$\boxed{3.3.1}$

$$A = \begin{bmatrix} 3 & 6 \\ 4 & 7 \end{bmatrix} \Rightarrow \det A = 21 - 24 = -3$$

$$A_1 = \begin{bmatrix} 9 & 6 \\ 14 & 7 \end{bmatrix} \Rightarrow \det A_1 = 63 - 84 = -21 \Rightarrow x_1 = \frac{\det A_1}{\det A} = \frac{-21}{-3} = \boxed{7}$$

$$A_2 = \begin{bmatrix} 3 & 9 \\ 4 & 14 \end{bmatrix} \Rightarrow \det A_2 = 42 - 36 = 6 \Rightarrow x_2 = \frac{6}{-3} = \boxed{-2}$$

$\boxed{3.3.3}$

$$[A \,\vdots\, b] = \left[\begin{array}{cc|c} 5 & 3 & 1 \\ 2 & 4 & -2 \end{array}\right]$$

$$A = \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix} \Rightarrow \det A = 20 - 6 = 14$$

$$A_1 = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix} \Rightarrow \det A_1 = 4 + 6 = 10 \Rightarrow x_1 = \frac{10}{14} = \boxed{\frac{5}{7}}$$

$$A_2 = \begin{bmatrix} 5 & 1 \\ 2 & -2 \end{bmatrix} \Rightarrow \det A_2 = -10 - 2 = -12 \Rightarrow x_2 = \frac{-12}{14}$$

$$\boxed{= -6/7}$$

$\boxed{3.3.7}$

$$\det(A) = \begin{vmatrix} 5s & 6 \\ 9 & 4s \end{vmatrix} = 20s^2 - 54$$

$$\det(A_1) = \begin{vmatrix} 6 & 6 \\ -3 & 4s \end{vmatrix} = 24s + 18$$

$$\det(A_2) = \begin{vmatrix} 5s & 6 \\ 9 & -3 \end{vmatrix} = -15s - 54$$

$\det(A) \neq 0 \Rightarrow 20s^2 - 54 \neq 0 \Rightarrow s^2 \neq \frac{27}{10} \Rightarrow s \neq \pm\sqrt{\frac{27}{10}}$

$$x_1 = \frac{24s + 18}{20s^2 - 54} = \frac{12s + 9}{10s^2 - 27}$$

$$x_2 = \frac{-15s - 54}{20s^2 - 54}$$

$\boxed{3.3.9}$

$$\det A = \begin{vmatrix} s & -4s \\ 3 & -12s \end{vmatrix} = -12s^2 + 12s, \ \det A \neq 0 \Rightarrow -12s^2 + 12s \neq 0$$

$\Rightarrow s(s-1) \neq 0$

$\Rightarrow s \neq 0, s \neq 1$

$$\det A_1 = \begin{vmatrix} 3 & -4s \\ 5 & -12s \end{vmatrix} = -36s + 20s = -16s$$

$$\det A_2 = \begin{vmatrix} s & 3 \\ 3 & 5 \end{vmatrix} = 5s - 9$$

$\boxed{3.3.11}$

$$A = \begin{bmatrix} 0 & -4 & -1 \\ 4 & 0 & 0 \\ -2 & 1 & 1 \end{bmatrix} \qquad \operatorname{adj}(A) = C^T$$

$C_{11} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} = 0 \ \Big| \ C_{12} = -\begin{bmatrix} 4 & 0 \\ -2 & 1 \end{bmatrix} = -4 \ \Big| \ C_{13} = \begin{bmatrix} 4 & 0 \\ -2 & 1 \end{bmatrix} = 4$

$C_{21} = \begin{bmatrix} -4 & -1 \\ 1 & 1 \end{bmatrix} = +3 \ \Big| \ C_{22} = \begin{bmatrix} 0 & -1 \\ -2 & 1 \end{bmatrix} = -2 \ \Big| \ C_{23} = \begin{bmatrix} 0 & -4 \\ -2 & 1 \end{bmatrix} = +8$

