Calculus III   Quiz #1   Fall 2017   NAME: Dangnhi Ngoc Ngo

10

1. (4 Pts) Given the points $P(-3,1,2)$, $Q(4,-1,-2)$ and $R(1,-2,3)$, determine the following:

$\vec{u} = \overrightarrow{PQ} = \langle 4-(-3), -1-1, -2-2 \rangle$
$= \langle 7, -2, -4 \rangle$ 1

$\vec{v} = \overrightarrow{PR} = \langle 1-(-3), -2-1, 3-2 \rangle = \langle 4, -3, 1 \rangle$ 1

$\vec{u} \cdot \vec{v} = 7 \cdot 4 + (-2) \cdot (-3) + (-4) \cdot 1 = 28 + 6 - 4 = 30$ 2

2. (2 Pts) Determine the magnitude of the vector $\vec{v} = \langle -2, 4, 6 \rangle$.

$|\vec{v}| = \sqrt{(-2)^2 + 4^2 + 6^2} = \sqrt{4+16+36} = \sqrt{56}$ 2

SEE PROBLEM #3 ON REVERSE

3. (4 Pts) If $\vec{u} = <2, 1, 3>$ and $\vec{v} = <-1, 2, -2>$ compute the scalar and vector projections of $\vec{u}$ in the direction of $\vec{v}$.

$$\mathbf{scal}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|} = \frac{2.(-1)+1.2+3.(-2)}{\sqrt{(-1)^2+2^2+(-2)^2}} = \frac{-2+2-6}{\sqrt{1+4+4}} = \frac{-6}{3} = -2$$

2

$$\mathbf{proj}_{\vec{v}}\vec{u} = \text{scal}_{\vec{v}}\,\vec{u}\,.\,\frac{\vec{v}}{|\vec{v}|}$$

$$= -2\cdot\left(\frac{1}{3}\right).<-1,2,-2>$$

$$= -\frac{2}{3}<-1,2,-2> = \left<\frac{2}{3}, \frac{-4}{3}, \frac{4}{3}\right>$$

2

(12) Very Good

Calculus III Quiz #2 Fall 2017 NAME: Dangnhi Ngoc Ngo

1. (4 Pts) Find a **unit** vector perpendicular to the vectors $\vec{u} = <-1,3,-2>$ **and** $\vec{v} = <1,2,-2>$.

$$\vec{w} = \vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ -1 & 3 & -2 \\ 1 & 2 & -2 \end{vmatrix} = \langle (-2).3 - 2.(-2), 1.(-2) - (-1).(-2), -1.2 - 1.3 \rangle$$

$$= \langle -6+4, -2-2, -2-3 \rangle$$

$$= \langle -2, -4, -5 \rangle$$

$\vec{w}$ is perpendicular to $\vec{u}$ and $\vec{v}$

$$|\vec{w}| = \sqrt{(-2)^2 + (-4)^2 + (-5)^2} = \sqrt{4+16+25} = \sqrt{45} = 3\sqrt{5}$$

Unit vector $\vec{w}$:

$$\frac{\vec{w}}{|\vec{w}|} = \frac{\langle -2,-4,-5 \rangle}{3\sqrt{5}} = \pm \left\langle \frac{-2}{3\sqrt{5}}, \frac{-4}{3\sqrt{5}}, \frac{-5}{3\sqrt{5}} \right\rangle$$

(4)

2. (4 Pts) Find the vector-valued function $\overrightarrow{r(t)}$ that parameterizes the line segment (not the infinite line) starting at the point P(1,-1,3) at time zero and ending at Q(3,-4,2) at time 3. Your parameterization should have a constant speed.

$$x(t) = 1 + (3-1)t = 1 + 2t$$
$$y(t) = -1 + (-4+1)t = -1 - 3t$$
$$z(t) = 3 + (2-3)t = 3 - t$$

Time starts from 0 to 3, therefore:

