

8.1

A decomposition  $\{R_1, R_2\}$  is a lossless-join decomposition if

$$R_1 \cap R_2 \rightarrow R_1 \text{ or } R_1 \cap R_2 \rightarrow R_2$$

Let  $R_1 = (A, B, C),$

$$R_2 = (A, D, E)$$

and  $R_1 \cap R_2 = A.$

Since  $A$  is a candidate key, therefore  $R_1 \cap R_2 \rightarrow R_1$

8.6

Compute closure of the following set  $F$  for relation schema  $r(A, B, C, D, E)$

$$A \rightarrow BC$$

$$CD \rightarrow E$$

$$B \rightarrow D$$

$$E \rightarrow A$$

Start with  $A$

$$\{A\}^+ = \{A, B, C, D, E\} \leftarrow A \text{ a candidate key}$$

$$\{B\}^+ = \{B, D\}$$

$$\{C\}^+ = \{C\}$$

$$\{D\}^+ = \{D\}$$

$$\{E\}^+ = \{A, B, C, D, E\} \leftarrow E \text{ is a candidate key.}$$

$$\{BC\}^+ = \{B, C, D\} = \{B, C, D, E\} = \{A, B, C, D, E\} \leftarrow \text{candidate key.}$$

$$\{BD\}^+ = \{B, D\}$$

$$\{CD\}^+ = \{C, D, E\} = \{A, B, C, D, E\} \leftarrow \text{candidate key.}$$

We don't have to compute and FD at the form  $A^*, BC^*, CD^*, E^*$  because they will return  $\alpha$  and  $\alpha$  is any subset of  $\{A, B, C, D, E\}$   
 $\Rightarrow$  The candidate key is  $\{A, E, BC, CD\}$

8.20

8/11/20

Find canonical cover of  $R$

$$A \rightarrow B$$

$$A \rightarrow C$$

$$CD \rightarrow E$$

$$B \rightarrow D$$

$$E \rightarrow A$$

- + Assume  $A \rightarrow B$  doesn't exist,  $(A)^+ = \{A, C\}$ ,  $B$  is not in the attributes of  $A$ , so  $A \rightarrow B$  is not redundant.
- + Assume  $A \rightarrow C$  doesn't exist,  $(A)^+ = \{A, B, D\}$ ,  $C$  is not in the attributes of  $A$ , so  $A \rightarrow C$  is not redundant.
- + Assume  $CD \rightarrow E$  doesn't exist,  $(CD)^+ = \{C, D\}$ ,  $E$  is not in the attributes of  $(CD)$ , so  $CD \rightarrow E$  is not redundant.
- + Assume  $B \rightarrow D$  doesn't exist,  $(B)^+ = \{B\}$ ,  $D$  is not in the attributes of  $B$ , so  $B \rightarrow D$  is not redundant.
- + Assume  $E \rightarrow A$  doesn't exist,  $(E)^+ = \{E\}$ ,  $A$  is not in the attributes of  $E$ , so  $E \rightarrow A$  is not redundant.

$\Rightarrow$  Canonical cover  $R'$  is  $\left\{ \begin{array}{l} A \rightarrow B, \\ A \rightarrow C \\ CD \rightarrow E \\ B \rightarrow D \\ E \rightarrow A \end{array} \right\}$

$\Rightarrow R' = \{(A, B, C), (C, D, E), (B, D), (E, A)\}$

$\Rightarrow R'$  is 3NF (third normal form)

using Chase Test

	A	B	C	D	E
$r_1(A, B, C)$	a	b	c	$d_1$	$e_1$
$r_2(C, D, E)$	$a_2$	$b_2$	c	d	e

FD:  $A \rightarrow BC$   
 $CD \rightarrow E$   
 $B \rightarrow D$   
 $E \rightarrow A$

Based on the functional dependencies, we can't remove any subscript. So the decomposition above is not a lossless.

8.29

a)  $(B)^+ = \{D, A, B, C, E\}$

b) Prove using Armstrong's axioms that AF is a super key.

$A \rightarrow BCD$  (given)

$ABCDE \rightarrow ABCDE$  (Augmentation)

$A \rightarrow ABCDE$  (Transitivity)

$AF \rightarrow ABCDEF$  (Augmentation)

c) Find the canonical cover of F

$A \rightarrow B$

$A \rightarrow C$

$A \rightarrow D$

$BC \rightarrow D$

$BC \rightarrow E$

$B \rightarrow D$

$D \rightarrow A$

\*  $A \rightarrow B \Rightarrow (A)^+ = \{A, C, D\}$  ✓

\*  $A \rightarrow C \Rightarrow (A)^+ = \{A, B, D\}$  ✓

\*  $A \rightarrow D \Rightarrow (A)^+ = \{A, B, C, D, E\} \rightarrow$  is redundant.

\*  $BC \rightarrow D \Rightarrow (BC)^+ = \{B, C, E, D, A\} \rightarrow$  is redundant.

\*  $BC \rightarrow E \Rightarrow (BC)^+ = \{B, C, D, A\}$  ✓

$$B \rightarrow D \Rightarrow (B)^+ = \{B\} \quad \checkmark$$

$$D \rightarrow A \Rightarrow (D)^+ = \{D\} \quad \checkmark$$

$\Rightarrow$  The canonical cover is:  $\{$

$$\begin{array}{l} A \rightarrow B \\ A \rightarrow C \\ BC \rightarrow E \\ B \rightarrow D \\ D \rightarrow A \end{array} \}$$

We minimize the number of relations

$$\Rightarrow \begin{array}{l} A \rightarrow BC \\ BC \rightarrow E \\ B \rightarrow D \\ D \rightarrow A \end{array} \quad \left. \vphantom{\begin{array}{l} A \rightarrow BC \\ BC \rightarrow E \\ B \rightarrow D \\ D \rightarrow A \end{array}} \right\} \text{minim}$$

From the FD above, we see that  $(B)^+ = \{D, A, B, C, E\}$ . So from B, we can also determine E.

$\Rightarrow$   $BC \rightarrow E$  and  $B \rightarrow D$  we can make  $B \rightarrow DE$

$\Rightarrow$  The final canonical cover is:  $\{$

$$\begin{array}{l} A \rightarrow BC \\ B \rightarrow DE \\ D \rightarrow A \end{array} \}$$

d) Find the 3NF based on c).

From c, we can break the canonical cover into its own relation.

$$r_1(A, B, C)$$

$$r_2(B, D, E)$$

$$r_3(D, A)$$

Since the relations above don't have F attribute from the original relation schema. So, we need to add another relation with a super key.

$$r_4(A, F)$$

Find BCNF.

We know that  $AF$  is superkey of relation  $r$ . Also, based on the FD, we can have the decomposing.

$r_1(A, B, C, D)$

$r_2(A, E, F)$

However, from the original FD, we know that from  $A$ , we can get to  $F$  ( $A \rightarrow E$ ) without the need of  $F$ . So this will violate the BCNF. We decompose the  $r_2$  into  $(A, E)$  and  $(A, F)$ .

→ Finally, we have the BCNF as follow:

$r_1(A, B, C, D)$

$r_2(A, E)$

$r_3(A, F)$

f) From the canonical cover, we can't get the BCNF as from  $e$ . From  $d$ , we can get BCNF as  $\{r_1(A, B, C), r_2(B, D, E), r_3(D, A), r_4(A, F)\}$ . So this is different from  $e$ .

However, if we can infer the canonical cover back to the original FD, then based on  $e$ , we can decompose it to BCNF same as above.

