1.(5 points) What is the language generated by the following CFG:

$$S \to AB$$
$$A \to aA \mid a$$
$$B \to bAB$$

2. For the following CFG:

$$S \to AB$$
$$A \to AA \mid a$$
$$B \to bA$$

(a) (5 points) Give a parse tree.

(b) (5 points) Show a leftmost derivation.

(c) (5 points) Show a rightmost derivation.

3.Give context-free grammars generating the following languages

(a) (5 points) The complement of the language $\{a^n b^n : n \geq 0\}$.

(b) (5 points) $\{x_1\#x_2\#\cdots\#x_k : k \geq 1$, each $x_i \in \{a,b\}^*$, and for some i and j, $x_i = x_j^R$.$\}$

4. Let

- $G = (V, \Sigma, R, S)$ be a grammar;
- $V = \{S, T, U\}$;
- $\Sigma = \{0,\#\}$; and
- R is the set of production rules:

$$S \to TT \mid U$$
$$T \to 0T \mid T0 \mid \#$$
$$U \to 0U00 \mid \#$$

(a) (10 points) Describe L(G) in plain language.

(b) (10 points) Is L(G) regular? Prove (yes/no).

5. (10 points) Convert the following CFG to Chomsky Normal Form:

$$A \to BAB \mid B \mid \varepsilon$$
$$B \to 00 \mid \varepsilon$$

6. Give "algorithmic" (i.e., steps) descriptions of Turing machines that decide the following languages over the alphabet $\{0,1\}$.

(a) (10 points) $\{w$: w contains twice as many 0s as 1s$\}$.

(b) (10 points) $\{w$: w does not contain twice as many 0s as 1s$\}$.

7. The Turing machine M has start state q s , state q 1 ; tape symbols 0, 1, and B; and the following transitions:

$\delta(q\ s,0) = (q\ s,0,R)$ $\delta(q\ s,1) = (q\ 1,1,L)$ $\delta(q\ s,B) = HALT$

$\delta(q\ 1,0) = (q\ 1,1,R)$ $\delta(q\ 1,1) = (q\ s,0,R)$ $\delta(q\ 1,B) = (q\ 1,B,L)$

M halts on the following input (2 points each):

000111	TRUE FALSE
10001	TRUE FALSE
0101	TRUE FALSE
0000	TRUE FALSE
B	TRUE FALSE

(2 points each) What is on the tape after M runs through the following input ?

(a) 0110011

(b) 001110

(c) 100100

(4 points) What does M do?

8. One of these languages is Turing-recognizable, the other is not. Which is which? For the recognizable one, describe briefly an algorithm that recognizes it. For the other one, explain.

(a) (5 points) {<M> : M is a TM that accepts 3 or more different inputs}.

(b) (5 points) {<M> : M is a TM that accepts 3 or fewer different inputs}.

9. (a) (12 points) Construct a Turing machine to do the following: The machine is started on the left of a tape that contains nothing but a string of $ signs. The machine is to divide the length of the string by three and leave on the tape result#remainder. No other symbol should be found on the tape at the end. In constructing the TM, show its formal description (states, transition function, etc.), and a few instantaneous descriptions (IDs) of its running.

(b) (12 points) What if we want to generalize to any arbitrary symbol (instead of $)? I.e., whenever any arbitrary symbol is repeated more than three times.

(c) (12 points) What if we want to generalize the division “by three” to any value?

10. Under what operations - if any - are the following closed? In each case justify.

(a) (5 points) P

(b) (5 points) NP

(c) (5 points) NPC

11. (10 points) Let $L_{\text{not-prime}}$ = {x : x is not a prime number }. Is $L_{\text{not-prime}}$ recursive, recursively-enumerable, or otherwise? Explain your answer.