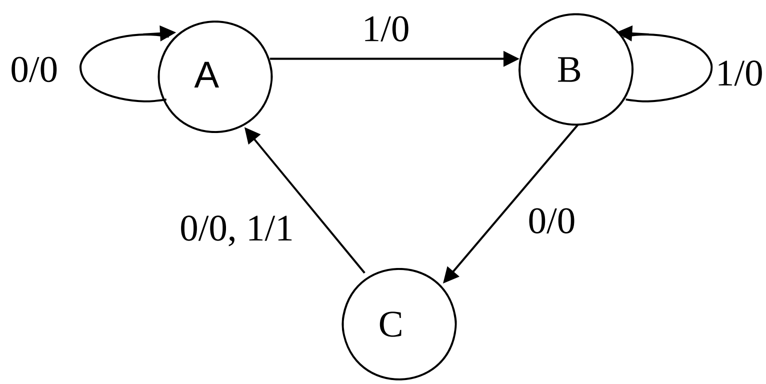


Table 10.19 State/output table for bit sequence detector.

Present state	Next state, Z	
	x = 0	x = 1
A	A, 0	B, 0
B	C, 0	B, 0
C	A, 0	A, 1

Table 10.20 Transition/output table for bit sequence detector.

	Q_2Q_1	$Q_2^+ Q_1^+$, Z	
		x = 0	x = 1
A	0 0	0 0, 0	1 0, 0
B	1 0	1 1, 0	1 0, 0
C	1 1	0 0, 0	0 0, 1
	0 1	d d, d	d d, d

Q_2^+

x \ Q_2Q_1	00	01	11	10
0	0	d	0	1
1	1	d	0	1

Q_1^+

x \ Q_2Q_1	00	01	11	10
0	0	d	0	1
1	0	d	0	0

Z

x \ Q_2Q_1	00	01	11	10
0	0	d	0	0
1	0	d	1	0

Figure 10.33 Next-state maps and output K-map.

$J_2 = x$ $K_2 = Q_1$ $J_1 = x'Q_2$ $K_1 = 1$ $Z = xQ_1$

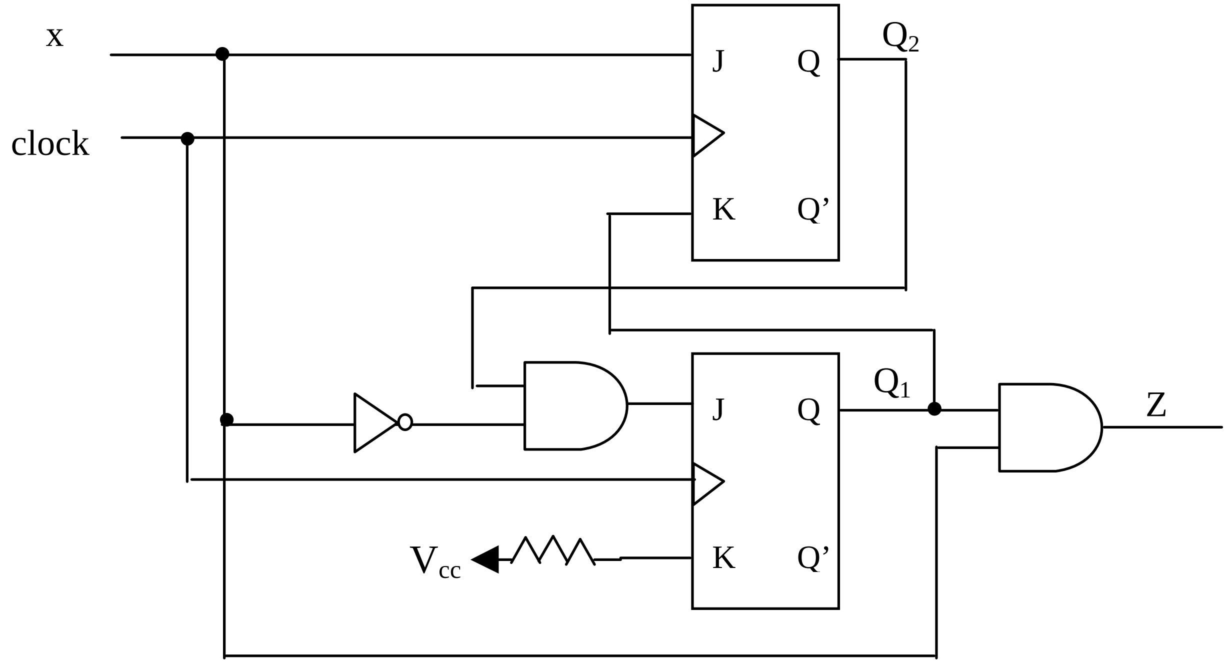


Figure 10.34 Sequential circuit of Mealy model to detect a sequence of 101.

