Student: Phong Vo **Date:** 04/24/20	**Instructor:** Erica Yankowskas **Course:** Linear Algebra I (Spring 2020)	**Assignment:** Section 4.7 Homework

1. Let $B = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $C = \{\mathbf{c}_1, \mathbf{c}_2\}$ be bases for a vector space V, and suppose $\mathbf{b}_1 = -2\mathbf{c}_1 + 3\mathbf{c}_2$ and $\mathbf{b}_2 = -7\mathbf{c}_1 + 6\mathbf{c}_2$.

a. Find the change-of-coordinates matrix from B to C.
b. Find $[\mathbf{x}]_C$ for $\mathbf{x} = 5\mathbf{b}_1 - 2\mathbf{b}_2$. Use part (a).

a. $\underset{C \leftarrow B}{P} = \begin{bmatrix} -2 & -7 \\ 3 & 6 \end{bmatrix}$

b. $[\mathbf{x}]_C = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

(Simplify your answers.)

2. Let $A = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ and $B = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ be bases for a vector space V, and suppose $\mathbf{b}_1 = 2\mathbf{a}_1 - 3\mathbf{a}_3$, $\mathbf{b}_2 = -\mathbf{a}_1 + \mathbf{a}_2$, $\mathbf{b}_3 = \mathbf{a}_1 + \mathbf{a}_2 + 6\mathbf{a}_3$.

a. Find the change-of-coordinates matrix from B to A.
b. Find $[\mathbf{x}]_A$ for $\mathbf{x} = \mathbf{b}_1 - 4\mathbf{b}_2 + 4\mathbf{b}_3$.

a. $\underset{A \leftarrow B}{P} = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 1 \\ -3 & 0 & 6 \end{bmatrix}$

b. $[\mathbf{x}]_A = \begin{bmatrix} 10 \\ 0 \\ 21 \end{bmatrix}$ (Simplify your answers.)

3. Let $A = \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ and $B = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ be bases for a vector space V, and suppose $\mathbf{a}_1 = 2\mathbf{b}_1 - \mathbf{b}_2$, $\mathbf{a}_2 = -\mathbf{b}_1 + 5\mathbf{b}_2 + \mathbf{b}_3$, $\mathbf{a}_3 = \mathbf{b}_2 - 6\mathbf{b}_3$.

a. Find the change-of-coordinates matrix from A to B.
b. Find $[\mathbf{x}]_B$ for $\mathbf{x} = 3\mathbf{a}_1 + 4\mathbf{a}_2 + \mathbf{a}_3$.

a. $\underset{B \leftarrow A}{P} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 5 & 1 \\ 0 & 1 & -6 \end{bmatrix}$

b. $[\mathbf{x}]_B = \begin{bmatrix} 2 \\ 18 \\ -2 \end{bmatrix}$ (Simplify your answers.)

4. Let $B = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $C = \{\mathbf{c}_1, \mathbf{c}_2\}$ be bases for $\mathbb{R}^2$. Find the change-of-coordinates matrix from B to C and the change-of-coordinates matrix from C to B.

$$\mathbf{b}_1 = \begin{bmatrix} -7 \\ -16 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} 3 \\ 7 \end{bmatrix}, \mathbf{c}_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \mathbf{c}_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Find the change-of-coordinates matrix from B to C.

$$\underset{C \leftarrow B}{P} = \begin{bmatrix} 5 & -2 \\ -12 & 5 \end{bmatrix}$$ (Simplify your answers.)

Find the change-of-coordinates matrix from C to B.

$$\underset{B \leftarrow C}{P} = \begin{bmatrix} 5 & 2 \\ 12 & 5 \end{bmatrix}$$ (Simplify your answers.)

5. The sets B and C are bases for a vector space V. Mark each statement true or false. Justify each answer.
a. The columns of $\underset{C \leftarrow B}{P}$ are linearly independent.
b. If $V = \mathbb{R}^2$, $B = \{\mathbf{b}_1, \mathbf{b}_2\}$, and $C = \{\mathbf{c}_1, \mathbf{c}_2\}$, then row reduction of $\begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 & \mathbf{b}_1 & \mathbf{b}_2 \end{bmatrix}$ to $\begin{bmatrix} I & P \end{bmatrix}$ produces a matrix P that satisfies $[\mathbf{x}]_B = P[\mathbf{x}]_C$ for all $\mathbf{x}$ in V.

a. Is the statement true or false?

- **A.** The statement is false. The columns of $\underset{C \leftarrow B}{P}$ are linearly dependent because they are the coordinate vectors of the linearly dependent set B.
- **B.** The statement is false. The columns of $\underset{C \leftarrow B}{P}$ are linearly dependent because they are the coordinate vectors of the linearly dependent set C.
- **C.** The statement is true. The columns of $\underset{C \leftarrow B}{P}$ are linearly independent because they are the coordinate vectors of the linearly independent set C.
- **D.** The statement is true. The columns of $\underset{C \leftarrow B}{P}$ are linearly independent because they are the coordinate vectors of the linearly independent set B.

b. Is the statement true or false?

- **A.** The statement is true. Left-multiplying $[\mathbf{x}]_C$ by P gives $[\mathbf{x}]_B$.
- **B.** The statement is false. Matrix P satisfies $[\mathbf{x}]_C = P[\mathbf{x}]_B$ for all $\mathbf{x}$ in V.
- **C.** The statement is false. Left-multiplying $[\mathbf{x}]_C$ by P gives $[\mathbf{x}]_B$.
- **D.** The statement is true. Matrix P is the change-of-coordinates matrix from C to B.

6. In $\mathbb{P}_2$, find the change-of-coordinates matrix from the basis $B = \{1 - 3t + t^2, 2 - 5t + 3t^2, 2 - 3t + 6t^2\}$ to the standard basis $C = \{1, t, t^2\}$. Then find the B-coordinate vector for $2 - 6t + 3t^2$.

In $\mathbb{P}_2$, find the change-of-coordinates matrix from the basis $B = \{1 - 3t + t^2, 2 - 5t + 3t^2, 2 - 3t + 6t^2\}$ to the standard basis $C = \{1, t, t^2\}$.

$$\underset{C \leftarrow B}{P} = \begin{bmatrix} 1 & 2 & 2 \\ -3 & -5 & -3 \\ 1 & 3 & 6 \end{bmatrix}$$ (Simplify your answers.)

Find the B-coordinate vector for $2 - 6t + 3t^2$.

$$[\mathbf{x}]_B = \begin{bmatrix} 6 \\ -3 \\ 1 \end{bmatrix}$$ (Simplify your answers.)

7. In $\mathbb{P}_2$, find the change-of-coordinates matrix from the basis $B = \{1 - 5t^2, 5 + t - 24t^2, 1 + 4t\}$ to the standard basis. Then write t^2 as a linear combination of the polynomials in B.

In $\mathbb{P}_2$, find the change-of-coordinates matrix from the basis B to the standard basis.

$$\underset{C \leftarrow B}{P} = \begin{bmatrix} 1 & 5 & 1 \\ 0 & 1 & 4 \\ -5 & -24 & 0 \end{bmatrix}$$

(Simplify your answers.)

Write t^2 as a linear combination of the polynomials in B.

$t^2 = 19 \left(1 - 5t^2\right) + -4 \left(5 + t - 24t^2\right) + 1 (1 + 4t)$

(Simplify your answers.)