Student: Phong Vo **Date:** 03/19/20	**Instructor:** Erica Yankowskas **Course:** Linear Algebra I (Spring 2020)	**Assignment:** Section 2.3 Homework

1. Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} 2 & 5 \\ -5 & -6 \end{bmatrix}$$

Choose the correct answer below.

- **A.** The matrix is invertible because its determinant is not zero.
- **B.** The matrix is not invertible because the matrix has 2 pivot positions.
- **C.** The matrix is invertible because its columns are multiples of each other. The columns of the matrix form a linearly dependent set.
- **D.** The matrix is not invertible because its determinant is zero.

2. Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} 16 & 4 \\ -12 & -3 \end{bmatrix}$$

Choose the correct answer below.

- **A.** The matrix is not invertible because the matrix has 2 pivot positions.
- **B.** The matrix is invertible because its determinant is not zero.
- **C.** The matrix is not invertible because its determinant is zero.
- **D.** The matrix is invertible because its columns are multiples of each other. The columns of the matrix form a linearly dependent set.

3. Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} 2 & 0 & 0 \\ -3 & -5 & 0 \\ 9 & 6 & -2 \end{bmatrix}$$

Choose the correct answer below.

- **A.** The matrix is invertible. If the given matrix is A, there is a 3×3 matrix C such that $CI = A$.
- **B.** The matrix is invertible. The given matrix has three pivot positions.
- **C.** The matrix is not invertible. The given matrix has two pivot positions.
- **D.** The matrix is not invertible. If the given matrix is A, the equation $A\mathbf{x} = \mathbf{b}$ has no solution for some $\mathbf{b}$ in $\mathbb{R}^3$.

4. Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} 4 & 0 & -4 \\ 3 & 0 & 5 \\ -4 & 0 & 9 \end{bmatrix}$$

Choose the correct answer below.

- **A.** The matrix is not invertible. If the given matrix is A, the equation A$\mathbf{x} = \mathbf{0}$ has only the trivial solution.
- **B.** The matrix is invertible. The given matrix has 2 pivot positions.
- **C.** The matrix is invertible. The columns of the given matrix span $\mathbb{R}^3$.
- **D.** The matrix is not invertible. If the given matrix is A, the columns of A do not form a linearly independent set.

5. Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} 1 & -3 & -6 \\ 0 & 4 & 3 \\ -2 & 3 & 0 \end{bmatrix}$$

Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

- **A.** The matrix is not invertible. In the given matrix the columns do not form a linearly independent set.
- **B.** The matrix is invertible. The given matrix has 3 pivot positions.
- **C.** The matrix is not invertible. If the given matrix is A, the equation A$\mathbf{x} = \mathbf{b}$ has no solution for at least one $\mathbf{b}$ in $\mathbb{R}^3$.
- **D.** The matrix is invertible. The given matrix is not row equivalent to the $n \times n$ identity matrix.

6. Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} -1 & -4 & 0 & 1 \\ 5 & 7 & 26 & -5 \\ -3 & -12 & 3 & 3 \\ 0 & -1 & 2 & 1 \end{bmatrix}$$

Choose the correct answer below.

- **A.** The matrix is invertible. The given matrix is not row equivalent to the $n \times n$ identity matrix.
- **B.** The matrix is invertible. The given matrix has 4 pivot positions.
- **C.** The matrix is not invertible. If the given matrix is A, the equation A$\mathbf{x} = \mathbf{b}$ has no solution for at least one $\mathbf{b}$ in $\mathbb{R}^4$.
- **D.** The matrix is not invertible. In the given matrix the columns do not form a linearly independent set.

7. Determine if the matrix below is invertible. Use as few calculations as possible. Justify your answer.

$$\begin{bmatrix} 3 & 5 & 7 & 5 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & 2 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Choose the correct answer below.

