Student: Phong Vo **Date:** 03/26/20	**Instructor:** Erica Yankowskas **Course:** Linear Algebra I (Spring 2020)	**Assignment:** Section 3.1 Homework

1. Compute the determinant using a cofactor expansion across the first row. Also compute the determinant by a cofactor expansion down the second column.

$$\begin{vmatrix} 3 & 0 & 4 \\ 3 & 3 & 3 \\ 0 & 5 & -2 \end{vmatrix}$$

Compute the determinant using a cofactor expansion across the first row. Select the correct choice below and fill in the answer box to complete your choice.

(Simplify your answer.)

- **A.** Using this expansion, the determinant is $-(3)(-21)+(0)(-6)-(4)(15)=$ _____.
- **B.** Using this expansion, the determinant is $-(0)(-6)+(3)(-6)-(5)(-3)=$ _____.
- **C.** Using this expansion, the determinant is $(0)(-6)-(3)(-6)+(5)(-3)=$ _____.
- **D.** Using this expansion, the determinant is $(3)(-21)-(0)(-6)+(4)(15)=$ -3.

Compute the determinant using a cofactor expansion down the second column. Select the correct choice below and fill in the answer box to complete your choice.

(Simplify your answer.)

- **A.** Using this expansion, the determinant is $-(3)(-21)+(0)(-6)-(4)(15)=$ _____.
- **B.** Using this expansion, the determinant is $(3)(-21)-(0)(-6)+(4)(15)=$ _____.
- **C.** Using this expansion, the determinant is $(0)(-6)-(3)(-6)+(5)(-3)=$ _____.
- **D.** Using this expansion, the determinant is $-(0)(-6)+(3)(-6)-(5)(-3)=$ -3.

2. Compute the determinant using a cofactor expansion across the first row. Also compute the determinant by a cofactor expansion down the second column.

$$\begin{vmatrix} 2 & -4 & 3 \\ 2 & 1 & 3 \\ 1 & 4 & -2 \end{vmatrix}$$

Compute the determinant using a cofactor expansion across the first row. Select the correct choice below and fill in the answer box to complete your choice.

(Simplify your answer.)

- **A.** Using this expansion, the determinant is $(2)(-14)-(-4)(-7)+(3)(7)=$ -35 .
- **B.** Using this expansion, the determinant is $(-4)(-7)-(1)(-7)+(4)(0)=$ ____ .
- **C.** Using this expansion, the determinant is $-(-4)(-7)+(1)(-7)-(4)(0)=$ ____ .
- **D.** Using this expansion, the determinant is $-(2)(-14)+(-4)(-7)-(3)(7)=$ ____ .

Compute the determinant using a cofactor expansion down the second column. Select the correct choice below and fill in the answer box to complete your choice.

(Simplify your answer.)

- **A.** Using this expansion, the determinant is $-(2)(-14)+(-4)(-7)-(3)(7)=$ ____ .
- **B.** Using this expansion, the determinant is $(2)(-14)-(-4)(-7)+(3)(7)=$ ____ .
- **C.** Using this expansion, the determinant is $(-4)(-7)-(1)(-7)+(4)(0)=$ ____ .
- **D.** Using this expansion, the determinant is $-(-4)(-7)+(1)(-7)-(4)(0)=$ -35 .

3. Compute the determinant of the following matrix using a cofactor expansion across the first row.

$$A = \begin{bmatrix} 3 & 7 & -3 \\ 5 & 0 & 6 \\ 4 & 5 & 3 \end{bmatrix}$$

Compute the determinant using a cofactor expansion across the first row. Select the correct choice below and fill in the answer box to complete your choice.

(Simplify your answer.)

- **A.** Using this expansion, the determinant is $(3)(-30)+(5)(-9)+(4)(25)=$ ____ .
- **B.** Using this expansion, the determinant is $(3)(-30)-(5)(-9)+(4)(25)=$ ____ .
- **C.** Using this expansion, the determinant is $(3)(-30)-(7)(-9)+(-3)(25)=$ -102 .
- **D.** Using this expansion, the determinant is $(3)(-30)+(7)(-9)+(-3)(25)=$ ____ .

