Section 6.2 HW / Discrete Structures II / Fall 2018

Instructions: Justify your answers with clear explanations! For most questions, you should use the (generalized) pigeonhole principle to justify your answers. (See the "hints" for exceptions.) In particular, this means that you should explain what the "boxes" are, and what objects are being placed in the boxes.

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1. Suppose that there are 70 students in a class. Each student comes from one of the 50 states of the U.S. Show that there must be at least two students from the same state.

2. Suppose that seven integers are selected at random. Show that there are at least two with the same remainder when divided by 4.

3. Suppose there are 100 students in a class. Explain why there must be at least four students whose last name begins with the same letter.

4. A bucket contains 20 marbles, each of which are one the following colors: red, yellow, or blue.

   (a) Explain why there must be at least seven red marbles, at least seven yellow marbles, or at least seven blue marbles.

   (b) Does it have to be true that there are at least seven red marbles? If so, explain why. If not, give a specific counterexample.

   (c) Explain why there must be at least five red marbles, at least seven blue marbles, or at least ten yellow marbles.

5. A bucket contains 25 marbles of each of the following colors: red, yellow, green, and blue. A person starts picking marbles at random from the bucket. What's the minimum number of marbles that must be selected in order to ensure that ...

   (a) ... there are at least three marbles of the same color?

   (b) ... there are at least three red marbles?

   Note: For each part, be sure to explain why the minimum number works, and also why you can't pick a smaller number of marbles.

6. A stack of 40 cards consists of cards that are one color on one side and another color on the other side. The possible colors are red, blue, yellow, or green.

   (a) Explain why there are at least seven cards with the same color combination in the deck. (*Note:* The order of the colors on a card doesn't matter. For example, a red-blue card is the same combination as a blue-red card.)

   (b) Show why it's not necessarily true that there are eight cards with the same color combination.

1. Assume for the sake of contradiction that each state has at most 1 student comes from. Then the total of the students at most = 1*50 = 50 students. This total contradicts the truth that there are 70 students. [70/50] = 2
2. We have 4 boxes with remainder r = {0,1,2,3}. Numbers of integers = 7.
   [7/4] = 2
3. There are 26 letters available as the first letter of a last name. There are 100 different last names.
   PP = [100/26] = 4. If each letter has at most 3 last names start with, there are 3*26 = 78 lastnames.
   This contradicts the fact that has 100 lastnames.
4. (a)Numbers of 'boxes' = # of colors = 3. There are 20 marbles. Apply PP, we have [20/3] = at leaset 7 marbles with 1 out of 3 colors. Assume for the sake of contradiction that each color has at most 6 marbles colored with. Then, there be at most 6*3 = 18 marbles, this amount contradicts with the fact has 20 marbles.
   (b)
5. (a) In case of there are at most 2 marbles of the same color, there are at most 2(marbles)*4(colors) = 8 marbles. So, there are at least 8+1=9 marbles to be picked up to ensure that there are at least 3 marbles of the same color.
   (b) Assume for the sake all of 25 marbles are same colors (except red).
   So, there are 25 yellow marbles + 25 green marbles 6 + 25 blue marbles = 75.
   To ensure there are at least 3 red marbles, there are 75 + 3 = 78 marbles to be picked.

Hints: You don't have to use the generalized pigeonhole principle on problems 4(bc), 5(b), or 6(b). However, give a clear explanation of your reasoning.

Answers:
4(b) no
5(a) 9
5(b) 78