

# Solution

Spring 2020

Name:

## Linear Algebra: Quiz 6

Show ALL work, as unjustified answers may receive no credit. Calculators are not allowed on any quiz or test paper. Make sure to exhibit skills discussed in class. Box all answers and clean up answers as much as possible.

### 1. Introduction to the Determinant (3.1)

[4pts] Find  $x$ , assuming:

$$\det \begin{bmatrix} x^2 & x & 2 \\ 2 & 1 & 1 \\ 0 & 0 & -5 \end{bmatrix} = 0 \iff \det(A) = 0$$

\*Factor Expanding Across  $R_3$ :

$$\det(A) = 0 + 0 + (-5)(-1)^{3+3} \det \begin{vmatrix} x^2 & x \\ 2 & 1 \end{vmatrix} = -5(x^2 - 2x)$$

\*Set  $\det(A) = 0$  & solve for  $x$ :

$$-5(x^2 - 2x) = 0$$

$$x(x - 2) = 0$$

$$\begin{cases} x = 0 \\ x = 2 \end{cases}$$

Ans

### 1. Properties of the Determinant (3.2)

[6pts, 2pts each] Let:

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \text{ and assume that } \det(A) = 10$$

Find the following:

$$\det(3A), \quad \det(2A^{-1}), \quad \& \quad \det \begin{bmatrix} a & g & d \\ b & h & e \\ c & i & f \end{bmatrix}$$

\*Part (a):  $\det(3A) = 3^3 \det(A) = 27(10) = \boxed{270}$  Ans

\*Part (b):  $\det(2A^{-1}) = 2^3 \det(A^{-1}) = 8 \left( \frac{1}{\det(A)} \right) = 8 \left( \frac{1}{10} \right) = \boxed{\frac{4}{5}}$  Ans

\*Part (c):  $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \xrightarrow{*R_2 \leftrightarrow R_3} \begin{bmatrix} a & b & c \\ g & h & i \\ d & e & f \end{bmatrix} = B; \det(B) = -\det(A)$

Let  $B = \begin{bmatrix} a & b & c \\ g & h & i \\ d & e & f \end{bmatrix} \Rightarrow B^T = \begin{bmatrix} a & g & d \\ b & h & e \\ c & i & f \end{bmatrix} \Rightarrow \therefore \det(B^T) = \det(B) = -\det(A) = \boxed{-10}$  Ans