Section 1.1: Systems of Linear Equations

Note: Here we are presented with systematic methods for solving Linear Equations → These algorithms will be used throughout the entire course.

Linear Equations

A **Linear Equation** in variables $x_1, \ldots, x_n$ is an equation of the form:

$$a_1x_1 + a_2x_2 + \cdots + a_{n-1}x_{n-1} = b,$$ where:

* $\{a_1, \ldots, a_n\}$ → coefficients
* b → constant (RHS of eq.)

*Both can be $\mathbb{R}$ or complex (known/defined)

* n → Subscript ($n \in \mathbb{Z}$ ST $n > 0$)

A **System of Linear Equations** -OR- **Linear System** is a collection of 1 or more linear equations involving the same variable:

$$a_1x_1 + a_2x_2 = b$$

$$a_3x_3 + a_4x_4 = b$$

*Note: The above "b"-values do NOT have to be equivalent.

Solutions & Types of Systems

A **Solution** of a system, $(s_1, s_2, \ldots, s_{n-1}, s_n)$, is a list of numbers that make each equation a true statement when the values $\{s_n\}_n^{n=1}$ are substituted in for $\{x_n\}_n^{n=1}$ respectively.

* $\{s_n\} \rightarrow$ Solution Set (Set of ALL possible sol.)

Note: A system may have one -or- infinitely many solution.

Two linear systems are called "**Equivalent**" if they have the same $\{s_n\}$

*IOW: Each sol. of the 1st system is a sol. of the 2nd & visa versa :)

Recall: Solving a System of Linear Equations in 2 variables (mainly x_1 & x_2) simpily amounts to finding the intersection of the two lines :)

*Two Types of Linear Systems:

1. **Consistent System:** The system has @ least one solution.
2. **Inconsistent System:** The system has NO solutions.

*Three Types of Solutions to Linear Systems:

1. Exactly ONE solution
2. NO solutions
3. Infinitely many solutions

3 Types of Solution Sets of a System of 2 Linear Eq.

Note: Each of the following cases can be easily verified w/ algebraic methods :) (i.e. Substitution, Elimination, Graphing)

Case 1: Exactly ONE Solution:

- Systems w/ one solution are consistent systems.
- The 2 linear equations that make up the system are then called "Independent Equations".

*IOW: The linear eq. are not the same line :)

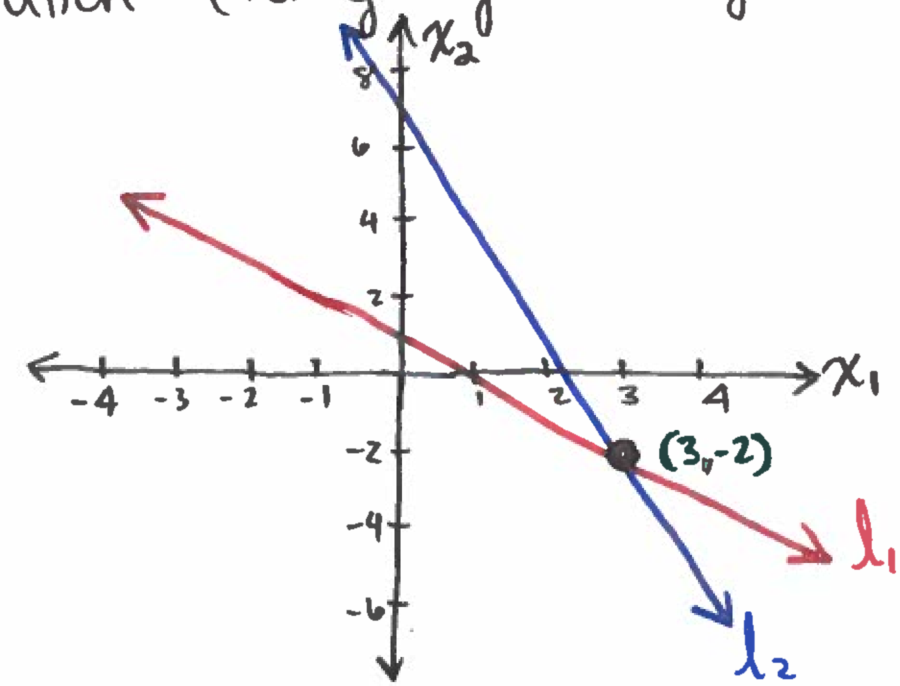
Example: (Intersecting Lines)

$x_1 + x_2 = 1 \quad (l_1)$

$3x_1 + x_2 = 7 \quad (l_2)$

*Notes:
- The graphs of these 2 equations are lines l_1 & l_2.
- A pair of #s (x_1, x_2) satisfies BOTH IFF the point lies on both l_1 & l_2.

