$\boxed{4.7.1}$

a.

$$b_1 = -2c_1 + 3c_2 = [\vec{c}_1 \;\; \vec{c}_2]\begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$\Rightarrow [\vec{b}_1]_C = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$b_2 = -7c_1 + 6c_2 = [\vec{c}_1 \;\; \vec{c}_2] = \begin{bmatrix} -7 \\ 6 \end{bmatrix}$$

$$\Rightarrow [\vec{b}_2]_C = \begin{bmatrix} -7 \\ 6 \end{bmatrix}$$

$$\Rightarrow \therefore \underset{C \leftarrow B}{P} = \begin{bmatrix} -2 & -7 \\ 3 & 6 \end{bmatrix}$$

b. $x = 5b_1 - 2b_2 = 5(-2c_1 + 3c_2) - 2(-7c_1 + 6c_2)$

$$\Rightarrow x = 4c_1 + 3c_2 = [\vec{c}_1 \;\; \vec{c}_2]\begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$\Rightarrow [x]_C = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$\boxed{4.7.5}$ $b_1 = 2a_1 - 3a_3 = [\vec{a}_1 \;\; \vec{a}_2 \;\; \vec{a}_3]\begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}$

$$b_2 = -a_1 + a_2 = [\vec{a}_1 \;\; \vec{a}_2 \;\; \vec{a}_3]\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$b_3 = a_1 + a_2 + 6a_3 = [\vec{a}_1\ \vec{a}_2\ \vec{a}_3]\begin{bmatrix}1\\1\\6\end{bmatrix}$$

$$\Rightarrow \underset{A\leftarrow B}{P} = \begin{bmatrix}2 & -1 & 1\\0 & 1 & 1\\-3 & 0 & 6\end{bmatrix}$$

<u>b.</u> $x = b_1 - 4b_2 + 4b_3$

$$x = \begin{bmatrix}2\\0\\-3\end{bmatrix} - 4\begin{bmatrix}-1\\1\\0\end{bmatrix} + 4\begin{bmatrix}1\\1\\6\end{bmatrix} = \boxed{\begin{bmatrix}10\\0\\21\end{bmatrix}}$$

$\boxed{4.7.5}$ $a_1 = 2b_1 - b_2 = [\vec{b}_1\ \vec{b}_2\ \vec{b}_3]\begin{bmatrix}2\\-1\\0\end{bmatrix}$

a.

$$a_2 = -b_1 + 5b_2 + b_3 = [\vec{b}_1\ \vec{b}_2\ \vec{b}_3]\begin{bmatrix}-1\\5\\1\end{bmatrix}$$

$$a_3 = b_2 - 6b_3 = [\vec{b}_1\ \vec{b}_2\ \vec{b}_3]\begin{bmatrix}0\\1\\-6\end{bmatrix}$$

The change-of-coordinates matrix from A to B

$$\underset{B\leftarrow A}{P} = \begin{bmatrix}2 & -1 & 0\\-1 & 5 & 1\\0 & 1 & -6\end{bmatrix}$$

b. $x = 3a_1 + 4a_2 + a_3 = 3\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + 4\begin{bmatrix} -1 \\ 5 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -6 \end{bmatrix}$

$$[x]_B = \begin{bmatrix} 2 \\ 18 \\ -2 \end{bmatrix}$$

$\boxed{4.7.8}$ $\left[\vec{c}_1 \;\; \vec{c}_2 \;\vdots\; \vec{b}_1 \;\; \vec{b}_2\right] = \left[\begin{array}{cc:cc} 1 & 1 & -7 & 3 \\ 4 & 3 & -16 & 7 \end{array}\right]$

$$\xrightarrow[= nR_2]{4R_1 - R_2} \left[\begin{array}{cc:cc} 1 & 1 & -7 & 3 \\ 0 & 1 & -12 & 5 \end{array}\right] \xrightarrow[= nR_1]{R_1 - R_2} \left[\begin{array}{cc:cc} 1 & 0 & 5 & -2 \\ 0 & 1 & -12 & 5 \end{array}\right]$$

$$\Rightarrow \begin{cases} [\vec{b}_1]_C = \begin{bmatrix} 5 \\ -12 \end{bmatrix} \\ [\vec{b}_2]_C = \begin{bmatrix} -2 \\ 5 \end{bmatrix} \end{cases} \Rightarrow \boxed{\underset{C \leftarrow B}{P} = \begin{bmatrix} 5 & -2 \\ -12 & 5 \end{bmatrix}}$$

