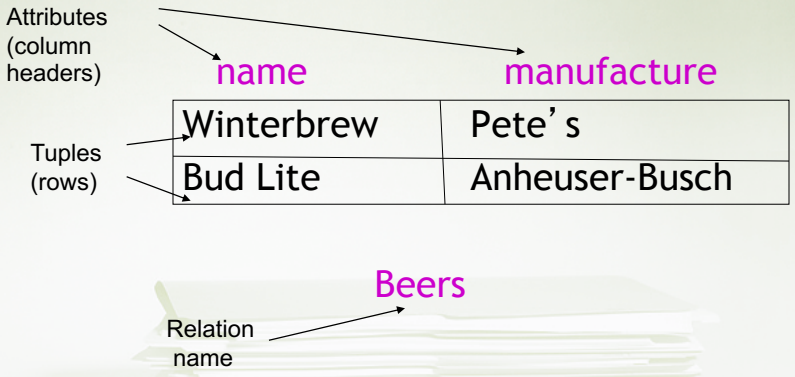
Relational Algebra

Slides adapted from http://infolab.stanford.edu/~ullman/fcdb.html

What is a Data Model?

1. Mathematical representation of data.
   - Examples: relational model = tables; semistructured model = trees/graphs.
2. Operations on data.
3. Constraints.

A Relation is a Table



3 of 42

Schemas

- *Relation schema* = relation name and attribute list.
  - Optionally: types of attributes.
  - Example: Beers(name, manf) or Beers(name: string, manf: string)
- *Database* = collection of relations.
- *Database schema* = set of all relation schemas in the database.

4 of 42

Why Relations?

- Very simple model.
- *Often* matches how we think about data.
- Abstract model that underlies SQL, the most important database language today.

A Running Example

Beers(name, manf)
Bars(name, addr, license)
Drinkers(name, addr, phone)
Likes(drinker, beer)
Sells(bar, beer, price)
Frequents(drinker, bar)

- Underline = *key* (tuples cannot have the same value in all key attributes).
  - Excellent example of a constraint.

Database Schemas in SQL

- SQL is primarily a query language, for getting information from a database.
- But SQL also includes a *data-definition* component for describing database schemas.

Creating (Declaring) a Relation

- Simplest form is:

```
CREATE TABLE <name> (
        <list of elements>
);
```

- To delete a relation:

```
DROP TABLE <name>;
```

Elements of Table Declarations

- Most basic element: an attribute and its type.
- The most common types are:
  - INT or INTEGER (synonyms).
  - REAL or FLOAT (synonyms).
  - CHAR(*n*) = fixed-length string of *n* characters.
  - VARCHAR(*n*) = variable-length string of up to *n* characters.

Example: Create Table

```
CREATE TABLE Sells (
    bar     CHAR(20),
    beer    VARCHAR(20),
    price   REAL
);
```

SQL Values

- Integers and reals are represented as you would expect.
- Strings are too, except they require single quotes.
  - Two single quotes = real quote, e.g., `'Joe''s Bar'`.
- Any value can be NULL.

11of 42

Dates and Times

- DATE and TIME are types in SQL.
- The form of a date value is:

  DATE 'yyyy-mm-dd'
  - Example: `DATE '2019-09-10'` for Sept. 10, 2019.

12of 42

Times as Values

- The form of a time value is:

  TIME 'hh:mm:ss'

with an optional decimal point and fractions of a second following.

  - Example: `TIME '11:00:02.5'` = two and a half seconds after 11:00AM.

13of 42

Declaring Keys

- An attribute or list of attributes may be declared PRIMARY KEY or UNIQUE.
- Either says that no two tuples of the relation may agree in all the attribute(s) on the list.
  1. There can be only one PRIMARY KEY for a relation, but several UNIQUE attributes.
  2. No attribute of a PRIMARY KEY can ever be NULL in any tuple. But attributes declared UNIQUE may have NULL's, and there may be several tuples with NULL.

14of 42

Declaring Single-Attribute Keys

- Place PRIMARY KEY after the type in the declaration of the attribute.
- Example:

```
CREATE TABLE Beers (
    name CHAR(20) PRIMARY KEY,
    manf CHAR(20)
);
```


Declaring Multiattribute Keys

- A key declaration can also be another element in the list of elements of a CREATE TABLE statement.
- This form is essential if the key consists of more than one attribute.
  - May be used even for one-attribute keys.

Example: Multiattribute Key

- The bar and beer together are the key for Sells:

```
CREATE TABLE Sells (
    bar     CHAR(20),
    beer    VARCHAR(20),
    price   REAL,
    PRIMARY KEY (bar, beer)
);
```

17of 42

What is an "Algebra"

- Mathematical system consisting of:
  - *Operands* - variables or values from which new values can be constructed.
  - *Operators* - symbols denoting procedures that construct new values from given values.

