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Instructor: Chuck Ormsby
Course: Multi-Variable and Vector Calculus -- **Assignment:** Section 13.6 Homework
Calculus III Spring 2018

1. The level curves of the surface $z = x^2 + y^2$ are circles in the xy-plane centered at the origin. Without computing the gradient, what is the direction of the gradient at (4,2) and (5, − 1) (determined up to a scalar multiple)?

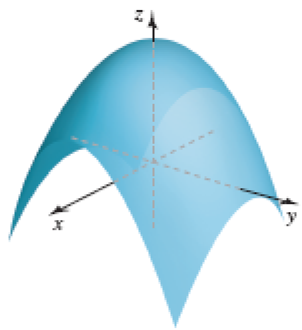
Determine the direction of the gradient at (4,2). Choose the correct answer below.

- [x] **A.** $\langle 4,2 \rangle$
- [ ] **B.** $\langle 2,-4 \rangle$
- [ ] **C.** $\langle -4,2 \rangle$
- [ ] **D.** $\langle 2,4 \rangle$
- [ ] **E.** $\langle 4,-2 \rangle$
- [ ] **F.** $\langle -2,4 \rangle$

Determine the direction of the gradient at (5, − 1). Choose the correct answer below.

- [ ] **A.** $\langle 5,1 \rangle$
- [ ] **B.** $\langle -1,5 \rangle$
- [ ] **C.** $\langle -5,-1 \rangle$
- [x] **D.** $\langle 5,-1 \rangle$
- [ ] **E.** $\langle -1,-5 \rangle$
- [ ] **F.** $\langle 1,5 \rangle$

2. Consider the function $f(x,y) = 1 - \frac{x^2}{4} - y^2$, whose graph is a paraboloid (see figure).



a. Find the value of the directional derivative at the point (1,1) in the direction $\langle \cos\theta, \sin\theta \rangle$ where $\theta = \frac{3\pi}{4}$.

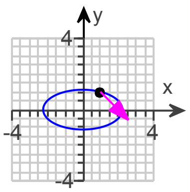
b. Sketch the level curve through the given point and indicate the direction of the directional derivative from part (a).

a. The directional derivative is $-3\frac{\sqrt{2}}{4}$.

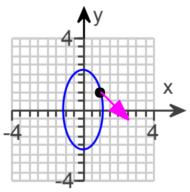
(Type an exact answer, using radicals as needed.)

b. Choose the correct sketch below.

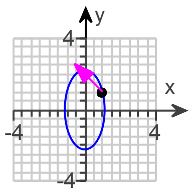
- [ ] **A.**



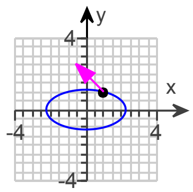
- [ ] **B.**



- [ ] **C.**



- [x] **D.**



3. First, compute the gradient of the function $f(x,y) = 2 + x^2 - y^2$. Then evaluate it at the point (3,3).

The gradient is $\nabla f(x,y) = \langle 2x, -2y \rangle$.

The gradient at (3,3) is $\langle 6, -6 \rangle$.

4. Compute the gradient of the following function and evaluate it at the given point P.

$g(x,y) = x^2 - 3x^2y - 9xy^2$; $P(-3,2)$

The gradient is $\nabla f(x,y) = \langle 2x - 6xy - 9y^2, -3x^2 - 18xy \rangle$.

The gradient at $(-3,2)$ is $\langle -6, 81 \rangle$.

5. First, compute the gradient of the following function. Then evaluate it at the given point P.

$F(x,y) = e^{-x^2 - y^2}$; $P(2,-1)$

The gradient is $\langle -2x\,e^{-x^2-y^2}, -2y\,e^{-x^2-y^2} \rangle$.

The gradient at $P(2,-1)$ is $\left\langle -\frac{4}{e^5}, \frac{2}{e^5} \right\rangle$.

