Student: Phong Vo **Date:** 03/26/20	**Instructor:** Erica Yankowskas **Course:** Linear Algebra I (Spring 2020)	**Assignment:** Section 3.2 Homework

1. State which property of determinants is illustrated in this equation.

$$\begin{vmatrix} 4 & -3 & 1 \\ -12 & 4 & 8 \\ -3 & -6 & 2 \end{vmatrix} = -\begin{vmatrix} -12 & 4 & 8 \\ 4 & -3 & 1 \\ -3 & -6 & 2 \end{vmatrix}$$

Choose the correct answer below.

- **A.** If a multiple of one row of A is added to another row to produce matrix B, then det B = det A.
- **B.** If two rows of A are interchanged to produce B, then det B = − det A.
- **C.** If one row of A is multiplied by k to produce B, then det B = k • det A.
- **D.** If A and B are square matrices, then det AB = (det A)(det B).

2. State which property of determinants is illustrated in this equation.

$$\begin{vmatrix} 8 & 4 & -2 \\ 24 & -5 & -3 \\ 4 & 6 & -5 \end{vmatrix} = \begin{vmatrix} 8 & 4 & -2 \\ 0 & -17 & 3 \\ 4 & 6 & -5 \end{vmatrix}$$

Choose the correct answer below.

- **A.** If A and B are square matrices, then det AB = (det A)(det B).
- **B.** If a multiple of one row of A is added to another row to produce matrix B, then det B = det A.
- **C.** If one row of A is multiplied by k to produce B, then det B = k • det A.
- **D.** If two rows of A are interchanged to produce B, then det B = − det A.

3. Combine the methods of row reduction and cofactor expansion to compute the determinant.

$$\begin{vmatrix} -1 & 3 & 6 & 0 \\ 3 & 3 & 5 & 0 \\ 7 & 6 & 8 & 6 \\ 5 & 3 & 5 & 3 \end{vmatrix}$$

The determinant is 99 .
(Simplify your answer.)

4. Find the determinant below, where $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4$.

$$\begin{vmatrix} a & b & c \\ d & e & f \\ 7g & 7h & 7i \end{vmatrix}$$

$\begin{vmatrix} a & b & c \\ d & e & f \\ 7g & 7h & 7i \end{vmatrix}$ = 28 (Simplify your answer.)

5. If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 19$, find the determinant of $\begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix}$.

The determinant is -19.

6. If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 3$, find $\begin{vmatrix} a & b & c \\ 8d+g & 8e+h & 8f+i \\ g & h & i \end{vmatrix}$.

$\begin{vmatrix} a & b & c \\ 8d+g & 8e+h & 8f+i \\ g & h & i \end{vmatrix} = 24$ (Simplify your answer.)

7. Use determinants to find out if the matrix is invertible.

$$\begin{bmatrix} 5 & 0 & -1 \\ 1 & -3 & -2 \\ 0 & -5 & -3 \end{bmatrix}$$

The determinant of the matrix is 0. (Simplify your answer.)

Is the matrix invertible? Choose the correct answer below.

- ○ The matrix is invertible.
- ● The matrix is not invertible.

8. Use determinants to find out if the matrix is invertible.

$$\begin{bmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ 3 & -1 & -2 & 4 \\ -1 & 2 & 8 & 5 \end{bmatrix}$$

The determinant of the matrix is -3. (Simplify your answer.)

Is the matrix invertible? Choose the correct answer below.

- ● **A.** The matrix is invertible.
- ○ **B.** The matrix is not invertible.

9. Use determinants to decide if the set of vectors is linearly independent.

$$\begin{bmatrix} 3 \\ 5 \\ -6 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -6 \\ 0 \\ 7 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ -2 \end{bmatrix}$$

The determinant of the matrix whose columns are the given vectors is 0 .
(Simplify your answer.)

Is the set of vectors linearly independent? Choose the correct answer below.

- **A.** The set of vectors is linearly independent.
- **B.** The set of vectors is linearly dependent.

10. Compute $\det B^4$ where $B = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 2 & 3 \\ 1 & 2 & 2 \end{bmatrix}$.

$\det B^4 =$ 16 (Simplify your answer.)

11. Show that if A is invertible, then $\det A^{-1} = \frac{1}{\det A}$.

What theorem(s) should be used to examine the quantity $\det A^{-1}$? Select all that apply.

- **A.** If one row of a square matrix A is multiplied by k to produce B, then $\det B = k \cdot (\det A)$.
- **B.** If A is an $n \times n$ matrix, then $\det A^T = \det A$.
- **C.** If A and B are $n \times n$ matrices, then $\det AB = (\det A)(\det B)$.
- **D.** A square matrix A is invertible if and only if $\det A \neq 0$.

Consider the quantity $(\det A)\left(\det A^{-1}\right)$. To what must this be equal?

- **A.** $\det A$
- **B.** $\det A^{-1}$
- **C.** $\det A^2$
- **D.** $\det I$

To what scalar must this new determinant be equal?

1 (Simplify your answer.)

Therefore, why is $\det A^{-1} = \frac{1}{\det A}$?

- **A.** Since $(\det A)\left(\det A^{-1}\right) = \det \left(A^T\right)^2$, $\det A^{-1}$ must be equal to $\det \left(A^T\right)^{-1}$.
- **B.** Since $(\det A)\left(\det A^{-1}\right) = 1$, it follows from algebra that $\det A^{-1} = \frac{1}{\det A}$.
- **C.** Since $(\det A)\left(\det A^{-1}\right) = \det A^2$, the previous theorem states that $\det A^{-1} = \det A$.
- **D.** Since $(\det A)\left(\det A^{-1}\right) = 0$, it follows from algebra that $\det A^{-1} = \frac{1}{\det A}$.

12. Find a formula for det(rA) when A is an $n \times n$ matrix.

Choose the correct answer below.

- **A.** $\det(rA) = r^n \cdot \det A$
- **B.** $\det(rA) = r \cdot \det A$
- **C.** $\det(rA) = \det r \cdot A$
- **D.** $\det(rA) = \det A$