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Exam2

Q1

$$\det(B^{-1}AB) = \det(B^{-1})\det(A)\det(B)$$

$$= \frac{1}{\det(B)} \times \det(A) \times \det(B)$$

$$= \det(A) = \boxed{-1}$$

Q2

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{vmatrix} = (1)\begin{vmatrix} 2 & 0 \\ 2 & 3 \end{vmatrix} = \boxed{6}$$

$$\det(A) = 6 \neq 0 \Rightarrow \boxed{T \text{ is an invertible transformation}}$$

$C_{11} = +\begin{vmatrix} 2 & 0 \\ 2 & 3 \end{vmatrix} = 6$	$C_{12} = -\begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} = -3$	$C_{13} = \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 0$
$C_{21} = -\begin{vmatrix} 0 & 0 \\ 2 & 3 \end{vmatrix} = 0$	$C_{22} = +\begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} = 3$	$C_{23} = -\begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix} = -2$
$C_{31} = +\begin{vmatrix} 0 & 0 \\ 2 & 0 \end{vmatrix} = 0$	$C_{32} = -\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} = 0$	$C_{33} = +\begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix} = 2$

$$\Rightarrow C = \begin{bmatrix} 6 & -3 & 0 \\ 0 & 3 & -2 \\ 0 & 0 & 2 \end{bmatrix} \Rightarrow adj(A) = C^T = \begin{bmatrix} 6 & 0 & 0 \\ -3 & 3 & 0 \\ 0 & -2 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A}\, adj(A) = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-1}{2} & \frac{1}{2} & 0 \\ 0 & -\frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

$$\boxed{\therefore T^{-1} = A^{-1}\vec{x} = \begin{bmatrix} x_1 \\ -\frac{1}{2}x_1 + \frac{1}{2}x_2 \\ -\frac{1}{3}x_2 + \frac{1}{3}x_3 \end{bmatrix}}$$

Q3

$$\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix}}_{\vec{b}}$$

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{vmatrix} = 1 \begin{vmatrix} 2 & 3 \\ 0 & 1 \end{vmatrix} = \boxed{2}$$

$$\det\left(A_1(\vec{b})\right) = \begin{vmatrix} 6 & 2 & 3 \\ 5 & 2 & 3 \\ 1 & 0 & 1 \end{vmatrix} = (1)\begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} + (1)\begin{vmatrix} 6 & 2 \\ 5 & 2 \end{vmatrix} = 0 + 2 = \boxed{2}$$

$$x_1 = \frac{\det\left(A_1(\vec{b})\right)}{\det(A)} = \frac{2}{2} = \boxed{1}$$

$$\det\left(A_2(\vec{b})\right) = \begin{vmatrix} 1 & 6 & 3 \\ 0 & 5 & 3 \\ 0 & 1 & 1 \end{vmatrix} = (1)\begin{vmatrix} 5 & 3 \\ 1 & 1 \end{vmatrix} = \boxed{2}$$

$$x_2 = \frac{\det\left(A_2(\vec{b})\right)}{\det(A)} = \frac{2}{2} = \boxed{1}$$

$$\det\left(A_3(\vec{b})\right) = \begin{vmatrix} 1 & 2 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 1 \end{vmatrix} = (1)\begin{vmatrix} 2 & 5 \\ 0 & 1 \end{vmatrix} = \boxed{2}$$

$$x_3 = \frac{\det\left(A_3(\vec{b})\right)}{\det(A)} = \frac{2}{2} = \boxed{1}$$

$$\Rightarrow \boxed{\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$$

$\boxed{Q4}$

$$A = \begin{bmatrix} x & 1 & 0 \\ 0 & x & 1 \\ 0 & 1 & x \end{bmatrix}$$

$$\det(A) = (x)\begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix} = x(x^2-1)$$

To let matrix A be invertible

$\Rightarrow \det(A) \neq 0 \iff x(x^2-1) \neq 0 \Rightarrow \therefore$ $\boxed{\begin{array}{l} x \neq 0 \\ \text{and } x \neq 1 \\ \text{and } x \neq -1 \end{array}}$

$\boxed{Q5}$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 4 & 5 \end{bmatrix},$$

$$\text{volume} = |\det(A)| = \begin{array}{cccccc} & & & 3 & 4 & 10 \\ \vert\; 1 & 2 & 1 & \vert\; 1 & 2 & \\ \vert\; 1 & 3 & 1 & \vert\; 1 & 3 & \\ \vert\; 1 & 4 & 5 & \vert\; 1 & 4 & \\ & & & 15 & 2 & 4 \end{array}$$

$$= \left|(15 + 2 + 4) - (3 + 4 + 10)\right| = \therefore \boxed{4}$$

cubic units