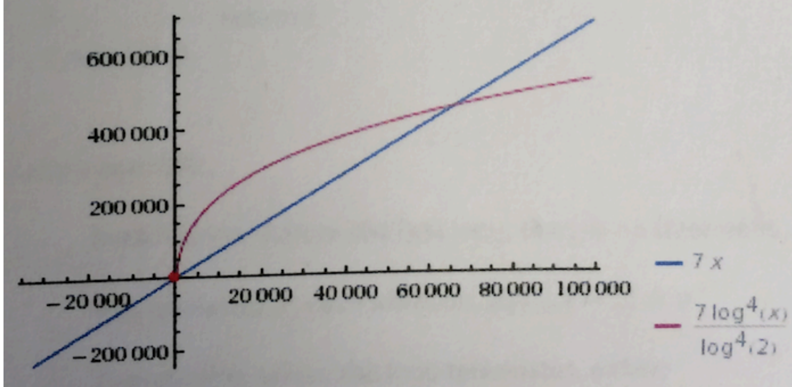
COMP.4040 HW1

1. **Solution** (credit from Denzel Pierre):

$7n > 7(\log_2 n)^4$



$n \geq 65536$

2. **Solution** (credit from Denzel Pierre):

```
Insertion_Sort(A, v)
for i = 1 to A.length
    if A[i] = v
        return i
return NIL
```

Loop Invariant:

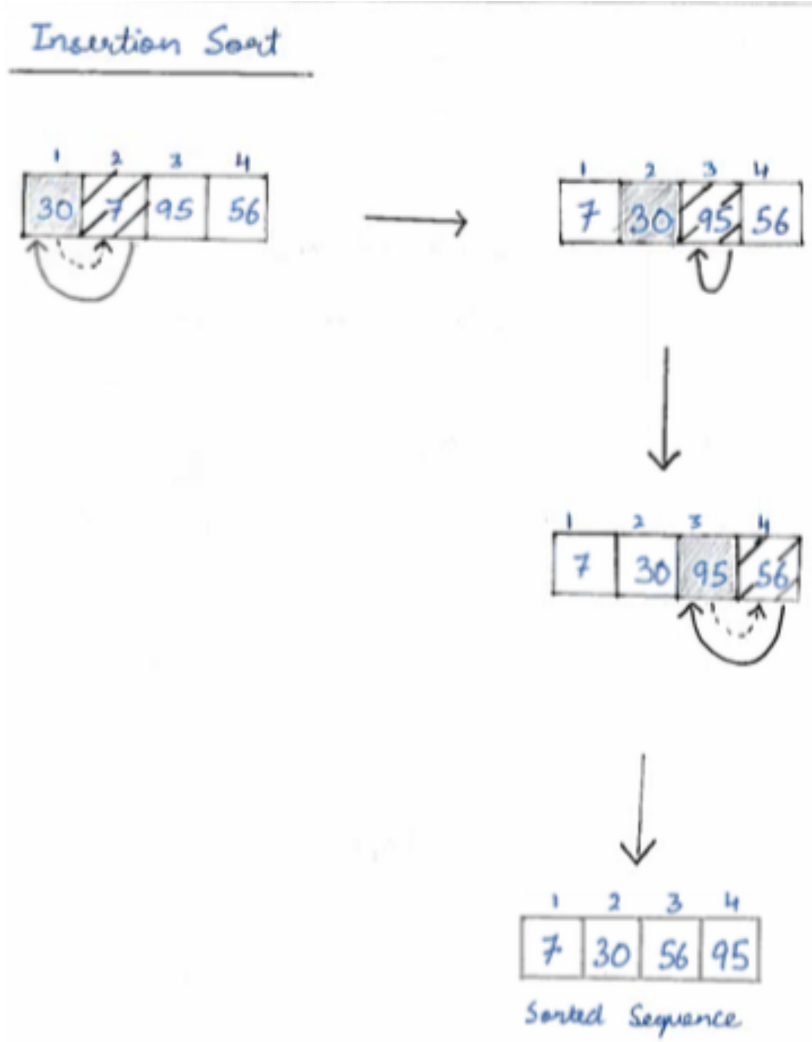
Initialization: Before the first loop, there is no statement.

