Name:

Linear Algebra 2: Exam 1 (Spring 2020)

Show ALL work, as unjustified answers may receive no credit. Calculators are not allowed on any quiz or test paper. *Make sure to exhibit skills discussed in class*. Box all answers and simplify answers as much as possible.

Good Luck! ☺

1. The Properties of Determinants (3.2)

[10pts] Let A and B be 4×4 matrices with $det(A) = -1$ and $det(B) = 2$. Find the following:

$$det(B^{-1}AB)$$

2. The Inverse of a Matrix (2.2) & Characteristics of Invertible Matrices (2.3)

[10pts] Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a Linear Transformation defined by:

$$T(\, x_1, x_2, x_3\,) = (\, x_1,\ x_1 + 2x_2,\ x_1 + 2x_2 + 3x_3\,)$$

Is T an invertible transformation? If it is, find a formula for T^{-1}.

3. Cramer's Rule, Volume, and Linear Transformations (3.3)

[10pts] Solve the linear system using *Cramer's Rule*:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 6 \\ 2x_2 + 3x_3 &= 5 \\ x_3 &= 1 \end{aligned}$$

4. Characteristics of Invertible Matrices (2.3)

[10pts] Use the *Invertible Matrix Theorem* to find the value(s) of x so that the matrix is invertible:

$$A = \begin{bmatrix} x & 1 & 0 \\ 0 & x & 1 \\ 0 & 1 & x \end{bmatrix}$$

5. Cramer's Rule, Volume, and Linear Transformations (3.3)

[10pts] Find the volume of the box formed by the triple of vectors in $\mathbb{R}^3$:

$$\vec{x} = (1,1,1)\,, \qquad \vec{y} = (2,3,4)\,, \qquad \vec{z} = (1,1,5)$$