

Due Date: May 3rd, 2019 (F), BEFORE the class starts

This assignment covers textbook Chapter 12&13.

1. BST: Using the definitions on p. 1177 of our textbook for *depth* of a tree node and *height* of a tree, consider the set of keys $K = \langle 20, 5, 1, 12, 7 \rangle$ and the different possible insertion orders for the keys in K . Based on the different possible insertion orders and their resulting Binary Search Trees, answer the following questions. (30 points)

a) What is the minimum height of a Binary Search Tree constructed from K ? Show an insertion order for the keys in K that generates a Binary Search Tree of minimum height. Draw the corresponding Binary Search Tree.

b) Are there any other insertion orders (beyond what you found in (a) above) for the keys in K that produce a Binary Search Tree of minimum height? If so, provide one such sample insertion order and its accompanying Binary Search Tree.

c) What is the maximum height of a Binary Search Tree constructed from K ? Show an insertion order for the keys in K that generates a Binary Search Tree of maximum height. Draw the corresponding Binary Search Tree.

2. Red-Black Tree: Exercise 13.1-1 on p311 in textbook (20 points)

3. Red-Black Tree: Exercise 13.2-3 on p314 in textbook (20 points)

4. Red-Black Tree: For the set of keys given above in Problem 1, show the sequence of red-black trees that result after successively inserting the keys into an initially empty red-black tree in the order given: $K = \langle 20, 5, 1, 12, 7 \rangle$. (*Show at least one tree resulting from each insertion*). State which case from the textbook applies. Assume that the root is always colored black.) (30 points)

Algorithms -- COMP.4040 Honor Statement
(Courtesy of Prof. Tom Costello and Karen Daniels with modifications)

Must be attached to each submission

Academic achievement is ordinarily evaluated on the basis of work that a student produces independently. Infringement of this Code of Honor entails penalties ranging from reprimand to suspension, dismissal or expulsion from the University.

Your name on any exercise is regarded as assurance and certification that what you are submitting for that exercise is the result of your own thoughts and study. Where collaboration is authorized, you should state very clearly which parts of any assignment were performed with collaboration and name your collaborators.

In writing examinations and quizzes, you are expected and required to respond entirely on the basis of your own memory and capacity, without any assistance whatsoever except such as what is specifically authorized by the instructor.

I certify that the work submitted with this assignment is mine and was generated in a manner consistent with this document, the course academic policy on the course website on Blackboard, and the UMass Lowell academic code.

Date: 05/03/2019

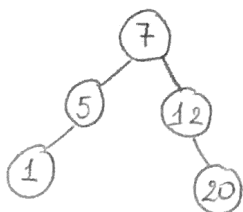
Name (please print): DANGNHI NGO

Signature: 

1/ BTS

$K = \langle 20, 5, 1, 12, 7 \rangle$

a/

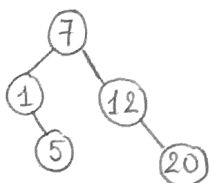


Minimum height of a BST constructed from K: 2

Insertion order for the keys in K:

$\langle 7, 5, 12, 1, 20 \rangle$

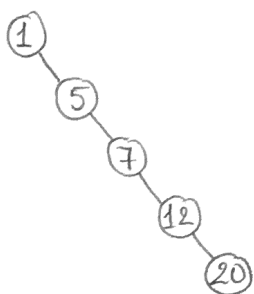
b/



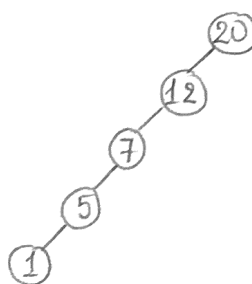
Different insertion order for the keys in K

$\langle 7, 1, 12, 5, 20 \rangle$

c/



or



Maximum height of a BTS constructed from K: 4

Insertion order for the keys in K: $\langle 1, 5, 7, 12, 20 \rangle$

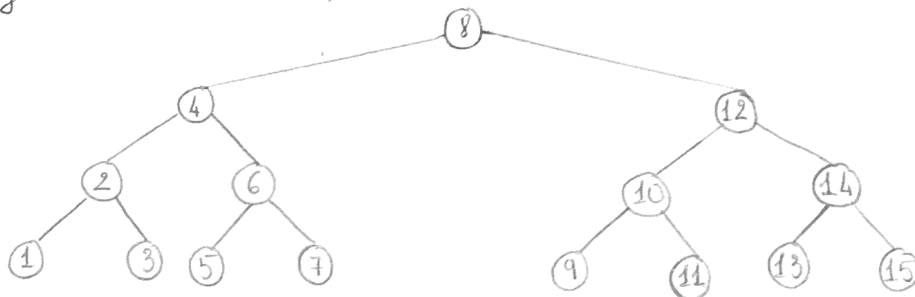
The maximum height is the linear to the left or to the right

2/ Red Black Tree

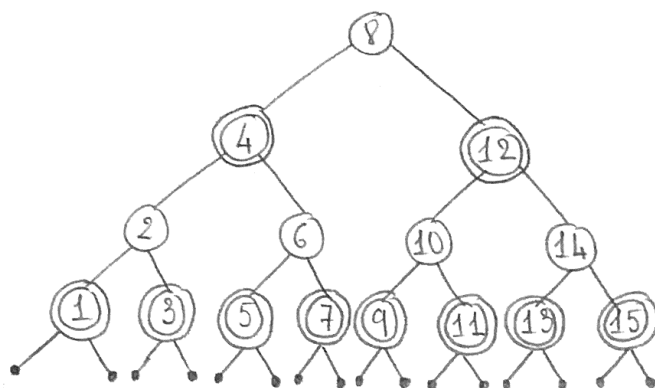
Exercise 13.1-1

Draw the complete binary search tree of height 3 on the keys $\{1, 2, \dots, 15\}$