$C_{31} = \begin{bmatrix} -4 & -1 \\ 0 & 0 \end{bmatrix} = 0 \ \Big| \ C_{32} = \begin{bmatrix} 0 & -1 \\ 4 & 0 \end{bmatrix} = 4 \ \Big| \ C_{33} = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix} = 16$

$$C = \begin{bmatrix} 0 & -4 & 4 \\ +3 & -2 & +8 \\ 0 & -4 & 16 \end{bmatrix} \Rightarrow C^T = \boxed{\begin{bmatrix} 0 & 3 & 0 \\ -4 & -2 & -4 \\ 4 & 8 & 16 \end{bmatrix} = \operatorname{adj}(A)}$$

$$A^{-1} = \frac{\operatorname{adj}(A)}{\det A} \Rightarrow$$

$$\det A = \begin{vmatrix} 0 & -4 & -1 \\ 4 & 0 & 0 \\ -2 & 1 & 1 \end{vmatrix} \begin{aligned} & 0(0) - (-4)4 + (-1)4 \\ =\; & 16 - 4 \\ =\; & 12 \end{aligned}$$

$$\Rightarrow A^{-1} = \frac{\operatorname{adj}(A)}{+12} = \boxed{\begin{bmatrix} 0 & \frac{1}{4} & 0 \\ -\frac{1}{3} & -\frac{1}{6} & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \end{bmatrix}} = A^{-1}$$

$\boxed{3.3.13}$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 1 \\ 3 & 1 & 1 \end{bmatrix} \qquad \operatorname{adj}(A) = C^T$$

$C_{11} = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1$	$C_{12} = -\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = 2$	$C_{13} = \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix} = 1$
$C_{21} = -\begin{vmatrix} 3 & 4 \\ 1 & 1 \end{vmatrix} = 1$	$C_{22} = +\begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix} = -10$	$C_{23} = -\begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = 7$
$C_{31} = +\begin{vmatrix} 3 & 4 \\ 0 & 1 \end{vmatrix} = 3$	$C_{32} = -\begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix} = +2$	$C_{33} = \begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} = -3$

$$C = \begin{bmatrix} -1 & 2 & 1 \\ 1 & -10 & 7 \\ 3 & 2 & -3 \end{bmatrix} \Rightarrow C^T = \boxed{\operatorname{adj}(A) = \begin{bmatrix} -1 & 1 & 3 \\ 2 & -10 & 2 \\ 1 & 7 & -3 \end{bmatrix}}$$

$$A^{-1} = \frac{\operatorname{adj}(A)}{\det A}$$

$$\det(A) = 2C_{11} + 3C_{12} + 4C_{13} = 2(-1) + 3(2) + 4(1)$$

$$= -2 + 6 + 4 = 8$$

$$\boxed{A^{-1} = \begin{bmatrix} -\frac{1}{8} & \frac{1}{8} & \frac{3}{8} \\ \frac{1}{4} & \frac{-5}{4} & \frac{1}{4} \\ \frac{1}{8} & \frac{7}{8} & -\frac{3}{8} \end{bmatrix}}$$

$\boxed{3.3.15}$

$$A = \begin{bmatrix} 2 & 0 & 0 \\ -3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix} \qquad \operatorname{adj} A = C^T$$

$$C_{11} = +\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = -2 \qquad C_{12} = -\begin{vmatrix} -3 & 1 \\ -1 & 1 \end{vmatrix} = 2 \qquad C_{13} = +\begin{vmatrix} -3 & 1 \\ -1 & 3 \end{vmatrix} = -8$$

$$C_{21} = -\begin{vmatrix} 0 & 0 \\ 3 & 1 \end{vmatrix} = 0 \qquad C_{22} = +\begin{vmatrix} 2 & 0 \\ -1 & 1 \end{vmatrix} = 2 \qquad C_{23} = -\begin{vmatrix} 2 & 0 \\ -1 & 3 \end{vmatrix} = -6$$

$$C_{31} = +\begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix} = 0 \qquad C_{32} = -\begin{vmatrix} 2 & 0 \\ -3 & 1 \end{vmatrix} = -2 \qquad C_{33} = +\begin{vmatrix} 2 & 0 \\ -3 & 1 \end{vmatrix} = 2$$

