Differential Form **Maxwell's Equations** **Integral Form**

$$\vec{\nabla}\bullet\vec{E} = \rho/\varepsilon_0 \qquad \oiint_S \vec{E}\bullet\vec{n}\,dS = Q_{encl}/\varepsilon_0$$

$$\vec{\nabla}\bullet\vec{B} = 0 \qquad \oiint_S \vec{B}\bullet\vec{n}\,dS = 0$$

$$\vec{\nabla}\times\vec{E} = -\partial\vec{B}/\partial t \qquad \oint_C \vec{E}\bullet d\vec{r} = -\iint_S \frac{\partial\vec{B}}{\partial t}\bullet\vec{n}\,dS$$

$$\vec{\nabla}\times\vec{B} = \mu_0(\vec{J}+\varepsilon_0\frac{\partial\vec{E}}{\partial t}) \qquad \oint_C \vec{B}\bullet d\vec{r} = \mu_0\iint_S \{\vec{J}+\varepsilon_0\frac{\partial\vec{E}}{\partial t}\}\bullet\vec{n}\,dS$$

$$= \mu_0(I_{encl}+\varepsilon_0\iint_S \frac{\partial\vec{E}}{\partial t}\bullet\vec{n}\,dS)$$

LET THERE BE LIGHT (set charge density, ρ , and current density, $\vec{J}$, equal to zero)

Therefore, $\vec{\nabla}\bullet\vec{E} = 0$ and $\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$. Now take the curl of $\vec{\nabla}\times\vec{E}$ and $\vec{\nabla}\times\vec{B}$ yielding:

$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = -\frac{\partial(\vec{\nabla}\times\vec{B})}{\partial t}$ and $\vec{\nabla}\times\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial(\vec{\nabla}\times\vec{E})}{\partial t}$. But for any vector field, $\vec{F}$, we

have the identity: $\vec{\nabla}\times\vec{\nabla}\times\vec{F} = \vec{\nabla}(\vec{\nabla}\bullet\vec{F}) - \nabla^2\vec{F}$ $where$ $\nabla^2 = (\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2})$.

Therefore:

$$\vec{\nabla}(\vec{\nabla}\bullet\vec{E}) - \nabla^2\vec{E} = -\frac{\partial(\vec{\nabla}\times\vec{B})}{\partial t} \quad \text{and} \quad \vec{\nabla}(\vec{\nabla}\bullet\vec{B}) - \nabla^2\vec{B} = \mu_0\varepsilon_0\frac{\partial(\vec{\nabla}\times\vec{E})}{\partial t}$$

Setting $\vec{\nabla}\bullet\vec{E} = 0$ and $\vec{\nabla}\bullet\vec{B} = 0$, yields:

$\nabla^2\vec{E} = \frac{\partial(\vec{\nabla}\times\vec{B})}{\partial t}$ and $\nabla^2\vec{B} = -\mu_0\varepsilon_0\frac{\partial(\vec{\nabla}\times\vec{E})}{\partial t}$. Finally, substituting for $\vec{\nabla}\times\vec{B}$ and $\vec{\nabla}\times\vec{E}$ yields:

Classic Wave Equations ...

$$\nabla^2\vec{E} = \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} \quad \textbf{and} \quad \nabla^2\vec{B} = \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$$

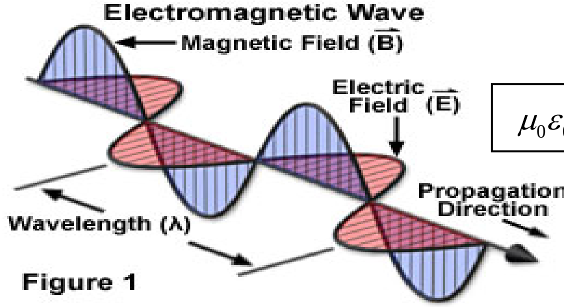

Figure 1

$$\boxed{\mu_0\varepsilon_0 = 1/c^2}$$