Name: (Print) DangNhi Ngo

6

1. (6 points) Summations: Give a closed form solution to: (1) $\sum_{i=1}^{n} 3n$ and (2) $\sum_{i=0}^{n} 3i$

Your answer should be a polynomial function of n.

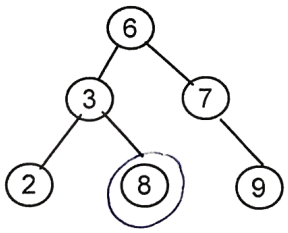
(1) $\sum_{i=1}^{n} 3n = 3n^2$

(2) $\sum_{i=0}^{n} 3i = \frac{3(n+0)(n+1)}{2} = \frac{3n(n+1)}{2} = \frac{3}{2}n^2 + \frac{3}{2}n$

4

2. (4 points) Trees: Circle TRUE if the statement below is true, and FALSE otherwise. Briefly explain your answer.

Statement: *This graph is a binary search tree.*



TRUE **FALSE**

Explanation: Because 8 is greater than 6, it should be on the right side of the tree

5

3. (5 points) Probability: If you toss a fair, 3-sided coin (with sides HEAD, MIDDLE, TAIL) two times, what is the probability of getting at most one TAIL?

$P = \binom{n}{k} p^k (1-p)^{n-k}$ $p = \frac{1}{3}$ $n = 2$

Getting at most one tail means that getting no tail or getting 1 tail:

$P = \binom{2}{0}\left(\frac{1}{3}\right)^0\left(1-\frac{1}{3}\right)^2 + \binom{2}{1}\left(\frac{1}{3}\right)^1\left(1-\frac{1}{3}\right)^1$

$= 1 \times 1 \times \frac{4}{9} + 2 \times \frac{1}{3} \times \frac{2}{3} = \frac{4}{9} + \frac{4}{9} = \frac{8}{9}$

(Continue to the back)

25/25

5 4. **(5 points) Logarithms**: Simplify and evaluate: $\dfrac{3^{(\log_4 64)}}{\log_2 8}$.

$$\frac{3^{(\log_4 64)}}{\log_2 8} = \frac{3^{(\log_4 2^6)}}{\log_2 2^3} = \frac{3^{6\log_{(2^2)} 2}}{3} = \frac{3^{(6\times\frac{1}{2})}}{3} = \frac{3^3}{3} = 3^2 = 9$$

5 5. **(5 points) Matrices**: Form the matrix product C = AB, where A and B are given as:

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 2 & 0 \\ 1 & 3 & 2 \end{pmatrix}.$$

3×2 2×3

$$C = AB = \begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 4 \end{pmatrix} \times \begin{pmatrix} 0 & 2 & 0 \\ 1 & 3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1\times 0 + 0\times 1 & 1\times 2 + 0\times 3 & 1\times 0 + 0\times 2 \\ 2\times 0 + 1\times 1 & 2\times 2 + 1\times 3 & 2\times 0 + 1\times 2 \\ 3\times 0 + 4\times 1 & 3\times 2 + 4\times 3 & 3\times 0 + 4\times 2 \end{pmatrix}$$

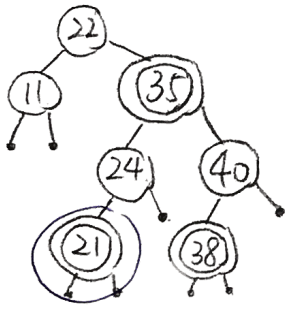
$$= \begin{pmatrix} 0 & 2 & 0 \\ 1 & 7 & 2 \\ 4 & 18 & 8 \end{pmatrix}$$

COMP.4040 Quiz – Red Black Tree

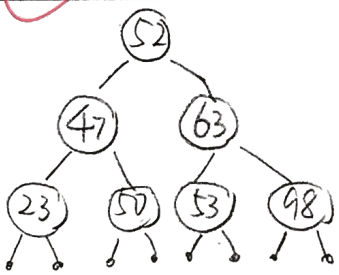
Name (PRINT): DANGNHI NGO

1. (20 points) Are the following legal Red-Black Trees? A node in single circle is a Black node and a node in double circle is a Red node.