For this given system, the single point $(3,-2)$ is the solution (verify algebraically :))



Case 2: No Solutions:

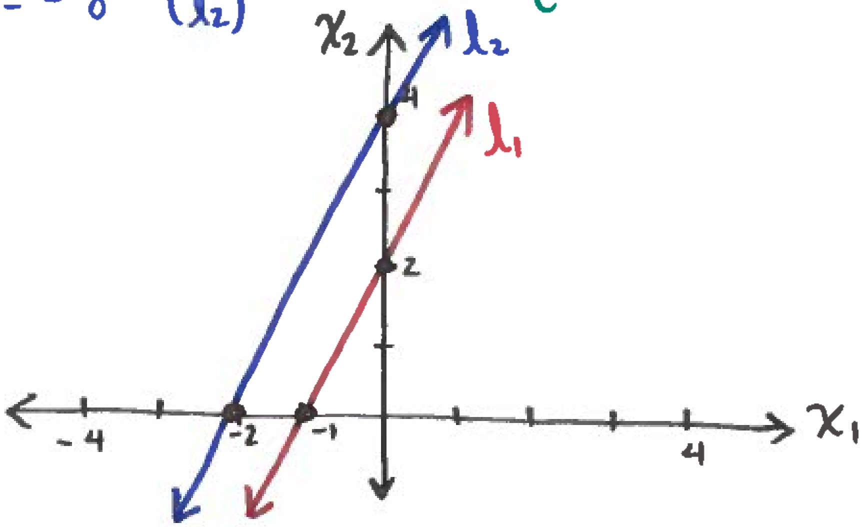
- Systems with NO solutions are <u>inconsistent systems</u>
- The two Linear Equations that make up the system are called "<u>Independent Equations</u>"

Example: (Parallel Lines):

$-2x_1 + x_2 = 2 \quad (l_1)$

$4x_1 - 2x_2 = -8 \quad (l_2)$

Note: There is NO solution point for these 2 equations (No intersection)



Case 3: Infinite Solutions:

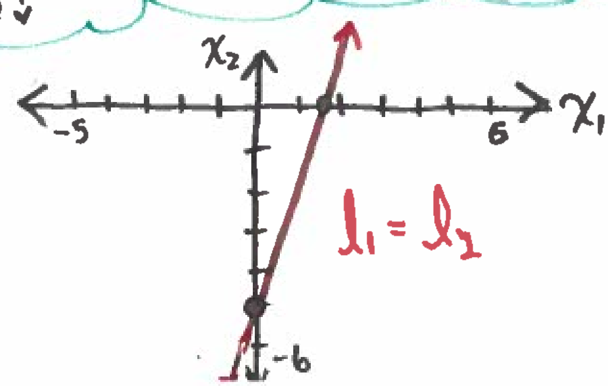
- Systems with infinitely many solutions are <u>consistent</u>.
- The two systems that make up the system are called "<u>Dependent Equations</u>"

⟹ Every point on the graph is a solution point b/c the equations are the same!

Example (Conciding Lines):

$3x_1 - x_2 = 5 \; (l_1)$

$3x_2 = 9x_1 - 15 \; (l_2)$



Solving a Linear System

Basic Strategy: Replace one system with an equivalent system that is easier to solve :)

*The Algorithm for Solving:

① Use the x_1 term in the first equation of the system to eliminate the x_1 terms in the other equations.

② Use the x_2 term in the second equation of the system to eleminate the x_2 terms in the other equations (ignoring equation 1 :))

⋮

*We continue on with this systematic procedure until we have obtained a simple equivalent system

Notes:

(i) The new, equivalent system will have a triangular form.

(ii) We can easily solve this new, equivalent system using back substitution.

