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1.

Compute each matrix sum or product if it is defined. If an expression is undefined, explain why. Let $A = \begin{bmatrix} 1 & 0 & -2 \\ 4 & -4 & 2 \end{bmatrix}$,

$B = \begin{bmatrix} 6 & -5 & 2 \\ 1 & -3 & -3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$, and $D = \begin{bmatrix} 3 & 5 \\ -1 & 5 \end{bmatrix}$.

$-3A$, $B-3A$, $AC$, $CD$

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Compute the matrix product $-3A$. Select the correct choice below and, if necessary, fill in the answer box within your choice.

- **A.** $-3A = \begin{bmatrix} -3 & 0 & 6 \\ -12 & 12 & -6 \end{bmatrix}$
  (Simplify your answer.)
- **B.** The expression $-3A$ is undefined because matrices cannot have negative coefficients.
- **C.** The expression $-3A$ is undefined because A is not a square matrix.
- **D.** The expression $-3A$ is undefined because matrices cannot be multiplied by numbers.

Compute the martrix sum $B-3A$. Select the correct choice below and, if necessary, fill in the answer box within your choice.

- **A.** $B-3A = \begin{bmatrix} 3 & -5 & 8 \\ -11 & 9 & -9 \end{bmatrix}$
  (Simplify your answer.)
- **B.** The expression $B-3A$ is undefined because B and $-3A$ have different sizes.
- **C.** The expression $B-3A$ is undefined because A is not a square matrix.
- **D.** The expression $B-3A$ is undefined because B and A have different sizes.

Compute the matrix product AC. Select the correct choice below and, if necessary, fill in the answer box within your choice.

- **A.** AC =
  (Simplify your answer.)
- **B.** The expression AC is undefined because the number of columns in A is not equal to the number of rows in C.
- **C.** The expression AC is undefined because the number of rows in A is not equal to the number of rows in C.
- **D.** The expression AC is undefined because the number of rows in A is not equal to the number of columns in C.

Compute the matrix product CD. Select the correct choice below and, if necessary, fill in the answer box within your choice.

- **A.** $CD = \begin{bmatrix} 0 & 20 \\ -7 & -5 \end{bmatrix}$
  (Simplify your answer.)
- **B.** The expression CD is undefined because the corresponding entries in C and D are not equal.
- **C.** The expression CD is undefined because square matrices cannot be multiplied.
- **D.** The expression CD is undefined because matrices with negative entries cannot be multiplied.

2.

Compute $A - 2I_3$ and $(2I_3)A$, where $A = \begin{bmatrix} 5 & -1 & 2 \\ -3 & 2 & -5 \\ -2 & 1 & 2 \end{bmatrix}$.

$A - 2I_3 = \begin{bmatrix} 3 & -1 & 2 \\ -3 & 0 & -5 \\ -2 & 1 & 0 \end{bmatrix}$

$(2I_3)A = \begin{bmatrix} 10 & -2 & 4 \\ -6 & 4 & -10 \\ -4 & 2 & 4 \end{bmatrix}$

3. Compute the product AB by the definition of the product of matrices, where $A\mathbf{b}_1$ and $A\mathbf{b}_2$ are computed separately, and by the row-column rule for computing AB.

$$A = \begin{bmatrix} -1 & 3 \\ 2 & 5 \\ 4 & -2 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix}$$

Set up the product $A\mathbf{b}_1$, where $\mathbf{b}_1$ is the first column of B.

$$A\mathbf{b}_1 = \begin{bmatrix} -1 & 3 \\ 2 & 5 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

(Use one answer box for A and use the other answer box for $\mathbf{b}_1$.)

Calculate $A\mathbf{b}_1$, where $\mathbf{b}_1$ is the first column of B.

$$A\mathbf{b}_1 = \begin{bmatrix} -7 \\ 3 \\ 18 \end{bmatrix}$$

(Type an integer or decimal for each matrix element.)