4. Compute the determinant using a cofactor expansion down the first column.

$$A = \begin{bmatrix} 7 & -5 & 2 \\ 5 & 1 & 3 \\ 0 & 4 & -2 \end{bmatrix}$$

Determine the value of the first term in the cofactor expansion. Substitute the value for a_{11} and complete the matrix for C_{11} below.

$$a_{11}C_{11} = (\;7\;)\det\begin{bmatrix} 1 & 3 \\ 4 & -2 \end{bmatrix}$$

Determine the value of the second term in the cofactor expansion. Substitute the value for a_{21} and complete the matrix for C_{21} below.

$$a_{21}C_{21} = -(\;5\;)\det\begin{bmatrix} -5 & 2 \\ 4 & -2 \end{bmatrix}$$

Determine the value of the third term in the cofactor expansion. Substitute the value for a_{31} and complete the matrix for C_{31} below.

$$a_{31}C_{31} = (\;0\;)\det\begin{bmatrix} -5 & 2 \\ 1 & 3 \end{bmatrix}$$

Complete the cofactor expansion to compute the determinant.

det A = -108

5. Compute the determinant by cofactor expansion. At each step, choose a row or column that involves the least amount of computation.

$$\begin{vmatrix} 5 & 0 & 0 & 5 \\ 4 & 8 & 3 & -2 \\ 2 & 0 & 0 & 0 \\ 9 & 2 & 1 & 4 \end{vmatrix}$$

$$\begin{vmatrix} 5 & 0 & 0 & 5 \\ 4 & 8 & 3 & -2 \\ 2 & 0 & 0 & 0 \\ 9 & 2 & 1 & 4 \end{vmatrix} = 20$$ (Simplify your answer.)

6. Compute the following determinant by cofactor expansions. At each step, choose the row or column that involves the least amount of computation.

$$\begin{vmatrix} 1 & -2 & 7 & 4 \\ 0 & 0 & 2 & 0 \\ 5 & -4 & -5 & 3 \\ 4 & 0 & 4 & 2 \end{vmatrix}$$

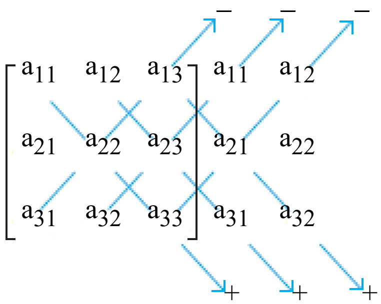
The determinant is -104 .

7. Compute the determinant by cofactor expansion. At each step, choose a row or column that involves the least amount of computation.

$$\begin{vmatrix} 2 & 3 & 3 & 4 & 0 \\ 4 & 0 & -4 & 1 & 0 \\ 6 & -3 & 7 & 4 & 1 \\ 4 & 0 & 0 & 0 & 0 \\ 6 & 3 & 4 & 2 & 0 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 3 & 3 & 4 & 0 \\ 4 & 0 & -4 & 1 & 0 \\ 6 & -3 & 7 & 4 & 1 \\ 4 & 0 & 0 & 0 & 0 \\ 6 & 3 & 4 & 2 & 0 \end{vmatrix} = 84$$ (Simplify your answer.)

8. The expansion of a 3×3 determinant can be remembered by this device. Write a second copy of the first two columns to the right of the matrix, and compute the determinant by multiplying entries on six diagonals. Add the downward diagonal products and subtract the upward products. Use this method to compute the following determinant.



$$\begin{vmatrix} 4 & -2 & -3 \\ 0 & 5 & -3 \\ -4 & -5 & 0 \end{vmatrix}$$

$$\begin{vmatrix} 4 & -2 & -3 \\ 0 & 5 & -3 \\ -4 & -5 & 0 \end{vmatrix} = -144$$

9. State the row operation performed below and describe how it affects the determinant.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}, \begin{bmatrix} a & b \\ 2c & 2d \end{bmatrix}$$

What row operation was performed?