6. Compute the directional derivative of the following function at the given point P in the direction of the given vector. Be sure to use a unit vector for the direction vector.

$f(x,y) = \sqrt{25 - x^2 - 5y}$; $P(5,-5)$; $\left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$

The directional derivative is $-\frac{2}{\sqrt{5}}$.

(Type an exact answer, using radicals as needed.)

7. Compute the directional derivative of the following function at the given point P in the direction of the given vector. Be sure to use a unit vector for the direction vector.

$f(x,y) = e^{-x-y}$; $P(\ln 3, \ln 4)$; $\langle 1,1 \rangle$

The directional derivative is $-\frac{1}{6\sqrt{2}}$.

(Type an exact answer, using radicals as needed.)

8. Compute the directional derivative of the following function at the given point P in the direction of the given vector. Be sure to use a unit vector for the direction vector.

$f(x,y) = \ln\left(6 + x^2 + 3y^2\right)$; $P(2,-2)$; $\langle 1,2 \rangle$

The directional derivative is $-\frac{10}{11\sqrt{5}}$.

(Type an exact answer, using radicals as needed.)

9. Consider the function $f(x,y) = x^4 - x^2y + 3y^2 + 9$ and the point $P(-1,1)$.
a. Find the unit vectors that give the direction of steepest ascent and steepest descent at P.
b. Find a vector that points in a direction of no change in the function at P.

a. What is the unit vector in the direction of steepest ascent at P?

$$\left\langle -\frac{2}{\sqrt{29}}, \frac{5}{\sqrt{29}} \right\rangle$$

(Type exact answers, using radicals as needed.)

What is the unit vector in the direction of steepest descent at P?

$$\left\langle \frac{2}{\sqrt{29}}, -\frac{5}{\sqrt{29}} \right\rangle$$

(Type exact answers, using radicals as needed.)

b. Which of the following vectors is in a direction of no change of the function at P?

- **A.** $\langle -5,-2 \rangle$
- **B.** $\langle -2,-5 \rangle$
- **C.** $\langle -5,2 \rangle$
- **D.** $\langle 2,5 \rangle$

10. Consider the function $F(x,y) = e^{-x^2/4 - y^2/4}$ and the point $P(-1,1)$.
a. Find the unit vectors that give the direction of steepest ascent and steepest descent at P.
b. Find a vector that points in a direction of no change in the function at P.

a. The direction of steepest ascent is $\left\langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right\rangle$.

The direction of steepest descent is $\left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$.

b. Which of the following vectors points in a direction of no change of the function at $P(-1,1)$?

- **A.** $\langle 1,-1 \rangle$
- **B.** $\langle 1,0 \rangle$
- **C.** $\langle -1,-1 \rangle$
- **D.** $\langle 0,1 \rangle$

11. Consider the function $f(x,y,z) = 1 + 3xyz$, the point $P(1,1,-1)$, and the unit vector $\mathbf{u} = \left\langle -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right\rangle$.

a. Compute the gradient of f and evaluate it at P.
b. Find the unit vector in the direction of maximum increase of f at P.
c. Find the rate of change of the function in the direction of maximum increase at P.
d. Find the directional derivative at P in the direction of the given vector.

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a. What is the gradient at the point $(1,1,-1)$?

$\langle -3, -3, 3 \rangle$

b. What is the unit vector in the direction of maximum increase?

$\left\langle -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$

(Type exact answers, using radicals as needed.)

c. What is the rate of change in the direction of maximum increase?

$3\sqrt{3}$ (Type an exact answer, using radicals as needed.)

d. What is the directional derivative in the direction of the given vector?

$\sqrt{3}$ (Type an exact answer, using radicals as needed.)

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12. Find the derivative of the function at P in the direction of the vector **A**.

$f(x,y,z) = xy + yz + zx$, $P(3,-3,1)$, $\langle 2,3,-6 \rangle$

$D_{\mathbf{u}}f(3,-3,1) = \frac{8}{7}$ (Simplify your answer.)