Set up the product $A\mathbf{b}_2$, where $\mathbf{b}_2$ is the second column of B.

$$A\mathbf{b}_2 = \begin{bmatrix} -1 & 3 \\ 2 & 5 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

(Use one answer box for A and use the other answer box for $\mathbf{b}_2$.)

Calculate $A\mathbf{b}_2$, where $\mathbf{b}_2$ is the second column of B.

$$A\mathbf{b}_2 = \begin{bmatrix} 13 \\ 18 \\ -12 \end{bmatrix}$$

(Type an integer or decimal for each matrix element.)

Determine the numerical expression for the first entry in the first column of AB using the row-column rule. Choose the correct answer below.

- **A.** $-1(4) + 3(-1)$
- **B.** $-1(4) - 3(-1)$
- **C.** $((-1) + (4)) \cdot ((3) + (-1))$
- **D.** $((-1) - (4)) \cdot ((3) - (-1))$

Determine the product AB.

$$AB = \begin{bmatrix} -7 & 13 \\ 3 & 18 \\ 18 & -12 \end{bmatrix}$$

(Use integers or decimals for any numbers in the expression.)

4. Compute the product AB by the definition of the product of matrices, where $A\mathbf{b}_1$ and $A\mathbf{b}_2$ are computed separately, and by the row-column rule for computing AB.

$$A = \begin{bmatrix} -2 & 3 \\ 3 & 4 \\ 4 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & -1 \\ -3 & 4 \end{bmatrix}$$

Set up the product $A\mathbf{b}_1$, where $\mathbf{b}_1$ is the first column of B.

$$A\mathbf{b}_1 = \begin{bmatrix} -2 & 3 \\ 3 & 4 \\ 4 & -4 \end{bmatrix} \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

(Use one answer box for A and use the other answer box for $\mathbf{b}_1$.)

Calculate $A\mathbf{b}_1$, where $\mathbf{b}_1$ is the first column of B.

$$A\mathbf{b}_1 = \begin{bmatrix} -19 \\ 3 \\ 32 \end{bmatrix}$$

(Type an integer or decimal for each matrix element.)

Set up the product $A\mathbf{b}_2$, where $\mathbf{b}_2$ is the second column of B.

$$A\mathbf{b}_2 = \begin{bmatrix} -2 & 3 \\ 3 & 4 \\ 4 & -4 \end{bmatrix} \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

(Use one answer box for A and use the other answer box for $\mathbf{b}_2$.)

Calculate $A\mathbf{b}_2$, where $\mathbf{b}_2$ is the second column of B.

$$A\mathbf{b}_2 = \begin{bmatrix} 14 \\ 13 \\ -20 \end{bmatrix}$$

(Type an integer or decimal for each matrix element.)

Determine the numerical expression for the first entry in the first column of AB using the row-column rule. Choose the correct answer below.

- **A.** $((-2)-(5)) \cdot ((3)-(-3)$
- **B.** $((-2)+(5)) \cdot ((3)+(-3)$
- **C.** $-2(5)+3(-3)$
- **D.** $-2(5)-3(-3)$

Determine the product AB.

$$AB = \begin{bmatrix} -19 & 14 \\ 3 & 13 \\ 32 & -20 \end{bmatrix}$$

(Use integers or decimals for any numbers in the expression.)

5. If a matrix A is $5 \times 5$ and the product AB is $5 \times 6$, what is the size of B?

The size of B is $5 \times 6$.

6. How many rows does B have if BC is a $9 \times 3$ matrix?

Matrix B has 9 rows.

7. Let $A = \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 8 \\ -4 & k \end{bmatrix}$. What value(s) of k, if any, will make $AB = BA$?

Select the correct choice below and, if necessary, fill in the answer box within your choice.

- **A.** $k = -2$ (Use a comma to separate answers as needed.)
- **B.** No value of k will make $AB = BA$

8.

