

Homework IV Solution

3.1) This exercise concerns TM M2, whose description and state diagram appear in Example 3.7. In each of the parts, give the sequence of configurations that M2 enters when started on the indicated input string.

a. 0

$q_1 0, \sqcup q_2 \sqcup, \sqcup \sqcup q_{accept}.$

c. 000

$q_1 00, \sqcup q_2 00, \sqcup x q_3 0, \sqcup x 0 q_4 \sqcup, \sqcup x 0 \sqcup q_{reject}.$

d. 000000

$q_1 000000, \sqcup q_2 000000, \sqcup x q_3 0000, \sqcup x 0 q_4 000, \sqcup x 0 x q_3 00, \sqcup x 0 x 0 q_4 0, \sqcup x 0 x 0 x q_3 \sqcup, \sqcup x 0 x 0 x q_3 \sqcup, \sqcup x 0 x 0 q_5 x \sqcup, \sqcup x 0 x q_5 0 x \sqcup, \sqcup x 0 q_5 x 0 x \sqcup, \sqcup x q_5 0 x 0 x \sqcup, \sqcup q_5 x 0 x 0 x \sqcup, q_5 \sqcup x 0 x 0 x \sqcup, \sqcup q_2 x 0 x 0 x \sqcup, \sqcup x q_2 0 x 0 x \sqcup, \sqcup x x q_3 x 0 x \sqcup, \sqcup x x x q_3 0 x \sqcup, \sqcup x x x 0 q_4 x \sqcup, \sqcup x x x 0 x q_4 \sqcup, \sqcup x x x 0 x \sqcup q_{reject}.$

plus. 00000000

$q_1 0000000, \sqcup q_2 0000000, \sqcup x q_3 000000, \sqcup x 0 q_4 00000, \sqcup x 0 x q_3 0000, \sqcup x 0 x 0 q_4 000, \sqcup x 0 x 0 x q_3 00, \sqcup x 0 x 0 x 0 q_4 0, \sqcup x 0 x 0 x 0 x q_3 \sqcup, \sqcup x 0 x 0 x 0 q_5 x \sqcup, \sqcup x 0 x 0 x q_5 0 x \sqcup, \sqcup x 0 x 0 q_5 x 0 x \sqcup, \sqcup x 0 x q_5 0 x 0 x \sqcup, \sqcup x 0 q_5 x 0 x 0 x \sqcup, \sqcup x q_5 0 x 0 x 0 x \sqcup, \sqcup q_5 x 0 x 0 x 0 x \sqcup, q_5 \sqcup x 0 x 0 x 0 x \sqcup, \sqcup q_2 x 0 x 0 x 0 x \sqcup, \sqcup x q_2 0 x 0 x 0 x \sqcup, \sqcup x x q_3 x 0 x 0 x \sqcup, \sqcup x x x q_3 0 x 0 x \sqcup, \sqcup x x x 0 q_4 x 0 x \sqcup, \sqcup x x x 0 x q_4 0 x \sqcup, \sqcup x x x 0 x x q_3 x \sqcup, \sqcup x x x 0 x x x q_3 \sqcup, \sqcup x x x 0 x x q_5 x \sqcup, \sqcup x x x 0 x q_5 x x \sqcup, \sqcup x x x 0 q_5 x x x \sqcup, \sqcup x x x q_5 0 x x x \sqcup, \sqcup x x q_5 x 0 x x x \sqcup, \sqcup x q_5 x x 0 x x x \sqcup, \sqcup q_5 x x x 0 x x x \sqcup, q_5 \sqcup x x x 0 x x x \sqcup, \sqcup q_2 x x x 0 x x x \sqcup, \sqcup x q_2 x x 0 x x x \sqcup, \sqcup x x q_2 x 0 x x x \sqcup, \sqcup x x x q_2 0 x x x \sqcup, \sqcup x x x x q_3 x x x \sqcup, \sqcup x x x x x q_3 x x \sqcup, \sqcup x x x x x x q_3 x \sqcup, \sqcup x x x x x x x q_3 \sqcup, \sqcup x x x x x x q_5 x \sqcup, \sqcup x x x x x q_5 x x \sqcup, \sqcup x x x x q_5 x x x \sqcup, \sqcup x x x q_5 x x x x \sqcup, \sqcup x q_5 x x x x x \sqcup, \sqcup q_5 x x x x x x \sqcup, q_5 \sqcup x x x x x x \sqcup, \sqcup q_2 x x x x x x \sqcup, \sqcup x q_2 x x x x x \sqcup, \sqcup x x q_2 x x x x x \sqcup, \sqcup x x x q_2 x x x x \sqcup, \sqcup x x x x q_2 x x x \sqcup, \sqcup x x x x x q_2 x x \sqcup, \sqcup x x x x x x q_2 x \sqcup, \sqcup x x x x x x x q_2 \sqcup, \sqcup x x x x x x \sqcup q_{accept}.$

