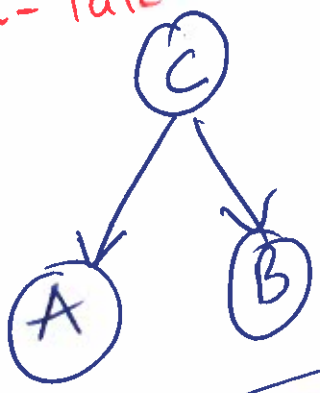
Conditional Independence BN

"Tail-Tail"



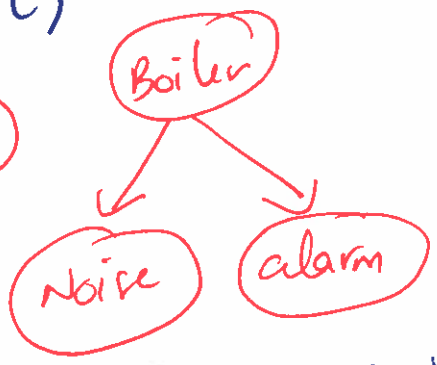
(1)

$$P(a,b,c) = P(a|c)\,P(b|c)\,P(c)$$

$$P(a,b|c) = \frac{P(a,b,c)}{P(c)} = P(a|c)\,P(b|c)$$

Conditioning on C

$P(c) > 0$



$$\Rightarrow A \perp B \mid C$$

(2)

(Head → Tail)

$$P(a,b,c) = P(a)\,P(c|a)\,P(b|c)$$

$$P(a,b|c) = \frac{P(a,b,c)}{P(c)}$$

$$= \frac{P(a)\,P(c|a)\,P(b|c)}{P(c)}$$

But $P(a,c) = P(a|c)\,P(c)$

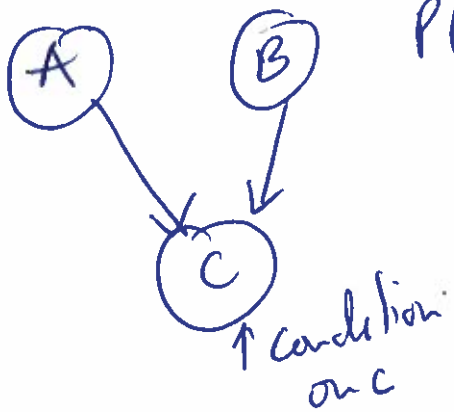
substituting $= \frac{P(a|c)\,P(c)\,P(b|c)}{P(c)}$

$$= P(a|c)\,P(b|c)$$

(1)

$\therefore \Rightarrow A \perp B \mid C$

(3) Head-head



↑ condition on C

$P(a,b,c) = P(a)\,P(b)\,P(c \mid a,b)$

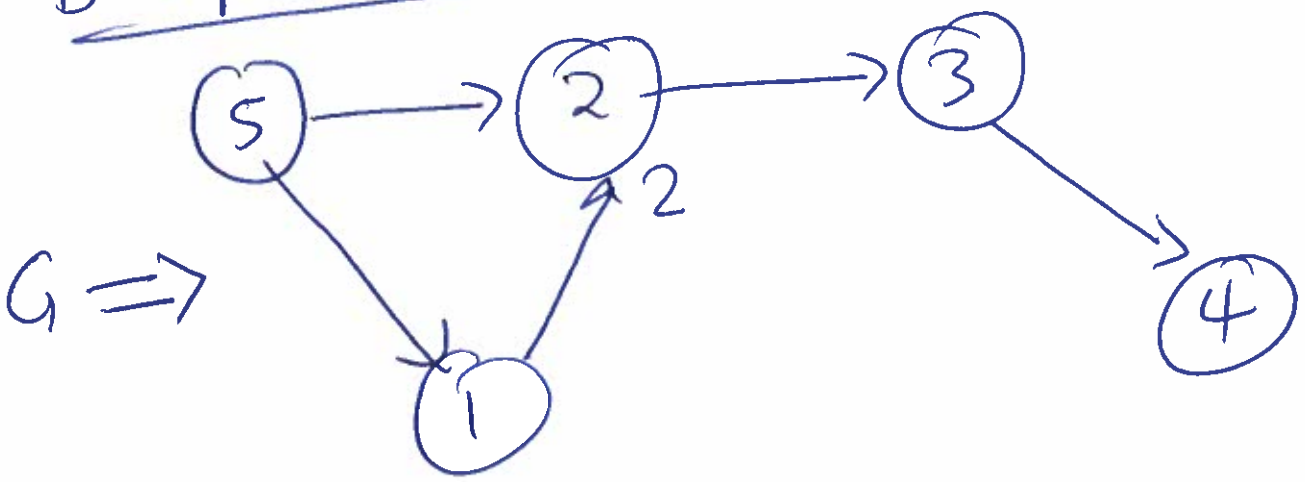
↑ won't resolve

$P(a,b,c) = \frac{P(a,b,c)}{P(c)}$

?