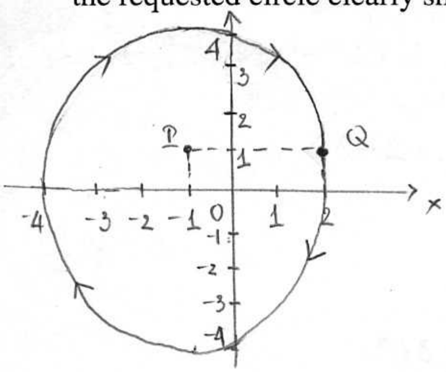
$$x(t) = 1 + \frac{2}{3}t$$
$$y(t) = -1 - t$$
$$z(t) = 3 - \frac{1}{3}t$$

$$\vec{r}(t) = \left\langle 1 + \frac{2}{3}t, -1 - t, 3 - \frac{1}{3}t \right\rangle, \text{ for } 0 \leqslant t \leqslant 3$$

(4)

SEE PROBLEM #3 ON REVERSE

3. (4 Pts) Parameterize a circle centered at P(-1,1) having radius 3. Have your parameterization start at Q(2,1) at time zero and travel clockwise taking 5 units of time to complete a full circle. Include a drawing of the requested circle clearly showing the starting position and direction of travel.



4

$0 \leqslant t \leqslant 5$ $\quad r = 3$

$0 \leqslant bt \leqslant -2\pi$

when $t = 5$, $5b = -2\pi \Rightarrow b = \frac{-2\pi}{5}$ ($b<0$ when traveling clockwise)

$x(t) = -1 + 3\cos\left(\frac{-2\pi}{5}t\right) = -1 + 3\cos\left(\frac{2\pi}{5}t\right)$

$y(t) = 1 + 3\sin\left(\frac{-2\pi}{5}t\right) = 1 - 3\sin\left(\frac{2\pi}{5}t\right)$

(for $0 \leqslant t \leqslant 5$)

Calculus III Quiz #3 Fall 2017 NAME: Dangnhi Ngo

9

1. (4 Pts) Provide the equation of the plane going through the point P(2,-1,3) having a normal vector $\vec{n} = <1, 4, -2>$. Then determine how far that plane is from the origin.

EQN of Plane:

$1(x-2) + 4(y+1) - 2(z-3) = 0$

$x + 4y - 2z - 2 + 4 + 6 = 0$

$x + 4y - 2z = -8$

2 1

$-8 / |\vec{n}|$

$|\vec{n}| = ?$

3

Distance from the origin: 8 -1

6

2. (6 Pts total, 3 Pts each) Answer the following questions for the surfaces specified.

2.a $4x^2 - 3y^2 + 2z^2 = -7$ Type: Hyperboloid 1 Axis of Symmetry: y-axis 1

Sub-type: Hyperboloid of two sheet 1

2.b $-2x^2 - y + 4z^2 = 16$ Type: Paraboloid 1 Axis of Symmetry: y - axis 1

Sub-type: Hyberbolid Paraboloid 1

Calculus III Quiz #4 Fall 2017 NAME: Dangnhi Ngo

11

1. (4 Pts) For the function $f(x,y) = x^2y - 2x\sqrt{y}$, find f_x , f_y, f_{xx}, and f_{yx}.

$f_x = 2xy - 2\sqrt{y}$ 1

$f_y = x^2 - \frac{x}{\sqrt{y}}$ 1

4

$f_{xx} = 2y$ 1

$f_{yx} = 2x - \frac{1}{\sqrt{y}}$ 1

2. (4 Pts) For the function in problem #1, find $\vec{\nabla}f(x,y)$ and then determine its value at the point (1,4).