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 6 & 5 \\ 1 & 5 & 7 \end{bmatrix}$ and $D = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. Compute AD and DA. Explain how the columns or rows of A change when A is multiplied by D on the right or on the left. Find a $3 \times 3$ matrix B, not the identity matrix or zero matrix, such that AB = BA.

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Compute AD.

$$AD = \begin{bmatrix} 7 & 3 & 2 \\ 7 & 18 & 10 \\ 7 & 15 & 14 \end{bmatrix}$$

Compute DA.

$$DA = \begin{bmatrix} 7 & 7 & 7 \\ 3 & 18 & 15 \\ 2 & 10 & 14 \end{bmatrix}$$

Explain how the columns or rows of A change when A is multiplied by D on the right or on the left. Choose the correct answer below.

- **A.** Right-multiplication (that is, multiplication on the right) by the diagonal matrix D multiplies each column of A by the corresponding diagonal entry of D. Left-multiplication by D multiplies each row of A by the corresponding diagonal entry of D.
- **B.** Both right-multiplication (that is, multiplication on the right) and left-multiplication by the diagonal matrix D multiplies each column entry of A by the corresponding diagonal entry of D.
- **C.** Both right-multiplication (that is, multiplication on the right) and left-multiplication by the diagonal matrix D multiplies each row entry of A by the corresponding diagonal entry of D.
- **D.** Right-multiplication (that is, multiplication on the right) by the diagonal matrix D multiplies each row of A by the corresponding diagonal entry of D. Left-multiplication by D multiplies each column of A by the corresponding diagonal entry of D.

Find a $3 \times 3$ matrix B, not the identity matrix or zero matrix, such that AB = BA. Choose the correct answer below.

- **A.** There is only one unique solution, $B = \begin{bmatrix} \; & \; & \; \\ \; & \; & \; \\ \; & \; & \; \end{bmatrix}$.

  (Simplify your answers.)
- **B.** There are infinitely many solutions. Any multiple of $I_3$ will satisfy the expression.
- **C.** There does not exist a matrix, B, that will satisfy the expression.

9. Suppose the second column of B is the sum of the fourth and fifth columns. What can be said about the second column of AB? Why?

What can be said about the second column of AB? Why?

A. The second column of AB is the sum of the fourth and fifth columns of B. If B is $\begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \cdots & \mathbf{b}_p \end{bmatrix}$, then the second column of AB is $A\mathbf{b}_2$ by definition. It is given that $\mathbf{b}_2 = \mathbf{b}_4 - \mathbf{b}_5$. By matrix-vector multiplication, $A\mathbf{b}_2 = (\mathbf{b}_4 - \mathbf{b}_5) = \mathbf{b}_4 - \mathbf{b}_5$.

B. The second column of AB is the sum of the fourth and fifth columns of AB. If B is $\begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \cdots & \mathbf{b}_p \end{bmatrix}$, then the second column of AB is $A\mathbf{b}_2$ by definition. It is given that $\mathbf{b}_2 = \mathbf{b}_4 - \mathbf{b}_5$. By matrix-vector multiplication, $A\mathbf{b}_2 = A(\mathbf{b}_4 - \mathbf{b}_5) = A\mathbf{b}_4 - A\mathbf{b}_5$.

C. The second column of AB is the sum of the fourth and fifth columns of AB. If B is $\begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \cdots & \mathbf{b}_p \end{bmatrix}$, then the second column of AB is $A\mathbf{b}_2$ by definition. It is given that $\mathbf{b}_2 = \mathbf{b}_4 + \mathbf{b}_5$. By matrix-vector multiplication, $A\mathbf{b}_2 = A(\mathbf{b}_4 + \mathbf{b}_5) = A\mathbf{b}_4 + A\mathbf{b}_5$.

