Student: Phong Vo **Date:** 02/19/20	**Instructor:** Erica Yankowskas **Course:** Linear Algebra I (Spring 2020)	**Assignment:** Section 1.7 Homework

1. Determine if the vectors are linearly independent. Justify your answer.

$$\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 9 \\ 3 \\ -12 \end{bmatrix}, \begin{bmatrix} 6 \\ 12 \\ -24 \end{bmatrix}$$

The vector equation has only the trivial solution, so the vectors are linearly independent.

2. Determine if the columns of the matrix form a linearly independent set. Justify your answer.

$$\begin{bmatrix} 0 & -3 & 9 \\ 2 & 1 & -7 \\ -1 & 4 & -8 \\ 1 & -4 & -2 \end{bmatrix}$$

Select the correct choice below and fill in the answer box within your choice.
(Type an integer or simplified fraction for each matrix element.)

A. If A is the given matrix, then the augmented matrix represents the equation A**x** = **0**. The reduced echelon form of this matrix indicates that A**x** = **0** has only the trivial solution. Therefore, the columns of A do not form a linearly independent set.

B. If A is the given matrix, then the augmented matrix represents the equation A**x** = **0**. The reduced echelon form of this matrix indicates that A**x** = **0** has more than one solution. Therefore, the columns of A do not form a linearly independent set.

C. If A is the given matrix, then the augmented matrix represents the equation A**x** = **0**. The reduced echelon form of this matrix indicates that A**x** = **0** has more than one solution. Therefore, the columns of A form a linearly independent set.

D. If A is the given matrix, then the augmented matrix $\begin{bmatrix} 0 & -3 & 9 & 0 \\ 2 & 1 & -7 & 0 \\ -1 & 4 & -8 & 0 \\ 1 & -4 & -2 & 0 \end{bmatrix}$ represents the equation A**x** = **0**. The reduced echelon form of this matrix indicates that A**x** = **0** has only the trivial solution. Therefore, the columns of A form a linearly independent set.

3. Determine if the columns of the matrix form a linearly independent set. Justify your answer.

$$\begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 1 & 1 & -4 \\ 2 & 1 & -8 \end{bmatrix}$$

Select the correct choice below and fill in the answer box within your choice.
(Type an integer or simplified fraction for each matrix element.)

- **A.** If A is the given matrix, then the augmented matrix represents the equation A**x** = **0**. The reduced echelon form of this matrix indicates that A**x** = **0** has only the trivial solution. Therefore, the columns of A do not form a linearly independent set.
- **B.** If A is the given matrix, then the augmented matrix represents the equation A**x** = **0**. The reduced echelon form of this matrix indicates that A**x** = **0** has more than one solution. Therefore, the columns of A form a linearly independent set.
- **C.** If A is the given matrix, then the augmented matrix represents the equation A**x** = **0**. The reduced echelon form of this matrix indicates that A**x** = **0** has more than one solution. Therefore, the columns of A do not form a linearly independent set.
- **D.** If A is the given matrix, then the augmented matrix $\begin{bmatrix} -4 & -3 & 0 & 0 \\ 0 & -1 & 4 & 0 \\ 1 & 1 & -4 & 0 \\ 2 & 1 & -8 & 0 \end{bmatrix}$ represents the equation A**x** = **0**. The reduced echelon form of this matrix indicates that A**x** = **0** has only the trivial solution. Therefore, the columns of A form a linearly independent set.

4. Determine if the columns of the matrix form a linearly independent set. Justify your answer.

$$\begin{bmatrix} 1 & -5 & 3 & 2 \\ -5 & 25 & -15 & 2 \end{bmatrix}$$

Choose the correct answer below.

- **A.** The columns of the matrix do form a linearly independent set because there are more entries in each vector than there are vectors in the set.
- **B.** The columns of the matrix do form a linearly independent set because the set contains more vectors than there are entries in each vector.
- **C.** The columns of the matrix do not form a linearly independent set because the set contains more vectors than there are entries in each vector.
- **D.** The columns of the matrix do not form a linearly independent set because there are more entries in each vector than there are vectors in the set.

5. Use the following vectors to answer parts (a) and (b).

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -4 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -3 \\ 12 \\ -6 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 4 \\ 7 \\ h \end{bmatrix}$$

(a) For what values of h is $\mathbf{v}_3$ in Span$\{\mathbf{v}_1, \mathbf{v}_2\}$?

(b) For what values of h is $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly dependent?

(a) For what values of h is $\mathbf{v}_3$ in Span$\{\mathbf{v}_1, \mathbf{v}_2\}$? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** h = ______ (Use a comma to separate answers as needed.)
- **B.** All values of h
- **C.** No values of h ✔

(b) For what values of h is $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly dependent? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** h = ______ (Use a comma to separate answers as needed.)
- **B.** All values of h ✔
- **C.** No values of h

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6. Find the value(s) of h for which the vectors are linearly dependent. Justify your answer.

$$\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ 7 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ h \end{bmatrix}$$

The value(s) of h which makes the vectors linearly dependent is(are) -4 because this will cause x_3 to be a free variable.

(Use a comma to separate answers as needed.)

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7. Find the value(s) of h for which the vectors are linearly dependent. Justify your answer.

$$\begin{bmatrix} 1 \\ -3 \\ -6 \end{bmatrix}, \begin{bmatrix} -4 \\ 13 \\ 8 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ h \end{bmatrix}$$

The value(s) of h which makes the vectors linearly dependent is(are) -232 because this will cause x_3 to be a free variable.

(Use a comma to separate answers as needed.)

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8. Determine by inspection whether the vectors are linearly independent. Justify your answer.

$$\begin{bmatrix} 6 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 9 \end{bmatrix}$$

Choose the correct answer below.

