Student: Phong Vo **Date:** 04/19/18	**Instructor:** Chuck Ormsby **Course:** Multi-Variable and Vector Calculus -- Calculus III Spring 2018	**Assignment:** Section 15.5 Homework

1. What is the value of $\nabla \cdot (\nabla \times \mathbf{F})$?

Choose the correct answer below.

- **A.** $f_x + g_y + h_z$
- **B.** $(h_y - g_z)\mathbf{i} + (f_z - h_x)\mathbf{j} + (g_x - f_y)\mathbf{k}$
- **C.** $-\mathbf{F}$
- **D.** $\mathbf{F}$
- **E.** $\frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial g}{\partial y}\mathbf{j} + \frac{\partial h}{\partial z}\mathbf{k}$
- ✔ **F.** 0

2. Find the divergence of the following vector field.

$$\mathbf{F} = \langle 6x^2 - 6y^2, y^2 - z^2, 2z^2 - 2x^2 \rangle$$

The divergence of **F** is $12x + 2y + 4z$.

3. Find the divergence of the following vector field.

$$\mathbf{F} = \frac{\langle x,y,z \rangle}{3 + 5x^2 + y^2}$$

The divergence of **F** is $\frac{5x^2 + y^2 + 9}{(3 + 5x^2 + y^2)^2}$.

4. Calculate the divergence of the following radial field. Express the result in terms of the position vector **r** and its length $|\mathbf{r}|$.

$$\mathbf{F} = \frac{\langle x,y,z \rangle}{x^2 + y^2 + z^2} = \frac{\mathbf{r}}{|\mathbf{r}|^2}$$

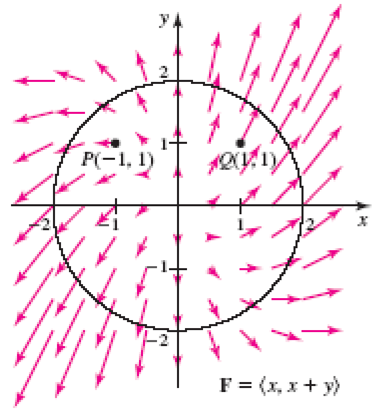
Choose the correct answer below.

- **A.** The divergence of **F** is $\frac{-1}{|\mathbf{r}|^2}$.
- **B.** The divergence of **F** is $\frac{-2}{|\mathbf{r}|^4}$.
- ✔ **C.** The divergence of **F** is $\frac{1}{|\mathbf{r}|^2}$.
- **D.** The divergence of **F** is 0.

5. Consider the following vector field, the circle C, and two points P and Q.

$\mathbf{F} = \langle x, x+y \rangle$

a. Without computing the divergence, does the graph suggest that the divergence is positive or negative at P and Q?
b. Compute the divergence and confirm your conjecture in part (a).
c. On what part of C is the flux outward? Inward?
d. Is the net flux across C positive or negative?



a. The graph suggests that the divergence at P is positive and at Q is positive.

b. At the point P the divergence is 2.

At the point Q the divergence is 2.

c. The flux is outward everywhere and flux is inward nowhere.

d. The net flux across C is positive.

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6. Consider the vector field $\langle 0,0,4 \rangle \times \mathbf{r}$, where $\mathbf{r} = \langle x,y,z \rangle$.
a. Compute the curl of the field and verify that it has the same direction as the axis of rotation.
b. Compute the magnitude of the curl of the field.

a. The curl of the field is $(0)\mathbf{i} + (0)\mathbf{j} + (8)\mathbf{k}$.

b. The magnitude of the curl of the field is 8.

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7. Consider the vector field $\langle 5,0,0 \rangle \times \mathbf{r}$, where $\mathbf{r} = \langle x,y,z \rangle$.
a. Compute the curl of the field and verify that it has the same direction as the axis of rotation.
b. Compute the magnitude of the curl of the field.

a. The curl of the field is $(10)\mathbf{i} + (0)\mathbf{j} + (0)\mathbf{k}$.

b. The magnitude of the curl of the field is 10.

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8. Consider the vector field $\langle -1,3,-5 \rangle \times \mathbf{r}$, where $\mathbf{r} = \langle x,y,z \rangle$.
a. Compute the curl of the field and verify that it has the same direction as the axis of rotation.
b. Compute the magnitude of the curl of the field.

a. The curl of the field is $(-2)\mathbf{i} + (6)\mathbf{j} + (-10)\mathbf{k}$.

b. The magnitude of the curl of the field is $\sqrt{140}$.
(Type an exact answer, using radicals as needed.)

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9. Compute the curl of the following vector field.

$\mathbf{F} = \langle 5x^2 - 5y^2, xy, 5z \rangle$

The curl of $\mathbf{F}$ is $(0)\mathbf{i} + (0)\mathbf{j} + (11y)\mathbf{k}$.

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10. Compute the curl of the following vector field.

$$\mathbf{F} = \left\langle 3xz^3 e^{y^2}, 2xz^3 e^{y^2}, 3xz^2 e^{y^2} \right\rangle$$

The curl of **F** is $\left(6xyz^2 e^{y^2} - 6xz^2 e^{y^2}\right)\mathbf{i} + \left(9xZ^2 e^{y^2} - 3z^2 e^{y^2}\right)\mathbf{j} + \left(2z^3 e^{y^2} - 6xyz^3 e^{y^2}\right)\mathbf{k}$.