Name: (Print) PHONG VO

1. **(6 points) Summations**: Give a closed form solution to: (1) $\sum_{i=1}^{n} 3n$ and (2) $\sum_{i=0}^{n} 3i$

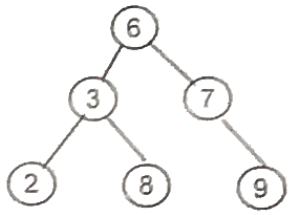
Your answer should be a polynomial function of n.

(1) $\sum_{i=1}^{n} 3n = 3n \times n = 3n^2$ ✓

(2) $\sum_{i=0}^{n} 3i = 3\,\frac{(n+1)n}{2}$ ✓

2. **(4 points) Trees**: Circle TRUE if the statement below is true, and FALSE otherwise. Briefly explain your answer.

Statement: *This graph is a binary search tree.*



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TRUE **FALSE**

Explanation: Left child < Right child

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3. **(5 points) Probability**: If you toss a fair, 3-sided coin (with sides HEAD, MIDDLE, TAIL) two times, what is the probability of getting at most one TAIL?

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The most is one TAIL $= \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$

no TAIL $= \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

$n = 2$

$p = \frac{1}{3}$

$\Rightarrow$ prob. of getting at most one TAIL $= \frac{1}{9} + \frac{2}{9} = \frac{1}{3}$

(Continue to the back)

4. **(5 points) Logarithms**: Simplify and evaluate: $\frac{3^{(\log_4 64)}}{\log_2 8}$.

$\log_4 64 = \log_4 4^3 = 3$

$\log_2 8 = 3$

Result $= \frac{3^3}{3} = 3^2 = 9$

5. **(10 points) Algorithm Analysis**: There is a mystery function called Mystery(n) and the pseudocode of the algorithm is shown as below (Assume that $n = 4^k$ for some positive integer $k \geq 1$.). Please analyze the worst-case asymptotic execution time of this algorithm (1) List the cost for executing each line of code and the number of executions for each line in the table; (2) then derive a recurrence of the running time. You do NOT need to solve the recurrence.

	Mystery(n)	Cost per line	Number of execution per line
1	**if** $n \leq 1$	C_1	1
2	**return** 1	C_2	1
3	Mystery ($n/4$)	C_3 $T(n/4)$	1
4	Mystery ($n/4$)	C_4 $T(n/4)$	1
5	**for** $i=1$ **to** n	C_5	$n+1$
6	print i	C_6	$\times n$

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$T(n) = C_1 + C_2 + C_3 + C_4 + C_5(n+1) + C_6$

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