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Date: 02/19/20	**Course:** Linear Algebra I (Spring 2020)	

1. Determine if the system has a nontrivial solution. Try to use as few row operations as possible.

$$\begin{aligned} 8x_1 - 4x_2 + 20x_3 &= 0 \\ -8x_1 - 4x_2 - 14x_3 &= 0 \\ 16x_1 + 8x_2 + 28x_3 &= 0 \end{aligned}$$

Choose the correct answer below.

- **A.** It is impossible to determine.
- **B.** The system has only a trivial solution.
- **C.** The system has a nontrivial solution.

2. Write the solution set of the given homogeneous system in parametric vector form.

$$\begin{aligned} 2x_1 + 2x_2 + 4x_3 &= 0 \\ -4x_1 - 4x_2 - 8x_3 &= 0 \\ -5x_2 + 15x_3 &= 0 \end{aligned} \qquad \text{where the solution set is } \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\mathbf{x} = x_3 \begin{bmatrix} -5 \\ 3 \\ 1 \end{bmatrix}$$

(Type an integer or simplified fraction for each matrix element.)

3. Write the solution set of the given homogeneous system in parametric vector form.

$$\begin{aligned} x_1 + 2x_2 + 18x_3 &= 0 \\ 2x_1 + x_2 + 18x_3 &= 0 \\ -x_1 + x_2 &= 0 \end{aligned} \qquad \text{where the solution set is } \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\mathbf{x} = x_3 \begin{bmatrix} -6 \\ -6 \\ 1 \end{bmatrix}$$

(Type an integer or simplified fraction for each matrix element.)

4. Describe all solutions of A**x = 0** in parametric vector form, where A is row equivalent to the given matrix.

$$\begin{bmatrix} 1 & 5 & -1 & 4 & 0 & -4 \\ 0 & 0 & 1 & 0 & 0 & 9 \\ 0 & 0 & 0 & 1 & 0 & -9 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{x} = x_2 \begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} -41 \\ 0 \\ -9 \\ 9 \\ 0 \\ 1 \end{bmatrix}$$

(Type an integer or fraction for each matrix element.)

5. Describe and compare the solution sets of $x_1 + 2x_2 - 3x_3 = 0$ and $x_1 + 2x_2 - 3x_3 = -7$.

Describe the solution set, $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, of $x_1 + 2x_2 - 3x_3 = 0$ in parametric vector form. Select the correct choice below and fill in the answer boxes within your choice.
(Type an integer or fraction for each matrix element.)

- **A.** $\mathbf{x} =$
- **B.** $\mathbf{x} = x_3$
- **C.** $\mathbf{x} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$
- **D.** $\mathbf{x} = \quad + x_2$

Describe the solution set, $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, of $x_1 + 2x_2 - 3x_3 = -7$ in parametric vector form. Select the correct choice below and fill in the answer boxes within your choice.
(Type an integer or fraction for each matrix element.)

- **A.** $\mathbf{x} =$
- **B.** $\mathbf{x} = \quad + x_3$
- **C.** $\mathbf{x} = \begin{bmatrix} -7 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$
- **D.** $\mathbf{x} = x_2 \quad + x_3$

Which option best compares the two equations?

- **A.** The solution set of the second equation is a plane parallel to the plane that is the solution set of the first equation.
- **B.** The solution set of the second equation is a plane parallel to the line that is the solution set of the first equation.
- **C.** The solution set of the second equation is a line parallel to the line that is the solution set of the first equation.
- **D.** The solution set of the second equation is a plane perpendicular to the line that is the solution set of the first equation.

6. Describe the solutions of the first system of equations below in parametric vector form. Provide a geometric comparison with the solution set of the second system of equations below.

$$\begin{aligned} 2x_1 + 2x_2 + 4x_3 &= 8 \\ -4x_1 - 4x_2 - 8x_3 &= -16 \\ -5x_2 + 15x_3 &= 15 \end{aligned} \qquad \begin{aligned} 2x_1 + 2x_2 + 4x_3 &= 0 \\ -4x_1 - 4x_2 - 8x_3 &= 0 \\ -5x_2 + 15x_3 &= 0 \end{aligned}$$

Describe the solution set, $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, of the first system of equations in parametric vector form. Select the correct choice below and fill in the answer box(es) within your choice.
(Type an integer or fraction for each matrix element.)