3.2) This exercise concerns TM M1, whose description and state diagram appear in Example 3.9. In each of the parts, give the sequence of configurations that M1 enters when started on the indicated input string.

b. 1#1.

$q_1 1 \# 1 \rightarrow x q_3 \# 1 \rightarrow x \# q_5 1 \rightarrow x q_6 \# x \rightarrow q_7 x \# x \rightarrow x q_1 \# x \rightarrow x \# q_8 x \rightarrow x \# x q_8 \sqcup \rightarrow x \# x \sqcup q_{accept}.$

c. 1##1.

$q_1 1 \# \# 1 \rightarrow x q_3 \# \# 1 \rightarrow x \# q_5 \# 1 \rightarrow x \# \# q_{reject} 1.$

d. 10#11.

$q_1 10 \# 11 \rightarrow x q_3 0 \# 11 \rightarrow x 0 q_3 \# 11 \rightarrow x 0 \# q_5 11 \rightarrow x 0 q_6 \# x 1 \rightarrow x q_7 0 \# x 1 \rightarrow q_7 x 0 \# x 1 \rightarrow x q_1 0 \# x 1 \rightarrow x x q_2 \# x 1 \rightarrow x x \# q_4 x 1 \rightarrow x x \# x q_4 1 \rightarrow x x \# x 1 q_{reject} \sqcup.$

e. $10\#10$.

$q_110\#10 \rightarrow xq_30\#10 \rightarrow x0q_3\#10 \rightarrow x0\#q_510 \rightarrow x0q_6\#x0 \rightarrow xq_70\#x0 \rightarrow$
 $q_7x0\#x0 \rightarrow xq_10\#x0 \rightarrow xxq_2\#x0 \rightarrow xx\#q_4x0 \rightarrow xx\#xq_40 \rightarrow xx\#q_6xx \rightarrow$
 $xxq_6\#xx \rightarrow xq_7x\#xx \rightarrow xxq_1\#xx \rightarrow xx\#q_8xx \rightarrow xx\#xq_8x \rightarrow xx\#xxq_8\sqcup \rightarrow$
 $xx\#xx\sqcup q_{accept}$.

plus. $01100\#01100$.

