Q1 Probability, Part I

14 Points

Below is a table listing the probabilities of three binary random variables. Fill in the correct values for each marginal or conditional probability below.

X_0	X_1	X_2	$P(X_0, X_1, X_2)$
0	0	0	0.160
1	0	0	0.100
0	1	0	0.120
1	1	0	0.040
0	0	1	0.180
1	0	1	0.200
0	1	1	0.120
1	1	1	0.080

Q1.1

7 Points

$P(X_0 = 1, X_1 = 0, X_2 = 1)$

.200

$P(X_0 = 0, X_1 = 1)$

.240

$P(X_2 = 0)$

.420

Q1.2

7 Points

$P(X_1 = 0 \mid X_0 = 1)$

.714

$P(X_0 = 1, X_1 = 0 \mid X_2 = 1)$

.345

$P(X_0 = 1 \mid X_1 = 0, X_2 = 1)$

.526

Q2 Probability, Part II

14 Points

You are given the prior distribution $P(X)$, and two conditional distributions $P(Y \mid X)$ and $P(Z \mid Y)$ as below (you are also given the fact that Z is independent from X given Y). All variables are binary variables. Compute the following joint distributions based on the chain rule.

X	$P(X)$
0	0.500
1	0.500

Y	X	$P(Y\|X)$
0	0	0.600
1	0	0.400
0	1	0.900
1	1	0.100

Z	Y	$P(Z\|Y)$
0	0	0.100
1	0	0.900
0	1	0.700
1	1	0.300

Q2.1

7 Points

$P(X = 0, Y = 0)$

.3

$P(X = 1, Y = 0)$

.45

$P(X = 0, Y = 1)$

.2

$P(X = 1, Y = 1)$

.05

Q2.2

7 Points

$P(X = 0, Y = 0, Z = 0)$

.03

$P(X = 1, Y = 1, Z = 0)$

.035

$P(X = 1, Y = 0, Z = 1)$

.405

$P(X = 1, Y = 1, Z = 1)$

.015

Q3 Probability, Part III

14 Points

For each of the following four subparts, you are given three joint probability distribution tables. For each distribution, please identify if the given independence / conditional independence assumption is true or false.

For your convenience, we have also provided some marginal and conditional probability distribution tables that could assist you in solving this problem.

X	Y	$P(X,Y)$
0	0	0.240
1	0	0.160
0	1	0.360
1	1	0.240

X	$P(X)$
0	0.600
1	0.400

Y	$P(Y)$
0	0.400
1	0.600

X is independent from Y.

- (selected) True
- False

X	Y	$P(X,Y)$
0	0	0.540
1	0	0.360
0	1	0.060
1	1	0.040

X	$P(X)$
0	0.600
1	0.400

X	Y	$P(X\|Y)$
0	0	0.600
1	0	0.400
0	1	0.600
1	1	0.400

X is independent from Y.

- (●) True
- ( ) False

X	Y	Z	$P(X,Y,Z)$
0	0	0	0.280
1	0	0	0.070
0	1	0	0.210
1	1	0	0.140
0	0	1	0.060
1	0	1	0.060
0	1	1	0.030
1	1	1	0.150

X	Z	$P(X\|Z)$
0	0	0.700
1	0	0.300
0	1	0.300
1	1	0.700

Y	Z	$P(Y\|Z)$
0	0	0.500
1	0	0.500
0	1	0.400
1	1	0.600

X	Y	Z	$P(X,Y\|Z)$
0	0	0	0.400
1	0	0	0.100
0	1	0	0.300
1	1	0	0.200
0	0	1	0.200
1	0	1	0.200
0	1	1	0.100
1	1	1	0.500

X is independent from Y given Z.