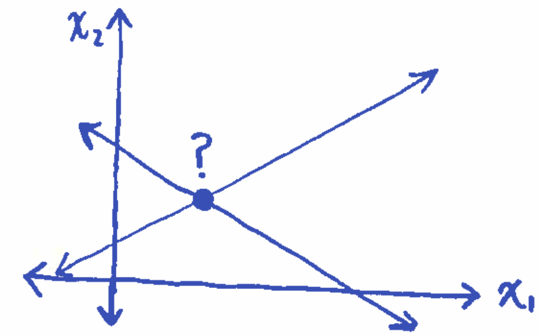
* 3 Basic Operations for Solving:

1) Replacement: Replace 1 eq. w/ the sum of itself & a multiple of another eq.

2) Interchange: Interchange 2 equations

3) Scaling: Multiply all terms of an eq. by a nonzero constant.

Example: Find the point (x_1, x_2) that lies on the line $x_1 + 2x_2 = 5$ and on the line $x_1 - x_2 = -1$.



Answer:

Solve the Linear System:

$$\begin{cases} x_1 + 2x_2 = 5 & (Eq(1)) \\ x_1 - x_2 = -1 & (Eq(2)) \end{cases}$$

Keep x_1 in Eq(1) & eliminate x_1 from Eq(2):

Add $-$(Eq 1) to Eq(2)

$$\begin{array}{l} -(Eq\,1) \\ +\ Eq\,2 \\ \hline \text{NEW } Eq(2) \end{array} \Longrightarrow + \begin{array}{r} \cancel{-x_1} - 2x_2 = -5 \\ \cancel{x_1} - x_2 = -1 \\ \hline -3x_2 = -6 \end{array} \Longrightarrow \boxed{x_2 = 2}$$

So, the NEW equivalent system is: $\begin{cases} x_1 + 2x_2 = 5 \\ x_2 = 2 \end{cases}$

Use Back-Substitution to find x_1:

Since $x_2 = 2 \Longrightarrow x_1 + 2(2) = 5$

$\Longrightarrow x_1 + 4 = 5$

$\Longrightarrow \boxed{x_1 = 1}$

∴ Solution Set:

$\boxed{(x_1, x_2) = (2,1)}$

Answer ✓

Example: Solve the system by using elementary row operations on the equations. Follow the systematic elimination procedure:

$$4x_1 + 8x_2 = -4$$
$$5x_1 + 6x_2 = 11$$

Answer:

* Simplify ($\mathcal{E}q.1$): To do this, divide both sides by "4"

$$\begin{aligned} 4(x_1 + 2x_2) &= -4 \\ 5x_1 + 6x_2 &= 11 \end{aligned} \quad \Longrightarrow \quad \begin{aligned} x_1 + 2x_2 &= -1 \quad (\mathcal{E}q\,1) \\ 5x_1 + 6x_2 &= 11 \quad (\mathcal{E}q\,2) \end{aligned}$$

* Keep x_1 in the 1st equation & eliminate it from $\mathcal{E}q$ (2):

To do this, add $-5(\mathcal{E}q\,1)$ to $\mathcal{E}q\,2$.

$$\begin{array}{l} -5(\mathcal{E}q\,1) \\ +\ (\mathcal{E}q\,2) \\ \hline \text{NEW } \mathcal{E}q\,2 \end{array} \quad \Longrightarrow \quad \begin{array}{rl} & -\cancel{5x_1} - 10x_2 = 5 \\ + & \cancel{5x_1} + 6x_2 = 11 \\ \hline & -4x_2 = 16 \end{array} \longrightarrow \boxed{x_2 = -4}$$

$\therefore$ The new, equivalent system is:

$$\left.\begin{aligned} x_1 + 2x_2 &= -1 \quad (\mathcal{E}q.1) \\ x_2 &= -4 \quad (\mathcal{E}q.2) \end{aligned}\right\}$$

Note: This is the triangular form ☺

Example² continued...

*Use back-substitution to solve for x_1:

Since $x_2 = -4 \Rightarrow x_1 + 2(-4) = -1$ & solve for x_1

$x_1 - 8 = -1 \Rightarrow \boxed{x_1 = 7}$

∴ Therefore, the solution of the system is:

$$\boxed{(x_1, x_2) = (7, -4)}$$ Answer.