- **A.** $\mathbf{x} =$ ______
- **B.** $\mathbf{x} = x_2$ ______
- ✔ **C.** $\mathbf{x} = \begin{bmatrix} 7 \\ -3 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -5 \\ 3 \\ 1 \end{bmatrix}$
- **D.** $\mathbf{x} = x_2$ ______ $+ x_3$ ______

Which option best compares the two systems?

- **A.** The solution set of the first system is a line perpendicular to the line that is the solution set of the second system.
- **B.** The solution set of the first system is a plane parallel to the plane that is the solution set of the second system.
- ✔ **C.** The solution set of the first system is a line parallel to the line that is the solution set of the second system.
- **D.** The solution set of the first system is a plane parallel to the line that is the solution set of the second system.

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7. Describe the solutions of the first system of equations below in parametric vector form. Provide a geometric comparison with the solution set of the second system of equations shown below.

$$\begin{aligned} x_1 - 3x_2 + 7x_3 &= 5 \\ 2x_1 + x_2 + 7x_3 &= 3 \\ -x_1 - 4x_2 &= 2 \end{aligned} \qquad \begin{aligned} x_1 - 3x_2 + 7x_3 &= 0 \\ 2x_1 + x_2 + 7x_3 &= 0 \\ -x_1 - 4x_2 &= 0 \end{aligned} \qquad \text{where the solution set is } \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Describe the solution set of the first system of equations in parametric vector form. Select the correct choice below and fill in the answer box(es) within your choice.
(Type an integer or fraction for each matrix element.)

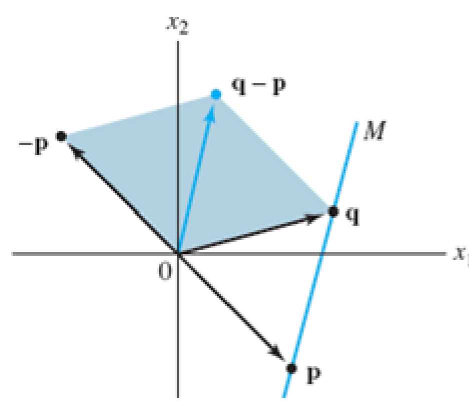
- **A.** $\mathbf{x} =$ ______
- **B.** $\mathbf{x} = x_2$ ______
- **C.** $\mathbf{x} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix}$
- **D.** $\mathbf{x} = x_2$ ______ $+ x_3$ ______

Which option best compares the two systems?

- **A.** The solution set of the first system is a line perpendicular to the line that is the solution set of the second system.
- **B.** The solution set of the first system is a plane parallel to the plane that is the solution set of the second system.
- **C.** The solution set of the first system is a plane parallel to the line that is the solution set of the second system.
- **D.** The solution set of the first system is a line parallel to the line that is the solution set of the second system.

8. Find a parametric equation of the line M through **p** and **q** for the given values of **p** and **q**. [Hint: M is parallel to the vector **q** − **p** shown in the figure. Note that the figure does not match the given vectors.]

$$\mathbf{p} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}, \mathbf{q} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$



Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice. Use the vector **q** − **p** in your response.

- **A.** $\mathbf{x} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$
- **B.** $\mathbf{x} = t\begin{bmatrix} -3 \\ 4 \end{bmatrix}$ (t in $\mathbb{R}$)
- **C.** $\mathbf{x} = \begin{bmatrix} -3 \\ 4 \end{bmatrix} + t\begin{bmatrix} 3 \\ -6 \end{bmatrix}$ (t in $\mathbb{R}$)
- **D.** $\mathbf{x} = t\begin{bmatrix} -3 \\ 4 \end{bmatrix} + t\begin{bmatrix} \_\_\_ \\ \_\_\_ \end{bmatrix}$ (t in $\mathbb{R}$)

9. Mark each statement True or False. Justify each answer.

a. A homogeneous equation is always consistent.

