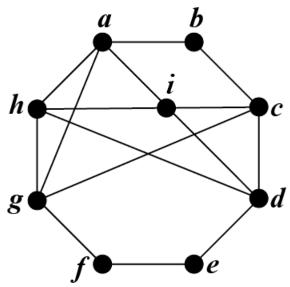
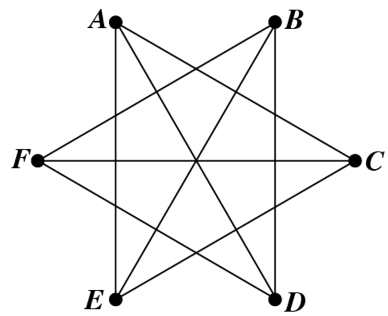
Section 10.7 Homework

1. For each of the following graphs, determine if the graph is planar. If it's planar, draw the planar representation, find the number of regions, and verify Euler's Theorem. If it's not planar, explain why not using the appropriate inequality.

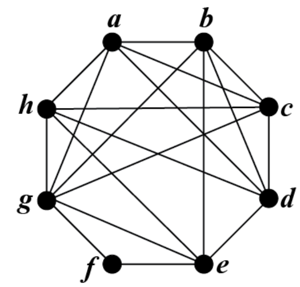
(a)



(b)



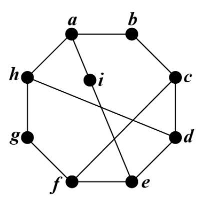
(c)



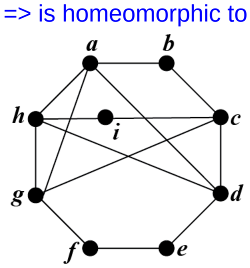
2. Let G be a connected planar graph. Suppose that G has 12 vertices which each have degree 5. Find the number of regions that must occur in any planar representation of G.

3. For each of the following graphs, determine if the graph is homeomorphic to $K_{3,3}$ or K_5. Explain why your answer works.

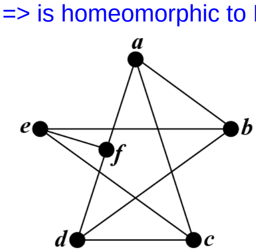
(a) max deg. = 3
=> is homeomorphic to K3,3



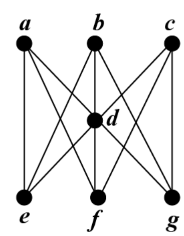
(b) max deg. = 4
=> is homeomorphic to K5



(c) max deg. = 3
=> is homeomorphic to K3,3

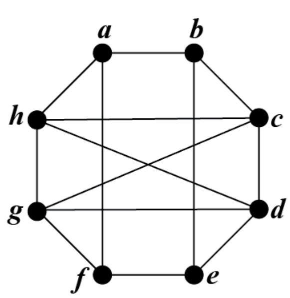


(d) planar => not homeomorphic to K3,3 or K5

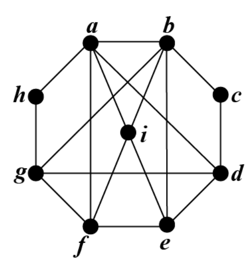


4. For each of the following graphs, determine if the graph is planar. If it's planar, draw the planar representation. If it's not planar, explain why not by using Kuratowski's Theorem. (*Hint:* This means you should find a subgraph that's homeomorphic to $K_{3,3}$ or K_5.)

(a)



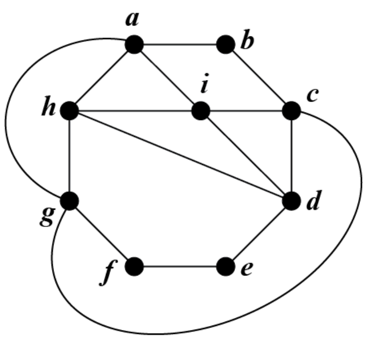
(b)



Theorem 1 (Kuratowski' s Theorem). Let G be a graph. Then G is nonplanar if and only if G contains a subgraph is homeomorphic to either K3,3 or K5.

Answers:

1. (a) planar:



   (b) planar

   (c) Not planar. (Use corollary 1.)

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3. (a) Yes, it's homeomorphic to $K_{3,3}$.

   (b) Yes, it's homeomorphic to K_5.

   (c) Yes. (*Hint:* To explain why, start by observing the graph is bipartite, and redrawing it as a "typical" bipartite graph.)

   (d) No. (Why not?) because it is planar

4. Neither of the graphs are planar! (The first graph has a subgraph that is homeomorphic to $K_{3,3}$, and the second graph has a subgraph homeomorphic to K_5.)