$$\therefore \underset{B \leftarrow C}{P} : \left[b_1 \;\; b_2 \;\vdots\; c_1 \;\; c_2\right] = \left[\begin{array}{cc:cc} -7 & 3 & 1 & 1 \\ -12 & 5 & 0 & 1 \end{array}\right]$$

$$\therefore \underset{C \leftarrow B}{P} = \left(\underset{B \leftarrow C}{P}\right)^{-1} = \underbrace{\begin{bmatrix} 5 & -2 \\ -12 & 5 \end{bmatrix}}_{A}{}^{-1}$$

$$\det A = 25 - 24 = 1 \neq 0$$

$$A^{-1} = \frac{1}{\underset{=1}{\det A}} \begin{bmatrix} 5 & 2 \\ 12 & 5 \end{bmatrix} = \boxed{\begin{bmatrix} 5 & 2 \\ 12 & 5 \end{bmatrix}}$$

$\boxed{4.7.13}$ $B = \{1-3t+t^2,\ 2-5t+3t^2,\ 2-3t+6t^2\}$

$$b_1 = \begin{bmatrix} 1 & t & t^2 \end{bmatrix}\begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}$$

$$b_2 = \begin{bmatrix} 1 & t & t^2 \end{bmatrix}\begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix} \Rightarrow \boxed{\underset{C\leftarrow B}{P} = \begin{bmatrix} 1 & 2 & 2 \\ -3 & -5 & -3 \\ 1 & 3 & 6 \end{bmatrix}}$$

$$b_3 = \begin{bmatrix} 1 & t & t^2 \end{bmatrix}\begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix}$$

$\therefore$ Find the β - coordinatevector for $2-6t+3t^2$

$$\left(\underset{C\leftarrow B}{P}\right)[\vec{p}]_\beta = [\vec{p}]_c$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ -3 & -5 & -3 & -6 \\ 1 & 3 & 6 & 3 \end{array}\right] \xrightarrow[R_3-R_1 = nR_3]{3R_1+R_2 = nR_2} \left[\begin{array}{ccc|c} 1 & 2 & 2 & 2 \\ 0 & 1 & 3 & 0 \\ 0 & 1 & 4 & 1 \end{array}\right]$$

$$\xrightarrow[R_3-R_2=nR_3]{R_1-2R_2 = nR_1} \left[\begin{array}{ccc|c} 1 & 0 & -4 & 2 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right] \xrightarrow[R_2-3R_3 =]{R_1+4R_3 = nR_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\Rightarrow \boxed{\therefore [\vec{p}]_B = \begin{bmatrix} 6 \\ -3 \\ 1 \end{bmatrix}}$$

$\boxed{4.7.14}$ $B = \{1 - 5t^2, 5 + t - 24t^2, 1 + 4t\}$

$$b_1 = \begin{bmatrix} 1 & t & t^2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 1 & t & t^2 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \\ -24 \end{bmatrix}$$

$$b_3 = \begin{bmatrix} 1 & t & t^2 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix} \Rightarrow \boxed{\underset{C \leftarrow B}{P} = \begin{bmatrix} 1 & 5 & 1 \\ 0 & 1 & 4 \\ -5 & -24 & 0 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 5 & 1 \\ 0 & 1 & 4 \\ -5 & -24 & 0 \end{bmatrix} [\vec{p}]_\beta = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Longleftrightarrow \left[\begin{array}{ccc|c} 1 & 5 & 1 & 0 \\ 0 & 1 & 4 & 0 \\ -5 & -24 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[\substack{R_1 - 5R_2 \\ = nR_1}]{\substack{5R_1 + R_3 \\ = nR3}} \left[\begin{array}{ccc|c} 1 & 0 & -19 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 1 & 5 & 1 \end{array}\right] \xrightarrow[\substack{R_3 - R_2 \\ = nR_3}]{} \left[\begin{array}{ccc|c} 1 & 0 & -19 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\xrightarrow[\substack{R_2 - 4R_3 \\ = nR_3}]{\substack{R_1 + 19R_3 \\ = nR_1}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 19 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 1 \end{array}\right] \Rightarrow \boxed{\therefore [\vec{p}]_\beta = \begin{bmatrix} 19 \\ -4 \\ 1 \end{bmatrix}}$$

$$\boxed{\text{Therefore, } t^2 = (19)(1 - 5t^2) - (4)(5 + t - 24t^2) + (1)(1 + 4t)}$$