Maintenance: At each iteration, $A[1 \ldots i-1] \neq v$

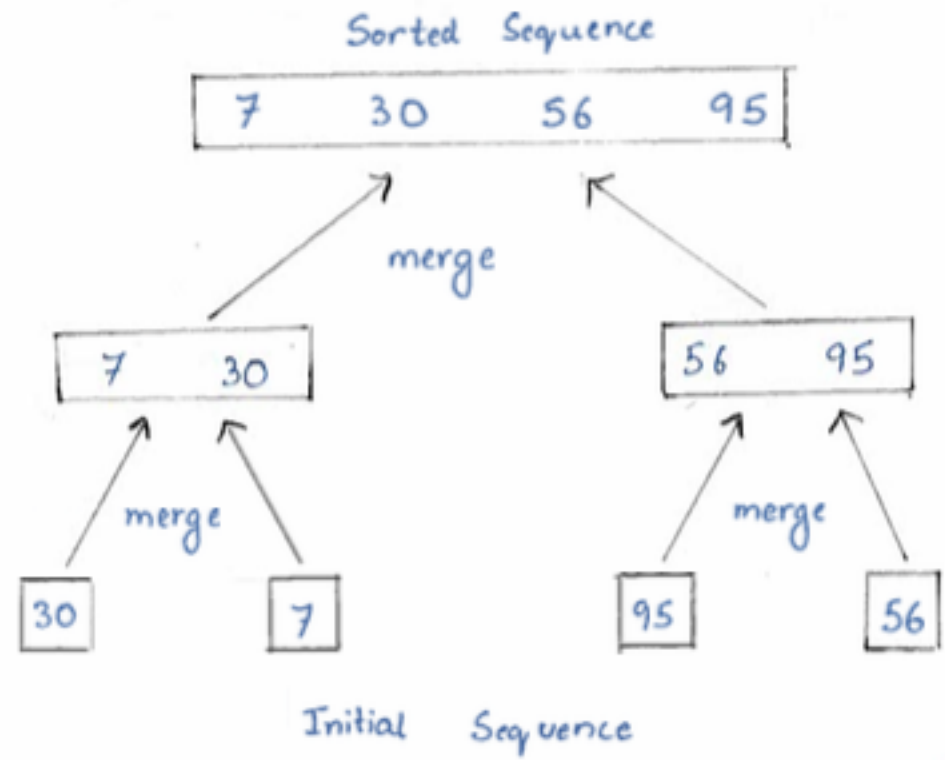
Termination: When the loop terminates, either:

- The for loop returns i, proving the if statement insures $A[i] = v$.
- The for loop returns NIL, proving that for $A[1 \ldots A.length]$, $A[i] \neq v$.

3. Solution (credit from Venkata Praneeth Mummaneni):



Merge Sort



Sorted Sequence
7 30 56 95
merge
7 30
56 95
merge
merge
30
7
95
56
Initial Sequence

4. Solution (credit from Denzel Pierre):

Mystery(n)

1 if $n \le 1$	c_1	1
2 return 1	c_2	1
3 for $i = 1$ to 5	c_3	6
4 for $j = 1$ to n^2	c_4	$5(n^2+1)$
5 print "this is a recursive call"	c_5	$5n^2$
6 Mystery$\left(\frac{n}{3}\right)$	$T\left(\frac{n}{3}\right)$	1
7 Mystery$\left(\frac{n}{3}\right)$	$T\left(\frac{n}{3}\right)$	1
8 Mystery $\left(\frac{n}{3}\right)$	$T\left(\frac{n}{3}\right)$	1

$$T(n) = c_1 + c_2 + 6c_3 + c_4(5n^2+5) + c_5(5n^2) + 3T\left(\frac{n}{3}\right)$$

$$T(n) = cn^2 + 3T\left(\frac{n}{3}\right)$$

$$T(n) = 3\left(3T\left(\frac{n}{9}\right) + \frac{cn^2}{3}\right) + cn^2$$

$$= 9T\left(\frac{n}{9}\right) + \frac{3cn^2}{3} + cn^2$$

$$T(n) = 9\left(3T\left(\frac{n}{27}\right) + \frac{cn^2}{3^2}\right) + cn^2$$

$$T(n) = \begin{cases} cn^2, if\ n = 1, c\ is\ constant > 0 \\ 3T\left(\frac{n}{3}\right) + cn^2, \quad if\ n > 1 \end{cases}$$

$$T(n) = \frac{cn^2}{3^0} + \frac{cn^2}{3^1} + \frac{cn^2}{3^2} + \frac{cn^2}{3^3} + \cdots = \sum_{k=0}^{n-1} \frac{1}{3} cn^2$$

$T(n) \le \theta(n^2)$ $T(n) = \Theta(n^2)$

It is not clear how to get T(n) in $\theta(n^2)$, should add the following analysis

$$\le cn^2 \sum_{i=0}^{\infty} \frac{1}{3^i}$$

The summation is geometric and converges to 3/2

$$\le \frac{3}{2} cn^2$$

5. **Solution** (credit from Venkata Praneeth Mummaneni):

a.

