Probability Identities

- Random variables (A)

  eg $P(A)$

  $\Rightarrow$ values; signifying the possibility of $\underline{\underline{A}}$ are in ~~stow~~ lowercase

  eg $P(A = a)$

- $P(a|b) = \dfrac{P(a \wedge b)}{P(b)}$ e.g. a = cloudy, b = spring

  so $P(\text{cloudy}|\text{spring}) = \dfrac{P(\text{cloudy} \wedge \text{spring})}{P(\text{spring})}$

  all occurrences

- $P(a \wedge b) = P(a|b)\,P(b)$ → (This is product rule)

$\boxed{\text{Bayes Rule}}$

- Then: $P(b|a) = \dfrac{P(b \wedge a)}{P(a)}$

  $P(b \wedge a) = P(b|a)\,(Pa)$

  commutative laws mean $P(b \wedge a) \equiv P(a \wedge b)$

Therefore: $P(b|a)\,P(a) = P(a|b)\,P(b)$

$$P(b|a) = \frac{P(a|b)\,P(b)}{P(a)}$$

Bayesian Networks

A technique to encode knowledge/uncertainty in a way a machine can understand.

Bayesian Network/Graph

e.g

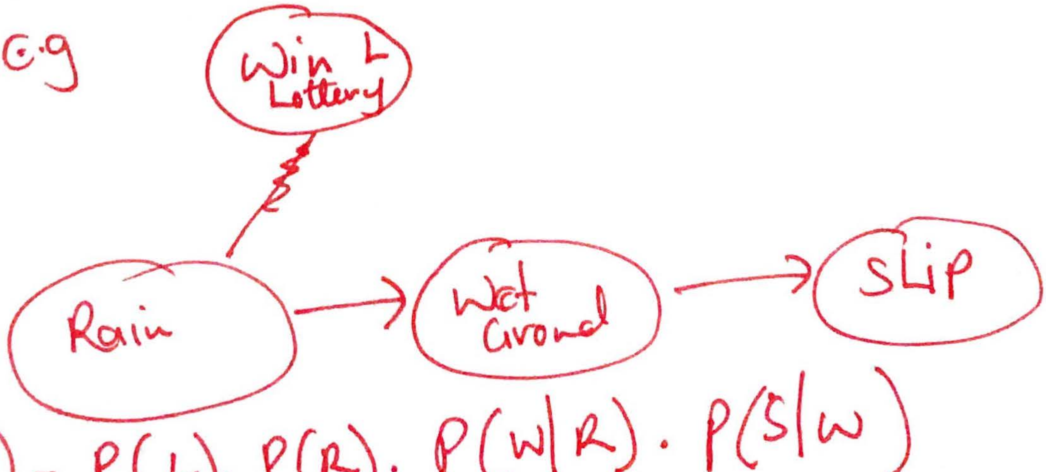
- Bayesian network will help represent the probabilities.