$q_101100\#01100 \rightarrow xq_21100\#01100 \rightarrow x1q_2100\#01100 \rightarrow x11q_200\#01100 \rightarrow$
 $x110q_20\#01100 \rightarrow x1100q_2\#01100 \rightarrow x1100\#q_401100 \rightarrow x1100\#xq_61100 \rightarrow$
 $x1100\#q_6x1100 \rightarrow x1100q_6\#x1100 \rightarrow x110q_70\#x1100 \rightarrow x11q_700\#x1100 \rightarrow$
 $x1q_7100\#x1100 \rightarrow xq_71100\#x1100 \rightarrow q_7x1100\#x1100 \rightarrow xq_11100\#x1100 \rightarrow$
 $xxq_3100\#x1100 \rightarrow xx1q_300\#x1100 \rightarrow xx10q_30\#x1100 \rightarrow xx100q_3\#x1100 \rightarrow$
 $xx100\#q_5x1100 \rightarrow xx100\#xq_51100 \rightarrow xx100\#q_6xx100 \rightarrow xx100q_6\#xx100 \rightarrow$
 $xx10q_70\#xx100 \rightarrow xx1q_700\#xx100 \rightarrow xxq_7100\#xx100 \rightarrow xq_7x100\#xx100 \rightarrow$
 $xxq_1100\#xx100 \rightarrow xxxq_300\#xx100 \rightarrow xxx0q_30\#xx100 \rightarrow xxx00q_3\#xx100 \rightarrow$
 $xxx00\#q_5xx100 \rightarrow xxx00\#xq_5x100 \rightarrow xxx00\#xxq_5100 \rightarrow xxx00\#xq_6xx00 \rightarrow$
 $xxx00\#q_6xxx00 \rightarrow xxx00q_6\#xxx00 \rightarrow xxx0q_70\#xxx00 \rightarrow xxxq_700\#xxx00 \rightarrow$
 $xxq_7x00\#xxx00 \rightarrow xxxq_100\#xxx00 \rightarrow xxxxxq_20\#xxx00 \rightarrow xxxxx0q_2\#xxx00 \rightarrow$
 $xxxxx0\#q_4xxx00 \rightarrow xxxxx0\#xq_4xx00 \rightarrow xxxxx0\#xxq_4x00 \rightarrow xxxxx0\#xxxq_400 \rightarrow$
 $xxxxx0\#xxq_6xx0 \rightarrow xxxxx0\#xq_6xxx0 \rightarrow xxxxx0\#q_6xxxx0 \rightarrow xxxxx0q_6\#xxxx0 \rightarrow$
 $xxxxxq_70\#xxxx0 \rightarrow xxxq_7x0\#xxxx0 \rightarrow xxxxxq_10\#xxxx0 \rightarrow xxxxxq_2\#xxxx0 \rightarrow$
 $xxxxxx\#q_4xxxx0 \rightarrow xxxxx\#xq_4xxx0 \rightarrow xxxxx\#xxq_4xx0 \rightarrow xxxxx\#xxxq_4x0 \rightarrow$
 $xxxxxx\#xxxxq_40 \rightarrow xxxxx\#xxxq_6xx \rightarrow xxxxx\#xxq_6xxx \rightarrow xxxxx\#xq_6xxxx \rightarrow$
 $xxxxxx\#q_6xxxxx \rightarrow xxxxxq_6\#xxxxx \rightarrow xxxxxq_7x\#xxxxx \rightarrow xxxxxq_1\#xxxxx \rightarrow$
 $xxxxxx\#q_8xxxxx \rightarrow xxxxx\#xq_8xxxx \rightarrow xxxxx\#xxq_8xxx \rightarrow xxxxx\#xxxq_8xx \rightarrow$
 $xxxxxx\#xxxxq_8x \rightarrow xxxxx\#xxxxxxq_8\rightarrow xxxxx\#xxxxxx \sqcup q_{accept} \sqcup$.

plus. $01101\#01100$.

