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Course: Linear Algebra I (Spring 2020)

Assignment: Section 2.2 Homework

1. Find the inverse of the matrix. $\begin{bmatrix} 5 & 8 \\ 7 & 6 \end{bmatrix}$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

A. $\begin{bmatrix} 5 & 8 \\ 7 & 6 \end{bmatrix}^{-1} = \begin{bmatrix} -\frac{3}{13} & \frac{4}{13} \\ \frac{7}{26} & -\frac{5}{26} \end{bmatrix}$ (Simplify your answers.)

B. The matrix is not invertible.

2. Find the inverse of the matrix.

$$\begin{bmatrix} 3 & 4 \\ -2 & -3 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

A. $\begin{bmatrix} 3 & 4 \\ -2 & -3 \end{bmatrix}^{-1} = \begin{bmatrix} 3 & 4 \\ -2 & -3 \end{bmatrix}$ (Simplify your answers.)

B. The matrix is not invertible.

3. Use the given inverse of the coefficient matrix to solve the following system.

$$\begin{aligned} 5x_1 + 2x_2 &= -4 \\ -6x_1 - 2x_2 &= 3 \end{aligned} \qquad A^{-1} = \begin{bmatrix} -1 & -1 \\ 3 & \frac{5}{2} \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

A. $x_1 = 1$ and $x_2 = -\frac{9}{2}$ (Simplify your answers.)

B. There is no solution.

4.

Let $A = \begin{bmatrix} 1 & 2 \\ 8 & 18 \end{bmatrix}$, $\mathbf{b}_1 = \begin{bmatrix} -3 \\ -20 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$, $\mathbf{b}_3 = \begin{bmatrix} 2 \\ 12 \end{bmatrix}$, and $\mathbf{b}_4 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$.

(a) Find A^{-1} and use it solve the four equations $A\mathbf{x} = \mathbf{b}_1$, $A\mathbf{x} = \mathbf{b}_2$, $A\mathbf{x} = \mathbf{b}_3$, and $A\mathbf{x} = \mathbf{b}_4$.

(b) The four equations in part (a) can be solved by the same set of operations, since the coefficient matrix is the same in each case. Solve the four equations in part (a) by row reducing the augmented matrix $[A\ \mathbf{b}_1\ \mathbf{b}_2\ \mathbf{b}_3\ \mathbf{b}_4]$.

Find A^{-1}. Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice

A. $A^{-1} = \begin{bmatrix} 9 & -1 \\ -4 & \frac{1}{2} \end{bmatrix}$ (Simplify your answers.)

B. The matrix is not invertible.

Solve $A\mathbf{x} = \mathbf{b}_1$.

$\mathbf{x} = \begin{bmatrix} -7 \\ 2 \end{bmatrix}$ (Simplify your answers.)

Solve $A\mathbf{x} = \mathbf{b}_2$.

$\mathbf{x} = \begin{bmatrix} 10 \\ -4 \end{bmatrix}$ (Simplify your answers.)

Solve $A\mathbf{x} = \mathbf{b}_3$.

$\mathbf{x} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$ (Simplify your answers.)

Solve $A\mathbf{x} = \mathbf{b}_4$.

$\mathbf{x} = \begin{bmatrix} 14 \\ -6 \end{bmatrix}$ (Simplify your answers.)

(b) Solve the four equations by row reducing the augmented matrix $[A\ \mathbf{b}_1\ \mathbf{b}_2\ \mathbf{b}_3\ \mathbf{b}_4]$. Write the augmented matrix $[A\ \mathbf{b}_1\ \mathbf{b}_2\ \mathbf{b}_3\ \mathbf{b}_4]$ in reduced echelon form.

$$\begin{bmatrix} 1 & 0 & -7 & 10 & 6 & 14 \\ 0 & 1 & 2 & -4 & -2 & -6 \end{bmatrix}$$ (Simplify your answers.)

Are the solutions the same in (a) and (b)?

- No
- Yes

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5. Use matrix algebra to show that if A is invertible and D satisfies $AD = I$, then $D = A^{-1}$.