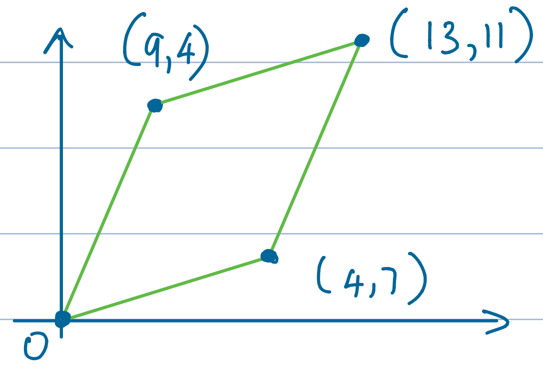
$$\Rightarrow C = \begin{vmatrix} -2 & 2 & -8 \\ 0 & 2 & -6 \\ 0 & -2 & 2 \end{vmatrix} \Rightarrow C^T = \boxed{\begin{vmatrix} -2 & 0 & 0 \\ 2 & 2 & -2 \\ -8 & -6 & 2 \end{vmatrix} = \text{adj} A}$$

$$A^{-1} = \frac{1}{\det A} \text{adj} A$$

$$\det A = 2 \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = 2(-2) = -4$$

$$\boxed{\Rightarrow A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 2 & \frac{3}{2} & -\frac{1}{2} \end{bmatrix}}$$

$\boxed{3.3.19}$



Area of the parallelogram :

$$\begin{vmatrix} 9 & 4 \\ 4 & 7 \end{vmatrix} = 63 - 16 = \boxed{47}$$

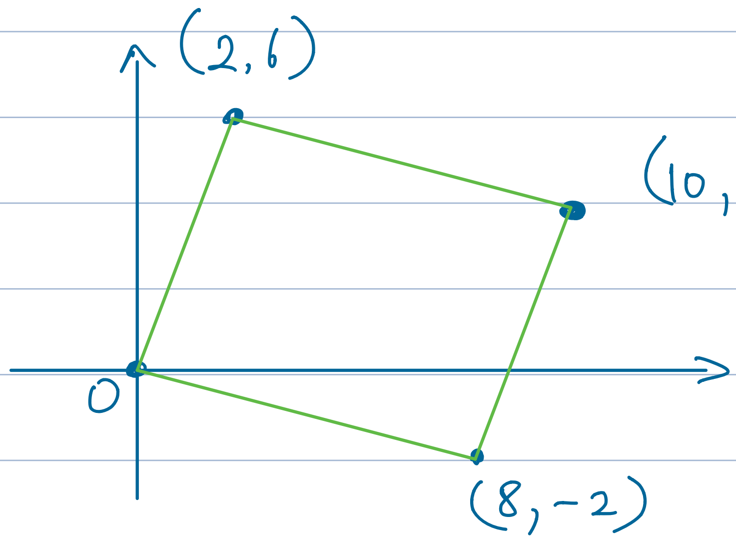
$\boxed{3.3.21}$ Set the point at $(-2,-3)$ as the origin

$(-2,-3) - (-2,-3) = (0,0)$

$(0,3) - (-2,-3) = (2,6)$

$(6,-5) - (-2,-3) = (8,-2)$

$(8,1) - (-2,-3) = (10,4)$



Area $=$

$$\begin{vmatrix} 8 & 2 \\ -2 & 6 \end{vmatrix}$$

$= 48 + 4 = \boxed{52}$

$\boxed{3.3.23}$ $(4,0,-2)$ $(1,2,6)$ $(7,1,0)$

Volume :

$$\begin{array}{ccc} -28 & 24 & 0 \end{array}$$

$$= \left|\begin{array}{ccc|cc} 4 & 1 & 7 & 4 & 1 \\ 0 & 2 & 1 & 0 & 2 \\ -2 & 6 & 0 & -2 & 6 \end{array}\right.$$

$$\begin{array}{ccc} 0 & -2 & 0 \end{array}$$

$= (-2) - (-28+24)$

$= -2 - (-4) = \boxed{2}$

$\boxed{3.3.27}$

$$\det B = \begin{vmatrix} -4 & -4 \\ 4 & 7 \end{vmatrix} = |-28+16| = 12$$

$$\det A = \begin{vmatrix} 3 & -3 \\ -6 & 3 \end{vmatrix} = |9-18| = 9$$

Area of $T(s) = (12)(9) = \boxed{108}$