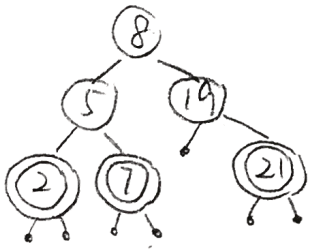
	Yes	No
A		✓
B	✓	
C	✓	
D		✓



(A)

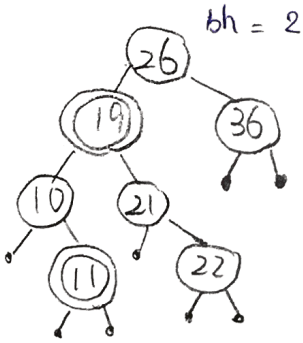


(B)



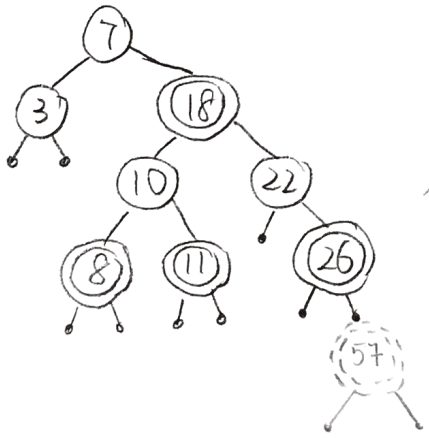
(C)

bh = 2



(D)

2. (5 points) If we use BST insertion algorithm to insert 57 to the following tree and color the node to red color, what tree property is going to be violated if there is any?



If inserting 57 and coloring the node to red color, it will violate the tree property: each red node has black parent and black children or two red nodes cannot be connected to each other.

COMP.4040 Quiz – Hash Table Sp2019

Name (PRINT): DANGNHI NGO

1. Given the following input (3412, 3413, 1741, 3269, 2909, 6291, 6373, 5129) and the hash function h(k) = k mod 10, which of the following statement(s) true? Choose all correct ones.

(A.) 3269, 2909, 5129 hash to the same value

B. 3412 and 3413 hash to the same value

(C.) 1741, 6291 hash to the same value

D. 3413, 3269, 6291, 6373 each hashes to a different value

2. The keys 14, 18, 33, 4, 3, 23, 25 and 5 are inserted into an initially empty hash table in this given order. The hash table has 10 slots and uses chaining with hash function h(k) = k mod 10. What is the hash table after inserting all keys? (multiple numbers in the same slot represents a linked list to chain the numbers together in that order)

0	
1	
2	
3	3, 23, 33
4	4, 14
5	5, 25
6	
7	
8	18
9	

A

0	
1	
2	
3	23, 3, 33
4	4, 14
5	5, 25
6	
7	
8	18
9	

(B)

0	
1	
2	
3	33, 23, 3
4	14, 4
5	25, 5
6	
7	
8	18
9	

C

0	
1	
2	
3	33, 3, 23
4	14, 4
5	25, 5
6	
7	
8	18
9	

D

$h(14) = 14 \bmod 10 = 4 \Rightarrow \#4$

$h(18) = 18 \bmod 10 = 8 \Rightarrow \#8$

$h(33) = 33 \bmod 10 = 3 \Rightarrow \#3$

$h(4) = 4 \bmod 10 = 4 \Rightarrow \#4$

$h(3) = 3 \bmod 10 = 3 \Rightarrow \#3$

$h(23) = 23 \bmod 10 = 3 \Rightarrow \#3$

$h(25) = 25 \bmod 10 = 5 \Rightarrow \#5$

$h(5) = 5 \bmod 10 = 5 \Rightarrow \#5$

3. The keys 14, 18, 33, 4, 3, 23, 25 and 5 are inserted into an initially empty hash table in this given order. The hash table has 10 slots and uses open addressing with hash function h(k) = k mod 10 and linear probing. What is the hash table after inserting all keys?