Check (For good luck!):

Eq(1): $4(7) + 8(-4) \stackrel{?}{=} -4$

$28 - 32 = -4$ ✓

Eq(2): $5(7) + 6(-4) \stackrel{?}{=} 11$

$35 - 24 = 11$ ✓

Woohoo!

Example: Solve the system

$$x_2 + 4x_3 = 2 \qquad Eq(1)$$
$$x_1 + 3x_2 + 3x_3 = -2 \qquad Eq(2)$$
$$3x_1 + 8x_2 + 5x_3 = -3 \qquad Eq(3)$$

Answer:

Note: The following order of solving is NOT unique ☺ Many possibilities ∃.

* Interchange Eq(1) & Eq(2):

$$\Rightarrow \quad \begin{aligned} x_1 + 3x_2 + 3x_3 &= -2 \qquad (1)\\ x_2 + 4x_3 &= 2 \qquad (2)\\ 3x_1 + 8x_2 + 5x_3 &= -3 \qquad (3)\end{aligned}$$

* Remove $3x_1$ from Eq(3) $\mapsto$ Add $-3[Eq 1]$ to Eq 3:

$$\begin{array}{l} -3[Eq(1)] \\ + \quad Eq(3) \\ \hline \text{NEW } Eq(3) \end{array} \quad \Rightarrow \quad \begin{array}{r} -3x_1 - 9x_2 - 9x_3 = +6 \\ + \quad 3x_1 + 8x_2 + 5x_3 = -3 \\ \hline -x_2 - 4x_3 = 3 \end{array}$$

So,

$$\begin{aligned} x_1 + 3x_2 + 3x_3 &= -2 \\ x_2 + 4x_3 &= 2 \\ -x_2 - 4x_3 &= 3 \end{aligned}$$

↘

Example continued...

*Remove $-x_2$ from Eq(3) → Add Eq(2) to Eq(3):

Eq(2)
+ Eq(3)
NEW Eq(3)

$$\Rightarrow \quad \begin{array}{r} x_2 + 4x_3 = 2 \\ + \quad -x_2 - 4x_3 = 3 \\ \hline 0 = 5 \end{array}$$

→ ←

* contradiction!

Answer:

$$\boxed{\text{Since } 0 \neq 5, \text{ the Linear System has NO Solution}}$$

Matrix Notation

The essential information of a Linear System can be converted to a compact form, in a rectangular array called "A matrix"

- A matrix with m-rows & n-columns is called an **m×n matrix**, where $m,n \in \mathbb{Z}$ ST $m,n > 0$.

- A **Coefficient Matrix** -or- **Matrix of Coefficients** is defined:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

*The Set of all elements, $\{a_{mn}\}$, are the coefficients of the variables.

- An **Augmented Matrix** contains an extra column for the constants on the RHS of a linear equation

Linear System:

$$x_1 - 2x_2 + 3x_3 = 1$$
$$x_2 - x_3 = 5$$
$$x_1 + 4x_3 = -3$$

$\Longleftrightarrow$

Augmented Matrix:

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 1 \\ 0 & 1 & -1 & 5 \\ 1 & 0 & 4 & -3 \end{array}\right]$$

Solving an Augmented Matrix

Note: The basic operations used for solving a system of linear equations correspond to solving an augmented matrix :

*Elementary Row Operations:

① **Replacement:** Replace one row w/ the sum of itself & a multiple of another row.

② **Interchange:** Interchange 2 rows.

③ **Scaling:** Multiply all elements of a row by a non-zero constant.

Notes:

(i) Row operations are reversible.

*IOW: If 2 rows are interchanged, scaled, or replaced, they can be returned to their original state by an interchange, scaling, or replacement (respectively)

(ii) Two matrices are called "Row Equivalent" if there is a sequence of elementary row operations that transforms one matrix into another.