18of 42

What is Relational Algebra?

- An algebra whose operands are relations or variables that represent relations.
- Operators are designed to do the most common things that we need to do with relations in a database.
  - The result is an algebra that can be used as a *query language* for relations.

Core Relational Algebra

- Union, intersection, and difference.
  - Usual set operations, but *both operands must have the same relation schema*.
- Selection: picking certain rows.
- Projection: picking certain columns.
- Products and joins: compositions of relations.
- Renaming of relations and attributes.

Selection

- R1 := σ_C (R2)
  - *C* is a condition (as in "if" statements) that refers to attributes of R2.
  - R1 is all those tuples of R2 that satisfy *C*.

Example: Selection

Relation Sells:

bar	beer	price
Joe's	Bud	2.50
Joe's	Miller	2.75
Sue's	Bud	2.50
Sue's	Miller	3.00

JoeMenu := $\sigma_{bar="Joe's"}$(Sells):

bar	beer	price
Joe's	Bud	2.50
Joe's	Miller	2.75

Projection

to select rows / tuples / records

- R1 := Π_L (R2)
  - *L* is a list of attributes from the schema of R2.
  - R1 is constructed by looking at each tuple of R2, extracting the attributes on list *L*, in the order specified, and creating from those components a tuple for R1.
  - Eliminate duplicate tuples, if any.

Example: Projection

to select column(s) / attributes

Relation Sells:

bar	beer	price
Joe's	Bud	2.50
Joe's	Miller	2.75
Sue's	Bud	2.50
Sue's	Miller	3.00

Prices := $\Pi_{beer,price}$(Sells):

beer	price
Bud	2.50
Miller	2.75
Miller	3.00

Product

- R3 := R1 X R2
  - Pair each tuple t1 of R1 with each tuple t2 of R2.
  - Concatenation t1t2 is a tuple of R3.
  - Schema of R3 is the attributes of R1 and then R2, in order.
  - But beware attribute *B* of the same name in R1 and R2: use R1.*B* and R2.*B*.

Example: R3 := R1 X R2

R1(

A,	B)
1	2
3	4

2 tuples

R2(

B,	C)
5	6
7	8
9	10

3 tuples

2 x 3 = 6 tuples

R3(

A,	R1.B,	R2.B,	C)
1	2	5	6
1	2	7	8
1	2	9	10
3	4	5	6
3	4	7	8
3	4	9	10

Theta-Join

- R3 := R1 $\bowtie_C$ R2
  - Take the product R1 X R2.
  - Then apply σ_C to the result.
- As for σ, *C* can be any boolean-valued condition.
  - Historic versions of this operator allowed only A θ B, where θ is =, <, etc.; hence the name "theta-join."

$\bowtie_C$: apply X, then apply condition σ_C

27of 42

Example: Theta Join

Sells(

bar,	beer,	price
Joe's	Bud	2.50
Joe's	Miller	2.75
Sue's	Bud	2.50
Sue's	Coors	3.00

)

Bars(

name,	addr
Joe's	Maple St.
Sue's	River Rd.

)

BarInfo := Sells $\bowtie_{\text{Sells.bar = Bars.name}}$ Bars

BarInfo(

bar,	beer,	price,	name,	addr
Joe's	Bud	2.50	Joe's	Maple St.
Joe's	Miller	2.75	Joe's	Maple St.
Sue's	Bud	2.50	Sue's	River Rd.
Sue's	Coors	3.00	Sue's	River Rd.

)

28 of 42

Joe

Natural Join

- A useful join variant (*natural* join) connects two relations by:
  - Equating attributes of the same name, and
  - Projecting out one copy of each pair of equated attributes.
- Denoted R3 := R1 ⋈ R2.

Example: Natural Join

Sells(

bar,	beer,	price
Joe's	Bud	2.50
Joe's	Miller	2.75
Sue's	Bud	2.50
Sue's	Coors	3.00

)

Bars(

bar,	addr
Joe's	Maple St.
Sue's	River Rd.

)

BarInfo := Sells ⋈ Bars

Note: *Bars.name* has become *Bars.bar* to make the natural join "work."

BarInfo(

bar,	beer,	price,	addr
Joe's	Bud	2.50	Maple St.
Joe's	Milller	2.75	Maple St.
Sue's	Bud	2.50	River Rd.
Sue's	Coors	3.00	River Rd.

