

1. Function Order of Growth: (20 points)

List the 4 functions below in non-decreasing asymptotic order of growth.

$$(\log n)^2 \quad n^{-2} \quad \lg(2^{\log(n^2)}) \quad n^2$$

Justify your answer mathematically by showing values of c and n_0 for each pair of functions that are adjacent in your ordering.

2. Pseudocode Analysis (25 points)

For the pseudocode below for procedure $\text{Mystery}(n)$, derive tight upper and lower bounds on its asymptotic worst-case running time $f(n)$. That is, for the set of inputs including those that force Mystery to work its hardest, find $g(n)$ such that $f(n) \in \Theta(g(n))$. Assume that the input n is a positive integer. Justify your answer.

$\text{Mystery}(n)$


1. if n is an even number
 2. for $i = 1$ to n
 3. for $j = n$ downto $n/2$
 4. print "even number"
5. else
 6. for $k = 1$ to $n/4$
 7. for $m = 1$ to n
 8. print "odd number"

3. True or False (25 points).

- a. $n \lg^2 n \in O(n^2)$
- b. $n \lg^2 n \in \Omega(n^{1.05})$
- c. $n^3 \in o(n^3)$
- d. The cost of the loop below is in $O(n)$

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for (i = 1; i <= n; i *= 2) { // n>=1
    constant work;
}
```

- e. The cost of the above loop is in $\Omega(\lg n)$

-  4. (20 points) For each of the following recurrences, give an expression for the runtime $T(n)$ if the recurrence can be solved with the Master Theorem. Otherwise, explain why the Master Theorem does not apply. Justify your answer.


(1) $T(n) = 3^n T\left(\frac{n}{3}\right) + n^3$

(2) $T(n) = 5T\left(\frac{n}{2}\right) + \sqrt{10}n^3$

(3) $T(n) = \frac{1}{4} T\left(\frac{n}{4}\right) + n \log n$

(4) $T(n) = T(n-1) + 2n$

(5) $T(n) = 16T\left(\frac{n}{4}\right) + n^2$

-  5. (10 points) Exercise 4.4-4. Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = 2T(n-1) + 1$. Use the substitution method to verify your answer.