Homework Set #2

1. (25 points) Rank the following 3 functions by order of asymptotic growth. That is, find an arrangement **$g_1(n)$, $g_2(n)$, $g_3(n)$** of the functions satisfying:

$$g_1(n) \in O(g_2(n)), \qquad g_2(n) \in O(g_3(n))$$

Justify your answer mathematically by showing values of c and n_o such that
$g_i(n) \le c g_{i+1}(n) \quad \forall n \ge n_0$

Functions:

$$\left(\frac{1}{2}\right)^{n^3} \qquad 3^{4\log_3 n} \qquad 5\lg n + n^2 \lg\lg n$$

2. (25 points) Suppose that for 3 (possibly different) functions of n: **$f_1(n)$, $f_2(n)$, $f_3(n)$** we know that:

i) $f_1(n) \in \Omega((1/2)^n)$ **ii)** $f_2(n) \in \Theta(n^2 \lg n)$ **iii)** $f_3(n) \in O(\lg^3 n)$

a) If statements **(i)-(iii)** are true, can we conclude that $f_3(n) \in O(f_2(n))$? **Why or why not?**

b) If statements **(i)-(iii)** are true, can we conclude that $f_2(n) \in \Omega(f_1(n))$? **Why or why not?**

3. True or False (25 points).

a. $n\lg^2 n \in O(n^2)$ [ ]

b. $n\lg^2 n \in \Omega(n^{1.05})$ [ ]

c. $n^3 \in o(n^3)$ [ ]

d. The cost of the loop below is in $O(n)$ [ ]

```
for (i = 1; i <= n; i *= 2) { // n>=1
    constant work;
}
```

e. The cost of the above loop is in $\Omega(\lg n)$ [ ]

4. (25 points) Pseudocode Analysis: For the pseudocode below for Mystery ($\boldsymbol{n}$), find tight upper and lower bounds on its asymptotic worst-case running time **f(*n*)**. That is, find **g(*n*)** such that **f(*n*)** $\in$ **Θ(g(*n*)).** (Assume that $\boldsymbol{n}$ is a positive integer.) Justify your answer.

```
Mystery (n )
        c ← 1
        for  i ← 1 to n
             do for  j ← i to n
                     do  for  k ← n  down to ⌊n/2⌋
                              do  c ← c + 1
         print c
```