Section 8.5 HW / Discrete Structures II / Fall 2018

Note: For problems 1 and 2, you can use Venn diagrams to visualize your answer, but be able to use the principle of inclusion-exclusion.

1. Set up the principle of inclusion-exclusion for two sets A_1, A_2. Then use your expression to answer the following questions:

   How many elements are there in $A_1 \cup A_2$ if there are 12 elements in A_1, 18 elements in A_2, and

   (a) $A_1 \cap A_2 = \varnothing$?

   (b) $|A_1 \cap A_2| = 1$?

   (c) $|A_1 \cap A_2| = 6$?

   (d) $A_1 \subseteq A_2$?

2. A language club contains 54 members, all of whom are taking French, German, or Spanish. The following information is known:

   - 27 students are taking French;
   - 22 students are taking German;
   - 28 students are taking Spanish;
   - 7 are taking both French and German;
   - 9 are taking both German and Spanish;
   - 4 are taking all three languages.

   Let F be the set of French students, let G be the set of German students, and let S be the set of Spanish students. Set up the principle of inclusion-exclusion for the three sets F, G, S. Then use your expression to determine how many students are taking both French and Spanish.

3. Write down the principle of inclusion-exclusion for the sets Q, R, S, T.

4. How many elements are in the union of four sets if each of the sets have 50, 60, 70, and 80 elements, each pair of the sets has 5 elements in common, each triple of sets has 1 common element, and no element is in all four sets?

Answers:

1. (a) 30
   (b) 29
   (c) 24
   (d) 18
2. 11
3. $|Q \cup R \cup S \cup T| = |Q| + |R| + |S| + |T| - |Q \cap R| - |Q \cap S| - |Q \cap T| - |R \cap S| - |R \cap T| - |S \cap T|$
   $+|Q \cap R \cap S| + |Q \cap R \cap T| + |Q \cap S \cap T| + |R \cap S \cap T| - |Q \cap R \cap S \cap T|$
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