Calculus III Quiz #9 Spring 2018 NAME: SOLUTION

1. a. (6 Pts) Find the scalar potential, $\emptyset(x, y, z)$, for the conservative vector field
$\vec{G} =< 4y + z\cos(xz), 4x + 2z, x\cos(xz) + 2y >$.

$$\phi_x = 4y + z\cos(xz) \Rightarrow \phi = \underline{4xy} + \underline{\sin(xz)} + C(y,z)$$

$$\phi_y = 4x + 2z \Rightarrow \phi = 4xy + \underline{2yz} + D(x,z)$$

$$\phi_z = x\cos(xz) + 2y \Rightarrow \phi = \sin(xz) + 2yz + E(x,y)$$

$$\Rightarrow \boxed{\phi(x,y,z) = 4xy + \sin(xz) + 2yz + C}$$

since $\vec{\nabla}\times\vec{G} = \vec{\nabla}\times\vec{\nabla}\phi = \vec{0}$ (Always!)

b. (1 Pt) Curl($\vec{G}$) = $\vec{0}$ [Hint: You should know it without computing it!]

See Second Problem on Reverse Side

Calculus III Quiz #9 Spring 2018 NAME:______________

1. a. (6 Pts) Find the scalar potential, $\emptyset(x, y, z)$, for the conservative vector field
$\vec{G} =< 4y + z\cos(xz), 4x + 2z, x\cos(xz) + 2y >$.

SAME AS ABOVE

b. (1 Pt) Curl($\vec{G}$) = ______________ [Hint: You should know it without computing it!]

See Second Problem on Reverse Side

2. Determine the divergence and curl of the vector field $\vec{F} = \langle xyz, e^{2xy}, \sin(xz) \rangle$.

a. (2 Pts) $\text{Div}(\vec{F}) =$ $yz + 2xe^{2xy} + x\cos(xz)$

b. (3 Pts) $\text{Curl}(\vec{F}) =$ $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xyz & e^{2xy} & \sin(xz) \end{vmatrix}$

$= \langle 0, xy - z\cos(xz), 2ye^{2xy} - xz \rangle$

2. Determine the divergence and curl of the vector field $\vec{F} = \langle \sin(xz), e^{2yz}, xyz \rangle$.

c. (2 Pts) $\text{Div}(\vec{F}) =$ $z\cos(xz) + 2ze^{2yz} + xy$

d. (3 Pts) $\text{Curl}(\vec{F}) =$ $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \sin(xz) & e^{2yz} & xyz \end{vmatrix}$

$= \langle xz - 2ye^{2yz}, x\cos(xz) - yz, 0 \rangle$