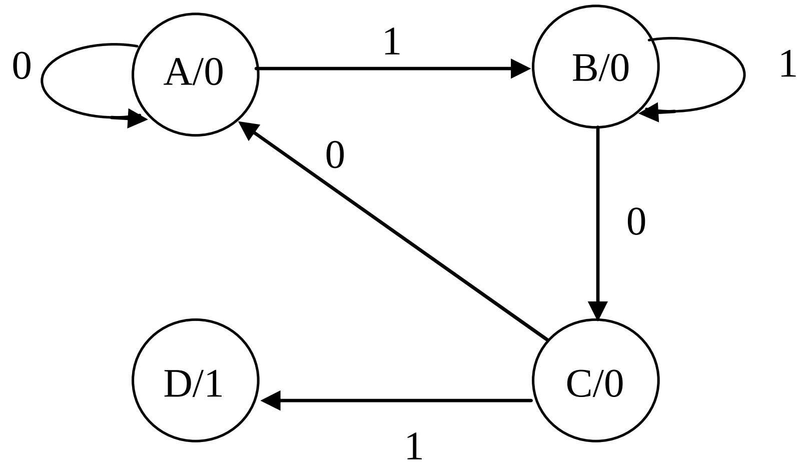


Figure 10.35 Moore model state diagram for overlapping sequences.

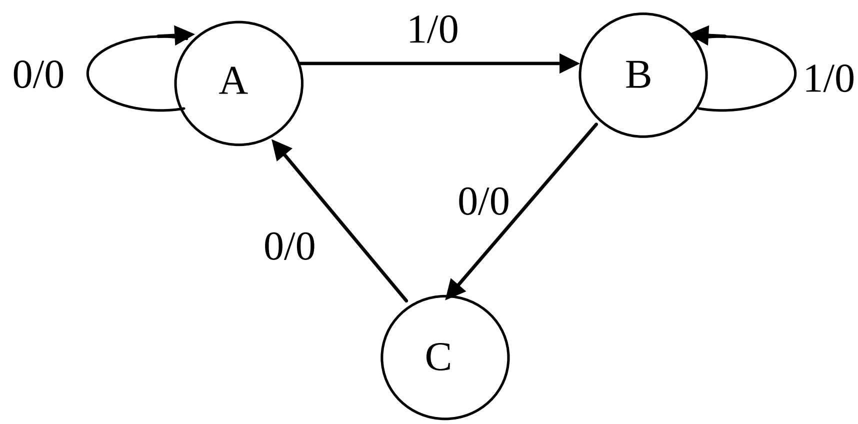


Figure 10.36 Mealy model state diagram for overlapping sequences.

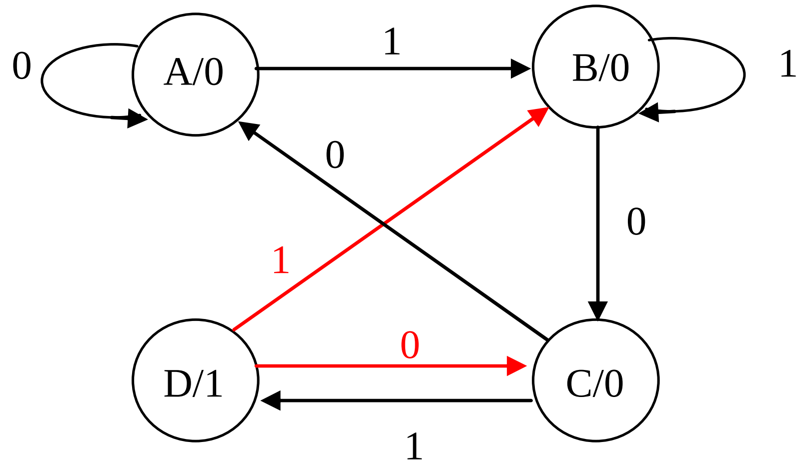


Figure 10.35 Moore model state diagram for overlapping sequences.

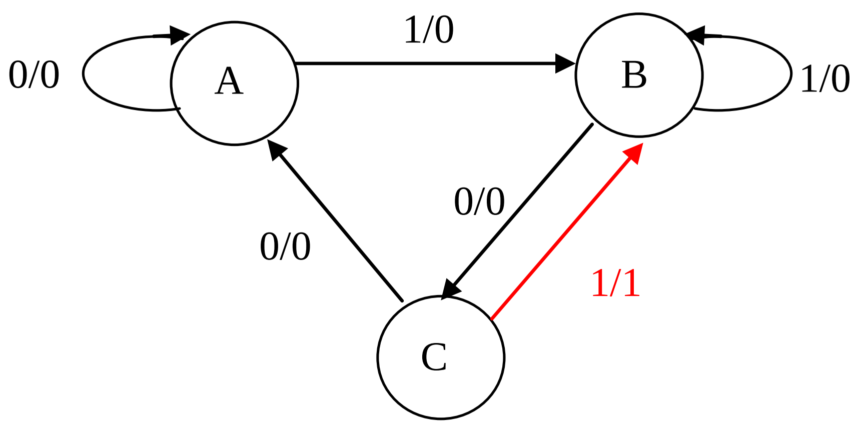


Figure 10.36 Mealy model state diagram for overlapping sequences.