MATH.3860

Prob and Stats – Test 1 Spring19 **Name:** Phong Vo

1. (20 points) Events A and B have respective probabilities $P(A) = \frac{1}{3}$ and $P(B) = \frac{2}{5}$, while $P(B|A) = \frac{3}{5}$.
   For each of the following, give an exact numerical answer as a **reduced fraction.**
   (a) What is $P(A \cap B)$?
   (b) What is $P(A \cup B)$?
   (c) What is $P(A - B)$?
   (d) What is $P(A|B)$?
   (e) What is $P(A|B^c)$?

2. (20 points) Two sections of a senior probability class are being taught. From what she has heard about the two instructors listed, Francesca estimates that her chances of passing course are 0.85 if she gets Professor A, and 0.60 if she gets Professor F. The section into which she is put is determined by the registrar. Suppose the chances of being assigned to Professor A are 4 out of 10. Fifteen weeks later we learn that Francesca passed the course. What is the probability she was enrolled in Professor A's section?

3. (20 points) At UML, 30% of the students are majoring in humanities, 50% in engineering, and 20% in science. Moreover, according to figures released by the registrar, the percentages of women majoring in humanities, engineering, and science are 75%, 45%, and 30% respectively. Suppose Jason meets Anna at a UML frat party. What is the probability that Anna is an engineering major?

4. (12 points) (a) If $P(A) = 1/4$ and $P(B) = 1/8$, determine $P(A \cup B)$ if A and B are mutually exclusive.
   (b) If $P(A) = 1/4$ and $P(B) = 1/8$, determine $P(A \cup B)$ if A and B are independent.
   (c) If $P(A) = 1/4$ and $P(B) = 1/8$, determine $P(A|B)$ if A and B are mutually exclusive.
   (d) If $P(A) = 1/4$ and $P(B) = 1/8$, determine $P(A|B)$ if A and B are independent.

5. (18 points) Consider a set of 10 urns, nine of which contain three white chips and three red chips each. The tenth urn contains five white chips and one red chip. An urn is picked at random. Then a sample of size three is drawn without replacement from that urn. If all three chips drawn are white, what is the probability that the urn being sampled is the one with five white chips?

6. (10 points) To qualify as a "three-of-a-kind" hand in a five card poker hand, the five cards must include three of the same denomination and two "single" cards — cards whose denominations match neither the triple nor each other. Compute the probability that a random poker hand draws a three-of-a-kind hand.

1.

(a) $p(A \cap B) = p(B|A)\,p(A)$

$= \frac{3}{5} \times \frac{1}{3} = \boxed{\frac{1}{5}}$

(b) $p(A \cup B) = p(A) + p(B) - p(A \cap B)$

$= \frac{1}{3} + \frac{2}{5} - \frac{1}{5}$

$= \frac{1}{3} + \frac{1}{5} = \boxed{\frac{8}{15}}$

c) $p(A - B) = p(A) - p(A \cap B)$

$= \frac{1}{3} - \frac{1}{5}$

$= \boxed{\frac{2}{15}}$

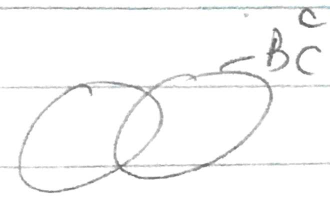
d) $p(A|B) = \frac{p(A \cap B)}{p(B)} = \frac{1/5}{2/5}$

$= \boxed{1/2}$

c) $p(A \mid B^c) = \dfrac{p(A \cap B^c)}{p(B^c)}$

$p(B^c) = 1 - p(B) = 1 - \dfrac{2}{5} = \dfrac{3}{5}$

$p(A \cap B^c) = p(A - B) = \dfrac{2}{15}$



$\Rightarrow \quad p(A \mid B^c) = \dfrac{2/15}{3/5} =$

20/20

$= \dfrac{2}{15} \times \dfrac{5}{3} = \boxed{\dfrac{2}{9}}$

4.

b) A and B are independent

$\Rightarrow \quad p(A \cap B) = p(A)\,p(B)$

$= \dfrac{1}{4} \times \dfrac{1}{8} = \dfrac{1}{32}$

$p(A \cup B) = p(A) + p(B) - p(A \cap B)$

$= \dfrac{1}{4} + \dfrac{1}{8} - \dfrac{1}{32} = \dfrac{8+4-1}{32}$

$= \boxed{11/32}$

d) $$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/32}{1/8} = \frac{1}{32} \times 8$$

