HW8

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X_{kl} : Indicator random variable associated with the event that two dinstinct keys k and l hash to the same slot.

$X_{kl} = I\{ k \neq l \text{ and } h(k) = h(l) \}$

$E[X_{kl}] = \Pr\{X_{kl}\}$

Assuming uniform hasing (i.e. each key is equally likely to be hashed to any slot of the table, independent of where other keys are hashing), then

$\Pr\{X_{kl}\} = \frac{1}{m}$

because once a key is hashed to a slot, the probability of a second key choosing that same slot, out of m possible slots, is $1/m$.

X : number of collisions

$$X = \sum_{k=1}^{n-1} \sum_{l=k+1}^{n} X_{kl}$$

$$E[X] = E\left[\sum_{k=1}^{n-1} \sum_{l=k+1}^{n} X_{kl}\right]$$

$$= \sum_{k=1}^{n-1} \sum_{l=k+1}^{n} E[X_{kl}]$$

$$= \sum_{k=1}^{n-1} \sum_{l=k+1}^{n} \frac{1}{m}$$

$$= \frac{1}{m} \sum_{k=1}^{n-1} \sum_{l=k+1}^{n} 1$$

$$= \frac{1}{m} \sum_{k=1}^{n-1} (n-k)$$

$$= \frac{1}{m}\left(\frac{1}{2}(n-1)n\right) = \frac{n(n-1)}{2m}$$

2(1)

n : number of elements

m : number of slots

$h(k) = h'(k) \bmod m$ **Using chaining**

$h(k) = k^2 \bmod 5$

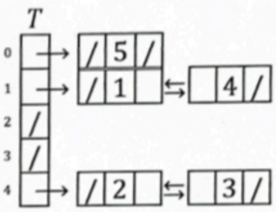
$h(3) = (3^2 \bmod 5) = 4$

$h(4) = (4^2 \bmod 5) = 1$

$h(2) = (2^2 \bmod 5) = 4$

$h(5) = (5^2 \bmod 5) = 0$

$h(1) = (1^2 \bmod 5) = 1$



$\alpha = \frac{n}{m} = \frac{5}{5} = 1$

(2) Using $h(k)$ as the primary hash function, illustrate the result of inserting these keys using <u>open addressing</u> with <u>linear probing</u>.

$h(k,i) = (h'(k) + i) \bmod m$

$h(k,i) = (k^2 + i) \bmod 5$

$h(3,0) = [(3^2 + 0) \bmod 5] = 4$

$h(4,0) = [(4^2 + 0) \bmod 5] = 1$

$h(2,0) = [(2^2 + 0) \bmod 5] = 4$ (collision)

$h(2,1) = [(2^2 + 1) \bmod 5] = 0$

$h(5,0) = [(5^2 + 0) \bmod 5] = 0$ (collision)

$h(5,1) = [(5^2 + 1) \bmod 5] = 1$ (collision)

$h(5,2) = [(5^2 + 2) \bmod 5] = 2$

$h(1,0) = [(1^2 + 0) \bmod 5] = 1$ (collision)

$h(1,1) = [(1^2 + 1) \bmod 5] = 2$ (collision)

$h(1,2) = [(1^2 + 2) \bmod 5] = 3$

	T
0	2
1	4
2	5
3	1
4	3

2(3)

$$h(k,i) = (h'(k) + c_1 i + c_2 i^2) \bmod m$$

$$h(k,i) = (k^2 + i + 2i^2) \bmod 5$$

Open addressing using quadratic probing

$h(3,0) = (3^2 + 0 + 0) \bmod 5 = 4$

$h(4,0) = (4^2 + 0 + 0) \bmod 5 = 1$

$h(2,0) = (2^2 + 0 + 0) \bmod 5 = 4$ (collision)

$h(2,1) = (2^2 + 1 + 2) \bmod 5 = 2$

$h(5,0) = (5^2 + 0 + 0) \bmod 5 = 0$

$h(1,0) = (1^2 + 0 + 0) \bmod 5 = 1$ (collision)

$h(1,1) = (1^2 + 1 + 2) \bmod 5 = 4$ (collision)

$h(1,2) = (1^2 + 2 + 8) \bmod 5 = 1$ (collision)

$h(1,3) = (1^2 + 3 + 18) \bmod 5 = 2$ (collision)

$h(1,4) = (1^2 + 4 + 32) \bmod 5 = 2$ (collision)

$h(1,0) = (1^2 + 0 + 0) \bmod 5 = 1$ (collision)

$\vdots$

No spot for $k = 1$

	T
0	5
1	4
2	2
3	
4	3

(4) What different values can the hash function $h(k) = k^2 \bmod m$ produce when $m = 11$? Carefully justify your answer in detail.

$0 \bmod 11 = 0$
$1 \bmod 11 = 1$
$4 \bmod 11 = 4$
$9 \bmod 11 = 9$
$16 \bmod 11 = 5$
$25 \bmod 11 = 3$
$36 \bmod 11 = 3$
$49 \bmod 11 = 5$
$64 \bmod 11 = 9$
$81 \bmod 11 = 4$
$100 \bmod 11 = 1$

$121 \bmod 11 = 0$
$144 \bmod 11 = 1$
$169 \bmod 11 = 4$
$196 \bmod 11 = 9$
$225 \bmod 11 = 5$
$256 \bmod 11 = 3$
$289 \bmod 11 = 3$
$324 \bmod 11 = 5$
$361 \bmod 11 = 9$
$400 \bmod 11 = 4$
$441 \bmod 11 = 1$

The pattern will continue repeating,

$\therefore \Rightarrow \langle 0,1,3,4,5,9 \rangle$

3.

```
h(k)
    return s − k

PAIR-EQUALS-SUM(A, n, s)
    Let T[0..s + 1] be a Chained Hash Table with          O(1)
    for i = 1 to n                                          O(n)
        if A[i] ≤ s                                         O(n) · O(1)
            CHAINED-HASH-INSERT(T, A[i])                    O(n) · O(1)
    for i = 1 to n                                          O(n)
        if A[i] ≤ s                                         O(n) · O(1)
            if CHAINED-HASH-SEARCH(T, s − A[i])             O(n) · O(1)
                return TRUE                                 O(n) · O(1)
    return FALSE                                            O(1)
```

Insert only elements less than the sum into hash table

Iterate through array, looking only at elements smaller than sum. Use the Hash to find an element that, when added to the current, yields the sum

(2) Justify the running time of the algorithm (5 points).

All CHAINED-HASH operations take $O(1)$ time.

$T(n) = O(n)$