- **A.** True. A homogenous equation can be written in the form $A\mathbf{x} = \mathbf{0}$, where A is an $m \times n$ matrix and $\mathbf{0}$ is the zero vector in $\mathbb{R}^m$. Such a system $A\mathbf{x} = \mathbf{0}$ always has at least one solution, namely, $\mathbf{x} = \mathbf{0}$. Thus a homogenous equation is always consistent.
- **B.** False. A homogenous equation can be written in the form $A\mathbf{x} = \mathbf{0}$, where A is an $m \times n$ matrix and $\mathbf{0}$ is the zero vector in $\mathbb{R}^m$. Such a system $A\mathbf{x} = \mathbf{0}$ always has at least one solution, namely, $\mathbf{x} = \mathbf{0}$. Thus a homogenous equation is always inconsistent.
- **C.** False. A homogenous equation can be written in the form $A\mathbf{x} = \mathbf{0}$, where A is an $m \times n$ matrix and $\mathbf{0}$ is the zero vector in $\mathbb{R}^m$. Such a system $A\mathbf{x} = \mathbf{0}$ always has at least one nontrivial solution. Thus a homogenous equation is always inconsistent.
- **D.** True. A homogenous equation can be written in the form $A\mathbf{x} = \mathbf{0}$, where A is an $m \times n$ matrix and $\mathbf{0}$ is the zero vector in $\mathbb{R}^m$. Such a system $A\mathbf{x} = \mathbf{0}$ always has at least one nontrivial solution. Thus a homogenous equation is always consistent.

b. The equation $A\mathbf{x} = \mathbf{0}$ gives an explicit description of its solution set.

- **A.** False. Since the equation is solved, $A\mathbf{x} = \mathbf{0}$ gives an implicit description of its solution set.
- **B.** True. Since the equation is solved, $A\mathbf{x} = \mathbf{0}$ gives an explicit description of the solution set.
- **C.** False. The equation $A\mathbf{x} = \mathbf{0}$ gives an implicit description of its solution set. Solving the equation amounts to finding an explicit description of its solution set.
- **D.** True. The equation $A\mathbf{x} = \mathbf{0}$ gives an explicit description of its solution set. Solving the equation amounts to finding an implicit description of its solution set.

c. The homogenous equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution if and only if the equation has at least one free variable.

- **A.** False. The homogeneous equation $A\mathbf{x} = \mathbf{0}$ always has the trivial solution.
- **B.** True. The homogenous equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution if and only if the equation has at least one free variable which implies that the equation has a nontrivial solution.
- **C.** False. The homogeneous equation $A\mathbf{x} = \mathbf{0}$ never has the trivial solution.
- **D.** True. The homogeneous equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution if and only if the matrix A has a row of zeros which implies the equation has at least one free variable.

d. The equation $\mathbf{x} = \mathbf{p} + t\mathbf{v}$ describes a line through $\mathbf{v}$ parallel to $\mathbf{p}$.

- **A.** False. The effect of adding $\mathbf{p}$ to $\mathbf{v}$ is to move $\mathbf{v}$ in a direction parallel to the plane through $\mathbf{p}$ and $\mathbf{0}$. So the equation $\mathbf{x} = \mathbf{p} + t\mathbf{v}$ describes a plane through $\mathbf{p}$ parallel to $\mathbf{v}$.
- **B.** False. The effect of adding $\mathbf{p}$ to $\mathbf{v}$ is to move $\mathbf{v}$ in a direction parallel to the line through $\mathbf{p}$ and $\mathbf{0}$. So the equation $\mathbf{x} = \mathbf{p} + t\mathbf{v}$ describes a line through $\mathbf{p}$ parallel to $\mathbf{v}$.
- **C.** False. The effect of adding $\mathbf{p}$ to $\mathbf{v}$ is to move $\mathbf{p}$ in a direction parallel to the plane through $\mathbf{v}$ and $\mathbf{0}$. So the equation $\mathbf{x} = \mathbf{p} + t\mathbf{v}$ describes a plane through $\mathbf{v}$ parallel to $\mathbf{p}$.
- **D.** True. The effect of adding $\mathbf{p}$ to $\mathbf{v}$ is to move $\mathbf{p}$ in a direction parallel to the line through $\mathbf{v}$ and $\mathbf{0}$. So the equation $\mathbf{x} = \mathbf{p} + t\mathbf{v}$ describes a line through $\mathbf{v}$ parallel to $\mathbf{p}$.