○ True

◉ False

X	Y	Z	$P(X,Y,Z)$
0	0	0	0.140
1	0	0	0.140
0	1	0	0.060
1	1	0	0.060
0	0	1	0.048
1	0	1	0.192
0	1	1	0.072
1	1	1	0.288

X	Z	$P(X\|Z)$
0	0	0.500
1	0	0.500
0	1	0.200
1	1	0.800

Y	Z	$P(Y\|Z)$
0	0	0.700
1	0	0.300
0	1	0.400
1	1	0.600

X	Y	Z	$P(X,Y\|Z)$
0	0	0	0.350
1	0	0	0.350
0	1	0	0.150
1	1	0	0.150
0	0	1	0.080
1	0	1	0.320
0	1	1	0.120
1	1	1	0.480

X is independent from Y given Z.

- (selected) True
- False

Q4 Chain Rule
16 Points

Select all expressions that are equivalent to the specified probability using the given independence assumptions.

Given no independence assumptions, $P(A, B \mid C)$ =

- [ ] $\frac{P(C|A)P(A|B)P(B)}{P(C)}$
- [ ] $\frac{P(B,C|A)P(A)}{P(B,C)}$
- [x] $P(A \mid B, C)P(B \mid C)$
- [ ] $\frac{P(A|C)P(B,C)}{P(C)}$

Given that A is independent of B given C, $P(A, B \mid C)$ =

- [ ] $\frac{P(C|A)P(A|B)P(B)}{P(C)}$
- [ ] $\frac{P(B,C|A)P(A)}{P(B,C)}$
- [x] $P(A \mid B, C)P(B \mid C)$
- [x] $\frac{P(A|C)P(B,C)}{P(C)}$

Given no independence assumptions, $P(A \mid B, C)$ =

- [ ] $\frac{P(C|A)P(A|B)P(B)}{P(C)}$
- [x] $\frac{P(B,C|A)P(A)}{P(B,C)}$
- [ ] $\frac{P(A|C)P(C|B)P(B)}{P(B,C)}$
- [x] $\frac{P(C|A,B)P(B|A)P(A)}{P(B|C)P(C)}$

Given that A is independent of B given C, $P(A \mid B, C)$ =

- [ ] $\frac{P(C|A)P(A|B)P(B)}{P(C)}$
- [x] $\frac{P(B,C|A)P(A)}{P(B,C)}$
- [x] $\frac{P(A|C)P(C|B)P(B)}{P(B,C)}$
- [x] $\frac{P(C|A,B)P(B|A)P(A)}{P(B|C)P(C)}$

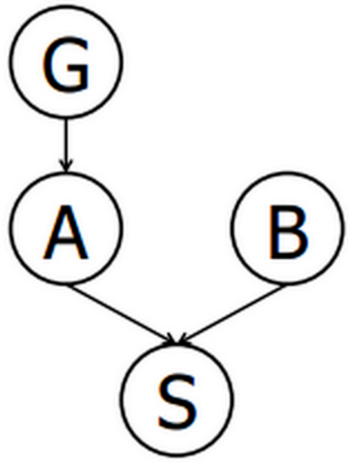
Q5 Bayes' Nets and Probability

16 Points

Suppose that a patient can have a symptom (S) that can be caused by two different diseases (A and B). It is known that the variation of gene G plays a big role in the manifestation of disease A. The Bayes' Net and corresponding probability tables for this situation are shown below.

G	$P(G)$
g	0.10
$\neg g$	0.90

G	A	$P(A \mid G)$
g	a	1.00
g	$\neg a$	0.00
$\neg g$	a	0.10
$\neg g$	$\neg a$	0.90



B	$P(B)$
b	0.40
$\neg b$	0.60

A	B	S	$P(S \mid A, B)$
a	b	s	1.00
a	b	$\neg s$	0.00
a	$\neg b$	s	0.90
a	$\neg b$	$\neg s$	0.10
$\neg a$	b	s	0.80
$\neg a$	b	$\neg s$	0.20
$\neg a$	$\neg b$	s	0.10
$\neg a$	$\neg b$	$\neg s$	0.90

Q5.1

11 Points

Compute P(g, a, b, s).

