2.2.1

$$\begin{bmatrix} 5 & 8 \\ 7 & 6 \end{bmatrix}^{-1} = \frac{1}{-26} \begin{bmatrix} 6 & -8 \\ -7 & 5 \end{bmatrix}$$

2.2.4

$$\begin{bmatrix} 3 & 4 \\ -2 & -3 \end{bmatrix}^{-1} = \frac{1}{-9+8} \begin{bmatrix} -3 & -4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -2 & -3 \end{bmatrix}$$

2.2.6

$$\begin{cases} 5x_1 + 2x_2 = -4 \\ -6x_1 - 2x_2 = 3 \end{cases} \Rightarrow \begin{cases} x_1 = 1 \\ x_2 = -9/2 \end{cases}$$

2.2.7

$$A^{-1} = \begin{bmatrix} 1 & 2 \\ 8 & 18 \end{bmatrix}^{-1} = \begin{bmatrix} 18 & -2 \\ -8 & 1 \end{bmatrix} \frac{1}{18-16} = \begin{bmatrix} 9 & -1 \\ -4 & 1/2 \end{bmatrix}$$

$$Ax = b_1 \Rightarrow x = A^{-1} b_1 = \begin{bmatrix} 9 & -1 \\ -4 & 1/2 \end{bmatrix} \begin{bmatrix} -3 \\ -20 \end{bmatrix} = \begin{bmatrix} -7 \\ 2 \end{bmatrix}$$

$$Ax = b_2 \Rightarrow x = A^{-1} b_2 = \begin{bmatrix} 9 & -1 \\ -4 & 1/2 \end{bmatrix} \begin{bmatrix} 2 \\ 8 \end{bmatrix} = \begin{bmatrix} 10 \\ -4 \end{bmatrix}$$

$$Ax = b_3 \Rightarrow x = A^{-1} b_3 = \begin{bmatrix} 9 & -1 \\ -4 & 1/2 \end{bmatrix} \begin{bmatrix} 2 \\ 12 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$$

$$Ax = b_4 \Rightarrow x = A^{-1} b_4 = \begin{bmatrix} 9 & -1 \\ -4 & 1/2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 14 \\ -6 \end{bmatrix}$$

(b) $\left[\begin{array}{cc|cccc} 1 & 2 & -3 & 2 & 2 & 2 \\ 8 & 18 & -20 & 8 & 12 & 4 \end{array}\right]$ $\begin{matrix} +24 & -16 & -16 \\ & & \end{matrix}$ $\xrightarrow{-8R_1+R_2 \to nR_2}$ $\left[\begin{array}{cc|cccc} 1 & 2 & -3 & 2 & 2 & 2 \\ 0 & 2 & 4 & -8 & -4 & -12 \end{array}\right]$

$\xrightarrow[R_2/2]{R_1-R_2 \to nR_1}$ $\left[\begin{array}{cc|cccc} 1 & 0 & -7 & 10 & 6 & 14 \\ 0 & 1 & 2 & -4 & -2 & -6 \end{array}\right]$

2.2.29

$$A^{-1} = \begin{bmatrix} 1 & -3 \\ 6 & -9 \end{bmatrix}^{-1} = \frac{1}{-9+18=9} \begin{bmatrix} -9 & 3 \\ -6 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1/3 \\ -2/3 & 1/9 \end{bmatrix}$$

2.2.30

$$A^{-1} = \begin{bmatrix} 4 & 4 \\ 2 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} 5 & -4 \\ -2 & 4 \end{bmatrix} \frac{1}{20-8=12} = \boxed{\frac{1}{12}\begin{bmatrix} 5 & -4 \\ -2 & 4 \end{bmatrix}}$$

2.2.31

$$[A \,|\, I] = \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 4 & 1 & 3 & 0 & 1 & 0 \\ -3 & -4 & 3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} 4R_1 - R_2 = nR_2 \\ \longrightarrow \\ 3R_1 + R_3 = nR_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & +1 & -5 & -4 & +1 & 0 \\ 0 & -4 & 9 & 3 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} R_2 + R_3 \\ = nR_3 \\ \longrightarrow \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -5 & -4 & 1 & 0 \\ 0 & 0 & -11 & -13 & 4 & 1 \end{array}\right] \frac{R_3}{-11} \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & -5 & -4 & 1 & 0 \\ 0 & 0 & 1 & \frac{13}{11} & \frac{-4}{11} & \frac{-1}{11} \end{array}\right]$$

$$\begin{array}{l} R_1 - 2R_3 \\ = nR_1 \\ \longrightarrow \\ R_2 + 5R_3 \\ = nR_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{-15}{11} & \frac{8}{11} & \frac{2}{11} \\ 0 & 1 & 0 & \frac{21}{11} & \frac{-9}{11} & \frac{-5}{11} \\ 0 & 0 & 1 & \frac{13}{11} & \frac{-4}{11} & \frac{-1}{11} \end{array}\right]$$

2.2.33

$$\begin{bmatrix} 11 & 0 & 0 \\ 11 & 11 & 0 \\ 11 & 11 & 11 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{11} & 0 & 0 \\ \frac{-1}{11} & \frac{1}{11} & 0 \\ 0 & \frac{-1}{11} & \frac{1}{11} \end{bmatrix}$$

$$\begin{bmatrix} 11 & 0 & 0 & 0 \\ 11 & 11 & 0 & 0 \\ 11 & 11 & 11 & 0 \\ 11 & 11 & 11 & 11 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{11} & 0 & 0 & 0 \\ \frac{-1}{11} & \frac{1}{11} & 0 & 0 \\ 0 & \frac{-1}{11} & \frac{1}{11} & 0 \\ 0 & 0 & \frac{-1}{11} & \frac{1}{11} \end{bmatrix}$$

2.2.38

$$AD = I_2 \Rightarrow D = A^{-1}I_2$$

$$[A \,|\, I_2] = \left[\begin{array}{cccc|cc} 0 & -1 & 1 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{bmatrix} 0 & -1 & 1 & 1 \\ 1 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \\ e & f \\ g & h \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Leftrightarrow \begin{cases} -\overset{0}{c} + \overset{1}{e} + \overset{0}{g} = 1 \\ -\overset{0}{d} + \overset{0}{f} + h^{0} = 0 \\ a^{1} + c^{0} - e^{1} = 0 \\ b^{1} + d^{0} - f^{0} = 1 \end{cases}$$

let $c=0, g=0 \Rightarrow e=1$

$d=0, b=1 \Rightarrow f=0$

$\Rightarrow h=0, a=1$

$$\Rightarrow D = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Verify:

$$AD = \begin{bmatrix} 0 & -1 & 1 & 1 \\ 1 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \therefore \boxed{\text{hold true}}$$