Student: Phong Vo **Date:** 02/19/20	**Instructor:** Erica Yankowskas **Course:** Linear Algebra I (Spring 2020)	**Assignment:** Section 1.3 Homework

1. Compute $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-2\mathbf{v}$.

$$\mathbf{u}=\begin{bmatrix} -5 \\ -9 \end{bmatrix}, \mathbf{v}=\begin{bmatrix} -8 \\ -6 \end{bmatrix}$$

Compute $\mathbf{u}+\mathbf{v}$.

$$\mathbf{u}+\mathbf{v}=\begin{bmatrix} -13 \\ -15 \end{bmatrix}$$

Compute $\mathbf{u}-2\mathbf{v}$.

$$\mathbf{u}-2\mathbf{v}=\begin{bmatrix} 11 \\ 3 \end{bmatrix}$$

2. Write a system of equations that is equivalent to the given vector equation.

$$x_1\begin{bmatrix} 4 \\ -3 \\ 9 \end{bmatrix}+x_2\begin{bmatrix} 7 \\ 0 \\ -9 \end{bmatrix}=\begin{bmatrix} 5 \\ -4 \\ 8 \end{bmatrix}$$

Choose the correct answer below.

A.
$$\begin{aligned} 4x_1 + 7x_2 &= 8 \\ -3x_1 &= -4 \\ 9x_1 - 9x_2 &= 5 \end{aligned}$$

B.
$$\begin{aligned} 4x_1 + 7x_2 &= 5 \\ -3x_1 + x_2 &= -4 \\ 9x_1 - 9x_2 &= 8 \end{aligned}$$

C.
$$\begin{aligned} 4x_1 + 7x_2 &= -4 \\ -3x_1 &= 5 \\ 9x_1 - 9x_2 &= 8 \end{aligned}$$

D. ✔
$$\begin{aligned} 4x_1 + 7x_2 &= 5 \\ -3x_1 &= -4 \\ 9x_1 - 9x_2 &= 8 \end{aligned}$$

3. Write a system of equations that is equivalent to the given vector equation.

$$x_1\begin{bmatrix} 3 \\ -5 \end{bmatrix}+x_2\begin{bmatrix} 8 \\ 2 \end{bmatrix}+x_3\begin{bmatrix} -2 \\ 1 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Choose the correct answer below.

A.
$$\begin{aligned} 3x_1 + 8x_2 - 2x_3 &= 0 \\ -5x_1 + 2x_2 &= 0 \end{aligned}$$

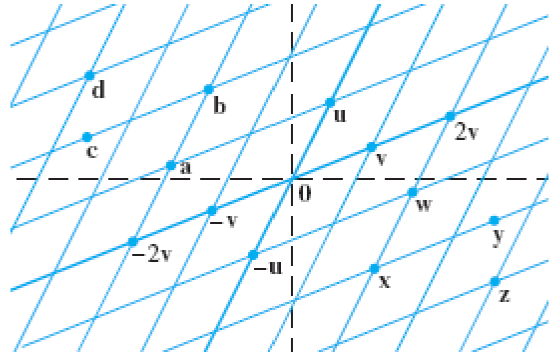
B.
$$\begin{aligned} 3x_1 + 8x_2 + 2x_3 &= 0 \\ 5x_1 + 2x_2 + x_3 &= 0 \end{aligned}$$

C.
$$\begin{aligned} 3x_1 + 8x_2 + 2x_3 &= 0 \\ 5x_1 + 2x_2 &= 0 \end{aligned}$$

D. ✔
$$\begin{aligned} 3x_1 + 8x_2 - 2x_3 &= 0 \\ -5x_1 + 2x_2 + x_3 &= 0 \end{aligned}$$

4. Use the accompanying figure to write each vector listed as a linear combination of **u** and **v**.

Vectors **b**, **c**, **w**, and **z**



Write **b** as a linear combination of **u** and **v**.

b = (2) **u** + (− 2) **v**
(Type integers or decimals.)

Write **c** as a linear combination of **u** and **v**.

c = (2) **u** + (− 3.5) **v**
(Type integers or decimals.)

Write **w** as a linear combination of **u** and **v**.

w = (− 1) **u** + (2) **v**
(Type integers or decimals.)

Write **z** as a linear combination of **u** and **v**.

z = (− 3) **u** + (4) **v**
(Type integers or decimals.)

5. Write a vector equation that is equivalent to the given system of equations.

$$\begin{aligned} x_2 + 2x_3 &= 0 \\ 3x_1 + 7x_2 - x_3 &= 0 \\ -x_1 + 6x_2 - 6x_3 &= 0 \end{aligned}$$

Choose the correct answer below.