.04

What is the probability that a patient has disease A?

.19

What is the probability that a patient has disease A given that they have disease B?

.19

What is the probability that a patient has disease A given that they have symptom S and disease B?

.2267

Q5.2

5 Points

What is the probability that a patient has the disease carrying gene variation G given that they have disease A?

.5263

What is the probability that a patient has the disease carrying gene variation G given that they have disease B?

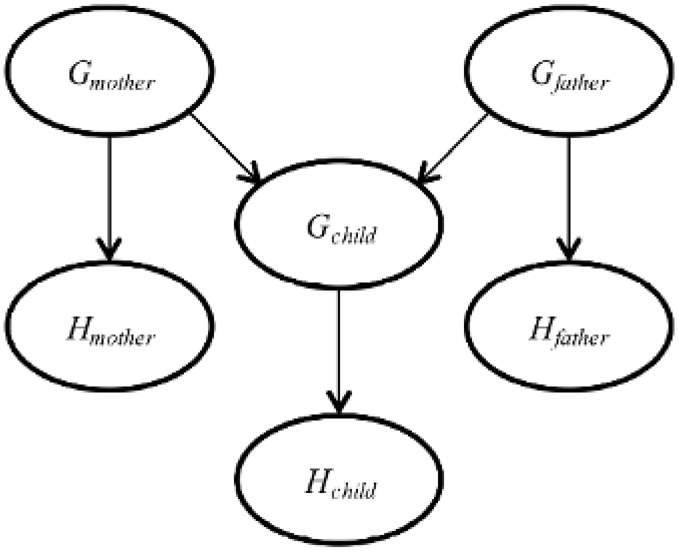
.1

Q6 Bayes' Nets Independence

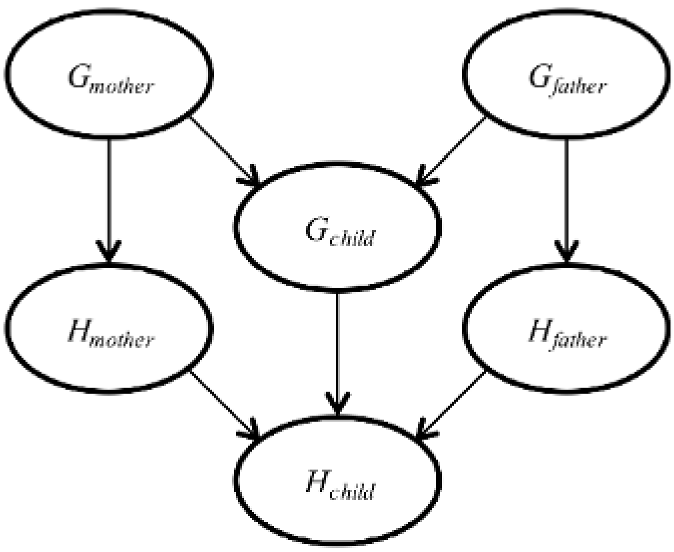
14 Points

Let H_x be a random variable denoting the handedness of an individual x, with possible values l or r. A common hypothesis is that left- or right-handedness is inherited by a simple mechanism; that is, perhaps there is a gene G_x, also with values l or r, and perhaps actual handedness turns out mostly the same (with some probability s) as the gene an individual possesses. Furthermore, perhaps the gene itself is equally likely to be inherited from either of an individual's parents, with a small nonzero probability m of a random mutation flipping the handedness.