1 1 1

$\vec{\nabla}f(x,y) = \langle f_x , f_y \rangle = \langle 2xy - 2\sqrt{y} , x^2 - \frac{x}{\sqrt{y}} \rangle$

4

$\vec{\nabla}f(1,4) = \langle 2xy - 2\sqrt{y}, x^2 - \frac{x}{\sqrt{y}} \rangle \Big|_{(1,4)}$ 1

$= \langle 2.1.4 - 2.\sqrt{4}, 1^2 - \frac{1}{\sqrt{4}} \rangle = \langle 4, \frac{1}{2} \rangle$

SEE PROBLEM #3 ON THE REVERSE SIDE

3. (4 Pts) Does the following limit exist: $\lim\limits_{(x,y)\to(-1,1)} \frac{x^2-y^2}{x+y}$? If not show why not. If it does exist, show that it does and determine the value of the limit.

$$\lim_{(x,y)\to(-1,1)} \frac{x^2-y^2}{x+y} = \lim_{(x,y)\to(-1,1)} (x-y) = -1-1 = -2$$

2 1

limit exists and the value of the limit is -2

3

$$\lim_{(x,y)\to(-1,1)} \frac{(x-y)\cancel{(x+y)}}{\cancel{(x+y)}}$$

$10\frac{1}{2}$ Good

Calculus III Quiz #5 Fall 2017 NAME: Dangnhi Ngo

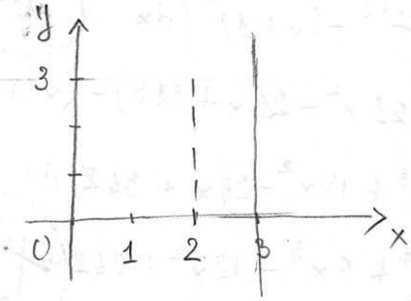
1. (4 Pts) Evaluate $\iint_R 3xe^{xy}\,dA$ where R is the region $0 \le x \le 2$ and $0 \le y \le 3$. NOTE: You must diagram the region of integration and show your intended order of integration with a "windshield wiper."

$$\iint_R 3xe^{xy}\,dA = \int_0^2\int_0^3 3xe^{xy}\,dy\,dx = 3\int_0^2 e^{xy}\Big|_0^3\,dx = 3\int_0^2 (e^{3x} - e^0)\,dx$$

(4)

$$= 3\int_0^2 (e^{3x} - 1)\,dx = 3\left[\frac{1}{3}e^{3x} - x\Big|_0^2\right] = 3\left(\frac{e^6}{3} - 2 - \frac{e^0}{3}\right)$$

$$= e^6 - 7$$



what is this line for? (windshield wiper?)

SEE SECOND PROBLEM ON REVERSE

$6\tfrac{1}{2}$ total

2. (6 Pts) Evaluate $\iint_R 2x^2 dA$ where R is the region bounded by $x = 0$, $y = x + 1$ and $y = -x + 7$.
~~NOTE: You must~~ diagram the region of integration and show your intended order of integration with a "windshield wiper."

Extra: Formula for the average value of the function $f(x,y)$ over the a region R: $\bar{f} = \frac{1}{\text{Area of R}} \iint_R f(x,y)\, dA$

+2

$\iint_R 2x^2 dA =$

Area: $\frac{1}{2} \times (7-1) \times 3 = 9$

$y = x + 1$
$y = -x + 7$ $\Rightarrow \begin{cases} 2y = 8 \\ y = x + 1 \end{cases} \Rightarrow \begin{cases} y = 4 \\ x = 3 \end{cases}$

$\bar{f} = \frac{1}{9} \times 693 = \boxed{77}$

$\iint_R 2x^2 dA = \int_0^3 \int_{x+1}^{-x+7} 2x^2\, dy\, dx = 2\int_0^3 \frac{1}{3} x^3 \Big|_{x+1}^{-x+7} dx$ ✓

$+4\tfrac{1}{2}$

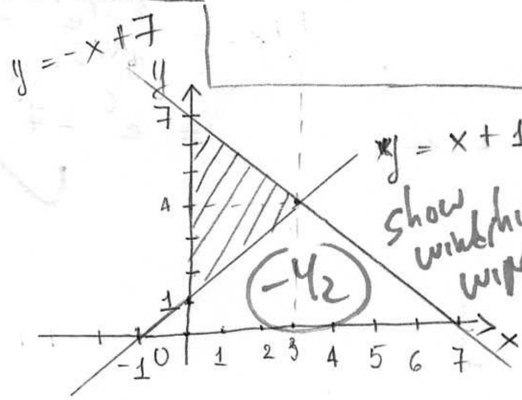
$= \frac{2}{3}\int_0^3 \left[(-x+7)^3 - (x+1)^3\right] dx$

