Student: Phong Vo **Date:** 02/06/18	**Instructor:** Chuck Ormsby **Course:** Multi-Variable and Vector Calculus -- Calculus III Spring 2018	**Assignment:** Section 12.3 Homework

1. Define the dot product of **u** and **v** in terms of their magnitudes and the angle between them.

Choose the correct answer below.

- ○ $\mathbf{u} \cdot \mathbf{v} = u_1 \cdot v_1 + u_2 \cdot v_2 + u_3 \cdot v_3$
- ◉ $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}|\ |\mathbf{v}| \cos\theta$ ✔
- ○ $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}| \cos\theta \left(\dfrac{\mathbf{v}}{|\mathbf{v}|}\right)$

2. Define the dot product of **u** and **v** in terms of the components of the vectors.

Choose the correct answer below.

- ○ $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}| \cos\theta \left(\dfrac{\mathbf{v}}{|\mathbf{v}|}\right)$
- ◉ $\mathbf{u} \cdot \mathbf{v} = u_1 \cdot v_1 + u_2 \cdot v_2 + u_3 \cdot v_3$ ✔
- ○ $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}|\ |\mathbf{v}| \cos\theta$

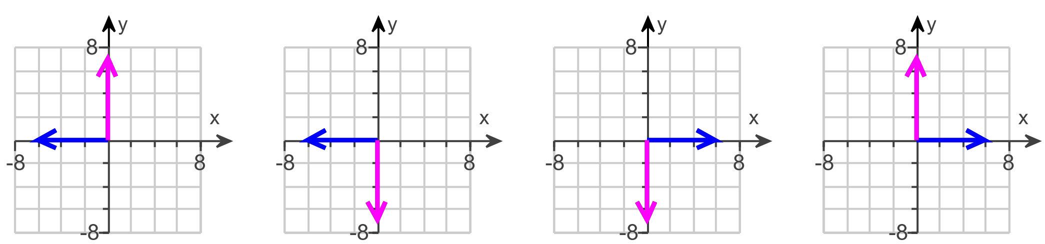
3. Compute $\langle -3,5,-6\rangle \cdot \langle -5,-4,-4\rangle$.

$\langle -3,5,-6\rangle \cdot \langle -5,-4,-4\rangle =$ 19 (Simplify your answer.)

4. Consider the vectors $\mathbf{u} = 6\mathbf{i}$ and $\mathbf{v} = -7\mathbf{j}$. Sketch the vectors, find the angle between the vectors, and compute the dot product using the definition $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}|\ |\mathbf{v}| \cos\theta$.

Sketch the vectors. Choose the correct graph below.

○ **A.** ○ **B.** ◉ **C.** ✔ ○ **D.**



From the sketch, what is the angle between the vectors?

$\theta = \dfrac{\pi}{2}$ (Type an exact answer, using π as needed.)

What is the dot product?

$6\mathbf{i} \cdot -7\mathbf{j} =$ 0 (Simplify your answer.)

5. Given $\mathbf{v} = 2\mathbf{i} + \mathbf{j}$ and $\mathbf{w} = \mathbf{i} + 2\mathbf{j}$
(a) find the dot product $\mathbf{v} \cdot \mathbf{w}$;
(b) find the angle between $\mathbf{v}$ and $\mathbf{w}$.

(a) $\mathbf{v} \cdot \mathbf{w} =$ 4 (Simplify your answer.)

(b) What is the angle between $\mathbf{v}$ and $\mathbf{w}$?

36.9 °
(Do not round until the final answer. Then round to the nearest tenth as needed.)

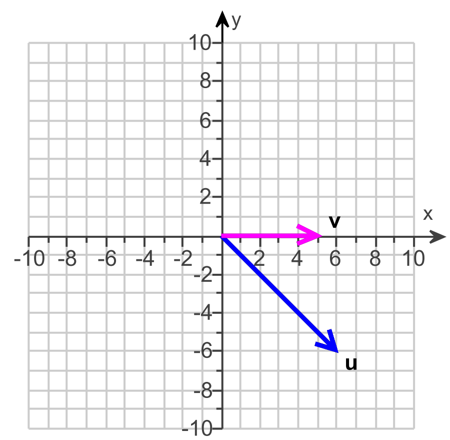
6. Find the dot product $\mathbf{v} \cdot \mathbf{w}$ and the angle between $\mathbf{v}$ and $\mathbf{w}$.

$\mathbf{v} = -\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$, $\mathbf{w} = 4\mathbf{i} + 4\mathbf{j} + \mathbf{k}$

$\mathbf{v} \cdot \mathbf{w} =$ 10
(Simplify your answer. Type an exact value, using radicals as needed.)

The angle between $\mathbf{v}$ and $\mathbf{w}$ is $\theta =$ 67.7 °.
(Do not round until the final answer. Then round to the nearest tenth as needed.)

7. Find $\text{proj}_{\mathbf{v}}\mathbf{u}$ and $\text{scal}_{\mathbf{v}}\mathbf{u}$ by inspection without using formulas.



$\text{proj}_{\mathbf{v}}\mathbf{u} =$ 6i
(Type your answer in terms of i and j.)

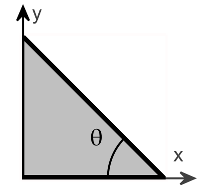
$\text{scal}_{\mathbf{v}}\mathbf{u} =$ 6

8. For the vectors $\mathbf{u} = \langle -4,5 \rangle$ and $\mathbf{v} = \langle -5,2 \rangle$, calculate $\text{proj}_{\mathbf{v}}\mathbf{u}$ and $\text{scal}_{\mathbf{v}}\mathbf{u}$.

$\text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle -\frac{150}{29}, \frac{60}{29} \right\rangle$

$\text{scal}_{\mathbf{v}}\mathbf{u} = \frac{30}{\sqrt{29}}$
(Type an exact answer, using radicals as needed.)

9. Find the components of the vertical force $\mathbf{F} = \langle 0, -4 \rangle$ in the directions parallel to and normal to the plane that makes an angle of $\theta = \tan^{-1}(1)$ with the positive x-axis as shown. Show that the total force is the sum of the two component forces.



What is the component of the force parallel to the plane?

$\langle$ __________ , __________ $\rangle$

What is the component of the force perpendicular to the plane?

$\langle$ __________ , __________ $\rangle$

Find the sum of these two forces.

$\langle$ __________ , __________ $\rangle$

10. Find three mutually orthogonal unit vectors in $\mathbf{R}^3$ besides $\pm\mathbf{i}$, $\pm\mathbf{j}$, and $\pm\mathbf{k}$.

There are multiple ways to do this and an infinite number of answers. For this problem, we choose a first vector **u** randomly, choose all but one component of a second vector **v** randomly, and choose the first component of a third vector **w** randomly. The other components x, y, and z are chosen so that **u**, **v**, and **w** are mutually orthogonal. Then unit vectors are found based on **u**, **v**, and **w**.

Start with $\mathbf{u} = \langle 1,0,3 \rangle$, $\mathbf{v} = \langle x,-1,1 \rangle$, and $\mathbf{w} = \langle 1,y,z \rangle$.

The unit vector based on **u** is $\langle$ __________ , __________ , __________ $\rangle$.
(Type exact answers, using radicals as needed.)

The unit vector based on **v** is $\langle$ __________ , __________ , __________ $\rangle$.
(Type exact answers, using radicals as needed.)

The unit vector based on **w** is $\langle$ __________ , __________ , __________ $\rangle$.
(Type exact answers, using radicals as needed.)