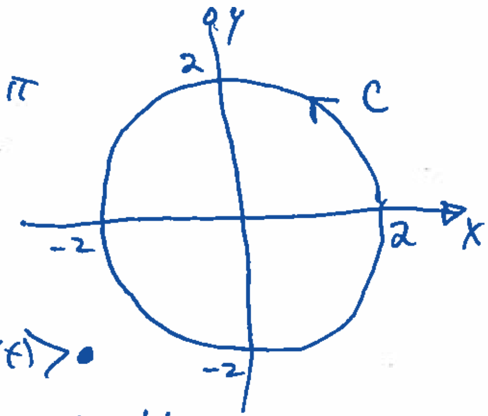
Calculus III Quiz #8 Spring 2018 NAME: SOLUTION

1. (6 Pts) Compute the flux integral $\int_C \vec{F}\cdot\vec{n}ds$ for the vector field $\vec{F} = < x-y, y-x >$ where C is a circle of radius 2 centered at the origin and oriented counter-clockwise.



$$\vec{r}(t) = \langle 2\cos(t), 2\sin(t)\rangle \quad 0 \le t \le 2\pi$$

$$\vec{r}\,'(t) = \langle -2\sin(t), 2\cos(t)\rangle$$

$$\int_C \vec{F}\cdot\vec{n}\,ds = \int_0^{2\pi} \overbrace{\langle 2\cos(t) - 2\sin(t), 2\sin(t) - 2\cos(t)\rangle}^{\vec{F}} \cdot \langle \underbrace{2\cos(t)}_{y'(t)}, \underbrace{2\sin(t)}_{-x'(t)}\rangle\, dt$$

$$= \int_0^{2\pi} [4\cos^2(t) - 4\sin(t)\cos(t) + 4\sin^2(t) - 4\sin(t)\cos(t)]\,dt$$

$$= \int_0^{2\pi} [4 - 8\sin(t)\cos(t)]\,dt = [4t - 2\sin^2(t)]\Big|_0^{2\pi}$$

$$= 8\pi - 0 = 8\pi$$

If you used the flux form of Green's Thm (not intended) your solution should look like:

$$\oint_C \vec{F}\cdot\vec{n}\,ds = \iint_R (f_x + g_y)\,dA = \iint_R (1+1)\,dA = \iint_R 2\,dA$$

$$= \text{twice the area of } A = 2\pi(2)^2$$

$$= 8\pi$$

SEE SECOND PROBLEM (over)

2. (6 Pts) Demonstrate that $\vec{F}(x,y,z) =< 2xy - z, x^2+1, -x >$ is a conservative vector field and then compute the work integral $\int_C \vec{F}\cdot\overrightarrow{dr}$ over the curve $\overrightarrow{r(t)} =< t+1, t^3, 4-t^3 >$ for $0 \le t \le 1$ [i.e., from P(1,0,4) to Q(2,1,3)].

Let $\vec{F} = < f, g, h >$

✓ $f_y = 2x \qquad g_x = 2x$

✓ $g_z = 0 \qquad h_y = 0$

✓ $f_z = -1 \qquad h_x = -1$

Passes all three tests so $\vec{F}$ is conservative. Now we will find $\phi(x,y,z)$.

$\phi_x = 2xy - z \Rightarrow \phi = x^2y - xz + C(y,z)$

$\phi_y = x^2 + C_y(y,z) \Rightarrow C_y = 1$ or $C = y + D(z)$

New model for ϕ is $\phi = x^2y - xz + y + D(z)$

$\phi_z = -x + D_z \Rightarrow D_z = 0$ or $D = \text{constant } (=0)$

Therefore $\boxed{\phi = x^2y - xz + y}$

and $\displaystyle\int_C \vec{F}\cdot\vec{dr} = \{x^2y - xz + y\}\Big|_{P(1,0,4)}^{Q(2,1,3)}$

$= (4 - 6 + 1) - (0 - 4 + 0)$

$= -1 + 4 = 3$