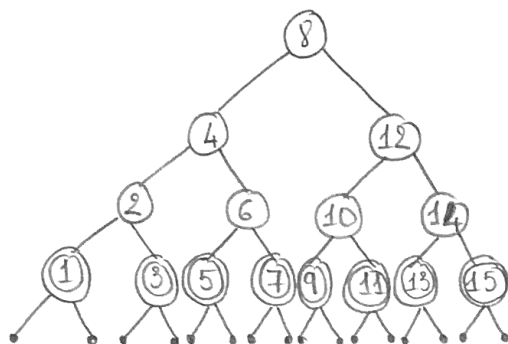
Add the NIL leaves & color the nodes in 3 different ways such that the black-heights of the resulting red-black trees are 2, 3 and 4



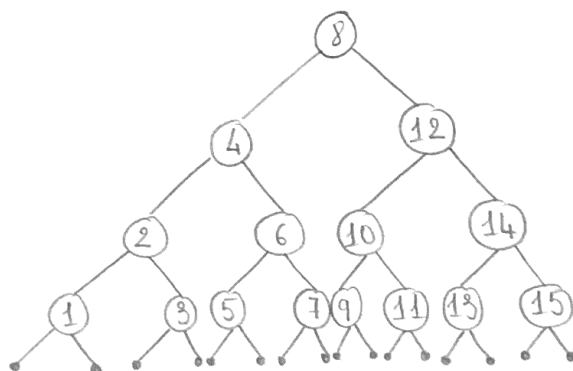
Black-height 2:



Black-height 3:



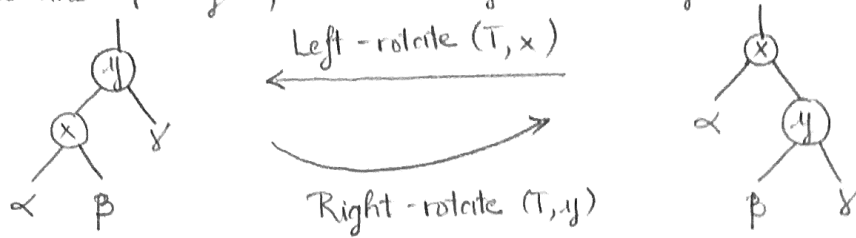
Black-height 4:



3/ Red-Black Tree

Exercise 13.2-3

Let a , b and c be arbitrary nodes in subtrees α , β and γ , respectively, in the left tree. How do the depths of a , b and c change when a left rotation is performed on node x ?



The depth of ' a ' increases by 1
 The depth of ' b ' remains same
 The depth of ' c ' decreases by 1

4/ Red-Black Tree:

$K = \langle 20, 5, 1, 12, 7 \rangle$

Show the sequence of red-black tree that results after successively inserting the keys into an initially empty red-black tree.

(1) Insert 20

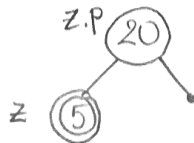
RED (20)

BLACK (20)

Color to
BLACK

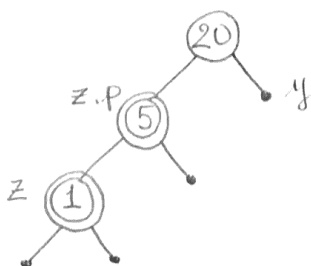
No case applied
 Root is always BLACK

(2) Insert 5



$z.p$ is not RED
 no fix-up
 no case applied

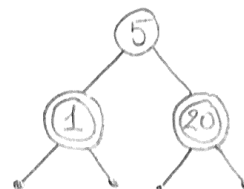
(3) Insert 1



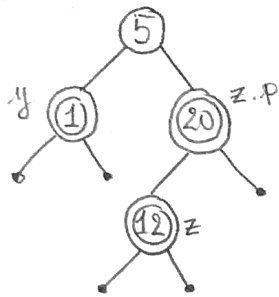
$z.p$ is a left child
 y is BLACK
 z is a left child
 <Case 3> applied

Recolor 5, 20

Right-rotate
 (20)



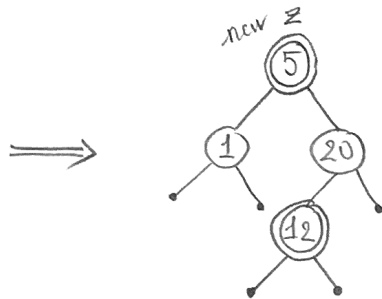
(4) Insert 12



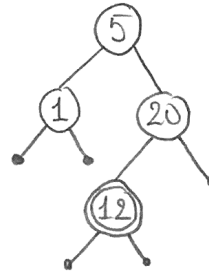
$z.p$ is a right child

y is RED

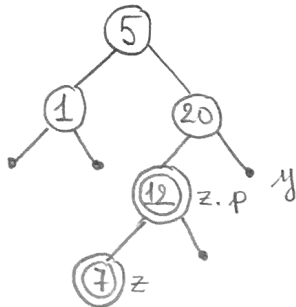
< Case 1' > applied



Recolor z



(5) Insert 7



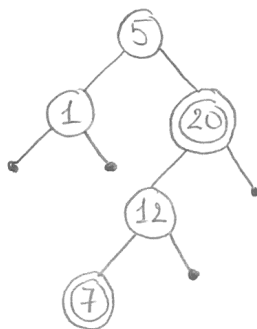
$z.p$ is a left child

y is BLACK

z is a left child

< Case 3 > applied

Recolor 12, 20



Right-rotate

(20)

