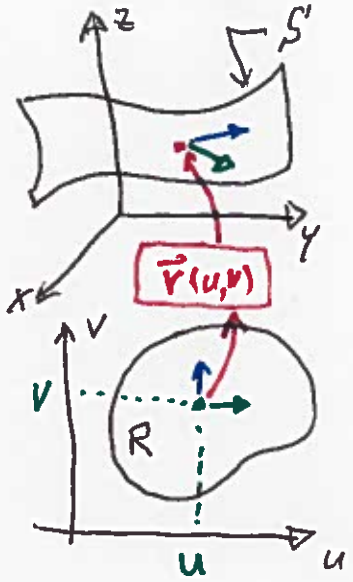
We wish to compute $\iint_S f(x,y,z)\,dS$

Parameterize surface $\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$



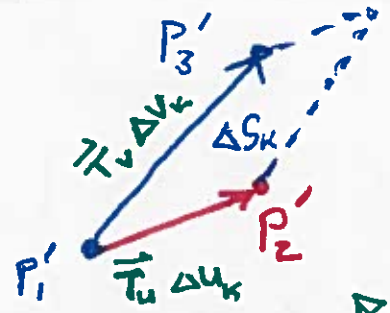
$\iint_S f\,dS = \iint_R f(u,v)\,\,?$ — we want differentials to be $du\,dv$ or $dv\,du$

But $dS \neq du\,dv$

We need replacement for dS

Answer: $dS = |\vec{T}_u \times \vec{T}_v|\,du\,dv$ (see below)

So $\iint_S f\,dS = \iint_R f(u,v)\,|\vec{T}_u \times \vec{T}_v|\,du\,dv$ (or $dv\,du$)



$\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle \quad (u,v) \in R$

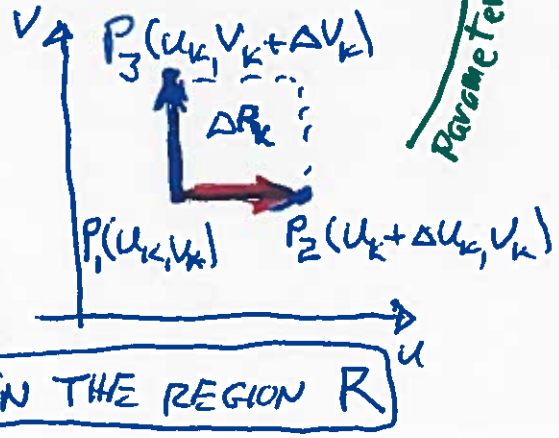
$\overrightarrow{P_1'P_2'} \cong \left\langle \frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u} \right\rangle_{(u_k,v_k)} \Delta u_k = \vec{T}_u \Delta u_k$

$\overrightarrow{P_1'P_3'} \cong \left\langle \frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v} \right\rangle_{(u_k,v_k)} \Delta v_k = \vec{T}_v \Delta v_k$

ON THE SURFACE S

Parameterization of S

$$\Delta S_k = |\vec{T}_u \Delta u_k \times \vec{T}_v \Delta v_k|$$



IN THE REGION R

$$= \underbrace{|\vec{T}_u \times \vec{T}_v|}_{\text{Amplification Factor}} \underbrace{\Delta u_k \Delta v_k}_{\Delta R_k}$$

$$\Rightarrow \frac{\Delta S_k}{\Delta R_k} = |\vec{T}_u \times \vec{T}_v|_{(u_k,v_k)}$$