i.e $P(L, R, W) = P(L) \cdot P(R) \cdot P(W|R)$

NOTE Winning lottery is independent $\equiv$ ~~Does~~ NOT influenced

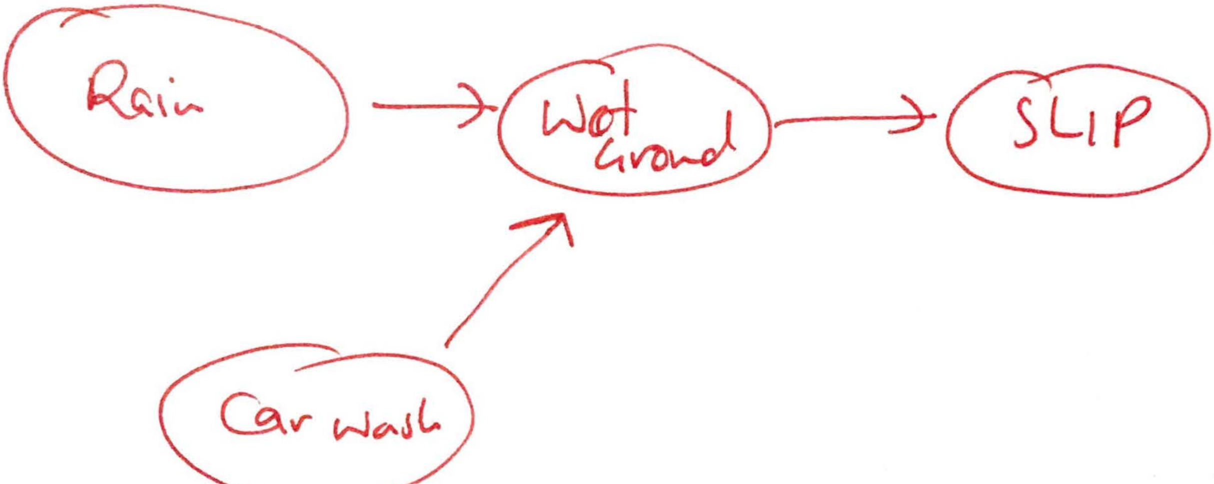
- **Note** more ~~dudes~~ variables can be added.

e.g



$P(L, R, W, S) = P(L) \cdot P(R) \cdot P(W|R) \cdot P(S|W)$

Example.



$$P(R,W,S,C) = P(R).\,P(C)\,\boxed{P(W|C,R)}\,P(S|W)$$

Wet ground is Influenced by Cash wash & Rain in this case

i.e if either $C=c$ or $R=r$ then $Wet = w$.

GENERAL RULE

$$P(X\,|\,Parents(X))$$

Inference in Bayesian Network

(1) Given a bayesia network

$P(X, Y, Z)$ what is $P(X)$

⇑ complete network with 3 variables

Question that we want to Reason about

Approach 1

Enumeration ≡ Exact method.

$$P(R, W, S, C) = P(R)\, P(C)\, P(W|C, R)\, P(S|W)$$

$$P(r|s) = \sum_{w} \sum_{c} P(r, w, s, c) / P(s)$$

Sum out over w, c ⇐ variables that are not part of the inquiry.

