MATH.3860.202
Prob and Stats – Final Spring 18 **Name:** _____

There are eight (8) questions on this exam. Box in your answer to each question. Good luck.

See problem 4.2.7

1. (10 points) In a certain stream of electronic data, errors occur at an average rate of twice every 10 seconds. Use a Poisson model for the number of errors in 30 seconds to determine the probability that more than 3 errors will occur during the next half-minute?

Ch 3.4

2. (10 points) The lifetime (in hours) of a random lightbulb is a continuous random variable with pdf $f_Y(y)$, given by

$$f_Y(y) = \left(\frac{1}{3000}\right) e^{-y/3000}, \quad y > 0$$

   (a) The manufacturer of the bulb offers a money back guarantee on bulbs that fail to last at least 300 hours. For what proportion of of his sales will he need to make a refund.

   (b) Compute the cumulative distribution function $F_Y(y)$.

Problem 2.5.9

3. (10 points) A coin with $P(head) = 2/3$ is tossed six times (independently). What is the probability of obtaining exactly four consecutive heads or exactly five consecutive heads?

4. (10 points) Two experimental drugs are being offered for acne treatments. From rumors about the treatments, Francesca estimates that her chances of successful treatment for acne are 0.85 if she gets Drug A, and 0.60 if she gets Drug B. The treatment regime into which Francesca is put is determined by Nurse Cratchett, who assigns Treatment with Drug A at random 4 out of 10 times. Fifteen weeks later we learn that Francesca had successful acne treatment. What is the probability Francesca was enrolled in Treatment with Drug A?

5. (20 points) A population has the continuous pdf $f_Y(y) = 3(1-y)^2$ for $0 \le y \le 1$.

Ch. 3.5 (a) Find $E(Y)$.

Ch. 3.6 (b) Compute $\text{Var}(Y)$.

book p. 243 (c) A random sample of size $n = 15$ is drawn from this population. Let $\bar{Y} = (1/15)\sum_{i=1}^{15} Y_i$ be the sample mean. Use the Central Limit Theorem to approximate

$$P(1/8 \le \bar{Y} \le 3/8).$$

6. (15 points) A biased coin has a 70% chance of coming up heads.

   (a) Write the binomial formula for the exact probability that out of the next 100 flips, there will be between 75 and 80 heads, inclusive.

   (b) Approximate the probability above using the normal approximation to the binomial.

7. (10 points) Records for the past several years show that the amount of money collected daily by a prominent televangelist is normally distributed with a mean (μ) of \$20,000 and a standard deviation (σ) of \$5,000. What are the chances that tomorrow's donations will exceed \$30,000.

Problem 4.3.23

8. (15 points) Events A and B have respective probabilities $P(A) = \frac{1}{3}$ and $P(B) = \frac{2}{5}$, while $P(B|A) = \frac{3}{5}$.
   For each of the following, give an exact numerical answer as a **reduced fraction.**
   (a) What is $P(A \cap B)$?
   (b) What is $P(A \cup B)$?
   (c) What is $P(A - B)$?
   (d) What is $P(A|B)$?
   (e) What is $P(A|B^c)$?