1. **Function Order of Growth**: (20 points)

   List the 4 functions below in non-decreasing asymptotic order of growth.

   $$(\log n)^2 \qquad n^{-2} \qquad \lg(2^{\log(n^2)}) \qquad n^2$$

   Justify your answer mathematically by showing values of c and n_0 for each pair of functions that are adjacent in your ordering.

2. **Pseudocode Analysis** (25 points)

   For the pseudocode below for procedure Mystery(n), derive tight upper and lower bounds on its asymptotic *worst-case* running time f **(**n**)**. That is, for the set of inputs including those that force Mystery to work its hardest, find g (n) such that f (n) $\in \Theta(g(n))$. Assume that the input n is a positive integer. Justify your answer.

```
Mystery (n)
if n is an even number
    for i = 1 to n
        for j = n downto n/2
            print "even number"
else
    for k = 1 to n/4
        for m = 1 to n
            print "odd number"
```

3. True or False (25 points).

a. $n\lg^2 n \in O(n^2)$

b. $n\lg^2 n \in \Omega(n^{1.05})$

c. $n^3 \in o(n^3)$

d. The cost of the loop below is in $O(n)$

```
for (i = 1; i <= n; i *= 2) { // n>=1
    constant work;
}
```

e. The cost of the above loop is in $\Omega(\lg n)$

4. (20 points) For each of the following recurrences, give an expression for the runtime T(n) if the recurrence can be solved with the Master Theorem. Otherwise, explain why the Master Theorem does not apply. Justify your answer.

(1) $T(n) = 3^n\, T(\frac{n}{3}) + n^3$

(2) $T(n) = 5T(\frac{n}{2}) + \sqrt{10}n^3$

(3) $T(n) = \frac{1}{4}\, T(\frac{n}{4}) + n \log n$

(4) $T(n) = T(n-1) + 2n$

(5) $T(n) = 16T\, (\frac{n}{4}) + n^2$

5. (10 points) Exercise 4.4-4. Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = 2T(n-1)+1$. Use the substitution method to verify your answer.