$$= \boxed{1/4}$$

6/12

③ H : 0.30 E : 0.50 S : 0.20

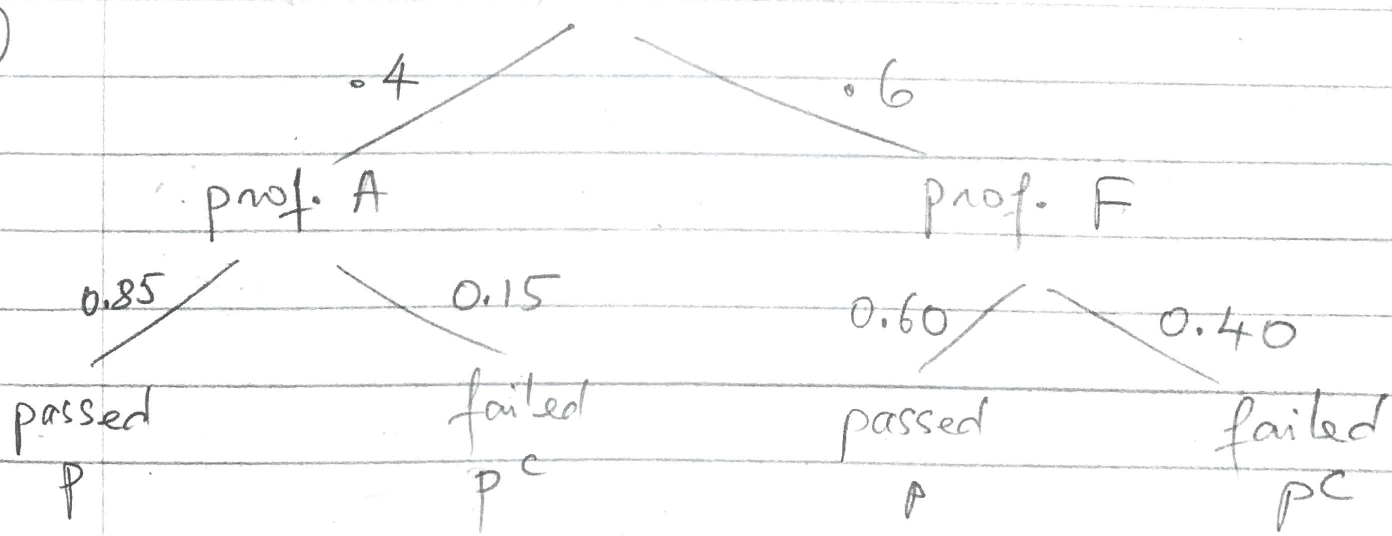
The prob. that Anna is an engineering major

$$= \frac{(.45)(.50)}{(0.30)(0.75)+(0.50)(0.45)+(0.20)(0.30)}$$

20/20

$$\frac{.225}{.225+.225+.06} = \frac{.225}{.51} = 0.441$$

②



We will find the prob. that Francesca passed the course when she enrolled in prof. A's section

A: is the event that she took class of prof. A

A^c: is the event that she took class of prof F

$p(A) = .40 \qquad p(A^c) = 1 - 0.40 = 0.60$

P: is the event that she passed the course

P^c: is the event that she failed the course

$$p(P \mid A) = \frac{p(A \mid P)\, p(P)}{p(A \mid P)\, p(P) + p(A \mid P^c)\, p(P^c)}$$

16/20

$$= \frac{(0.85)(0.4)}{(0.85)(0.4) + (0.40)(0.6)} \quad X$$

$$= \frac{.34}{.34 + .24} = \frac{.34}{.58}$$

$$= \boxed{0.5862}$$

4. a) $p(A \cap B) = p($

5. $\boxed{1} \to \boxed{9}$: $3W + 3R$

$\boxed{10^{th}}$: $5W + 1R$

$= \boxed{W_1}\ \boxed{W_2}\ \boxed{W3}\ \boxed{W4}\ \boxed{W5}\ \boxed{R}$

prob that picking the 10^{th} urn which has 5 chips $= 1/10$

prob that picking 3 white chips from urn $1 \to 9$

$$= \frac{1}{\binom{6}{3}} = \frac{1}{\frac{6!}{3!3!}} = \frac{3!\,3!}{6!}$$

$$= \frac{3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4} = \frac{1}{20}$$

prob that picking 3 white chips from urn 10^{th}

$$= \frac{\binom{5}{3}}{\binom{6}{3}} = \frac{\frac{5!}{3!\,2!}}{\frac{6!}{3!\,3!}}$$

$$= \frac{5!}{3!\,2!}\ \frac{3!\,3!}{6!} = \frac{\cancel{5!}\ 3!}{2!\ \cancel{6!}_{6}}$$

$$= \frac{3}{6} = \frac{1}{2}$$

The prob that size three is drawn provided they are from the 10th urn

$$= \frac{p(\text{size three and from } 10^{th} \text{ urn})}{p(\text{size three in 9 urn}) + p(\text{size three in } 10^{th} \text{ urn})}$$

$$= \frac{\frac{1}{2} \cdot \frac{1}{10}}{9\left(\frac{1}{20} \cdot \frac{1}{10}\right) + 1 \cdot \left(\frac{1}{2}\right)\left(\frac{1}{10}\right)}$$

$$= \frac{\frac{1}{20}}{\frac{1}{10}\left(\frac{9}{20} + \frac{1}{2}\right)} =$$

$$= \frac{1}{2\left(\frac{9}{20} + \frac{1}{2}\right)} = \frac{1}{\frac{9}{10} + 1}$$

$$= \frac{1}{\frac{19}{10}} = \boxed{\frac{10}{19}}$$

18/18

6

1st 2nd 3rd 4th 5th

The 1st card has 13 choices ⇒
2nd 1 choice
3rd 1 choice
4th card has 12 choices
5th card has 11 choices

⇒ Number of outcomes: $|E| = 13 \times 12 \times 11$

Sample space $= \binom{52}{5} = \frac{52!}{5!\,47!}$ ✓

⇒ prob. that a random poker hand draws of a three-of-a-kind hand

$$= \boxed{\frac{13 \times 12 \times 11}{5!\ 47!}}$$ ✗

=

5/10