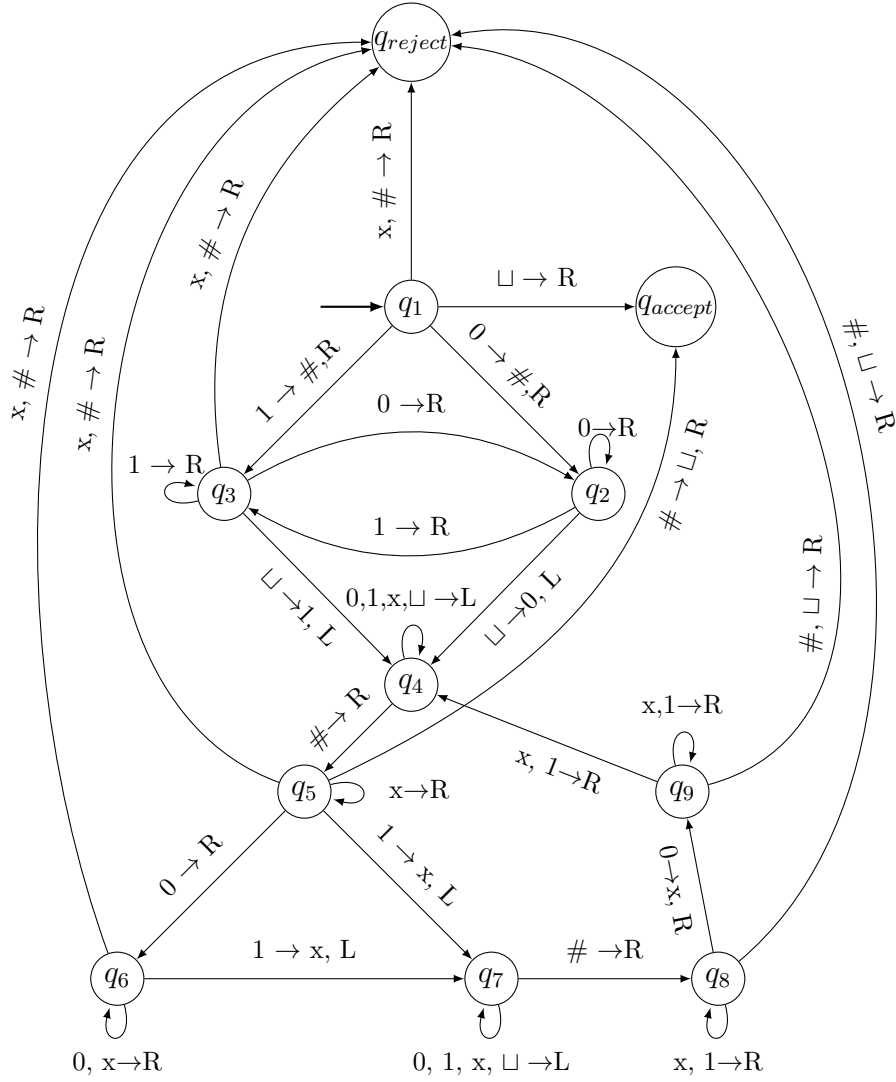
$q_101101\#01100 \rightarrow xq_21101\#01100 \rightarrow x1q_2101\#01100 \rightarrow x11q_201\#01100 \rightarrow$
 $x110q_21\#01100 \rightarrow x1101q_2\#01100 \rightarrow x1101\#q_401100 \rightarrow x1101\#xq_61100 \rightarrow$
 $x1101\#q_6x1100 \rightarrow x1101q_6\#x1100 \rightarrow x110q_71\#x1100 \rightarrow x11q_701\#x1100 \rightarrow$
 $x1q_7101\#x1100 \rightarrow xq_71101\#x1100 \rightarrow q_7x1101\#x1100 \rightarrow xq_11101\#x1100 \rightarrow$
 $xxq_3101\#x1100 \rightarrow xx1q_301\#x1100 \rightarrow xx10q_31\#x1100 \rightarrow xx101q_3\#x1100 \rightarrow$
 $xx101\#q_5x1100 \rightarrow xx101\#xq_51100 \rightarrow xx101\#q_6xx100 \rightarrow xx101q_6\#xx100 \rightarrow$
 $xx10q_71\#xx100 \rightarrow xx1q_701\#xx100 \rightarrow xxq_7101\#xx100 \rightarrow xq_7x101\#xx100 \rightarrow$
 $xxq_1101\#xx100 \rightarrow xxxq_301\#xx100 \rightarrow xxx0q_31\#xx100 \rightarrow xxx01q_3\#xx100 \rightarrow$
 $xxx01\#q_5xx100 \rightarrow xxx01\#xq_5x100 \rightarrow xxx01\#xxq_5100 \rightarrow xxx01\#xq_6xx00 \rightarrow$
 $xxx01\#q_6xxx00 \rightarrow xxx01q_6\#xxx00 \rightarrow xxx0q_71\#xxx00 \rightarrow xxxq_701\#xxx00 \rightarrow$
 $xxq_7x01\#xxx00 \rightarrow xxxq_101\#xxx00 \rightarrow xxxxxq_21\#xxx00 \rightarrow xxxxx1q_2\#xxx00 \rightarrow$
 $xxxxx1\#q_4xxx00 \rightarrow xxxxx1\#xq_4xx00 \rightarrow xxxxx1\#xxq_4x00 \rightarrow xxxxx1\#xxxq_400 \rightarrow$
 $xxxxx1\#xxq_6xx0 \rightarrow xxxxx1\#xq_6xxx0 \rightarrow xxxxx1\#q_6xxxx0 \rightarrow xxxxx1q_6\#xxxx0 \rightarrow$
 $xxxxxq_71\#xxxx0 \rightarrow xxxq_7x1\#xxxx0 \rightarrow xxxxxq_11\#xxxx0 \rightarrow xxxxxq_3\#xxxx0 \rightarrow$
 $xxxxxx\#q_5xxxx0 \rightarrow xxxxx\#xq_5xxx0 \rightarrow xxxxx\#xxq_5xx0 \rightarrow xxxxx\#xxxq_5x0 \rightarrow$
 $xxxxxx\#xxxxq_50 \rightarrow xxxxx\#xxxx0q_{reject} \sqcup$.