$= \frac{2}{3}\int_0^3 \left[(-x^3 + 21x^2 - 21x + 343) - (x^3 + 3x^2 + 3x + 1)\right] dx$

$= \frac{2}{3}\int_0^3 (-2x^3 + 18x^2 - 24x + 342)\, dx$

$= \frac{2}{3}\left(-\frac{1}{2}x^4 + 6x^3 - 12x^2 + 342x \Big|_0^3\right)$

$= -\frac{1}{3}\cdot 3^4 + 4\cdot 3^3 - 8\cdot 3^2 + 228\cdot 3 = -27 + 108 - 72 + 684 = \boxed{693}$



$-\tfrac{1}{2}$

show windshield wiper!

Calculus III   Quiz #6   Fall 2017 9½ NAME: Dangnhi Ngo

1. (5 Pts) Find the volume under the upper sheet of the two-sheet hyperboloid $z = \sqrt{16 + x^2 + y^2}$ that is in the first quadrant and where $x^2 + y^2 \le 9$. You MUST draw a neat diagram of the region of integration showing a windshield wiper that illustrates your intended integration method.

4½

Let $x = r\cos\theta$ $\quad x^2+y^2 \le 9 \quad z = \sqrt{16+x^2+y^2}$

$y = r\sin\theta \quad r^2 \le 9 \quad \Rightarrow z = \sqrt{16+r^2}$

$r^2 = x^2+y^2 \quad r \le 3 \quad dA = r\,dr\,d\theta$

? Region Diagr

? windshield wiper?

$$V = \int_0^{\pi/2}\int_0^3 \sqrt{16+r^2}\, r\,dr\,d\theta$$

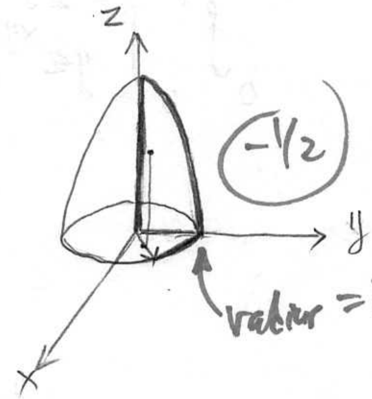
Let $t = 16+r^2 \Rightarrow dt = 2r\,dr$

$$\int \sqrt{16+r^2}\, r\,dr = \frac{1}{2}\int \sqrt{t}\,dt = \frac{1}{3}t^{3/2} + C$$

$$\text{So } V = \int_0^{\pi/2} \frac{1}{3}(16+r^2)^{3/2}\Big|_0^3 \,d\theta$$

$$V = \frac{1}{3}\int_0^{\pi/2}\left[(16+9)^{3/2} - 16^{3/2}\right]d\theta = \frac{1}{3}\int_0^{\pi/2}(125-64)\,d\theta$$

$$= \frac{61}{3}\theta\Big|_0^{\pi/2} = \boxed{\frac{61\pi}{6}}$$



(−½)

wiper = ?

SEE SECOND PROBLEM ON REVERSE

2. (5 Pts) Change the order of integration for the following iterated integral: $\int_0^2 \int_0^{4-2x} ye^{xy}\,dydx$. You MUST draw a neat diagram of the region of integration showing the REVISED windshield wiper NOT the original one. You do NOT have to perform the resulting integration.