*IOW: If the augmented matrices of 2 linear systems are row equivalent, then the 2 systems have the same solution set

Example: Solve the following augmented matrix using appropriate row operations & describe the solution of the original system:

$$\left[\begin{array}{ccc|c} 1 & 6 & 3 & -5 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Answer:

* Note: R_3 is producing a contradiction!

$$\Rightarrow \quad 0 \neq -2$$

$\therefore$ The given system is inconsistent & NO solution $\exists$. ✓

Answer ✓

Example: Solve the following augmented matrix using appropriate row operations & then describe the solution set of the original system:

$$\begin{array}{c c}
\begin{bmatrix} 1 & -3 & 0 & 0 & -5 \\ 0 & 1 & -2 & 0 & -6 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} & \begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{matrix}
\end{array}$$

Answer:

Note: "x_1" has already been removed from all rows except R_1 :)

*Remove $-3x_2$ from R_1: Add $3(R_2)$ to R_1:

$$\begin{array}{l} 3(R_2) \\ +\ R_1 \\ \hline \text{NEW } R_1 \end{array} \Longrightarrow \begin{array}{rl} & 0x_1 + \cancel{3x_2} - 6x_3 + 0x_4 = -18 \\ + & x_1 - \cancel{3x_2} + 0x_3 + 0x_4 = -5 \\ \hline & x_1 + 0x_2 - 6x_3 + 0x_4 = -23 \end{array}$$

So,

$$\begin{bmatrix} 1 & 0 & -6 & 0 & -23 \\ 0 & 1 & -2 & 0 & -6 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

✓ ✓

*Now need to remove $-6x_3$ from R_1 & $-2x_3$ from R_2 :)

Example continued...

*Remove $-6X_3$ from $R_1 \rightarrow$ Add $6(R_3)$ to R_1:

$$\begin{array}{l} 6(R_3) \\ + \quad R_1 \\ \hline \text{NEW } R_1 \end{array} \Longrightarrow \begin{array}{rl} & 0X_1 + 0X_2 + 6X_3 - 6X_4 = 18 \\ + & X_1 + 0X_2 - 6X_3 + 0X_4 = -23 \\ \hline & X_1 + 0X_2 + 0X_3 - 6X_4 = -5 \quad \checkmark \end{array}$$

*Remove $-2X_3$ from $R_2 \rightarrow$ Add $2(R_3)$ to R_2:

$$\begin{array}{l} 2(R_3) \\ + \quad R_2 \\ \hline \text{NEW } R_2 \end{array} \Longrightarrow \begin{array}{rl} & 0X_1 + 0X_2 + 2X_3 - 2X_4 = 6 \\ + & 0X_1 + X_2 - 2X_3 + 0X_4 = -6 \\ \hline & 0X_1 + X_2 + 0X_3 - 2X_4 = 0 \quad \checkmark \end{array}$$

So,

$$\begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & -6 & -5 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \\ \begin{array}{ccccc} \checkmark & \checkmark & \checkmark & * & \end{array} \end{array}$$

*Remove $-6X_4$ from $R_1 \rightarrow$ Add $6(R_4)$ to R_1:

$$\begin{array}{l} 6(R_4) \\ + \quad R_1 \\ \hline \text{NEW } R_1 \end{array} \Longrightarrow \begin{array}{rl} & 0X_1 + 0X_2 + 0X_3 + 6X_4 = 12 \\ + & X_1 + 0X_2 + 0X_3 - 6X_4 = -5 \\ \hline & X_1 + 0X_2 + 0X_3 + 0X_4 = 7 \quad \checkmark \end{array}$$

$\searrow$

Example[2] continued...

* Remove $-2x_4$ from R_2 → Add $2(R_4)$ to R_2:

$$\begin{array}{l} 2(R_4) \\ +\ R_2 \\ \hline \text{NEW } R_2 \end{array} \Rightarrow \begin{array}{rl} & 0x_1 + 0x_2 + 0x_3 + 2x_4 = 4 \\ + & 0x_1 + x_2 + 0x_3 - 2x_4 = 0 \\ \hline & 0x_1 + x_2 + 0x_3 + 0x_4 = 4 \ \checkmark \end{array}$$

* Remove $-x_4$ from R_3 → Add R_4 to R_3:

$$\begin{array}{l} R_4 \\ +\ R_3 \\ \hline \text{NEW } R_3 \end{array} \Rightarrow \begin{array}{rl} & 0x_1 + 0x_2 + 0x_3 + x_4 = 2 \\ + & 0x_1 + 0x_2 + x_3 - x_4 = 3 \\ \hline & 0x_1 + 0x_2 + x_3 + 0x_4 = 5 \ \checkmark \end{array}$$

So,
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 7 \\ 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Answer ✓

∴ The Solution Set of the System is:

$$(x_1, x_2, x_3, x_4) = (7, 4, 5, 2)$$

* Consistent System w/ one, unique solution :)

Existence & Uniqueness Questions

Recall: A solution set for a system of linear equations contains either ONE solution, NO solutions, or infinitely many solutions (*We explore this further in next section :)

To Determine which type of Solution Set is true for a given Linear System, we ask the following...