$A \not\perp B \mid C$

D-separation



$G \Rightarrow$

G is DAG.

Let A, B, C – be disjoint subsets of vertices.

i.e $(A \cup B \cup C)$ → not necessary all vertices in G.

A path → not necessary between 2 vertices is blocked (wrt C) if it passes through a vertex v such that either:

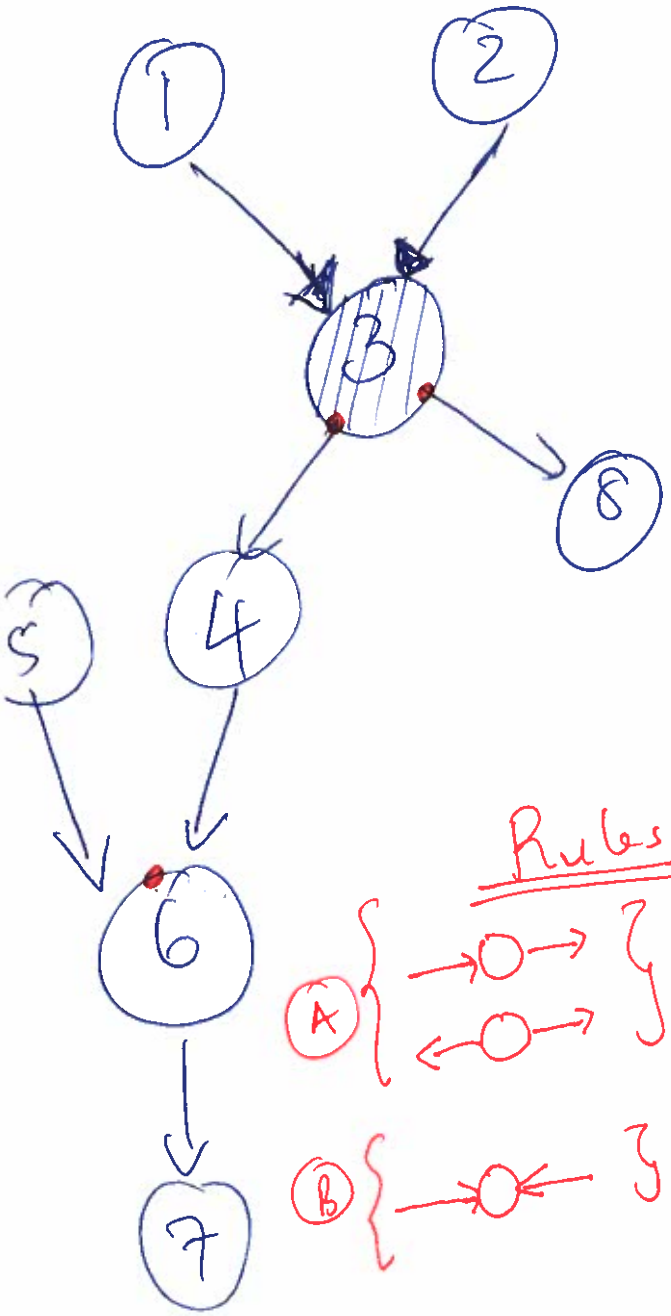
a) Arrow are head → to → ~~head~~ tail or tail → tail and $v \in C$

OR

b) arrows are head → to → head and $v \notin C$ and none of the descendants of v are in C

Defn: A & B are d-separated by C if all paths from a vertex of A to a vertex of B are blocked w.r.t C.

Theorem: (D-separation) i.e Directed separation.
If A & B are d-separated by C
then $A \perp B \mid C$



$C = \{3\}$

→ conditioning on $\underline{3}$

$X_i \perp X_j \mid X_3$?

