## Solution

Name:

Linear Algebra: Quiz 4

Show ALL work, as unjustified answers may receive no credit. Calculators are not allowed on any quiz or test paper. Make sure to exhibit skills discussed in class. Box all answers and clean up answers as much as possible.

1. [4pts] Assume that T is a Linear Transformation. Find the Standard Matrix of T:

 $T: \mathbb{R}^2 \to \mathbb{R}^2$  first performs a horizontal shear that transforms  $\overrightarrow{e_1}$  to  $\overrightarrow{e_1} - 2\overrightarrow{e_2}$  (leaving  $\overrightarrow{e_2}$  unchanged), and then reflects points through the line  $x_2 = -x_1$ 

$$\overrightarrow{e}, -2\overrightarrow{e}_2 \mapsto T(\overrightarrow{e}_1 - 2\overrightarrow{e}_2) = -\overrightarrow{e}_2 - 2(-\overrightarrow{e}_1) = -\overrightarrow{e}_1 + 2\overrightarrow{e}_1 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

 $\overline{e_2} \longmapsto T(\overline{e_2}) = -\overline{e_1} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ 

: Standard Matrix of T: 
$$A = [T(\vec{e_1} - 2\vec{e_2}) \ T(\vec{e_2})] = \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix}$$

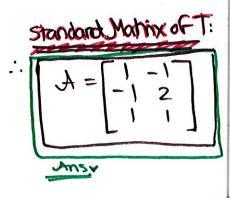
2. Define a Linear Transformation  $T: \mathbb{R}^3$ 

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mapsto \begin{bmatrix} x_1 - x_2 \\ -x_1 + 2x_2 \\ x_1 + x_2 \end{bmatrix}$$

(a) [2pts] Find the Standard Matrix of T.

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \mapsto X_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + X_2 \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$\overrightarrow{X}$$

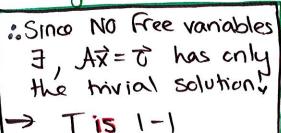


(b) [2pts] Is T one-to-one? Explain.

Recall: T is a 1-1 mapping when 
$$T(x) = \vec{0}$$
 has only the trivial solv

$$[A : \vec{c}] = \begin{bmatrix} 1 & -1 \\ -1 & Z \end{bmatrix} \sim \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

Since NO Free variables  $\vec{d}$ ,  $\vec{d}$ ,  $\vec{d}$ ,  $\vec{d}$  is a conly the trivial solution.



(c) [2pts] Is T onto? Explain.

Recall: T maps Rn onto Rm when columns of A span Rm. Answ

⇒ A pirot position I in each row (By Equivalence Thm 1.4)

Since the #of pivot positions < the # of rows → T is NOT onto.