

Q₁

$$\det \begin{vmatrix} x^2 & x & 2 \\ 2 & 1 & 1 \\ 0 & 0 & -5 \end{vmatrix} = (-5) \begin{vmatrix} x^2 & x \\ 2 & 1 \end{vmatrix} = (-5)(x^2 - 2x)$$

$$\det A = 0 \Leftrightarrow (-5)(x^2 - 2x) = 0$$

$$\Leftrightarrow x(x-2) = 0$$

$$\Leftrightarrow \begin{cases} x = 0 \\ x = 2 \end{cases}$$

$$x = 0 : \det \begin{vmatrix} 0 & 0 & 2 \\ 2 & 1 & 1 \\ 0 & 0 & -5 \end{vmatrix} = -5 \begin{vmatrix} 0 & 0 \\ 2 & 1 \end{vmatrix} = 0 \quad \checkmark$$

$$x = 2 : \det \begin{vmatrix} 4 & 2 & 2 \\ 2 & 1 & 1 \\ 0 & 0 & -5 \end{vmatrix} = (-5) \begin{vmatrix} 4 & 2 \\ 2 & 1 \end{vmatrix} = 0 \quad \checkmark$$

• • $x = 0 \text{ OR } x = 2$

Q₂

$$(a) \det(3A) = 3^3 \det(A) = 27 \times 10 = \boxed{270}$$

$$(b) \det(2A^{-1}) = 2^3 \det(A^{-1}) = 2^3 \times \frac{1}{\det A} = 8 \times \frac{1}{10} = \boxed{\frac{4}{5}}$$

$$\begin{aligned}
 & \text{(c)} \quad \det \begin{vmatrix} a & g & d \\ b & h & e \\ c & i & f \end{vmatrix} = \begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix} = - \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \boxed{-10} \\
 & \qquad \underbrace{\hspace{1.5cm}}_A \qquad \underbrace{\hspace{1.5cm}}_{A^T}
 \end{aligned}$$