A. $x_1\begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix} + x_2\begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix} + x_3\begin{bmatrix} 2 \\ -1 \\ -6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

B. $x_1\begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix} + x_2\begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix} + x_3\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -6 \end{bmatrix}$

C. $x_1\begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix} + x_2\begin{bmatrix} 2 \\ -1 \\ -6 \end{bmatrix} + x_3\begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

D. $x_1\begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix} + x_2\begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix} + x_3\begin{bmatrix} 2 \\ -1 \\ -6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

6. Determine if b is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$.

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ -4 \\ 0 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 4 \\ -6 \\ 40 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 3 \\ -3 \\ 36 \end{bmatrix}$$

Choose the correct answer below.

- **A.** Vector b is not a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$.
- **B.** Vector b is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$. The pivots in the corresponding echelon matrix are in the first entry in the first column and the second entry in the second column.
- **C.** Vector b is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$. The pivots in the corresponding echelon matrix are in the first entry in the first column, the second entry in the second column, and the third entry in the fourth column.
- **D.** Vector b is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$. The pivots in the corresponding echelon matrix are in the first entry in the first column, the second entry in the second column, and the third entry in the third column.

7. Determine if **b** is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$.

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} -5 \\ 3 \\ -5 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} -4 \\ 9 \\ 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 13 \\ -5 \\ 8 \end{bmatrix}$$

Choose the correct answer below.

- **A.** Vector **b** is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$. The pivots in the corresponding echelon matrix are in the first entry in the first column and the third entry in the second column, and the third entry in the third column.
- **B.** Vector **b** is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$. The pivots in the corresponding echelon matrix are in the first entry in the first column, the second entry in the second column, and the third entry in the third column.
- **C.** Vector **b** is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$. The pivots in the corresponding echelon matrix are in the first entry in the first column, the second entry in the second column, and the third entry in the fourth column.
- **D.** Vector **b** is not a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$.

8. Determine if **b** is a linear combination of the vectors formed from the columns of the matrix A.

$$A = \begin{bmatrix} 1 & -5 & 3 \\ 0 & 4 & 7 \\ -3 & 15 & -9 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 4 \\ -7 \\ -4 \end{bmatrix}$$

Choose the correct answer below.

- **A.** Vector **b** is not a linear combination of the vectors formed from the columns of the matrix A.
- **B.** Vector **b** is a linear combination of the vectors formed from the columns of the matrix A. The pivots in the corresponding echelon matrix are in the first entry in the first column and the third entry in the second column, and the third entry in the third column.
- **C.** Vector **b** is a linear combination of the vectors formed from the columns of the matrix A. The pivots in the corresponding echelon matrix are in the first entry in the first column, the second entry in the second column, and the third entry in the third column.
- **D.** Vector **b** is a linear combination of the vectors formed from the columns of the matrix A. The pivots in the corresponding echelon matrix are in the first entry in the first column, the second entry in the second column, and the third entry in the fourth column.

9. Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} -6 \\ -11 \\ 2 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 5 \\ -13 \\ h \end{bmatrix}$. For what value(s) of h is **b** in the plane spanned by $\mathbf{a}_1$ and $\mathbf{a}_2$?

The value(s) of h is(are) 11 . (Use a comma to separate answers as needed.)

10. Construct a 3×3 matrix A, with nonzero entries, and a vector **b** in $\mathbb{R}^3$ such that **b** is not in the set spanned by the columns of A.

Choose the correct answer below.

- **A.** $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$ $\mathbf{b} = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$
- **B.** $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix}$ $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}$
- **C.** $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$ $\mathbf{b} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$
- **D.** $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 3 & 3 & 2 \end{bmatrix}$ $\mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

11. Mark each statement True or False. Justify each answer. Complete parts a through e below.

a. When **u** and **v** are nonzero vectors, Span{**u**,**v**} contains only the line through **u** and the line through **v** and the origin.

- **A.** False. Span{**u**,**v**} will not contain the origin.
- **B.** True. Span{**u**,**v**} is the set of all scalar multiples of **u** and all scalar multiples of **v**.
- **C.** False. Span{**u**,**v**} includes linear combinations of both **u** and **v**.

b. Any list of five real numbers is a vector in $\mathbb{R}^5$.

- **A.** False. A list of numbers is not enough to constitute a vector.
- **B.** True. $\mathbb{R}^5$ denotes the collection of all lists of five real numbers.
- **C.** False. A list of five real numbers is a vector in $\mathbb{R}^6$.
- **D.** False. A list of five real numbers is a vector in $\mathbb{R}^n$.

c. Asking whether the linear system corresponding to an augmented matrix $\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{b} \end{bmatrix}$ has a solution amounts to asking whether **b** is in Span $\{\mathbf{a}_1,\mathbf{a}_2,\mathbf{a}_3\}$.

- **A.** True. An augmented matrix has a solution when the last column can be written as a linear combination of the other columns. A linear system augmented has a solution when the last column of its augmented matrix can be written as a linear combination of the other columns.
- **B.** False. An augmented matrix having a solution does not mean **b** is in Span $\{\mathbf{a}_1,\mathbf{a}_2,\mathbf{a}_3\}$.
- **C.** False. If **b** corresponds to the origin then it cannot be in Span $\{\mathbf{a}_1,\mathbf{a}_2,\mathbf{a}_3\}$.

d. The vector **v** results when a vector $\mathbf{u}-\mathbf{v}$ is added to the vector **v**.