3.8) Give implementation-level descriptions of Turing machines that decide the following languages over the alphabet $\{0,1\}$ /

b. $\{w \mid w \text{ contains twice as many 0s as 1s}\}$

- 1.** Begin scanning the tape for any unmarked 0s. Mark the first zero that is found. If there are no zeroes that have been marked, move the head back to the front of the tape and scan the tape for any unmarked 1s. If none are found, move to the accept state. Otherwise, move to reject state.
- 2.** Continue scanning the tape for the next unmarked 0. Mark it once found. If none are found, move to the reject state. Else, move the head back to the front of the tape.
- 3.** Begin scanning the tape for any unmarked 1s. Mark it once found. If none exist, move to the reject state. Else, move on to the next step.
- 4.** Move the head back to the beginning of the tape and restart scanning the tape for any unmarked 0s. Mark the next 0 that is found. If there are no zeroes that have been marked, move the head back to the front of the tape and scan for any unmarked 1s. If none are found, move to the accept state, otherwise move to the reject state.

State diagrams:



plus. 010100

$q_1 010100 \rightarrow \#q_2 10100 \rightarrow \#0q_3 0100 \rightarrow \#01q_2 100 \rightarrow \#010q_3 00 \rightarrow \#0101q_2 0$
 $\rightarrow \#01010q_2 \sqcup \rightarrow \#0101q_4 00 \rightarrow \#010q_4 100 \rightarrow \#01q_4 0100 \rightarrow \#0q_4 10100$
 $\rightarrow \#q_4 010100 \rightarrow q_4 \#010100 \rightarrow \#q_5 010100 \rightarrow \#0q_6 10100 \rightarrow \#q_7 0x0100$
 $\rightarrow q_7 \#0x0100 \rightarrow \#q_8 0x0100 \rightarrow \#xq_9 0100 \rightarrow \#xxq_9 0100 \rightarrow \#xq_4 xx100$
 $\rightarrow \#q_4 xxx100 \rightarrow q_4 \#xxx100 \rightarrow \#q_5 xxx100 \rightarrow \#xq_5 xx100 \rightarrow \#xxq_5 x100$
 $\rightarrow \#xxxq_5 100 \rightarrow \#xxq_7 xx00 \rightarrow \#xq_7 xxx00 \rightarrow \#q_7 xxxx00 \rightarrow q_7 \#xxxx00$
 $\rightarrow \#q_8 xxxx00 \rightarrow \#xq_8 xxx00 \rightarrow \#xxq_8 xx00 \rightarrow \#xxxq_8 x00 \rightarrow \#xxxxq_8 00$
 $\rightarrow \#xxxxxq_9 0 \rightarrow \#xxxxq_4 xx \rightarrow \#xxxq_4 xxx \rightarrow \#xxq_4 xxxx \rightarrow \#xq_4 xxxxx$
 $\rightarrow \#q_4 xxxxxx \rightarrow q_4 \#xxxxxx \rightarrow \#q_5 xxxxxx \rightarrow \#xq_5 xxxxx \rightarrow \#xxq_5 xxxxx$
 $\rightarrow \#xxxq_5 xxx \rightarrow \#xxxxq_5 xx \rightarrow \#xxxxxq_5 x \rightarrow \#xxxxxxq_5 \sqcup \rightarrow$
 $\#xxxxxx \sqcup q_{accept}$

