Spring 2020

Solution

Name:

Linear Algebra: Quiz 2

Show ALL work, as unjustified answers may receive no credit. Calculators are not allowed on any quiz or test paper. Make sure to exhibit skills discussed in class. Box all answers and clean up answers as much as possible.

1. [5pts] Is $\vec{b} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$ a linear combination of the vectors $\vec{a_1} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $\vec{a_2} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $\vec{a_3} = \begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix}$?

*Row-Reduce the Augmented Matrix, $[\vec{a}_1\ \vec{a}_2\ \vec{a}_3 \mid \vec{b}]$:

$$\left[\begin{array}{ccc|c} 0 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & 2 & -2 & 3 \end{array}\right] \overset{R_1 \leftrightarrow R_2}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & -1 & -1 \\ 0 & 1 & -1 & 1 \\ 1 & 2 & -2 & 3 \end{array}\right] \underset{\text{NEW } R_3}{\overset{-R_1 + R_3}{\sim}} \left[\begin{array}{ccc|c} 1 & 1 & -1 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & 4 \end{array}\right] \underset{\text{NEW } R_3}{\overset{-R_2 + R_3}{\sim}}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 3 \end{array}\right] \quad 0 \neq 3 \quad \rightarrow\leftarrow$$

*Note: The last augmented matrix corresponds to an inconsistent matrix (NO Solution).

$$\boxed{\therefore \text{The Vector Eq. } x_1\vec{a}_1 + x_2\vec{a}_2 + x_3\vec{a}_3 = \vec{b} \text{ has NO solution} \Rightarrow \vec{b} \text{ is NOT a linear combination of } \vec{a}_1, \vec{a}_2, \vec{a}_3.}$$

Answer.

2. [5pts] For the following vectors, what value(s) of h is $\vec{y} \in span\{\vec{v_1}, \vec{v_2}\}$:

$$\vec{v_1} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \quad \vec{v_2} = \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}, \quad \vec{y} = \begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix}$$

Note:

To answer this, we need to solve the Vector Eq.: $x_1\vec{v}_1 + x_2\vec{v}_2 = \vec{y}$:)

*Row-Reduce the Augmented Matrix, $[\vec{v}_1\ \vec{v}_2 \mid \vec{y}]$:

$$\left[\begin{array}{cc|c} 1 & -3 & h \\ 0 & 1 & -5 \\ -2 & 8 & -3 \end{array}\right] \underset{\text{NEW } R_3}{\overset{2R_1 + R_3}{\sim}} \left[\begin{array}{cc|c} 1 & -3 & h \\ 0 & 1 & -5 \\ 0 & 2 & 2h-3 \end{array}\right] \underset{\text{NEW } R_3}{\overset{-2R_2 + R_3}{\sim}} \left[\begin{array}{cc|c} 1 & -3 & h \\ 0 & 1 & -5 \\ 0 & 0 & 2h+7 \end{array}\right]$$

*Note: Since we want $\vec{y} \in span\{\vec{v}_1, \vec{v}_2\}$, we want a consistent system.

$$\boxed{\begin{array}{l} \therefore \text{The Linear System is consistent when } 2h+7=0. \\ \Rightarrow \vec{y} \in span\{\vec{v}_1, \vec{v}_2\} \text{ when } h = -7/2 \end{array}}$$

Answer.