$$P(r|s) \propto \sum_{w} \sum_{c} P(r) \cdot P(c)\, P(w|c, r)\, P(s|w)$$

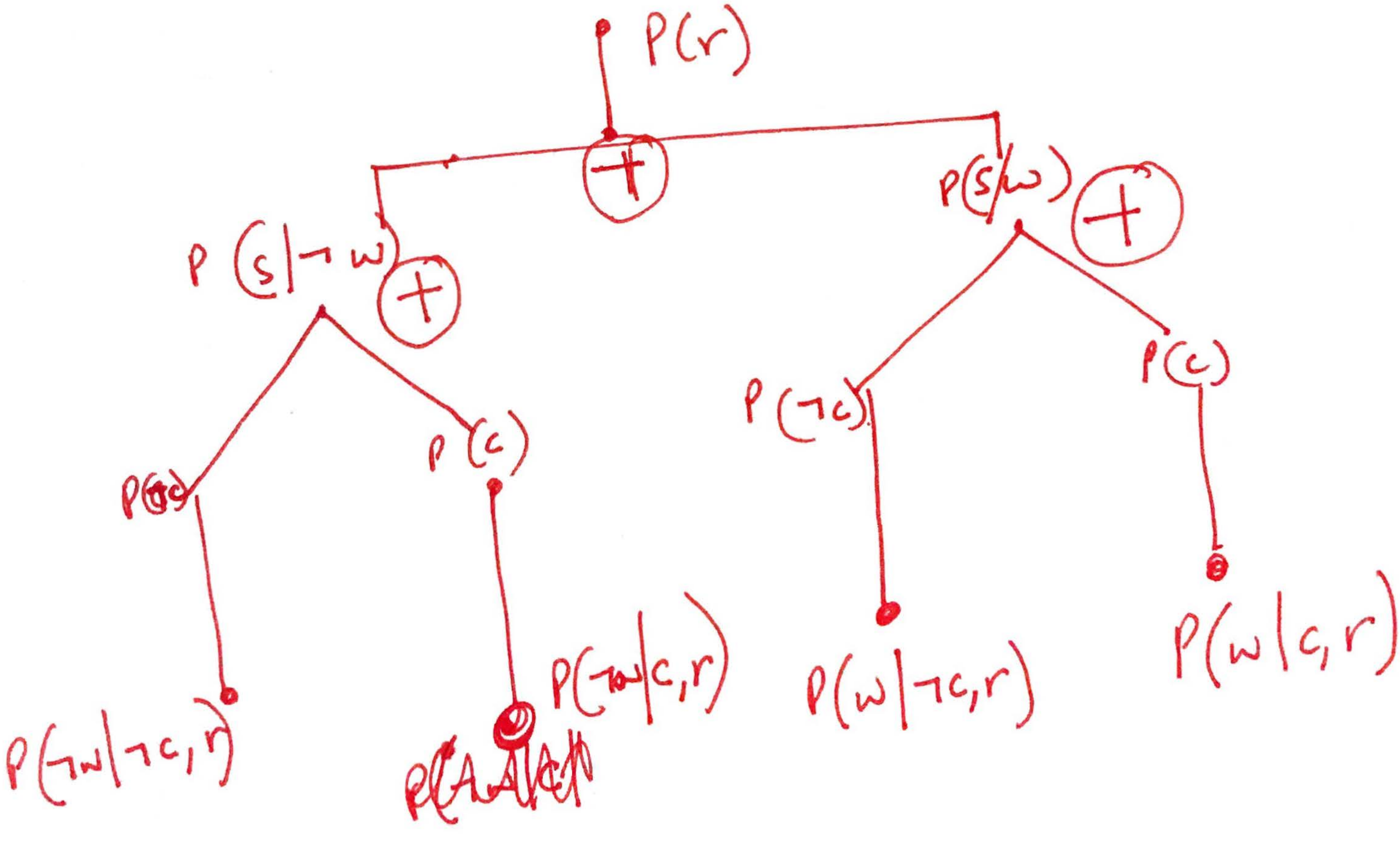
⟺ if we leave out the denominator; we shall have to normalize by looking at r & $\neg r$

$$P(R,W,S,C) = P(R)\,P(C)\,P(W|C,R)\,P(S|W)$$

$$P(r|s) = \sum_w \sum_c P(r,w,s,c) / P(s)$$

$$P(r|s) \propto \sum_w \sum_c P(r) \cdot P(c) \cdot P(w|c,r)\,P(s|w)$$

$$P(r|s) \propto P(r) \sum_w P(s|w) \cdot \sum_c P(c) \cdot P(w|c,r)$$



Variable Elimination

$$P(r|s) \propto \sum_w \sum_c P(r)\, P(c)\, P(w|c,r)\, P(s|w)$$

What terms depend on $\underline{\underline{c}}$: $P(c)\, P(w|c,r)$ recall in this case $\underline{r}$ is $\underline{\underline{\text{fixed}}}$

<u>Then</u>

$$f_c(w) = \sum_c P(c)\, P(w|c,r)$$

Thus: $P(r|s) \propto \sum_w P(r)\, P(s|w)\, f_c(w)$

Example

$$P(W,X,Y,Z) = P(W)\,P(X|W)\,P(Y|X)\,P(Z|Y)$$

$$P(Y) = ?$$

Start by summing out all unrelevant variables.

$$P(Y) = \sum_{W} \sum_{x} \sum_{Z} P(W)\,P(X|W)\,P(Y|X)\,P(Z|Y)$$

Note $P(Y)$ shall be a table of probabilities because of the many variables involved.

$$f_W(X) = \sum_{W} P(W)\,P(X|W)$$