plus. 010101

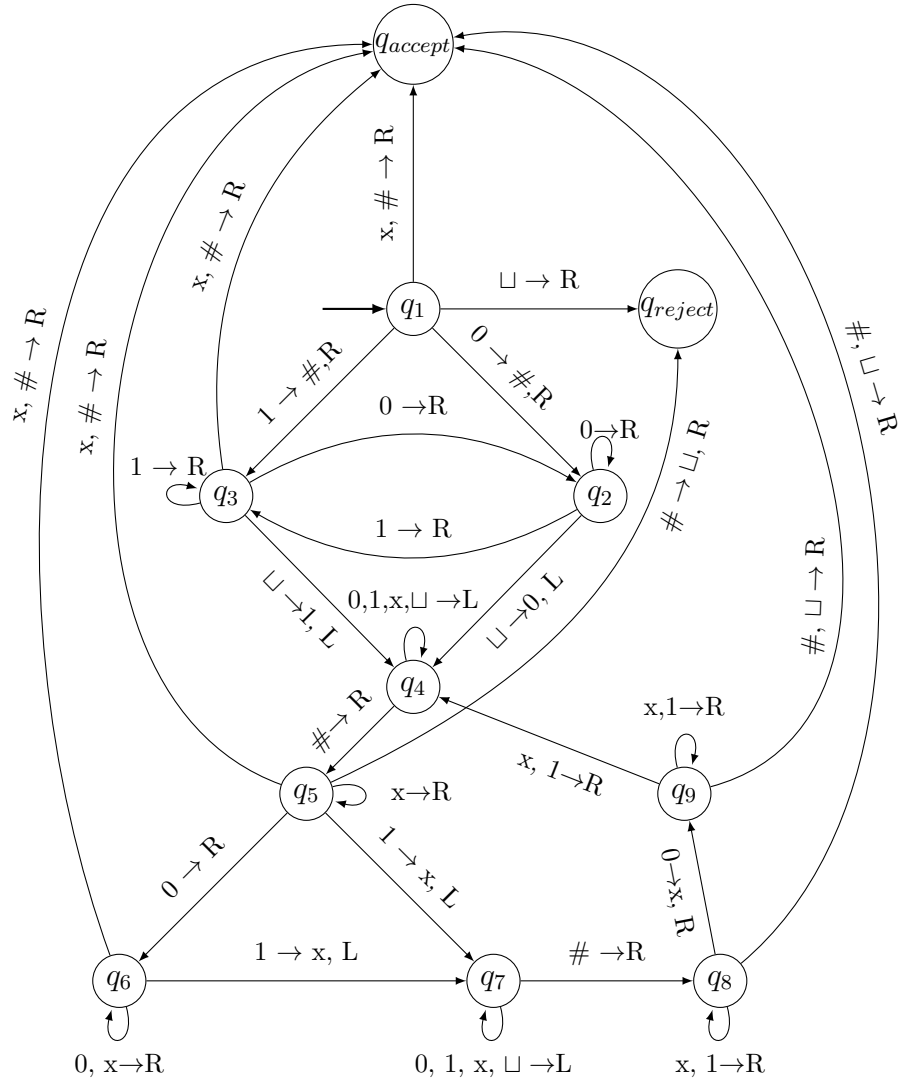
$q_1 010101 \rightarrow \#q_2 10101 \rightarrow \#0q_3 0101 \rightarrow \#01q_2 101 \rightarrow \#0101q_3 01 \rightarrow \#0101q_2 1$
 $\rightarrow \#01010q_3 \sqcup \rightarrow \#0101q_4 01 \rightarrow \#010q_4 101 \rightarrow \#01q_4 0101 \rightarrow \#0q_4 10101$

$\rightarrow \#q_4010101 \rightarrow q_4\#010101 \rightarrow \#q_5010101 \rightarrow \#0q_610101 \rightarrow \#q_70x0101$
 $\rightarrow q_7\#0x0101 \rightarrow \#q_80x0101 \rightarrow \#xq_9x0101 \rightarrow \#xxq_90101 \rightarrow \#xq_4xx101$
 $\rightarrow \#q_4xxx101 \rightarrow q_4\#xxx101 \rightarrow \#q_5xxx101 \rightarrow \#xq_5xx101 \rightarrow \#xxq_5x101$
 $\rightarrow \#xxxq_5101 \rightarrow \#xxq_7xx01 \rightarrow \#xq_7xxx01 \rightarrow \#q_7xxxx01 \rightarrow q_7\#xxxx01$
 $\rightarrow \#q_8xxxx01 \rightarrow \#xq_8xxx01 \rightarrow \#xxq_8xx01 \rightarrow \#xxxq_8x01 \rightarrow \#xxxxq_801$
 $\rightarrow \#xxxxxq_91 \rightarrow \#xxxxx1q_9\sqcup \rightarrow \#xxxxx1\sqcup q_{reject}$

c. $\{w \mid w \text{ does not contain twice as many 0s as 1s}\}$

1. Begin scanning the tape for any unmarked 0s. Mark the first zero that is found. If there are no unmarked zeroes, move the head back to the front of the tape and re scan for unmarked zeroes.
2. Mark the next unmarked zero. If there are none, move to the accept state. Else, move the head back to the front of the tape.
3. Scan the tape for 1s and mark the first one that is found. If there are no 1s that have been marked, move to the accept state.
4. Move the head to the start of the tape and scan again for any unmarked 0s. If there are none, move on to step 2.
5. Move the head to the start of the tape again to scan for any unmarked 1s. If there are none, move to the reject state. Else, move to the accept state.

