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**Course:** Linear Algebra I (Spring 2020)

**Assignment:** Section 2.5 Homework

1. Solve the equation A**x** = **b** by using the LU factorization given for A. Also solve A**x** = **b** by ordinary row reduction.

$$A = \begin{bmatrix} 4 & -7 & -3 \\ -4 & 4 & 2 \\ 8 & -5 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} 4 & -7 & -3 \\ 0 & -3 & -1 \\ 0 & 0 & -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ -8 \\ 28 \end{bmatrix}$$

Let L**y** = **b** and U**x** = **y**. Solve for **x** and **y**.

$$\mathbf{y} = \begin{bmatrix} 2 \\ -6 \\ 6 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} 3 \\ 4 \\ -6 \end{bmatrix}$$

Row reduce the augmented matrix [A **b**] and use it to find **x**.

The reduced row echelon form of [A **b**] is $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -6 \end{bmatrix}$, yielding $\mathbf{x} = \begin{bmatrix} 3 \\ 4 \\ -6 \end{bmatrix}$.

YOU ANSWERED: $\begin{bmatrix} 3 \\ \\ -6 \end{bmatrix}$

2. Solve the equation A**x** = **b** by using the LU factorization given for A.

$$A = \begin{bmatrix} 2 & -6 & 2 \\ -4 & 9 & 1 \\ 4 & -9 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -6 & 2 \\ 0 & -3 & 5 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 16 \\ -17 \\ 17 \end{bmatrix}$$

Let L**y** = **b** and U**x** = **y**. Solve for **x** and **y**.

$$\mathbf{y} = \begin{bmatrix} 16 \\ 15 \\ 0 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} -7 \\ -5 \\ 0 \end{bmatrix}$$

3. Solve the equation $A\mathbf{x} = \mathbf{b}$ by using the LU factorization given for A.

$$A = \begin{bmatrix} 1 & 2 & 2 & 0 \\ -2 & -2 & -4 & 10 \\ 2 & 0 & 2 & -20 \\ -4 & 0 & -6 & 41 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 2 & -2 & 1 & 0 \\ -4 & 4 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 & 0 \\ 0 & 2 & 0 & 10 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 17 \\ -24 \\ 10 \\ -21 \end{bmatrix}$$

Let $L\mathbf{y} = \mathbf{b}$ and $U\mathbf{x} = \mathbf{y}$. Solve for $\mathbf{x}$ and $\mathbf{y}$.

$$\mathbf{y} = \begin{bmatrix} 17 \\ 10 \\ -4 \\ 3 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} 33 \\ -10 \\ 2 \\ 3 \end{bmatrix}$$

4. Find an LU factorization of the matrix A (with L unit lower triangular).

$$A = \begin{bmatrix} 5 & 3 \\ -2 & -3 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 \\ -\frac{2}{5} & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 5 & 3 \\ 0 & -\frac{9}{5} \end{bmatrix}$$

5. Find an LU factorization of the matrix A (with L unit lower triangular).

$$A = \begin{bmatrix} 6 & 8 \\ 12 & 13 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 6 & 8 \\ 0 & -3 \end{bmatrix}$$

6. Find an LU factorization of the matrix A (with L unit lower triangular).

$$A = \begin{bmatrix} -2 & 0 & 4 \\ 6 & 3 & -8 \\ 6 & 12 & 12 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -3 & 4 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} -2 & 0 & 4 \\ 0 & 3 & 4 \\ 0 & 0 & 8 \end{bmatrix}$$

7. When A is invertible, MATLAB finds $A^{-1}$ by factoring LU (where L may be permuted lower triangular), inverting L and U, and then computing $U^{-1}L^{-1}$. Use this method to compute the inverse of the given matrix A.

$$A = \begin{bmatrix} 3 & -9 & 3 \\ -12 & 33 & -6 \\ 0 & -3 & 3 \end{bmatrix}, \text{ where } A = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & -9 & 3 \\ 0 & -3 & 6 \\ 0 & 0 & -3 \end{bmatrix}$$

Compute $U^{-1}$ and $L^{-1}$.

$$U^{-1} = \begin{bmatrix} \frac{1}{3} & -1 & -\frac{5}{3} \\ 0 & -\frac{1}{3} & -\frac{2}{3} \\ 0 & 0 & -\frac{1}{3} \end{bmatrix} \qquad L^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ -4 & -1 & 1 \end{bmatrix}$$

Compute $A^{-1}$.

$$A^{-1} = \begin{bmatrix} 3 & \frac{2}{3} & -\frac{5}{3} \\ \frac{4}{3} & \frac{1}{3} & -\frac{2}{3} \\ \frac{4}{3} & \frac{1}{3} & -\frac{1}{3} \end{bmatrix}$$

8. Suppose $A = BC$, where B is invertible. Show that any sequence of row operations that reduces B to I also reduces A to C. The converse is not true, since the zero matrix may be factored as $0 = B \cdot 0$.

Which of the following pieces of information in the problem statement are relevant for showing that any sequence of row operations that reduces B to I also reduces A to C? Select all that apply.

- [ ] **A.** The converse is not true.
- [x] **B.** $A = BC$.
- [x] **C.** B is invertible.
- [ ] **D.** The zero matrix may be factored as $0 = B \cdot 0$.

Given the relevant pieces of information from the previous step, there exist elementary matrices $E_1, \ldots, E_p$ corresponding to row operations that reduce B to I, in the sense that $E_p \ldots E_1 \; B = I$.

Applying the same sequence of row operations to A amounts to left-multiplying A by the product $E_p \ldots E_1$.

The proof is complete because $E_p \ldots E_1 \; A = E_p \ldots E_1 \; B \; C = IC = C$.