- A. The matrix is invertible. The given matrix has 4 pivot positions.
- B. The matrix is not invertible. In the given matrix the columns do not form a linearly independent set.
- C. The matrix is invertible. The given matrix is not row equivalent to the $n \times n$ identity matrix.
- D. The matrix is not invertible. If the given matrix is A, the equation $A\mathbf{x} = \mathbf{b}$ has no solution for at least one $\mathbf{b}$ in $\mathbb{R}^4$.

8. An $m \times n$ upper triangular matrix is one whose entries below the main diagonal are zeros, as is shown in the matrix to the right. When is a square upper triangular matrix invertible? Justify your answer.

$$\begin{bmatrix} 3 & 4 & 7 & 4 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & 2 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Choose the correct answer below.

- A. A square upper triangular matrix is invertible when the matrix is equal to its own transpose. For such a matrix A, $A = A^T$ means that the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^n$.
- B. A square upper triangular matrix is invertible when all entries above the main diagonal are zeros as well. This means that the matrix is row equivalent to the $n \times n$ identity matrix.
- C. A square upper triangular matrix is invertible when all entries on the main diagonal are ones. If any entry on the main diagonal is not one, then the equation $A\mathbf{x} = \mathbf{b}$, where A is an $n \times n$ square upper triangular matrix, has no solution for at least one $\mathbf{b}$ in $\mathbb{R}^n$.
- D. A square upper triangular matrix is invertible when all entries on its main diagonal are nonzero. If all of the entries on its main diagonal are nonzero, then the $n \times n$ matrix has n pivot positions.

9. Explain why the columns of A^2 span $\mathbb{R}^n$ whenever the columns of an $n \times n$ matrix A are linearly independent.

Choose the correct answer below. Note that the invertible matrix theorem is abbreviated IMT.

- A. If the columns of A are linearly independent, then it directly follows that the columns of A^2 span $\mathbb{R}^n$.
- B. If the columns of A are linearly independent and A is square, then A is invertible, by the IMT. Thus, A^2, which is the product of invertible matrices, is not invertible. So, the columns of A^2 span $\mathbb{R}^n$.
- C. If the columns of A are linearly independent and A is square, then A is not invertible. Thus, A^2, which is the product of non invertible matrices, is also not invertible. So, the columns of A^2 span $\mathbb{R}^n$.
- D. If the columns of A are linearly independent and A is square, then A is invertible, by the IMT. Thus, A^2, which is the product of invertible matrices, is also invertible. So, by the IMT, the columns of A^2 span $\mathbb{R}^n$.

10. Let A and B be $n \times n$ matrices. Show that if AB is invertible so is B.

Choose the correct answer below. Note that the invertible matrix theorem is abbreviated IMT.

- **A.** Since AB is invertible then by the IMT AB^T is an invertible matrix. Therefore, matrix B is invertible by part (l) of the IMT.
- **B.** Let W be the inverse of AB. Then $WAB = I$ and $(WA)B = I$. Therefore, matrix B is invertible by part (j) of the IMT.
- **C.** Since AB is invertible, then it directly follows that $A = B^{-1}$ and $B = A^{-1}$ by the IMT. Therefore, matrix B is invertible.
- **D.** Let W be the inverse of AB. Then $WAB = B$. Therefore, since B is the product of two invertible matrices, W and AB, matrix B is invertible.

11. The given T is a linear transformation from $\mathbb{R}^2$ into $\mathbb{R}^2$. Show that T is invertible and find a formula for T^{-1}.

$$T(x_1,x_2) = (5x_1 - 9x_2, -5x_1 + 7x_2)$$

To show that T is invertible, calculate the determinant of the standard matrix for T. The determinant of the standard matrix is

-10 .

(Simplify your answer.)

$$T^{-1}(x_1,x_2) = \left(-\frac{7}{10}x_1 - \frac{9}{10}x_2, -\frac{1}{2}x_1 - \frac{1}{2}x_2\right)$$

(Type an ordered pair. Type an expression using x_1 and x_2 as the variables.)