- **A.** The set is linearly dependent because there are four vectors but only two entries in each vector.
- **B.** The set is linearly independent because at least one of the vectors is a multiple of another vector.
- **C.** The set is linearly dependent because at least one of the vectors is a multiple of another vector.
- **D.** The set is linearly independent because there are four vectors in the set but only two entries in each vector.

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9. Determine by inspection whether the vectors are linearly independent. Justify your answer.

$$\begin{bmatrix} 10 \\ 20 \\ -5 \end{bmatrix}, \begin{bmatrix} -2 \\ -4 \\ -1 \end{bmatrix}$$

Choose the correct answer below.

- **A.** The set is linearly independent because neither vector is a multiple of the other vector. Two of the entries in the first vector are -5 times the corresponding entry in the second vector. But this multiple does not work for the third entries.
- **B.** The set is linearly dependent because the first vector is a multiple of the other vector. The entries in the first vector are -5 times the corresponding entry in the second vector.
- **C.** The set is linearly independent because the first vector is a multiple of the other vector. The entries in the first vector are -5 times the corresponding entry in the second vector.
- **D.** The set is linearly dependent because neither vector is a multiple of the other vector. Two of the entries in the first vector are -5 times the corresponding entry in the second vector. But this multiple does not work for the third entries.

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10. In parts (a) to (d) below, mark the statement True or False.

a. Two vectors are linearly dependent if and only if they lie on a line through the origin. Choose the correct answer below.

- **A.** False. If two vectors are linearly dependent then the graph of one will be orthogonal, or perpendicular, to the other.
- **B.** False. Two vectors are linearly dependent if one of the vectors is a multiple of the other. The larger vector will be further from the origin than the smaller vector.
- **C.** True. Two vectors are linearly dependent if one of the vectors is a multiple of the other. Two such vectors will lie on the same line through the origin.
- **D.** True. Linearly dependent vectors must always intersect at the origin.

b. If a set contains fewer vectors than there are entries in the vectors, then the set is linearly independent. Choose the correct answer below.

- **A.** True. If a set contains fewer vectors than there are entries in the vectors, then there are less variables than equations, so there cannot be any free variables in the equation $A\mathbf{x} = \mathbf{0}$.
- **B.** True. There exists a set that contains fewer vectors than there are entries in the vectors that is linearly independent. One example is a set consisting of two vectors where one of the vectors is not a scalar multiple of the other vector.
- **C.** False. A set of vectors is linearly independent if none of the vectors can be written as a linear combination of the others. If there are fewer vectors than entries in the vectors, then at least one of the vectors must be written as a linear combination of the others.
- **D.** False. There exists a set that contains fewer vectors than there are entries in the vectors that is linearly dependent. One example is a set consisting of two vectors where one of the vectors is a scalar multiple of the other vector.

c. If $\mathbf{x}$ and $\mathbf{y}$ are linearly independent, and if $\mathbf{z}$ is in Span$\{\mathbf{x}, \mathbf{y}\}$, then $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent. Choose the correct answer below.

- **A.** False. Since $\mathbf{z}$ is in Span$\{\mathbf{x}, \mathbf{y}\}$, $\mathbf{z}$ cannot be written as a linear combination of $\mathbf{x}$ and $\mathbf{y}$. The set $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly independent.
- **B.** False. Vector $\mathbf{z}$ is in Span$\{\mathbf{x}, \mathbf{y}\}$ and $\mathbf{x}$ and $\mathbf{y}$ are linearly independent, so $\mathbf{z}$ must also be linearly independent of $\mathbf{x}$ and $\mathbf{y}$. The set $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly independent.
- **C.** True. Since $\mathbf{z}$ is in Span$\{\mathbf{x}, \mathbf{y}\}$, $\mathbf{z}$ is a linear combination of $\mathbf{x}$ and $\mathbf{y}$. Since $\mathbf{z}$ is a linear combination of $\mathbf{x}$ and $\mathbf{y}$, the set $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent.
- **D.** True. Vector $\mathbf{z}$ is in Span$\{\mathbf{x}, \mathbf{y}\}$ and $\mathbf{x}$ and $\mathbf{y}$ are linearly independent, so $\mathbf{z}$ is a scalar multiple of $\mathbf{x}$ or of $\mathbf{y}$. Since $\mathbf{z}$ is a multiple of $\mathbf{x}$ or $\mathbf{y}$, the set $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent.

d. If a set in $\mathbb{R}^n$ is linearly dependent, then the set contains more vectors than there are entries in each vector. Choose the correct answer below.

- **A.** False. If a set in $\mathbb{R}^n$ is linearly dependent, then the set contains more entries in each vector than vectors.
- **B.** True. For a set in $\mathbb{R}^n$ to be linearly dependent, it must contain more than n vectors.
- **C.** True. There exists a set in $\mathbb{R}^n$ that is linearly dependent and contains more than n vectors. One example is a set in $\mathbb{R}^2$ consisting of three vectors where one of the vectors is a scalar multiple of another.
- **D.** False. There exists a set in $\mathbb{R}^n$ that is linearly dependent and contains n vectors. One example is a set in $\mathbb{R}^2$ consisting of two vectors where one of the vectors is a scalar multiple of the other.

11. Given $A = \begin{bmatrix} 2 & 2 & 4 \\ -4 & 2 & -2 \\ -4 & -1 & -5 \\ 5 & 0 & 5 \end{bmatrix}$, observe that the third column is the sum of the first and second columns. Find a nontrivial solution of A**x** = 0 without performing row operations. [Hint: Write A**x** = **0** as a vector equation.]

$\mathbf{x} = \quad x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$