Section 6.5 HW / Discrete Structures II / Fall 2018

by: PHONG VO

Warning: Some problems require techniques from previous sections.

1. How many ways are there to assign three jobs to five employees if ...
   (a) ... each employee can be given more than one job? Perm with repetition; each of three jobs can be done by 5 choices : 5*5*5 = 5^3 = 125
   (b) ... each job must be assigned to a different employee? Pick 3/5 employees to get jobs in way (x1,x2, x3)=>P(5,3)=5*4*3=60 ways
2. A bagel shop has seven types of bagels (plain (x1), onion (x2), poppy (x3), everything (x4), rye (x5), raisin (x6), and blueberry (x7)). How many ways are there to choose five bagels ...
   (a) ... with no restrictions? x1+x2+x3+x4+x5+x6+x7=5 => # ways = C(5+6,6) = C(11,6) =11*10*9*8*7*6/6!=11*10*9*8*7*6/6*5*4*3*2*1=462
   (b) ... with at least one plain bagel and at least one rye bagel? (x'1=1,x2,x3,x4,x'5=1,x6,x7)=>C(5-x'1-x'5+6,6) =C(5-1-1+6,6)=C(9,6) =9*8*7*6*5*4/6*5*4*3*2*1=84
   (c) ... so that all the bagels are of different types? Pick 5 bagels out of 7 types=>C(7,5)=7*6*5*4*3/5*4*3*2*1=21
   (d) ... so that the selection only contains some combination of poppy and onion bagels? x2+x3=5=> C(5+1,1)=C(6,1)=6
   (e) ... so that the selection contains exactly two different types? Pick 2 types out of 7 types = C(7,2) = 7*6/2! = 21 ways => wrong
3. How many solutions are there for the equation

$$x_1 + x_2 + x_3 + x_4 = 10$$

   where x_1, x_2, x_3, x_4 are nonnegative integers such that ...
   (a) ... there are no additional restrictions? C(10+3,3) = C(13,3) = 13*12*11/3*2*1 = 286
   (b) ... $x_4 \geq 3$? (x1, x2, x3, x'4) where x1+x2+x3+x'4=7 => ans = C(10,3) = 120
   (c) ... $x_4 < 3$? = Total - [(b)] = 286 - 120 = 166
   (d) ... $x_i \geq 1$ for all i? =(1,1,1,1) + (x'1,x'2,x'3,x'4) where x'i 's >=0, sum of x'1 + x'2 + x'3 + x'4 = 6 => C(6+3,3) = C(9,3) = 9*8*7/3! = 9*8*7/3*2*1 = 3*4*7 = 12*7 = 84
4. How many strings of letters consist of exactly 3 As, 4 Bs, and 3 Cs? (e.g. BAACABBCCB) =10!/(3!4!3!)=10*3*4**7*5 =4,200 ways
5. How many ways are there to distribute five different pieces of fruit among three children? 3*3*3*3*3=243 ways
6. How many ways are there to distribute five identical apples among three children? x1+x2+x3=5=> C(5+2,2)=C(7,2) =7*6/2! = 21
7. (a) In how many ways can a professor assign grades to the ten students in a class if she plans to give exactly 3 As, 4 Bs and 3 Cs? 10!/(3!4!3!)=4,200 ways
   (b) How many ways are there to distribute ten different books among three children if Alice gets three books, Bob gets four books, and Candice gets three books? 10!/(3!4!3!)=4,200 ways
   (c) A student has ten granola bars: three apple bars, four blueberry bars, and three chocolate bars. He is planning to consume exactly one bar each day for ten days. How many ways can he do this? 10!/(3!4!3!)=4,200 ways

Answers:

1. (a) 125
   (b) 60
2. (a) 462
   (b) 84
   (c) 21
   (d) 6 (It's easy to list the possibilities in this case!)
   (e) 84
3. (a) 286
   (b) 120
   (c) 166 (Compare with parts (a) and (b).)
   (d) 84
4. 4200
5. 243
6. 21
7. All answers are the same as question #4!

   (The objects you're trying to count can be "encoded" as strings consisting of 3 As, 4 Bs, and 3 Cs. Try to figure out how to do this in each part.)