State diagrams:



plus. 000111

$q_1 000111 \rightarrow \#q_2 00111 \rightarrow \#0q_2 00111 \rightarrow \#00q_2 0111 \rightarrow \#000q_2 111$
 $\rightarrow \#0000q_3 11 \rightarrow \#00001q_3 1 \rightarrow \#00001q_4 11 \rightarrow \#0000q_4 111 \rightarrow \#000q_4 0111$
 $\rightarrow \#00q_4 00111 \rightarrow \#0q_4 000111 \rightarrow \#q_4 0000111 \rightarrow q_4 \#0000111 \rightarrow$
 $\#q_5 0000111$
 $\rightarrow \#0q_6 000111 \rightarrow \#00q_6 00111 \rightarrow \#000q_6 0111 \rightarrow \#0000q_6 111 \rightarrow$
 $\#000q_7 0x11$
 $\rightarrow \#00q_7 00x11 \rightarrow \#0q_7 000x11 \rightarrow \#q_7 0000x11 \rightarrow q_7 \#0000x11 \rightarrow$
 $\#q_8 0000x11$
 $\rightarrow \#xq_9 000x11 \rightarrow \#q_4 xx00x11 \rightarrow q_4 \#xx00x11 \rightarrow \#q_5 xx00x11 \rightarrow$
 $\#xq_5 x00x11$
 $\rightarrow \#xxq_5 00x11 \rightarrow \#xx0q_6 0x11 \rightarrow \#xx00q_6 x11 \rightarrow \#xx00xq_6 11 \rightarrow$
 $\#xx00q_7 xx1$
 $\rightarrow \#xx0q_7 0xx1 \rightarrow \#xxq_7 00xx1 \rightarrow \#xq_7 x00xx1 \rightarrow \#q_7 xx00xx1 \rightarrow$
 $q_7 \#xx00xx1$

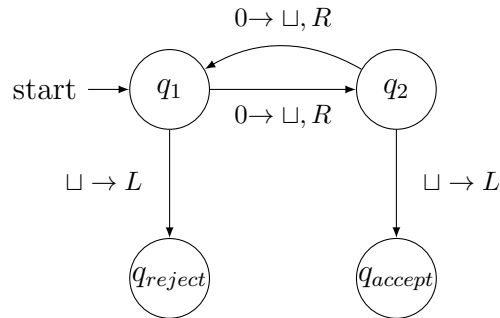
$\rightarrow \#q_8xx00xx1 \rightarrow \#xq_8x00xx1 \rightarrow \#xxq_800xx1 \rightarrow \#xxxq_90xx1 \rightarrow$
 $\#xxq_4xxxx1$
 $\rightarrow \#xq_4xxxxx1 \rightarrow \#q_4xxxxxx1 \rightarrow q_4\#xxxxxx1 \rightarrow \#q_5xxxxxx1 \rightarrow$
 $\#xq_5xxxxx1$
 $\rightarrow \#xxq_5xxxx1 \rightarrow \#xxxq_5xxx1 \rightarrow \#xxxxq_5xx1 \rightarrow \#xxxxxq_5x1 \rightarrow$
 $\#xxxxxxq_51$
 $\rightarrow \#xxxxxq_7xx \rightarrow \#xxxxq_7xxx \rightarrow \#xxxq_7xxxx \rightarrow \#xxq_7xxxxx \rightarrow$
 $\#xq_7xxxxxx$
 $\rightarrow \#q_7xxxxxxx \rightarrow q_7\#xxxxxxx \rightarrow \#q_8xxxxxxx \rightarrow \#xq_8xxxxxx \rightarrow$
 $\#xxq_8xxxxx$
 $\rightarrow \#xxxq_8xxxx \rightarrow \#xxxxq_8xxx \rightarrow \#xxxxxq_8xx \rightarrow \#xxxxxxq_8x \rightarrow$
 $\#xxxxxxxq_8\sqcup$
 $\rightarrow \#xxxxxxx\sqcup q_{accept}$