)

Renaming

- The ρ operator gives a new schema to a relation.
- R1 := $\rho_{R1(A1,\ldots,An)}$(R2) makes R1 be a relation with attributes A1,…,A*n* and the same tuples as R2.
- Simplified notation: R1(A1,…,A*n*) := R2.

take attributes $A_1, A_2, \ldots A_n$ of R_1 to make a relation R_2

31of 42

Example: Renaming

Bars(

name,	addr
Joe's	Maple St.
Sue's	River Rd.

)

R(bar, addr) := Bars

to take attributes 'bar' and 'addr' of R to create relation 'Bars'

R(

bar,	addr
Joe's	Maple St.
Sue's	River Rd.

)

32 of 42

Building Complex Expressions

- Combine operators with parentheses and precedence rules.
- Three notations, just as in arithmetic:
  1. Sequences of assignment statements.
  2. Expressions with several operators.
  3. Expression trees.

33of 42

Sequences of Assignments

- Create temporary relation names.
- Renaming can be implied by giving relations a list of attributes.
- Example: R3 := R1 $\bowtie_C$ R2 can be written:

  R4 := R1 X R2

  R3 := σ_C (R4)

34of 42

Expressions in a Single Assignment

- Example: the theta-join R3 := R1 $\bowtie_C$ R2 can be written: R3 := σ_C (R1 X R2)
- Precedence of relational operators:
  1. [σ, π, ρ] (highest)
  2. [x, $\bowtie$]
  3. $\cap$
  4. [$\cup$, $-$]

35of 42

Expression Trees

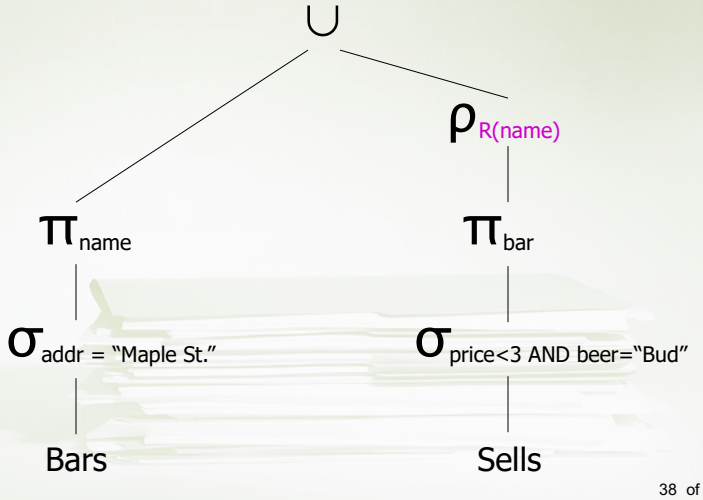
- Leaves are operands – either variables standing for relations or particular, constant relations.
- Interior nodes are operators, applied to their child or children.

36of 42

Example: Tree for a Query

- Using the relations Bars(name, addr) and Sells(bar, beer, price), find the names of all the bars that are either on Maple St. or sell Bud for less than $3.

As a Tree:

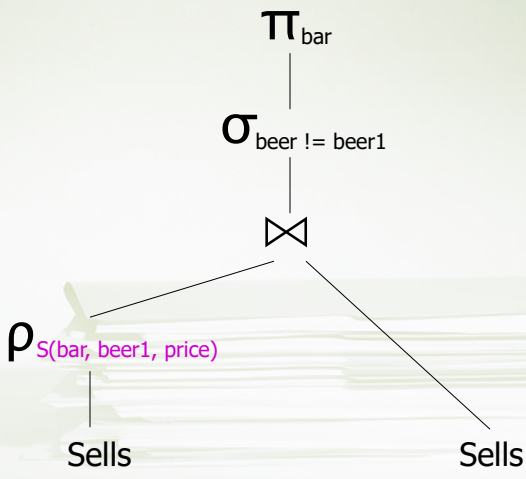


Example: Self-Join

- Using Sells(bar, beer, price), find the bars that sell two different beers at the same price.
- Strategy: by renaming, define a copy of Sells, called S(bar, beer1, price). The natural join of Sells and S consists of quadruples (bar, beer, beer1, price) such that the bar sells both beers at this price.

39of 42

The Tree



40 of 42

Schemas for Results

- Union, intersection, and difference: the schemas of the two operands must be the same, so use that schema for the result.
- Selection: schema of the result is the same as the schema of the operand.
- Projection: list of attributes tells us the schema.

Schemas for Results --- (2)

- Product: schema is the attributes of both relations.
  - Use R1.*A* and *R2.A*, etc., to distinguish two attributes named *A*.
- Theta-join: same as product.
- Natural join: union of the attributes of the two relations.
- Renaming: the operator tells the schema.