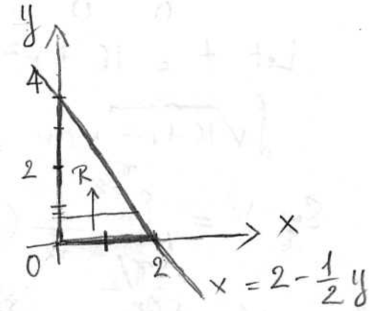
$$\int_0^2 \int_0^{4-2x} ye^{xy}\,dy\,dx \qquad \begin{array}{l} 0 \leq y \leq 4-2x \\ 0 \leq x \leq 2 \end{array}$$

5

$$y = 4-2x \Rightarrow 2x = 4-y \Rightarrow x = 2 - \tfrac{1}{2}y$$

$$2 - \tfrac{1}{2}y = 0 \Rightarrow \tfrac{1}{2}y = 2 \Rightarrow y = 4$$

$$\int_0^4 \int_0^{2-\frac{1}{2}y} ye^{xy}\,dx\,dy$$



Calculus III Quiz #8 Fall 2017 NAME: Dangnhi Ngo

13

1. (5 Pts) Compute the work integral $\int_C \vec{F} \cdot \overrightarrow{dr}$ for the vector field $F = < 2y, x + y >$ over the curve C: $y = x - 2$ from P(1,-1) to Q(5,3).

1 $\vec{r}(t) = \langle t, t-2 \rangle$ for $1 \leq t \leq 5$

1 $\vec{r}'(t) = \langle 1, 1 \rangle \Rightarrow |\vec{r}'(t)| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$$\int_C \vec{F} \cdot d\vec{r} = \int_1^5 \langle 2(t-2), t + (t-2) \rangle \cdot \langle 1, 1 \rangle \, dt$$

1 1

5

$$= \int_1^5 \langle 2t - 4, 2t - 2 \rangle \cdot \langle 1, 1 \rangle \, dt$$

$$= \int_1^5 (2t - 4 + 2t - 2)\, dt = \int_1^5 (4t - 6)\, dt = 2t^2 - 6t \Big|_1^5 = 20 - (-4) = 24$$

1

2. (3 Pts) Show that the vector field $\vec{F} = < f, g, h > = < 2x + yz, xz - 2, xy >$ is a conservative vector field. Be explicit (using f, g, h) in explaining the tests you are making.

$f_y = z = g_x$ ✓ 1 Therefore, F is a conservative

$g_z = x = h_y$ ✓ 1 1

$f_z = y = h_x$ ✓

3

SEE THIRD PROBLEM ON REVERSE

3. (5 Pts) Find the scalar potential function, $\emptyset(x, y, z)$, for the conservative vector field given in problem #2: $\vec{F} = < 2x + yz, xz - 2, xy >$.

$\phi_x = 2x + yz$ 1 $\Rightarrow \phi = x^2 + xyz + C(y, z)$ 1

Derivative ϕ to y : $\phi_y = xz + C_y(y,z)$ 1

We have : $\phi_y = xz - 2$ 1/2

$\Rightarrow C_y(y,z) = -2 \Rightarrow C(y,z) = -2y$

Now, we have : $\phi = x^2 + xyz - 2y + d(z)$

Derivative ϕ to z : $\phi_z = xy + d'(z)$ 1/2

We have : $\phi_z = xy$ 1/2

$\Rightarrow d'(z) = 0 \Rightarrow d(z)$ is a real number

Therefore : $\boxed{\phi = x^2 + xyz - 2y}$ 1/2

5

Calculus III Quiz #9 13 Fall 2017 NAME: Dangnhi Ngo

1. (5 Pts) **Use the circulation form of Green's Theorem** to calculate the value of $\int_C \vec{F} \cdot \vec{dr}$ for the vector field $\vec{F} = < x-y, 3x+2y >$ over the path C shown below.