Fundamental Questions About Linear Systems

① Is the System Consistent?

⇒ IOW, does the system have @ least one solution?

② If a solution exists, is it unique?

⇒ IOW, does the system have 1 solution or infinitely many?

Note: Here we look @ answering these questions via the row operations of an augment matrix, but these 2 specific questions follow us throughout the entire course!

Example: Determine whether the following system is (!
consistent. Do NOT completely solve the system:

$$2x_1 - 8x_4 = -12 \qquad Eq(1)$$

$$2x_2 + 2x_3 = 0 \qquad Eq(2)$$

$$x_3 + 8x_4 = 4 \qquad Eq(3)$$

$$-5x_1 + 3x_2 + 5x_3 + x_4 = 3 \qquad Eq(4)$$

Answer:

Recall: "Consistent" implies the system has @ least 1 solution :)

* Convert system to an Augmented Matrix (easier to read :)):

$$\Rightarrow \begin{bmatrix} 2 & 0 & 0 & -8 & -12 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 8 & 4 \\ -5 & 3 & 5 & 1 & 3 \end{bmatrix}$$

* Simplify: Divide R_1 by 2

* Simplify: Divide R_2 by 2

ok

ok

$$= \begin{bmatrix} 1 & 0 & 0 & -4 & -6 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 8 & 4 \\ -5 & 3 & 5 & 1 & 3 \end{bmatrix}$$

* Note: To verify if the system is consistent (w/o solving), we want to attain "triangle" form

$\Rightarrow$ Change/Edit R_4 :)

Example' Continued...

*Remove $-5X_1$ from $R_4 \rightarrow$ Add $5(R_1)$ to R_4:

$$\begin{array}{l} 5(R_1) \\ +\ R_4 \\ \hline \text{NEW } R_4 \end{array} \Rightarrow \begin{array}{r} 5X_1 + 0X_2 + 0X_3 - 20X_4 = -30 \\ + -5X_1 + 3X_2 + 5X_3 + X_4 = 3 \\ \hline 0X_1 + 3X_2 + 5X_3 - 19X_4 = -27 \end{array} \checkmark$$

*Remove $3X_2$ from $R_4 \rightarrow$ Add $-3(R_2)$ to R_4:

$$\begin{array}{l} -3(R_2) \\ +\ R_4 \\ \hline \text{NEW } R_4 \end{array} \Rightarrow \begin{array}{r} 0X_1 - 3X_2 - 3X_3 + 0X_4 = 0 \\ + \ 0X_1 + 3X_2 + 5X_3 - 19X_4 = -27 \\ \hline 0X_1 + 0X_2 + 2X_3 - 19X_4 = -27 \end{array}$$

*Remove $2X_3$ from $R_4 \rightarrow$ Add $-2(R_3)$ to R_4:

$$\begin{array}{l} -2(R_3) \\ +\ R_4 \\ \hline \text{NEW } R_4 \end{array} \Rightarrow + \begin{array}{r} 0X_1 + 0X_2 - 2X_3 - 16X_4 = -8 \\ 0X_1 + 0X_2 + 2X_3 - 19X_4 = -27 \\ \hline 0X_1 + 0X_2 + 0X_3 - 35X_4 = -35 \end{array}$$

*Simplify R_4: Divide R_4 by -35

$$\Rightarrow 0X_1 + 0X_2 + 0X_3 + X_4 = 1 \checkmark$$

So, $\begin{bmatrix} 1 & 0 & 0 & -4 & -6 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 8 & 4 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

$\boxed{\therefore \text{Since triangular form attained, system is consistent!}}$ Ans.