Choose the correct answer below.

- **A.** Left-multiply each side of the equation $AD = I$ by A^{-1} to obtain $A^{-1}AD = A^{-1}I$, $ID = A^{-1}$, and $D = A^{-1}$.
- **B.** Right-multiply each side of the equation $AD = I$ by A^{-1} to obtain $ADA^{-1} = IA^{-1}$, $DI = A^{-1}$, and $D = A^{-1}$.
- **C.** Add A^{-1} to both sides of the equation $AD = I$ to obtain $AD + A^{-1} = I + A^{-1}$, $DI = A^{-1}$, and $D = A^{-1}$.
- **D.** Add A^{-1} to both sides of the equation $AD = I$ to obtain $A^{-1} + AD = A^{-1} + I$, $ID = A^{-1}$, and $D = A^{-1}$.

6. Suppose A is $n \times n$ and the equation $Ax = 0$ has only the trivial solution. Explain why A has n pivot columns and A is row equivalent to I_n.

Choose the correct answer below.

- **A.** Suppose A is $n \times n$ and the equation $Ax = 0$ has only the trivial solution. Then there are n free variables in this equation, thus A ha
the main diagonal. Hence A is row equivalent to the $n \times n$ identity matrix, I_n.
- **B.** Suppose A is $n \times n$ and the equation $Ax = 0$ has only the trivial solution. Then there are no free variables in this equation, thus A h
diagonal. Hence A is the $n \times n$ identity matrix, I_n.
- **C.** Suppose A is $n \times n$ and the equation $Ax = 0$ has only the trivial solution. Then there are no free variables in this equation, thus A h
on the main diagonal. Hence A is row equivalent to the $n \times n$ identity matrix, I_n.

7. Suppose A is $n \times n$ and the equation $Ax = b$ has a solution for each b in $\mathbb{R}^n$. Explain why A must be invertible. [Hint: Is A row equivalent to I_n?]

Choose the correct answer below.

- **A.** If the equation $Ax = b$ has a solution for each b in $\mathbb{R}^n$, then A has one pivot position. It follows that A is row equivalent to I_n. There
- **B.** If the equation $Ax = b$ has a solution for each b in $\mathbb{R}^n$, then A has a pivot position in each row. Since A is square, the pivots must
- **C.** If the equation $Ax = b$ has a solution for each b in $\mathbb{R}^n$, then A has a pivot position in each row. Since A is square, the pivots must
- **D.** If the equation $Ax = b$ has a solution for each b in $\mathbb{R}^n$, then A does not have a pivot position in each row. Since A is square, and I

8. Find the inverse of the matrix, if it exists.

$$A = \begin{bmatrix} 1 & -3 \\ 6 & -9 \end{bmatrix}$$

Find the inverse. Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- **A.** $A^{-1} = \begin{bmatrix} -1 & \frac{1}{3} \\ -\frac{2}{3} & \frac{1}{9} \end{bmatrix}$ (Type an integer or simplified fraction for each matrix element.)
- **B.** The matrix A does not have an inverse.

9. Find the inverse of the matrix, if it exists.

$$A = \begin{bmatrix} 4 & 4 \\ 2 & 5 \end{bmatrix}$$

Find the inverse. Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

A. $A^{-1} = \begin{bmatrix} \frac{5}{12} & -\frac{1}{3} \\ -\frac{1}{6} & \frac{1}{3} \end{bmatrix}$ (Simplify your answers.)

B. The matrix is not invertible.

10. Find the inverse of the given matrix, if it exists.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 4 & 1 & 3 \\ -3 & -4 & 3 \end{bmatrix}$$

Find the inverse. Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

A. $A^{-1} = \begin{bmatrix} -\frac{15}{11} & \frac{8}{11} & \frac{2}{11} \\ \frac{21}{11} & -\frac{9}{11} & -\frac{5}{11} \\ \frac{13}{11} & -\frac{4}{11} & -\frac{1}{11} \end{bmatrix}$ (Type integers or simplified fractions.)