means blocked

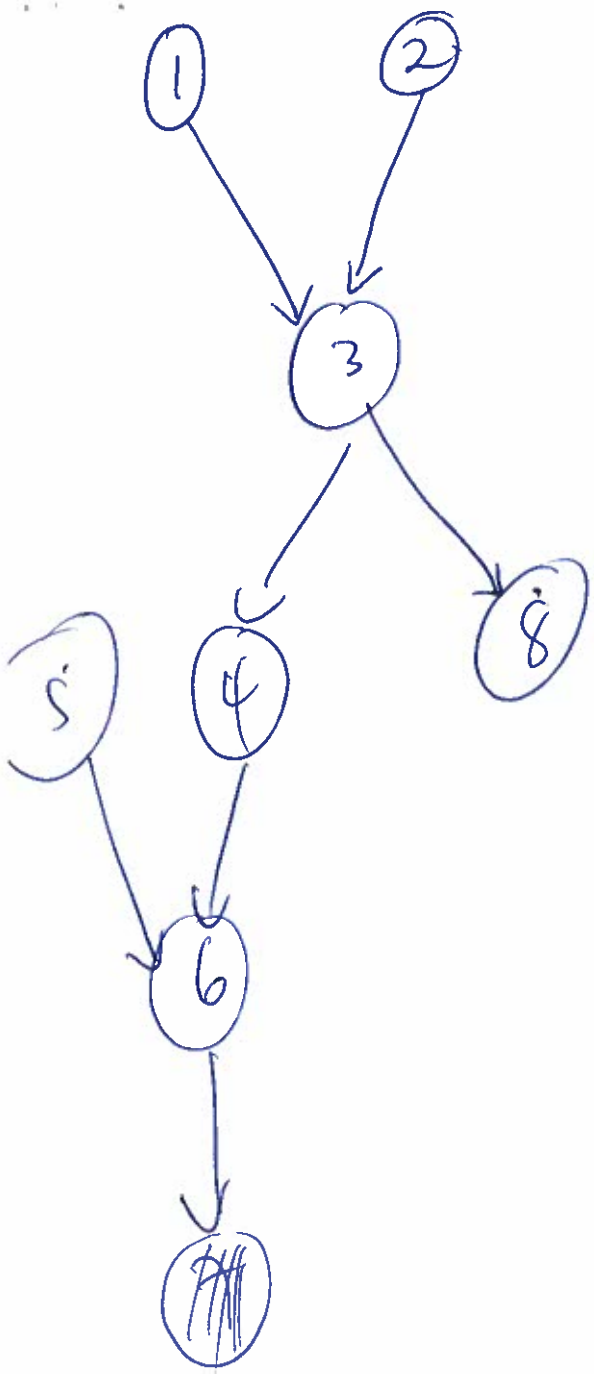
i, j	d-separated
1 4	yes
1 2	no
4 5	yes
4 7	no
4 8	yes
4 6	no
2 7	yes
2 5	yes
5 8	yes.

Rules

(A) → ○ → } blocked if ●
(A) ← ○ → } blocked if ●

(B) → ⊗ ← } blocked if vertex $v \notin C$ & no descendants in $\underline{C}$

5



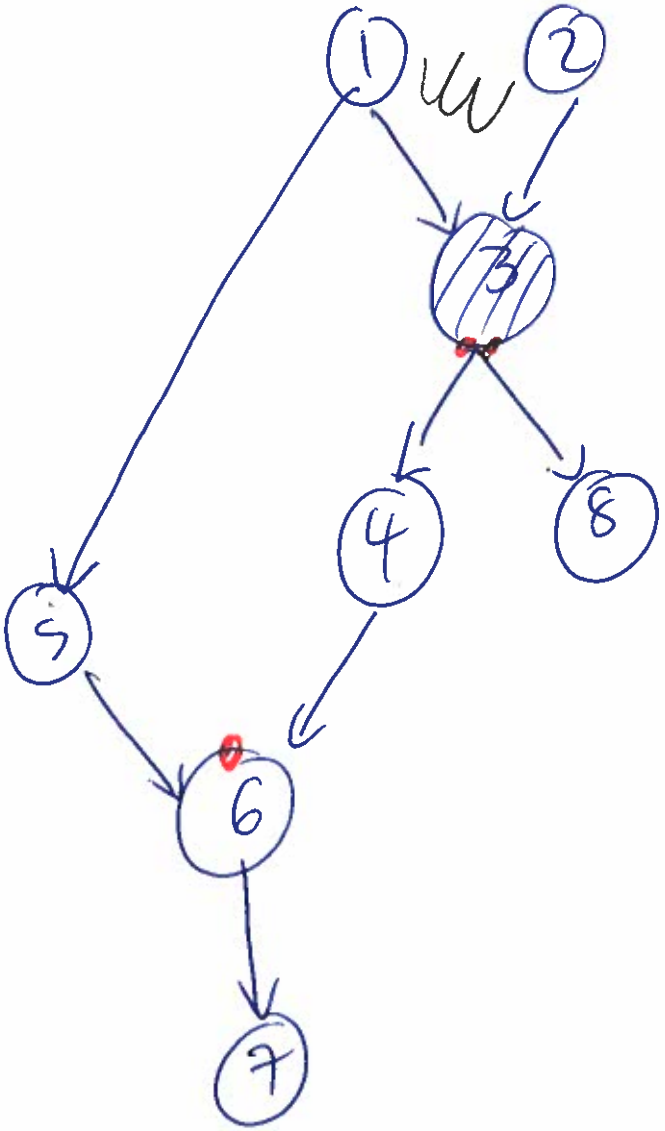
$C = \{7\}$

is $x_i \perp x_j \mid x_7$?

i	j	d

Nothing given

7



i	j	(blocked) d separate
2	5	no
2	7	no
2	4	yes
7	8	yes.

Blocked — means d separated

Open → means NOT deseparated