h(25,2) = (25+2) mod 10 = 7 (collision)
h(25,3) = (25+3) mod 10 = 8 (collision)
h(25,4) = (25+4) mod 10 = 9
h(5,0) = 5 mod 10 = 5 (collision)
h(5,1) = (5+1) mod 10 = 6 (collision)
h(5,2) = (5+2) mod 10 = 7 (collision)
h(5,3) = (5+3) mod 10 = 8 (collision)
h(5,4) = (5+4) mod 10 = 9 (collision)
h(5,5) = (5+5) mod 10 = 0

0	
1	
2	
3	23
4	4
5	5
6	
7	
8	18
9	

A

0	
1	
2	5
3	33
4	14
5	4
6	3
7	23
8	18
9	25

B

0	
1	
2	
3	33
4	14
5	25
6	
7	
8	18
9	

C

0	5
1	
2	
3	33
4	14
5	4
6	3
7	23
8	18
9	25

(D)

h(14,0) = 14 mod 10 = 4
h(18,0) = 18 mod 10 = 8
h(33,0) = 33 mod 10 = 3
h(4,0) = 4 mod 10 = 4 (collision)
h(4,1) = (4+1) mod 10 = 5
h(3,0) = 3 mod 10 = 3 (collision)
h(3,1) = (3+1) mod 10 = 4 (collision)

h(3,2) = (3+2) mod 10 = 5 (collision)
h(3,3) = (3+3) mod 10 = 6
h(23,0) = 23 mod 10 = 3
h(23,1) = (23+1) mod 10 = 4 (collision)
h(23,2) = (23+2) mod 10 = 5 (collision)
h(23,3) = (23+3) mod 10 = 6 (collision)
h(23,4) = (23+4) mod 10 = 7
h(25,0) = 25 mod 10 = 5 (collision)
h(25,1) = (25+1) mod 10 = 6 (collision)

4. (1) What is the load factor in Problem 2 above?

$\alpha = n/m = 8/10 = 4/5$ ✓

(2) What is the load factor in Problem 3 above?

$\alpha = n/m = 8/10 = 4/5$ ✓

20/20

COMP.4040 Quiz – Probabilistic Analysis Sp2019

Name (PRINT): DangNhi Ngo

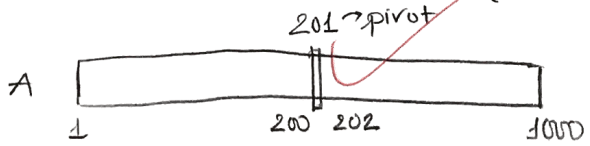
Please show your work to get partial credits.

(20 points) Use the randomized quick sort algorithm below to answer question.

```
RANDOMIZED-QUICKSORT(A, p, r)
1  if p < r
2      q = RANDOMIZED-PARTITION(A, p, r)        q = 201
3      RANDOMIZED-QUICKSORT(A, p, q − 1)        (A, 1, 200)
4      RANDOMIZED-QUICKSORT(A, q + 1, r)        (A, 202, 1000)

RANDOMIZED-PARTITION(A, p, r)
1  i = RANDOM(p, r)
2  exchange A[r] with A[i]
3  return PARTITION(A, p, r)
```

Array A contains 1000 distinct elements. After the first call to RANDOMIZED-PARTITION, the return value is 201 (i.e., the pivot location is 201). When call RANDOMIZED-QUICKSORT to sort the first sub-array recursively, what is the expected value returned from RANDOM function (the result can be a fraction)?



$Pr\{X\} = \frac{1}{200}$ (Because sorting the first sub-array)

Expected value is returned from RANDOM function:

$$E(X) = \sum_{i=1}^{200} X_i \cdot Pr\{X\}$$

$$= \frac{1}{200} \times \sum_{i=1}^{200} X_i$$

$$= \frac{1}{200} \times (1 + 2 + \ldots + 200)$$

$$= \frac{1}{200} \times \frac{200 \times (200+1)}{2} = \frac{1}{200} \times \frac{200 \times 201}{2} = \frac{201}{2}$$

$$\boxed{E(X) = \frac{201}{2}}$$