e. The solution set of $A\mathbf{x} = \mathbf{b}$ is the set of all vectors of the form $\mathbf{w} = \mathbf{p} + \mathbf{v}_h$, where $\mathbf{v}_h$ is any solution of the equation $A\mathbf{x} = \mathbf{0}$.

- **A.** False. The solution set could be empty. The statement is only true when the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent for some given $\mathbf{b}$, and there exists a vector $\mathbf{p}$ such that $\mathbf{p}$ is a solution.
- **B.** True. The equation $A\mathbf{x} = \mathbf{b}$ is always consistent and there always exists a vector $\mathbf{p}$ that is a solution.

C. False. The solution set could be empty. The statement is only true when the equation A**x** = **b** is consistent for some given **b**, and there exists a vector **p** such that **p** is a solution.

D. False. The solution set could be the trivial solution. The statement is only true when the equation A**x** = **b** is inconsistent for some given **b**, and there exists a vector **p** such that **p** is a solution.

10. Mark each statement True or False. Justify each answer.

a. A homogeneous system of equations can be inconsistent. Choose the correct answer below.

- **A.** False. A homogeneous equation cannot be written in the form A**x** = **0**, where A is an m × n matrix and **0** is the zero vector in $\mathbb{R}^m$. Such a system A**x** = **0** does not have the solution **x** = **0**. Thus, a homogeneous system of equations cannot be inconsistent.
- **B.** True. A homogeneous equation cannot be written in the form A**x** = **0**, where A is an m × n matrix and **0** is the zero vector in $\mathbb{R}^m$. Such a system A**x** = **0** does not have the solution **x** = **0**. Thus, a homogeneous system of equations can be inconsistent.
- **C.** False. A homogeneous equation can be written in the form A**x** = **0**, where A is an m × n matrix and **0** is the zero vector in $\mathbb{R}^m$. Such a system A**x** = **0** always has at least one solution, namely **x** = **0**. Thus, a homogeneous system of equations cannot be inconsistent.
- **D.** True. A homogeneous equation can be written in the form A**x** = **0**, where A is an m × n matrix and **0** is the zero vector in $\mathbb{R}^m$. Such a system A**x** = **0** always has at least one solution, namely **x** = **0**. Thus, a homogeneous system of equations can be inconsistent.

b. If **x** is a nontrivial solution of A**x** = **0**, then every entry in **x** is nonzero. Choose the correct answer below.

- **A.** True. A nontrivial solution of A**x** = **0** is a nonzero vector **x** that satisfies A**x** = **0**. Thus, a nontrivial solution **x** can have some zero entries so long as not all of its entries are zero.
- **B.** True. A nontrivial solution of A**x** = **0** is a nonzero vector **x** that satisfies A**x** = **0**. Thus, a nontrivial solution **x** cannot have any zero entries.
- **C.** False. A nontrivial solution of A**x** = **0** is the zero vector. Thus, a nontrivial solution **x** must have all zero entries.
- **D.** False. A nontrivial solution of A**x** = **0** is a nonzero vector **x** that satisfies A**x** = **0**. Thus, a nontrivial solution **x** can have some zero entries so long as not all of its entries are zero.

c. The effect of adding **p** to a vector is to move the vector in a direction parallel to **p**. Choose the correct answer below.