D. The second column of AB is the sum of the fourth and fifth columns of B. If B is $\begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \cdots & \mathbf{b}_p \end{bmatrix}$, then the second column of AB is $A\mathbf{b}_2$ by definition. It is given that $\mathbf{b}_2 = \mathbf{b}_4 + \mathbf{b}_5$. By matrix-vector multiplication, $A\mathbf{b}_2 = (\mathbf{b}_4 + \mathbf{b}_5) = \mathbf{b}_4 + \mathbf{b}_5$.

10. Let A be an $m \times n$ matrix, and let B and C have sizes for which the indicated sums and products are defined. Prove that $A(B + C) = AB + AC$ and that $(B + C)A = BA + CA$. Use the row-column rule. The (i,j)-entry in $A(B + C)$ can be written in either of the two ways below.

$$a_{i1}(b_{1j} + c_{1j}) + \ldots + a_{in}(b_{nj} + c_{nj}) \quad \text{or} \quad \sum_{k=1}^{n} a_{ik}(b_{kj} + c_{kj})$$

Prove that $A(B + C) = AB + AC$. Choose the correct answer below.

A. The (i,j)-entry of $A(B + C)$ equals the (i,j)-entry of $AB + AC$, because

$$\sum_{k=1}^{n} a_{ik}(b_{kj} + c_{kj}) = \sum_{k=1}^{n} a_{ik}b_{kj} - \sum_{k=1}^{n} a_{ik}c_{kj}.$$

B. The (i,j)-entry of $A(B + C)$ equals the (i,j)-entry of $AB + AC$, because

$$\sum_{k=1}^{n} a_{ik}(b_{kj} + c_{kj}) = \sum_{k=1}^{n} a_{ik}b_{kj} + \sum_{k=1}^{n} a_{ik}c_{kj}.$$

C. The (i,j)-entry of $A(B + C)$ equals the (i,j)-entry of $AB + AC$, because

$$\sum_{k=1}^{n} a_{ik}(b_{kj} + c_{kj}) = \sum_{k=1}^{n} a_{ki}b_{jk} + \sum_{k=1}^{n} a_{ki}c_{jk}.$$

Prove that $(B + C)A = BA + CA$. Choose the correct answer below.

A. The (i,j)-entry of $(B + C)A$ equals the (i,j)-entry of $BA + CA$, because

$$\sum_{k=1}^{n} (b_{ik} + c_{ik})a_{kj} = \sum_{k=1}^{n} b_{ik}a_{kj} - \sum_{k=1}^{n} c_{ik}a_{kj}.$$

B. The (i,j)-entry of $(B + C)A$ equals the (i,j)-entry of $BA + CA$, because

$$\sum_{k=1}^{n} (b_{ik} + c_{ik})a_{kj} = \sum_{k=1}^{n} b_{ki}a_{jk} + \sum_{k=1}^{n} c_{ki}a_{jk}.$$

C. The (i,j)-entry of $(B + C)A$ equals the (i,j)-entry of $BA + CA$, because

$$\sum_{k=1}^{n} (b_{ik} + c_{ik})a_{kj} = \sum_{k=1}^{n} b_{ik}a_{kj} + \sum_{k=1}^{n} c_{ik}a_{kj}.$$

11. Prove the theorem $(AB)^T = B^T A^T$. [Hint: Consider the ith row of $(AB)^T$.]

Complete the first step of the proof by filling in the blank.

The (i,j)-entry of $(AB)^T$ is the (j,i)-entry of AB, which is $a_{j1} b_{1i} + \dots + a_{jn} b_{ni}$.

Complete the second step of the proof by filling in the blank.

The entries in row i of $B^T$ are $b_{1i}, \dots, b_{ni}$.

Complete the third step of the proof by filling in the blank.

The entries in column j of $A^T$ are $a_{j1}, \dots, a_{jn}$.

Complete the fourth step of the proof by filling in the blank.

The (i,j)-entry in $B^T A^T$ is $a_{j1} b_{1i} + \dots + a_{jn} b_{ni}$.

Write a conclusion by filling in the blank.

Therefore, $(AB)^T = B^T A^T$.