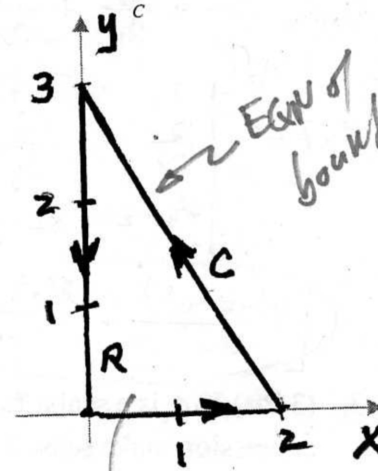
$$\int_C \vec{F} \cdot \vec{dr} = \iint_R \frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \, dA$$

$$= \int_0^3 \int_0^2 (3+1)\, dx\, dy = \int_0^3 \int_0^2 4 \, dx \, dy$$

$$= \int_0^3 4x \Big|_0^2 dy$$

$$= \int_0^3 8 \, dy = 8y \Big|_0^3 = 8 \times 3 = \boxed{24}$$

wrong limits -1



EQN of boundary?

windshield wiper?

4

SEE PROBLEMS ON REVERSE

2. (6 Pts) Compute the divergence and curl of the vector field $\vec{F} = < x^2y - z, xz - y^2, x - y + z >$

6

All this writing needed?

$$\vec{\nabla}\cdot\vec{F} = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z} = 2xy + (-2y) + 1 = 2y(x-1) + 1$$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y - z & xz - y^2 & x - y + z \end{vmatrix} = \left[\frac{\partial(x-y+z)}{\partial y} - \frac{\partial(xz-y^2)}{\partial z}\right] i + \left[\frac{\partial(x^2y-z)}{\partial z} - \frac{\partial(x-y+z)}{\partial x}\right] j$$

$$+ \left[\frac{\partial(xz-y^2)}{\partial x} - \frac{\partial(x^2y-z)}{\partial y}\right] k = (-1-x) i + (-1-1) j + (z - x^2) k = \boxed{(-1-x)\vec{i} - 2\vec{j} + (z-x^2)\vec{k}}$$

3. (3 Pts) If ϕ is a scalar function and $\vec{F}$ is a vector field (both defined in $\mathbb{R}^3$), does the following expression make sense.? Whether yes or no, indicate which operations are valid and which are not valid.

$\vec{\nabla}\times[\vec{\nabla}\cdot(\vec{F}\times\vec{\nabla}\phi)]$ The overall expression is not valid.

3

Operations: $\vec{\nabla}\cdot(\vec{F}\times\vec{\nabla}\phi)$ is valid

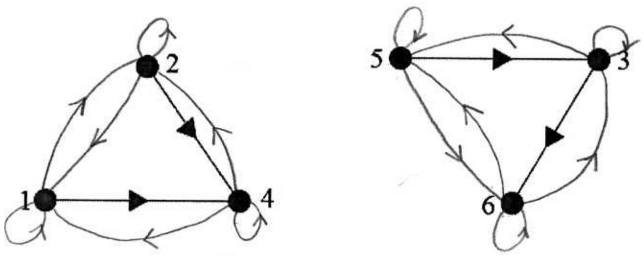
$\vec{\nabla}\times[\vec{\nabla}\cdot(\vec{F}\times\vec{\nabla}\phi)]$ is not valid

Section 6.3: Equivalence Relations

Let r be an equivalence relation on the set $A = \{1, 2, \ldots, 6\}$.

Suppose that some of the edges of the digraph of r are given below.

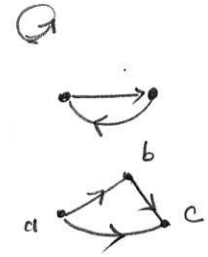
(a) What other edges must be in the digraph? Draw them below. (Hint: Use the fact that r is an equivalence relation, which means that it is reflexive, symmetric, and transitive.)



Reflexive:

Symmetric:

Transitive:



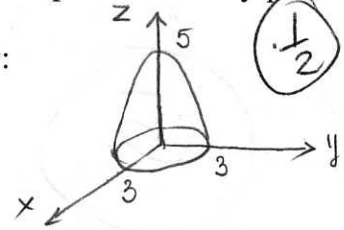
(b) Compute the equivalence class $c(k)$ for $k = 1, 2, 3, 4, 5, 6$.