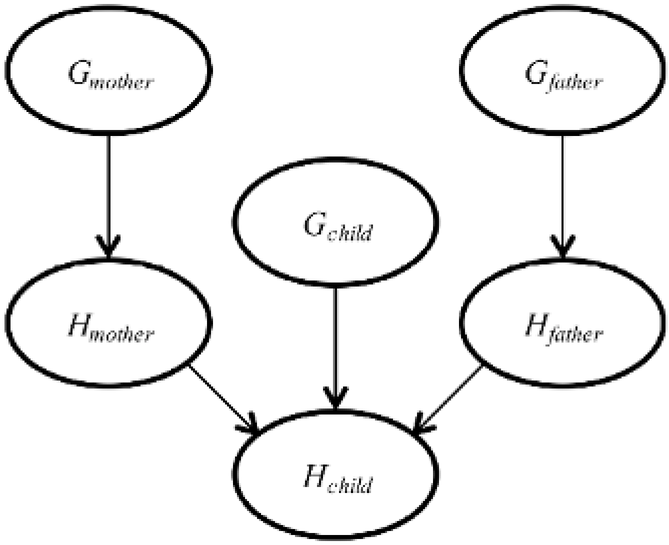
The following three images are possible models involving the genes G and handednesses H.


G_{mother}
G_{father}
G_{child}
H_{mother}
H_{father}
H_{child}

(a)


G_{mother}
G_{father}
G_{child}
H_{mother}
H_{father}
H_{child}

(b)



(c)

Which of the three networks above claim that
$P(G_{father}, G_{mother}, G_{child}) = P(G_{father})P(G_{mother})P(G_{child})$?

- [ ] (a)
- [ ] (b)
- [x] (c)

Which of the three networks make independence claims that are consistent with the hypothesis about the inheritance of handedness?

- [x] (a)
- [x] (b)
- [ ] (c)

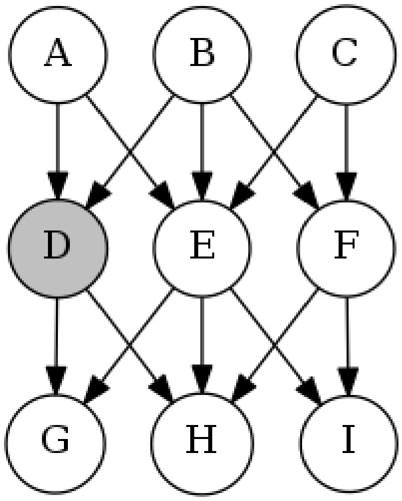
Which of the three networks is the best description of the hypothesis?

- (●) (a)
- ( ) (b)
- ( ) (c)

Q7 D-Separation

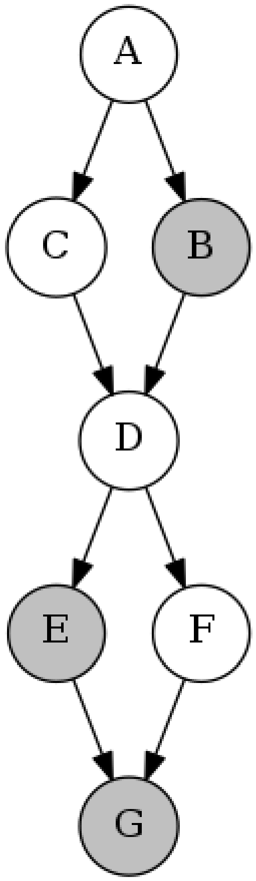
12 Points

You are given several graphical models below, and each graphical model is associated with an independence (or conditional independence) assertion. Please specify if the assertion is true or false.