Array with elements in the order $n, n-1, n-2, \dots 3, 2, 1$ has the most number of inversions.

This array has $(n-1) + (n-2) + \dots + 3 + 2 + 1$ inversions

$\Rightarrow$ Number of inversions $= \frac{n(n-1)}{2}$

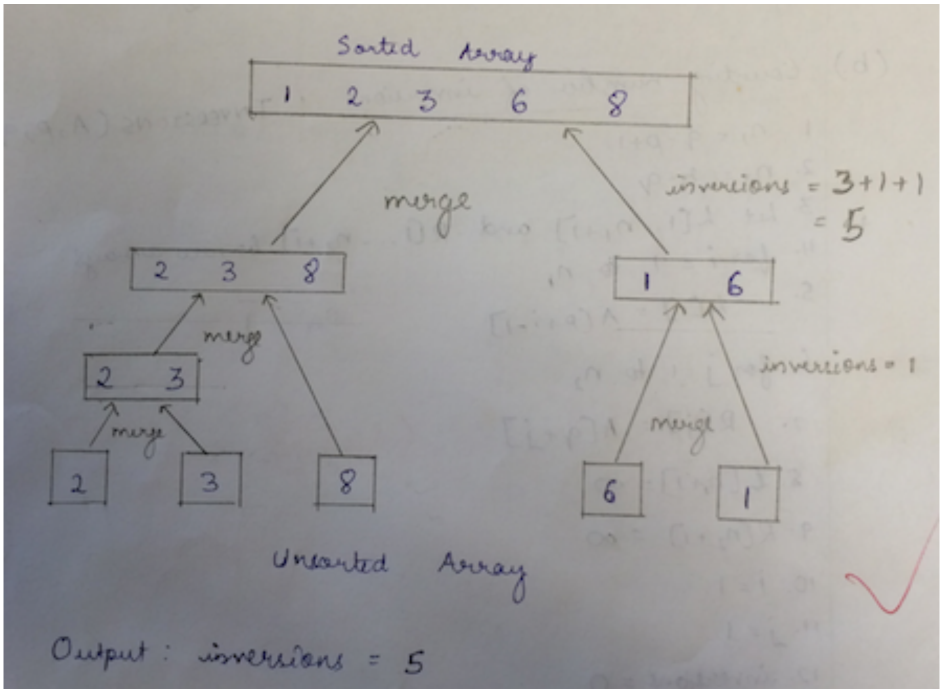
b.

```
COUNT-INVERSIONS(A, p, r)
  inversions = 0
  if p<r
     q = ⌊(p+r)/2⌋
     inversions = inversions + COUNT-INVERSIONS(A, p, q)
     inversions = inversions + COUNT-INVERSIONS(A, q+1, r)
     inversions = inversions + INVERSIONS(A, p, q, r)
  return inversions.
```

Counting number of inversions : INVERSIONS (A, p, q, r)

```
n1 = q - p + 1
n2 = r - q
Let L[1..n1+1] and R[1...n2+1] be new arrays
for i = 1 to n1
    L[i] = A[p+i-1]
for j = 1 to n2
    R[j] = A[q+j]
L[n1+1] = ∞
R[n2+1] = ∞
i = 1
j = 1
inversions = 0
iscounted = FALSE
for k = p to r
    if iscounted == FALSE and R[j] < L[i]
        inversions = inversions + n1 - i + 1
        iscounted = TRUE
    if L[i] ≤ R[j]
        A[k] = L[i]
        i = i + 1
    else A[k] = R[j]
        j = j + 1
        iscounted = FALSE
return inversions
```

c.


Sorted Array
1 2 3 6 8
merge
inversions = 3+1+1
= 5
2 3 8
1 6
merge
inversions = 1
2 3
merge
merge
2
3
8
6
1
Unsorted Array
Output : inversions = 5