- **A.** True. Adding $\mathbf{u}-\mathbf{v}$ to **v** results in **v**.
- **B.** False. Adding $\mathbf{u}-\mathbf{v}$ to **v** results in $\mathbf{u}-2\mathbf{v}$.
- **C.** False. Adding $\mathbf{u}-\mathbf{v}$ to **v** results in $2\mathbf{v}$.
- **D.** False. Adding $\mathbf{u}-\mathbf{v}$ to **v** results in **u**.

e. The weights $c_1,\ldots,c_p$ in a linear combination $c_1\mathbf{v}_1+\ldots+c_p\mathbf{v}_p$ cannot all be zero.

- **A.** True. Setting all the weights equal to zero results in the vector **0**.
- **B.** False. Setting all the weights equal to zero results in the vector **0**.
- **C.** False. Setting all the weights equal to zero does not result in the vector **0**.
- **D.** True. Setting all the weights equal to zero does not result in the vector **0**.

12. Let $A = \begin{bmatrix} 1 & 0 & -7 \\ 0 & 2 & -4 \\ -2 & 4 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix}$. Denote the columns of A by $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$, and let $W = \text{Span}\{\mathbf{a}_1,\mathbf{a}_2,\mathbf{a}_3\}$.

a. Is **b** in $\{\mathbf{a}_1,\mathbf{a}_2,\mathbf{a}_3\}$? How many vectors are in $\{\mathbf{a}_1,\mathbf{a}_2,\mathbf{a}_3\}$?
b. Is **b** in W? How many vectors are in W?
c. Show that $\mathbf{a}_3$ is in W. [Hint: Row operations are unnecessary.]

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a. Is **b** in $\{\mathbf{a}_1,\mathbf{a}_2,\mathbf{a}_3\}$?

- Yes
- ✔ No

How many vectors are in $\{\mathbf{a}_1,\mathbf{a}_2,\mathbf{a}_3\}$?

- **A.** Two
- **B.** One
- **C.** Infinitely many
- ✔ **D.** Three

b. Set up the appropriate augmented matrix for determining if **b** is in W.

$$\begin{bmatrix} 1 & 0 & -7 & 4 \\ 0 & 2 & -4 & 1 \\ -2 & 4 & 3 & -4 \end{bmatrix}$$

(Simplify your answers.)

Is **b** in W?

- **A.** No, because the row-reduced form of the augmented matrix has a pivot in the rightmost column.
- **B.** Yes, because the row-reduced form of the augmented matrix has a pivot in the rightmost column.
- **C.** No, because the row-reduced form of the augmented matrix does not have a pivot in the rightmost column.
- ✔ **D.** Yes, because the row-reduced form of the augmented matrix does not have a pivot in the rightmost column.

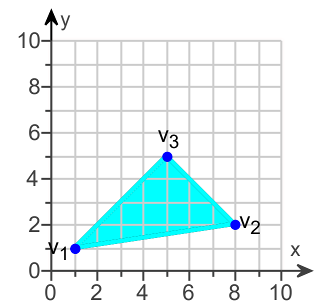
How many vectors are in W?

- **A.** One
- **B.** Three
- ✔ **C.** Infinitely many
- **D.** Two

c. The vector $\mathbf{a}_3$ is in $W = \text{Span}\{\mathbf{a}_1,\mathbf{a}_2,\mathbf{a}_3\}$ because $\mathbf{a}_3$ can be written as a linear combination $c_1\mathbf{a}_1 + c_2\mathbf{a}_2 + c_3\mathbf{a}_3$ where c_1, c_2, and c_3 are scalars.

Thus, $\mathbf{a}_3$ is in W because $\mathbf{a}_3 =$ 0 $\mathbf{a}_1 +$ 0 $\mathbf{a}_2 +$ 1 $\mathbf{a}_3$.
(Simplify your answers.)

13. A thin triangular plate of uniform density and thickness has vertices at $v_1 = (1,1)$, $v_2 = (8,2)$, $v_3 = (5,5)$, as in the figure to the right, and the mass of the plate is 3 g. Complete parts a and b below.



a. Find the (x,y)-coordinates of the center of mass of the plate. This "balance point" of the plate coincides with the center of mass of a system consisting of three 1-gram point masses located at the vertices of the plate.

The center of mass of the plate is located at $\left(\frac{14}{3},\frac{8}{3}\right)$.

(Type an ordered pair. Type integers or simplified fractions.)

b. Determine how to distribute an additional mass of 6 g at the three vertices of the plate to move the balance point of the plate to (4,2). [Hint: Let w_1, w_2, w_3 denote the masses added at the three vertices, so that $w_1 + w_2 + w_3 = 6$.]

Add 3.5 g at (1,1), add 2 g at (8,2) and add 0.5 g at (5,5).

(Type integers or decimals rounded to one decimal place as needed.)

YOU ANSWERED: 3

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