Example: Determine the value(s) of h such that the following linear system is consistent:

$$\begin{cases} x_1 + hx_2 = 4 \\ 3x_1 + 15x_2 = 8 \end{cases}$$

Answer:

* Rewrite the System as an Augmented Matrix:

$$\begin{cases} x_1 + hx_2 = 4 \\ 3x_1 + 15x_2 = 8 \end{cases} \iff \left[\begin{array}{cc|c} 1 & h & 4 \\ 3 & 15 & 8 \end{array}\right]$$

* Use the Systematic Procedure to show the matrix is consistent:

$$\begin{array}{r} \bullet\ -3R_1 \\ +\ R_2 \\ \hline \text{new } R_2 \end{array} \longrightarrow \left[\begin{array}{cc|c} 1 & h & 4 \\ 0 & -3h+15 & -4 \end{array}\right] \overset{\frac{1}{-3h+15}R_2}{\sim} \left[\begin{array}{cc|c} 1 & h & 4 \\ 0 & 1 & \frac{-4}{(-3h+15)} \end{array}\right]$$

* Solve for h:

i) Row 1: Since R_1 contain x_1, $h \in \mathbb{R}$ ✓

ii) Row 2: Since R_2 contains x_2 contain $\Rightarrow$ Must prevent contradictions/undefined solution!

Answer

$\boxed{\therefore\ h \text{ can be any } \mathbb{R} \text{ except}: -3h+15 \neq 0,\quad h \neq 5}$

TLDR: The system is consistent $\forall\, h$ except $h = 5$:)

Example: Determine the value(s) of h such that the following matrix is an augmented matrix of a consistent system:

$$\left[\begin{array}{cc|c} 1 & 5 & -4 \\ 2 & h & -8 \end{array}\right]$$

Answer:

Note: Here we already have a "1" in the x_1 position of R_1 :)

$$\begin{array}{l} \bullet\ -2R_1 \\ \quad + R_2 \\ \hline \text{new } R_2 \end{array} \longrightarrow \left[\begin{array}{cc|c} 1 & 5 & -4 \\ 0 & -10+h & 0 \end{array}\right] \overset{\frac{1}{(-10+h)} R_2}{\sim} \left[\begin{array}{cc|c} 1 & 5 & -4 \\ 0 & 1 & 0 \end{array}\right]$$

$\boxed{\therefore h \text{ can be any } \mathbb{R} \text{ to produce a consistent system.}}$

Ans.

Example 7: Determine the value(s) of h ST the matrix is an augmented matrix of a consistent system:

$$\begin{bmatrix} -15 & 18 & h \\ 5 & -6 & 4 \end{bmatrix}$$

Answer:

Recall: A linear system of eq. is consistent if @ least one solution $\exists$

***Note:** R_1 is a scalar multiple of R_2 !

$$\begin{bmatrix} -15 & 18 & h \\ 5 & -6 & 4 \end{bmatrix} = \begin{bmatrix} -3(5) & -3(-6) & h \\ 5 & -6 & 4 \end{bmatrix} = \begin{bmatrix} 5 & -6 & \frac{-h}{3} \\ 5 & -6 & 4 \end{bmatrix}$$

$\Rightarrow$ The equations are dependent & infinitely many solutions $\exists$ if $-\frac{h}{3} = 4 \Rightarrow h = -12$

Answer:

$$\boxed{\text{∴ The augmented matrix is a consistent linear system if } h = -12.}$$

Example: Find an equation for g, h, & K such that the following augmented matrix corresponds to a consistent system:

$$\left[\begin{array}{ccc|c} 1 & -6 & 7 & g \\ 0 & 18 & -20 & h \\ -4 & 6 & -8 & K \end{array}\right]$$

Answer:

* Rewrite the system in its equivalent, simplified form:

• $4R_1 + R_3$ / new R_3 $\rightarrow$ $\left[\begin{array}{ccc|c} 1 & -6 & 7 & g \\ 0 & 18 & -20 & h \\ 0 & -18 & 20 & 4g+K \end{array}\right]$

* Note: R_2 & R_3 are scalar multiples!

• $R_2 + R_3$ / new R_3 $\rightarrow$ $\left[\begin{array}{ccc|c} 1 & -6 & 7 & g \\ 0 & 18 & -20 & h \\ 0 & 0 & 0 & h+4g+K \end{array}\right]$

Caution: R_3 has the potential to produce a contradiction!

∴ $\boxed{\text{The augmented matrix represents a consistent system IFF : } h + 4g + K = 0}$

Answer.