B. The matrix A does not have an inverse.

11. Use the algorithm for finding A^{-1} to find the inverses of the matrices shown to the right. Let A be the corresponding $n \times n$ matrix, and let B be its inverse. Guess the form of B, and then show that AB = I.

$$\begin{bmatrix} 11 & 0 & 0 \\ 11 & 11 & 0 \\ 11 & 11 & 11 \end{bmatrix} \text{ and } \begin{bmatrix} 11 & 0 & 0 & 0 \\ 11 & 11 & 0 & 0 \\ 11 & 11 & 11 & 0 \\ 11 & 11 & 11 & 11 \end{bmatrix}$$

$$\begin{bmatrix} 11 & 0 & 0 \\ 11 & 11 & 0 \\ 11 & 11 & 11 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{11} & 0 & 0 \\ -\frac{1}{11} & \frac{1}{11} & 0 \\ 0 & -\frac{1}{11} & \frac{1}{11} \end{bmatrix}$$

$$\begin{bmatrix} 11 & 0 & 0 & 0 \\ 11 & 11 & 0 & 0 \\ 11 & 11 & 11 & 0 \\ 11 & 11 & 11 & 11 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{11} & 0 & 0 & 0 \\ -\frac{1}{11} & \frac{1}{11} & 0 & 0 \\ 0 & -\frac{1}{11} & \frac{1}{11} & 0 \\ 0 & 0 & -\frac{1}{11} & \frac{1}{11} \end{bmatrix}$$

If A is the corresponding $n \times n$ matrix and B is its inverse, which of the following is B?

A.

$$\begin{bmatrix} \frac{1}{11} & 0 & 0 & \cdots & 0 & 0 \\ \frac{1}{11} & -\frac{1}{11} & 0 & & 0 & 0 \\ 0 & \frac{1}{11} & -\frac{1}{11} & & 0 & 0 \\ \vdots & & & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & \frac{1}{11} & -\frac{1}{11} \end{bmatrix}$$

For $j = 1,2,\ldots,n$, let $\mathbf{a}_j$, $\mathbf{b}_j$, and $\mathbf{e}_j$ denote the jth columns of A, B, and I, respectively. Evaluate $A\mathbf{b}_j$.

$$A\mathbf{b}_j = A\left(\frac{1}{11}\mathbf{e}_j - \frac{1}{11}\mathbf{e}_{j+1}\right)$$

$$A\mathbf{b}_j = \frac{1}{11}A\mathbf{e}_j - \frac{1}{11}A\mathbf{e}_{j+1}$$ Rewrite without parentheses.

$$A\mathbf{b}_j = \frac{1}{11}\mathbf{a}_j - \frac{1}{11}\mathbf{a}_{j+1}$$ Multiply.

Because the result from the previous step is equal to $\mathbf{e}_j$, it follows that AB = I.

YOU ANSWERED: $\mathbf{a}_j$,

12. Let $A = \begin{bmatrix} 0 & -1 & 1 & 1 \\ 1 & 1 & -1 & 0 \end{bmatrix}$. Construct a 4×2 matrix D, using only 1 and 0 as entries, such that $AD = I_2$. Is it possible that $CA = I_4$ for some 4×2 matrix C? Explain.

$$D = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Is it possible that $CA = I_4$ for some 4×2 matrix C? Explain. Choose the correct answer below.

- **A.** No, because neither C nor A are invertible. When writing I_m as the product of two matrices, since I_m is invertible, those two mat
- **B.** Yes, because C is a 4×2 matrix and A is a 2×4 matrix, making CA a 4×4 matrix. For every $m \times n$ matrix, there exists an $n \times m$
- **C.** No, because if it were true, then $CA\mathbf{x}$ would equal $\mathbf{x}$ for all $\mathbf{x}$ in $\mathbb{R}^4$. Since the columns of A are linearly dependent, $A\mathbf{x} = \mathbf{0}$ for so
- **D.** Yes, if $C = A^T$. The product of any $m \times n$ matrix and its transpose is I_m.