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1. (5 points) Given the following input (3412, 3413, 1741, 3269, 2909, 6291, 6373, 5129) and the hash function $h(k) = k \bmod 10$, which of the following statement(s) are true? Choose all correct ones.

(A) 3269, 2909, 5129 hash to the same value $\text{output} = 9$

B. 3412 and 3413 hash to the same value

(C) 1741, 6291 hash to the same value $\text{output} = 1$

D. 3413, 3269, 6291, 6373 each hashes to a different value

2. (5 points) The keys 14, 18, 33, 4, 3, 23, 25 and 5 are inserted into an initially empty hash table in this given order. The hash table has 10 slots and uses chaining with hash function $h(k) = k \bmod 10$. What is the hash table after inserting all keys? (multiple numbers in the same slot represents a linked list to chain the numbers together in that order)

0	
1	
2	
3	3, 23, 33
4	4, 14
5	5, 25
6	
7	
8	18
9	

A

0	
1	
2	
3	23, 3, 33 ✓
4	4, 14 ✓
5	5, 25 ✓
6	
7	
8	18 ✓
9	

(B)

0	
1	
2	
3	33, 23, 3
4	14, 4
5	25, 5
6	
7	
8	18
9	

C

0	
1	
2	
3	33, 3, 23
4	14, 4
5	25, 5
6	
7	
8	18
9	

D

$$h(14) = h(4) = 4$$

$$h(18) = 8$$

$$h(33) = h(3) = h(23) = 3$$

$$h(25) = h(5) = 5$$

3. (5 points) The keys 14, 18, 33, 4, 3, 23, 25 and 5 are inserted into an initially empty hash table in this given order. The hash table has 10 slots and uses open addressing with hash function $h(k) = k \bmod 10$ and linear probing. What is the hash table after inserting all keys?

$$h(23,4) = (23+4) \bmod 10 = 7$$

$$h(25,0) = (25+0) \bmod 10 = 5 \text{ (collision)}$$

$$h(25,1) = (25+1) \bmod 10 = 6 \text{ (collision)}$$

$$h(25,2) = (25+2) \bmod 10 = 7 \text{ (collision)}$$

$$h(25,3) = (25+3) \bmod 10 = 8 \text{ (collision)}$$

$$h(25,4) = (25+4) \bmod 10 = 9 \text{ A}$$

0	
1	
2	
3	23
4	4
5	5
6	
7	
8	18
9	

0	
1	
2	5
3	33
4	14
5	4
6	3
7	23
8	18
9	25

0	
1	
2	
3	33
4	14
5	25
6	
7	
8	18
9	

0	5
1	
2	
3	33
4	14
5	4
6	3
7	23
8	18
9	25

$h(5,0) = (5+0) \bmod 10 = 5 \text{ (collision)}$
 $h(5,1) = (5+1) \bmod 10 = 6 \text{ (collision)}$
 $h(5,2) = (5+2) \bmod 10 = 7 \text{ (collision)}$
 $h(5,3) = (5+3) \bmod 10 = 8 \text{ (collision)}$
 $h(5,4) = (5+4) \bmod 10 = 9 \text{ (collision)}$
 $h(5,5) = (5+5) \bmod 10 = 0$

$h(14,0) = (14+0) \bmod 10 = 4$
 $h(18,0) = (18+0) \bmod 10 = 8$
 $h(33,0) = (33+0) \bmod 10 = 3$
 $h(4,0) = (4+0) \bmod 10 = 4 \text{ (collision)}$
 $h(4,1) = (4+1) \bmod 10 = 5$
 $h(3,0) = (3+0) \bmod 10 = 3 \text{ (collision)}$

$h(3,1) = (3+1) \bmod 10 = 4 \text{ (collision)}$
 $h(3,2) = (3+2) \bmod 10 = 5 \text{ (collision)}$
 $h(3,3) = (3+3) \bmod 10 = 6$
 $h(23,0) = 23 \bmod 10 = 3 \text{ (collision)}$
 $h(23,1) = (23+1) \bmod 10 = 4 \text{ (collision)}$
 $h(23,2) = (23+2) \bmod 10 = 5 \text{ (collision)}$
 $h(23,3) = (23+3) \bmod 10 = 6 \text{ (collision)}$

4. (5 points) (1) What is the load factor in Problem 2 above?

$$\alpha = \frac{n}{m} = \frac{8}{10} = .8$$

(2) What is the load factor in Problem 3 above?

$$\alpha = \frac{n}{m} = \frac{8}{10} = .8$$

4. (20 points) Design and Analysis of an Algorithm

Consider an unsorted array A of n integers; design an efficient algorithm that accepts A , n and s as the inputs and determines if the array contains two integers such that they add up to a specific target number s . That is: if we can find $A[i] + A[j] == s$ ($1 \leq i, j \leq n$, $i \neq j$), the algorithm should return TRUE, otherwise return FALSE.

Design requirement:

- the *efficient* algorithm you are going to design should provide an $O(n \lg n)$ running time, rather than an $O(n^2)$ running-time solution.
- To keep your answers brief, you may use any algorithms that we have learned from lectures and the textbook as subroutines (this means you do NOT need to re-write those algorithms, just call them with the proper input/output).

mergesort
heap sort

(1) (12 points) Algorithm Pseudocode (please use textbook conventions):

Sum-seeking (A, n, s)

// sort first

MERGE_SORT($A, 1, n$) ✓ } $T(n) = O(n \lg n)$

-3

for ($i = 1$ to n)

for ($j = i$ to n)

if ($A[i] + A[j] == s$)

return TRUE

return FALSE

$T(n) = O(n)$

this is $O(n^2)$
not $O(n)$

Seeking $(s - A[i])$ vs. ~~every~~ element. If $\begin{cases} ==, \rightarrow \text{return TRUE} \\ \neq, \rightarrow \text{return FALSE} \end{cases}$



(2) (8 points) What is the running time of the algorithm that you designed? Justify your answer.

The total running time

$$T(n) = \cancel{O(n)}^X + O(n \lg n)^{\checkmark}$$

$$\Rightarrow T(n) = \cancel{O(n \lg n)}^X \quad -5$$

32/40