

Quiz #10 Spr 2018

2. Evaluating $\iint_S (\nabla \times \vec{F}) \cdot \vec{n} dS$ directly

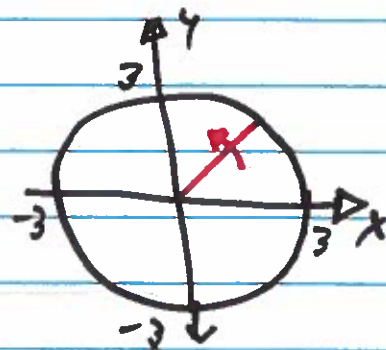
$$\vec{F} = \langle 2x, 3y+x, y+z \rangle \quad \nabla \times \vec{F} = \langle 1, 0, 1 \rangle$$

$$S: z = x^2 + y^2 + 1 \quad 1 \leq z \leq 10$$

$$\iint_S (\nabla \times \vec{F}) \cdot \vec{n} dS = \iint_R (\nabla \times \vec{F}) \cdot \langle -2x, -2y, 1 \rangle dA$$

$$= \iint_R \langle 1, 0, 1 \rangle \cdot \langle -2x, -2y, 1 \rangle dA$$

$$= \iint_R (1 - 2x) dA$$



Use polar coordinates

$$= \int_0^{2\pi} \int_0^3 (1 - 2r \cos \theta) r dr d\theta$$

$$= \int_0^{2\pi} \left[\frac{1}{2} r^2 \right]_0^3 d\theta + \int_0^{2\pi} \cancel{\cos(\theta)} d\theta \left(-\frac{2}{3} r^3 \right) \Big|_0^3$$

$$= 2\pi \left(\frac{9}{2} \right) = 9\pi$$