- **A.** True. Given **v** and **p** in $\mathbb{R}^2$ or $\mathbb{R}^3$, the effect of adding **p** to **v** is to move **v** in a direction parallel to the line through **p** and **0**.
- **B.** False. Given **v** and **p** in $\mathbb{R}^2$ or $\mathbb{R}^3$, the effect of adding **p** to **v** is to move **v** in a direction parallel to the plane through **v** and **0**.
- **C.** False. Given **v** and **p** in $\mathbb{R}^2$ or $\mathbb{R}^3$, the effect of adding **p** to **v** is to move **v** in a direction parallel to the plane through **p** and **0**.
- **D.** False. Given **v** and **p** in $\mathbb{R}^2$ or $\mathbb{R}^3$, the effect of adding **p** to **v** is to move **v** in a direction parallel to the line through **v** and **0**.

d. The equation A**x** = **b** is homogeneous if the zero vector is a solution. Choose the correct answer below.

- **A.** False. A system of linear equations is said to be homogeneous if it can be written in the form A**x** = **b**, where A is an m × n matrix and **b** is a nonzero vector in $\mathbb{R}^m$. Thus, the zero vector is never a solution of a homogeneous system.
- **B.** False. A system of linear equations is said to be homogeneous if it can be written in the form A**x** = **0**, where A is an m × n matrix and **0** is the zero vector in $\mathbb{R}^m$. If the zero vector is a solution, then **b** = A**x** = A**0** = **0**, which is false.
- **C.** True. A system of linear equations is said to be homogeneous if it can be written in the form A**x** = **b**, where A is an m × n matrix and **b** is a nonzero vector in $\mathbb{R}^m$. If the zero vector is a solution, then **b** = **0**.
- **D.** True. A system of linear equations is said to be homogeneous if it can be written in the form

A**x** = **0**, where A is an m × n matrix and **0** is the zero vector in $\mathbb{R}^m$. If the zero vector is a

e. If A**x** = **b** is consistent, then the solution set of A**x** = **b** is obtained by translating the solution set of A**x** = **0**. Choose the correct answer below.

- **A.** True. Suppose the equation A**x** = **b** is consistent for some given **b**, and let **p** be a solution. Then the solution set of A**x** = **b** is the set of all vectors of the form $\mathbf{w} = \mathbf{p} + \mathbf{v}_h$, where $\mathbf{v}_h$ is any solution of the homogeneous equation A**x** = **0**.
- **B.** True. Suppose the equation A**x** = **b** is consistent for some given **b**. Then the solution set of A**x** = **b** is the set of all vectors of the form $\mathbf{w} = \mathbf{p} + \mathbf{v}_h$, where $\mathbf{v}_h$ is not a solution of the homogeneous equation A**x** = **0**.
- **C.** False. Suppose the equation A**x** = **b** is consistent for some given **b**. Then the solution set of A**x** = **b** is the set of all vectors of the form $\mathbf{w} = \mathbf{p} + \mathbf{v}_h$, where $\mathbf{v}_h$ is not a solution of the homogeneous equation A**x** = **0**.
- **D.** False. Suppose the equation A**x** = **b** is consistent for some given **b**, and let **p** be a solution. Then the solution set of A**x** = **b** is the set of all vectors of the form $\mathbf{w} = \mathbf{p} + \mathbf{v}_h$, where $\mathbf{v}_h$ is any solution of the homogeneous equation A**x** = **0**.

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11. A is a 3 × 3 matrix with three pivot positions.

(a) Does the equation A**x** = **0** have a nontrivial solution?
(b) Does the equation A**x** = **b** have at least one solution for every possible **b**?

(a) Does the equation A**x** = **0** have a nontrivial solution?

- No
- Yes

(b) Does the equation A**x** = **b** have at least one solution for every possible **b**?

- No
- Yes

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12. A is a 2 × 5 matrix with two pivot positions.

(a) Does the equation A**x** = **0** have a nontrivial solution?
(b) Does the equation A**x** = **b** have at least one solution for every possible **b**?

(a) Does the equation A**x** = **0** have a nontrivial solution?

- No
- Yes

(b) Does the equation A**x** = **b** have at least one solution for every possible **b**?

- Yes
- No