Name:

Linear Algebra I: Exam 2 (Summer 2019)

<u>Show ALL work, as unjustified answers may receive no credit</u>. Calculators are not allowed on any quiz or test paper. *<u>Make sure to exhibit skills discussed in class</u>*. Box all answers and simplify answers as much as possible.

Good Luck! ☺

1. <u>The Inverse of a Matrix</u>

[8 pts] Use the *<u>Algorithm for Finding A^{-1}</u>* to find the inverse of the given matrix, if it exists:

$$A = \begin{bmatrix} 1 & 0 & -4 \\ 2 & 1 & 4 \\ -3 & -2 & 4 \end{bmatrix}$$

2. ***Characteristics of Invertible Matrices***

Define a Linear Transformation $\mathrm{T}: \mathbb{R}^2 \to \mathbb{R}^2$ *by* $\mathrm{T}\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - 3x_2 \\ 5x_1 + 2x_2 \end{bmatrix}$.

(a) [3 pts] Is T an invertible Linear Transformation? Explain.

(b) [2 pts] If T is invertible, find the formula for T^{-1}.

3. ***Matrix Factorizations***

[5 pts] Find the LU Factorization of the matrix A (with L unit lower triangular):

$$A = \begin{bmatrix} 6 & 9 \\ 18 & 25 \end{bmatrix}$$

4. ***Introduction to Determinants***

[8 pts] Compute the determinant by *Cofactor Expansion*. At each step, choose a row or column that involves the least amount of computation:

$$A = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & -1 & 1 & 1 & 0 \end{bmatrix}$$

5. ***The Properties of Determinants***

[8 pts] Find the determinant of the provided matrix. Specify whether the matrix has an inverse without trying to compute the inverse:

$$A = \begin{bmatrix} 2 & -2 & -2 & -2 \\ -2 & 2 & 3 & 0 \\ -2 & -2 & 2 & 0 \\ 1 & -1 & -3 & -1 \end{bmatrix}$$

6. ***Cramer's Rule, Volume, and Linear Transformations***

[8 pts] Find the inverse of the following matrix using the *Inverse Formula*:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$$

7. Cramer's Rule, Volume, and Linear Transformations

[8 pts] Find the volume of the box with one vertex at the origin and adjacent vertices $(1, 0, -2)$, $(1, 2, 4)$, $(7, 1, 0)$.

Bonus Question: ***<u>Cramer's Rule, Volume, and Linear Transformations</u>***

[5 pts] Solve the linear system using *<u>Cramer's Rule</u>*:

$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 2 \\ 3x_1 - 2x_2 + x_3 &= -1 \\ -5x_1 - 4x_2 + 2x_3 &= 3 \end{aligned}$$

Scratch Work (Not Graded)