- **A.** The row operation subtracts 2 from row 2.
- **B.** The row operation scales row 2 by 2.
- **C.** The row operation adds 2 to row 2.
- **D.** The row operation scales row 2 by one-half.

How does this affect the determinant?

- **A.** The determinant is 0.
- **B.** The determinant is divided by 2.
- **C.** The determinant is multiplied by 2.
- **D.** The determinant is unchanged.

10. Explore the effects of an elementary row operation on the determinant of a matrix. State the row operation and describe how it affects the determinant.

$$\begin{bmatrix} 8 & 5 \\ 5 & 4 \end{bmatrix}, \begin{bmatrix} 8 & 5 \\ 5+8k & 4+5k \end{bmatrix}$$

What is the elementary row operation?

- A. Replace row 2 with k times row 2.
- B. Replace row 2 with k times row 1 plus row 2.
- C. Replace row 2 with row 1 plus k times row 2.
- D. Replace row 2 with k times row 1.

How does the row operation affect the determinant?

- A. The determinant is increased by 80k.
- B. The determinant is decreased by 40k.
- C. The determinant is increased by 40k.
- D. The determinant does not change.

11. Explore the effects of an elementary row operation on the determinant of a matrix. State the row operation and describe how it affects the determinant.

$$\begin{bmatrix} -4 & 7 & -4 \\ 4 & -3 & 5 \\ 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} -4 & 7 & -4 \\ 4 & -3 & 5 \\ k & k & k \end{bmatrix}$$

What is the elementary row operation?

- A. Replace row 3 with k plus row 3.
- B. Replace row 3 with k times row 3.
- C. Replace row 3 with row 3 divided by k.
- D. Replace row 3 with row 3 minus k.

How does the row operation affect the determinant?

- A. The determinant is multiplied by k.
- B. The determinant is increased by 3k.
- C. The determinant is decreased by 3k.
- D. The determinant does not change.

12. Compute the determinant of the following elementary matrix.

$$\begin{bmatrix} 1 & -x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1 & -x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$ (Simplify your answer.)

13. Compute the determinant of the following elementary matrix.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -1$$ (Simplify your answer.)

14. Let A be an $n \times n$ matrix. Mark each statement True or False. Justify each answer.

a. The cofactor expansion of det A down a column is the negative of the cofactor expansion along a row.
b. The determinant of a triangular matrix is the sum of the entries on the main diagonal.

a. Choose the correct answer below.

- **A.** False, because the determinant of A can be computed by cofactor expansion across any row or down any column. Since the determinant of A is well defined, both of these cofactor expansions will be equal.
- **B.** True, because cofactor expansion across a row adds each of the cofactors together. Cofactor expansion down a column subtracts each cofactor from one another. This causes the two cofactor expansions to have opposite signs.
- **C.** False, because the determinant of A can only be calculated by cofactor expansion across a row. Cofactor expansion down a column has no relation to the determinant.
- **D.** True, because the plus or minus sign of the (i,j)-cofactor depends on the position of a_{ij} in matrix A. Cofactor expansion down a column switches the order of i and j, thereby switching the sign of the cofactor expansion across a row.

b. Choose the correct answer below.

- **A.** True, because cofactor expansion along the row (or column) with the most zeros of a triangular matrix produces a determinant equal to the sum of the entries along the main diagonal.
- **B.** True, because the determinant of A is the following finite series.

  $$\det A = \sum_{j=1}^{n} (-1)^{1+j} a_{1j} \det A_{1j}$$

  In a triangular matrix, this series simplifies to the sum of the entries along the main diagonal.
- **C.** False, because the determinant of a matrix is the arithmetic mean of the entries along the main diagonal.
- **D.** False, because the determinant of a triangular matrix is the product of the entries along the main diagonal.