Due Date: Sept. 20, 2016

1. **Compare Functions**: (10 points) What is the smallest integer value of $n > 3$ such that an algorithm whose running time is $7n$ runs *slower than* an algorithm whose running time is $7(\log_2 n)^4$ on the same machine? Justify your answer. (Hint: You may write a program, draw a plot, or/and proof)

2. **Pseudocode and Loop Invariant**: (15 points) textbook, Exercise2.1-3, p22, Searching Problem

3. **Sorting Algorithms**: (20 points) Using textbook Figure 2.2 and Figure 2.4 as models to illustrate the operations of Insertion_Sort and Merge_Sort on the array A = <30, 7, 95, 56>

4. **Analysis**: (20 points) There is a mystery function called Mystery(n) and the pseudocode of the algorithm is shown as below. Please analyze the worst-case asymptotic execution time of this algorithm using the method we learn in the class. Express the execution time as a function of the input value n. Assume that $n = 3^k$ for some positive integer $k \geq 1$. Justify your answer.
   Hint:
   > (a) Draw a recursion tree to help with your analysis.
   > (b) Appendix A may help with your calculation

   > Mystery(n)
   > 1   **if** $n \leq 1$
   > 2   **return** 1
   > 3   **for** i=1 **to** 5
   > 4   **for** j = 1 **to** n^2
   > 5   print "this is a recursive call."
   > 6   Mystery ($n/3$)
   > 7   Mystery ($n/3$)
   > 8   Mystery ($n/3$)

5. **Algorithm Design** (35 points)
   Input: array A contains n distinct numbers from 1 to n, in arbitrary order.
   Output: *number of inversions (*defined as the number of pair(i, j) of array indices with i < j and A[i] > A[j]).
   (a) (5 points) What array with elements from the set {1, 2, …, n} has the most inversions? How many does it have?
   (b) (25 points) Create an algorithm using divide-and-conquer approach that determines the number of inversions in any permutation on n elements in Θ(nlgn) worst-case time (Hint: modify the merge sort).
   (c) (5 points) Use array <2, 3, 8, 6, 1> as an example to briefly explain how your algorithm works.

Algorithms -- COMP.4040 Honor Statement
(Courtesy of Prof. Tom Costello and Karen Daniels with modifications)

Must be attached to each submission

Academic achievement is ordinarily evaluated on the basis of work that a student produces independently. Infringement of this Code of Honor entails penalties ranging from reprimand to suspension, dismissal or expulsion from the University.

Your name on any exercise is regarded as assurance and certification that what you are submitting for that exercise is the result of your own thoughts and study. Where collaboration is authorized, you should state very clearly which parts of any assignment were performed with collaboration and name your collaborators.

In writing examinations and quizzes, you are expected and required to respond entirely on the basis of your own memory and capacity, without any assistance whatsoever except such as what is specifically authorized by the instructor.

I certify that the work submitted with this assignment is mine and was generated in a manner consistent with this document, the course academic policy on the course website on Blackboard, and the UMass Lowell academic code.

Date: _____

Name (please print):   _____

Signature: _____