plus. 000110

$q_1000110 \rightarrow \#q_200110 \rightarrow \#0q_20110 \rightarrow \#00q_2110 \rightarrow \#000q_310 \rightarrow$
 $\#00011q_2\sqcup$
 $\rightarrow \#0001q_410 \rightarrow \#000q_4110 \rightarrow \#00q_40110 \rightarrow \#0q_400110 \rightarrow \#q_4000110$
 $\rightarrow q_4\#000110 \rightarrow \#q_5000110 \rightarrow \#0q_500110 \rightarrow \#00q_50110 \rightarrow \#000q_5110$
 $\rightarrow \#000q_5110 \rightarrow \#00q_70x10 \rightarrow \#0q_700x10 \rightarrow \#q_7000x10 \rightarrow q_7\#000x10$
 $\rightarrow \#q_8000x10 \rightarrow \#xq_900x10 \rightarrow \#q_4xx0x10 \rightarrow q_4\#xx0x10 \rightarrow \#q_5xx0x10$
 $\rightarrow \#xq_5x0x10 \rightarrow \#xxq_50x10 \rightarrow \#xx0q_6x10 \rightarrow \#xx0xq_610 \rightarrow \#xx0q_7xx0$
 $\rightarrow \#xxq_70xx0 \rightarrow \#xq_7x0xx0 \rightarrow \#q_7xx0xx0 \rightarrow q_7\#xx0xx0 \rightarrow \#q_8xx0xx0$
 $\rightarrow \#xq_8x0xx0 \rightarrow \#xxq_80xx0 \rightarrow \#xxxq_9xx0 \rightarrow \#xxxxq_9x0 \rightarrow \#xxxxxq_90$
 $\rightarrow \#xxxxq_4xx \rightarrow \#xxxq_4xxx \rightarrow \#xxq_4xxxx \rightarrow \#xq_4xxxxx \rightarrow \#q_4xxxxxx$
 $\rightarrow q_4\#xxxxxx \rightarrow \#q_5xxxxxx \rightarrow \#xq_5xxxxx \rightarrow \#xxq_5xxxx \rightarrow \#xxxq_5xxx$
 $\rightarrow \#xxxxq_5xx \rightarrow \#xxxxxq_5x \rightarrow \#xxxxxxq_5\sqcup \rightarrow \#xxxxxx\sqcup q_{reject}$

Modified M2:

b. To recognize odd numbers of 0s



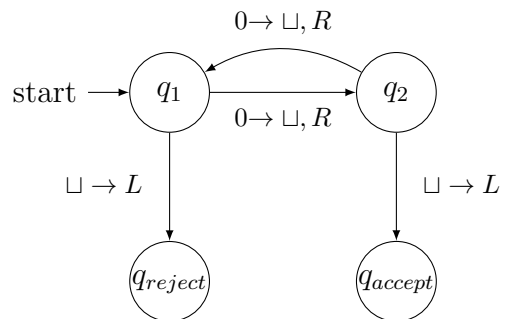
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$q_1000 \rightarrow \sqcup q_200 \rightarrow \sqcup\sqcup q_10 \rightarrow \sqcup\sqcup\sqcup q_2\sqcup \rightarrow \sqcup\sqcup\sqcup q_{accept}\sqcup\sqcup$

0000

$q_10000 \rightarrow \sqcup q_2000 \rightarrow \sqcup\sqcup q_100 \rightarrow \sqcup\sqcup\sqcup q_20 \rightarrow \sqcup\sqcup\sqcup\sqcup q_1\sqcup \rightarrow \sqcup\sqcup\sqcup q_{reject}\sqcup$

c. To recognize even numbers of 0s



000

$q_1 000 \rightarrow \sqcup q_2 00 \rightarrow \sqcup \sqcup q_1 0 \rightarrow \sqcup \sqcup \sqcup q_2 \sqcup \rightarrow \sqcup \sqcup q_{reject} \sqcup \sqcup$

0000

$q_1 0000 \rightarrow \sqcup q_2 000 \rightarrow \sqcup \sqcup q_1 00 \rightarrow \sqcup \sqcup \sqcup q_2 0 \rightarrow \sqcup \sqcup \sqcup \sqcup q_1 \sqcup \rightarrow \sqcup \sqcup \sqcup q_{accept} \sqcup$