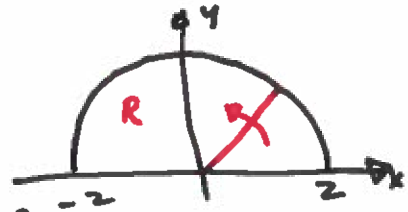
Calculus III Quiz #10 Spring 2018 NAME: SOLUTION

PICK EITHER PROBLEM 1 or PROBLEM 2 – Do NOT do both. IF you do both, I will pick which to grade. If you try both, CROSS out the one you do not want graded.

1. (12 Points) **SURFACE INTEGRAL OF SCALAR FUNCTION:** Evaluate the surface integral $\iint_S f dS$ where $f = x^2 + y^2$ and S is the paraboloid $z = x^2 + y^2$ where $0 \le z \le 4$ and $y \ge 0$. Focus on expressing the answer as an integral in polar coordinates. Final evaluation of this integral is worth the final two points.

2. (12 Points) **STOKES' THEOREM:** Evaluate $\iint_S (\vec{\nabla} \times \vec{F}) \cdot \vec{n} dS$ where $\vec{F}$=<2x,3y+x,y+z> and S is the paraboloid $z = x^2 + y^2 + 1$ where $1 \le z \le 10$. Drawing a NEAT diagram of the surface showing either the boundary curve C (if you use it) or any alternate surface S\` (if you use it) is important.

1. $$\iint_S f\,dS = \iint_R f(x,y)\left(g_x^2+g_y^2+1\right)^{1/2} dA$$



$$= \int_0^{\pi} \int_0^2 r^2\left((2x)^2+(2y)^2+1\right)^{1/2} r\,dr\,d\theta$$

$$= \int_0^{\pi} d\theta \int_0^2 (4r^2+1)^{1/2} r^3\,dr$$

$u = 4r^2+1$
$du = 8r\,dr$
$r^2 = \frac{1}{4}(u-1)$
$u(0) = 1$
$u(2) = 17$

$$= \pi \int_1^{17} u^{1/2}\,\frac{1}{4}(u-1)\,\frac{1}{8}\,du$$

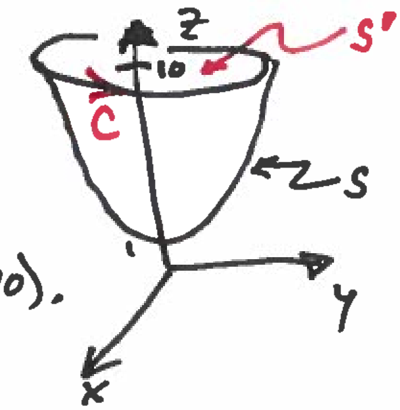
$$= \frac{\pi}{32}\int_1^{17}\left(u^{3/2}-u^{1/2}\right)du$$

$$= \frac{\pi}{32}\left(\frac{2}{5}u^{5/2}-\frac{2}{3}u^{3/2}\right)\Bigg|_1^{17}$$

$$= \frac{\pi}{32}\left[\left(\frac{2}{5}17^{5/2}-\frac{2}{3}17^{3/2}\right)-\left(\frac{2}{5}-\frac{2}{3}\right)\right]$$

$$= \frac{\pi}{16}\left[\frac{17^{5/2}}{5}-\frac{17^{3/2}}{3}+\frac{2}{15}\right]$$

2. $\displaystyle\iint_S (\vec{\nabla}\times\vec{F})\cdot\vec{n}\, dS = \oint_C \vec{F}\cdot d\vec{r}$



A. Do surface integral but change $S \to S'$ where S' is circular disk inside C (@ $z=10$).

$\vec{F} = \langle 2x,\ 3y+x,\ y+10\rangle$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x & 3y+x & y+10 \end{vmatrix} = \langle 1, 0, 1\rangle$$

$\vec{n} = \langle 0,0,1\rangle$ the unit normal for S' is $\vec{n} = \vec{k}$

$$\iint_S (\vec{\nabla}\times\vec{F})\cdot\vec{n}\, dS = \iint (\vec{\nabla}\times\vec{F})\cdot\vec{k}\, dS' = \iint_{S'} dS' = \text{Area of } S' = 9\pi$$

B. Evaluate line integral over C. $\vec{F} = \langle 2x, 3y+x, y+10\rangle$

$\vec{r}(t) = \langle 3\cos(t),\ 3\sin(t),\ 10\rangle \quad \vec{r}\,'(t) = \langle -3\sin(t),\ 3\cos(t),\ 0\rangle$

$$\oint_C \vec{F}\cdot d\vec{r} = \int_0^{2\pi} \langle 6\cos(t),\ 9\sin(t)+3\cos(t),\ 3\sin(t)+10\rangle \cdot \langle -3\sin(t),\ 3\cos(t),\ 0\rangle\, dt$$

$$= \int_0^{2\pi} [-18\sin(t)\cos(t) + 27\sin(t)\cos(t) + 9\cos^2(t)]\, dt$$

$$= \int_0^{2\pi} \overbrace{\cancel{9\sin(t)\cos(t)}}^{=0 \text{ over } 2\pi}\, dt + \int_0^{2\pi} 9\cos^2(t)\, dt$$

$$= \frac{9}{2}\int_0^{2\pi} [1 + \overbrace{\cancel{\cos(2t)}}^{=0 \text{ over } 2\pi}]\, dt = \frac{9}{2}\int_0^{2\pi} dt = 9\pi$$