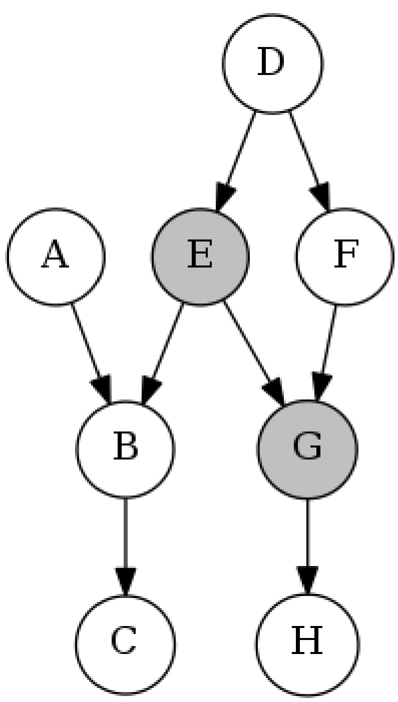
It is guaranteed that G is independent of H given D

- ○ True
- ◉ False



It is guaranteed that A is independent of D given E, B, G

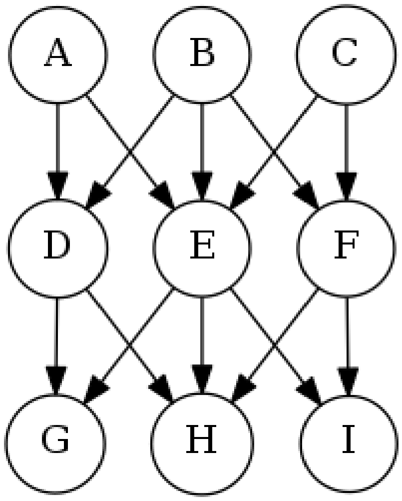
- True
- False



It is guaranteed that H is independent of B given G, E

True (selected)

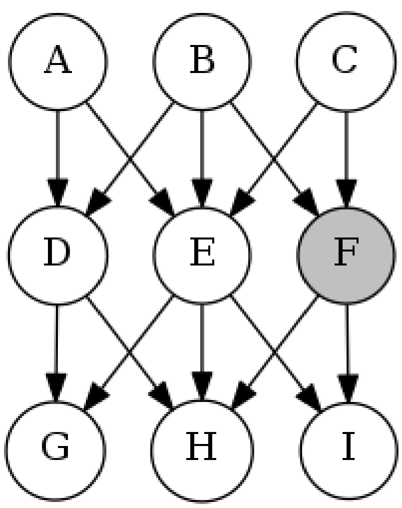
False



It is guaranteed that A is independent of C

True (selected)

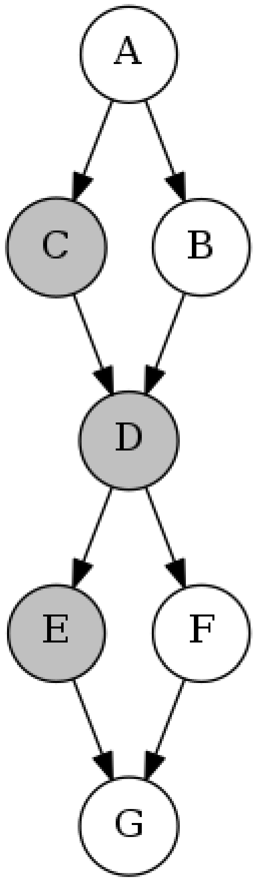
False



It is guaranteed that D is independent of C given F

True

False (selected)



It is guaranteed that G is independent of B given C, E, D

- (●) True
- ( ) False

HW 6 (Electronic Component)

● GRADED

STUDENT

[redacted]

TOTAL POINTS

100 / 100 pts

QUESTION 1

Probability, Part I 14 pts

1.1 (no title) **7** / 7 pts

1.2 (no title) **7** / 7 pts

QUESTION 2

Probability, Part II 14 pts

2.1 (no title) **7** / 7 pts

2.2 (no title) **7** / 7 pts

QUESTION 3

Probability, Part III **14** / 14 pts

QUESTION 4

Chain Rule **16** / 16 pts

QUESTION 5

Bayes' Nets and Probability 16 pts

5.1 (no title) **11** / 11 pts

5.2 (no title) **5** / 5 pts

QUESTION 6

Bayes' Nets Independence **14** / 14 pts

QUESTION 7

D-Separation **12** / 12 pts