Calculus III Quiz #10 Fall 2017 NAME: Dangnhi Ngo 12 1/2

1. Your goal is to evaluate the integral $\iint_S (\vec{\nabla} \times \vec{F}) \cdot \vec{n}\, dS$ over that portion of the explicitly-defined surface S given by $z = \sqrt{25 - x^2 - y^2}$ that is above the region $x^2 + y^2 \leq 9$ (the cap of a hemisphere of radius 5 where $4 \leq z \leq 5$). Assume that $\vec{n}$ is an upward normal (positive z component) and $\vec{F} = < y,\ -x,\ 2z >$. Part c can be done on the reverse side if desired.

   a. (3 points) Before beginning the evaluation, sketch a **NEAT** side-view of the surface (z-up) and the closed path C in the xy-plane. Also, compute the curl of $\vec{F}$.

Side View:



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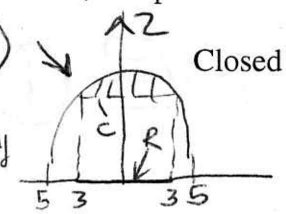
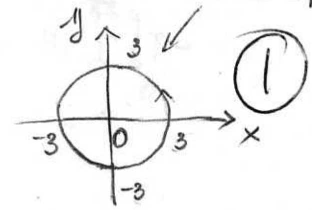


Closed Path, C:

C is up at z = 4



1

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & -x & 2z \end{vmatrix} = \langle 0, 0, -2 \rangle$$ 1

   b. (5 points) Compute the surface integral directly using: $\iint_S \overline{F} \cdot \vec{n}\, dS = \iint_R \overline{F} \cdot < -g_x, -g_y, 1 > dA$

   CAUTION: Be sure to use $\vec{\nabla} \times \vec{F}$ for $\vec{F}$.

   c. (5 points) Compute the surface integral using **Stokes' Thm** $\iint_S (\nabla \times \overline{F}) \cdot \vec{n}\, dS = \oint_C \overline{F} \cdot d\vec{r}$

b/ $g(x,y) = (25 - x^2 - y^2)^{1/2}$

$g_x = \frac{1}{\cancel{2}}(25 - x^2 - y^2)^{-1/2}(-\cancel{2}x) = \dfrac{-x}{(25 - x^2 - y^2)^{1/2}}$

$g_y = \frac{1}{\cancel{2}}(25 - x^2 - y^2)^{-1/2}(-\cancel{2}y) = \dfrac{-y}{(25 - x^2 - y^2)^{1/2}}$

$\iint_R \vec{F} \cdot \langle -g_x, -g_y, 1 \rangle dA = \iint_R \langle 0, 0, -2 \rangle \cdot \left\langle \dfrac{x}{(25 - x^2 - y^2)^{1/2}}, \dfrac{y}{(25 - x^2 - y^2)^{1/2}}, 1 \right\rangle dA$

4

$= \iint_R -2\, dA = -2 \cdot (\pi \cdot 3^2) = \boxed{-18\pi}$

1

c/ $\vec{r}(t) = \langle 3\cos t, 3\sin t, \cancel{5}^{4} \rangle$ 2 for $0 \leq t \leq 2\pi$

$\vec{r}'(t) = \langle -3\sin t, 3\cos t, 0 \rangle$

$$\oint_C \vec{F} \cdot d\vec{r} = \int_0^{2\pi} \langle 3\sin t, -3\cos t, \cancel{10}^{8} \rangle \cdot \langle -3\sin t, 3\cos t, 0 \rangle \, dt$$

2

$$= \int_0^{2\pi} (-9\sin^2 t - 9\cos^2 t) \, dt = \int_0^{2\pi} -9(\sin^2 t + \cos^2 t) \, dt$$

$$= -9 \int_0^{2\pi} 1 \, dt = -9 \cdot 2\pi = \boxed{-18\pi}$$ 1

5