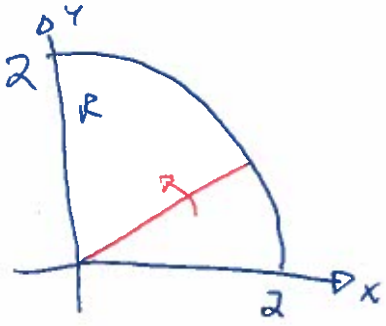
Calculus III Quiz #6_v.2 Spring 2018 NAME: SOLUTION

1. (6 Pts) Evaluate the double integral $\iint_R xydA$ where R is the quarter circle of radius $r=2$ in the first quadrant (i.e., $x \geq 0$ and $y \geq 0$). Be sure to draw the region R and the windshield wiper you are using to perform the integration.



$$\iint_R xy\,dA = \int_0^{\pi/2}\int_0^2 r\cos\theta \cdot r\sin\theta \cdot r\,dr\,d\theta$$

$$= \int_0^{\pi/2} \sin\theta\cos\theta\,d\theta \int_0^2 r^3\,dr$$

$$= \left(\frac{1}{2}\sin^2\theta\Big|_0^{\pi/2}\right)\left(\frac{1}{4}r^4\Big|_0^2\right) = \frac{1}{2}\cdot 4 = 2$$

SEE SECOND PROBLEM (over)

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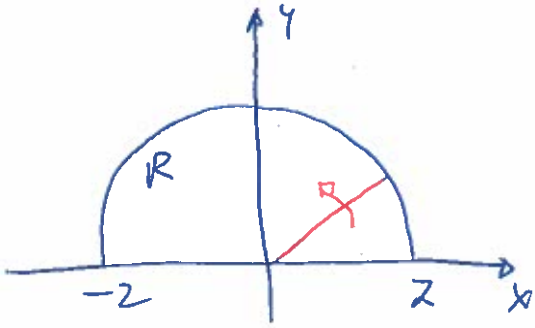
SAME AS ABOVE

SEE SECOND PROBLEM (over)

2. (6 Pts) Evaluate the triple integral $\iiint_D \frac{\sin(z)}{(3+x^2+y^2)}dV$ where D is the region defined by $0 \le x^2+y^2 \le 4$, $0 \le z \le \pi$ and $y \ge 0$. Provide an appropriate figure that illustrates either D or a relevant region, R, in the xy-plane.

$$\iiint_D \frac{\sin(z)\,dV}{(3+x^2+y^2)} = \int_0^{\pi}\int_0^{2}\int_0^{\pi} \frac{\sin(z)\,dz\,r\,dr\,d\theta}{(3+r^2)}$$



$$= \int_0^{\pi} d\theta \int_0^{2} \frac{r\,dr}{(3+r^2)} \int_0^{\pi} \sin(z)\,dz$$

$$= \pi\left[\frac{1}{2}\ln(3+r^2)\right]_0^{2}\left(-\cos(z)\Big|_0^{\pi}\right)$$

$$= \pi\,\frac{1}{2}\left(\ln(7)-\ln(3)\right)(2) = \pi\ln(7/3)$$

2. (6 Pts) Evaluate the triple integral $\iiint_D \frac{\sin(z)}{(4+x^2+y^2)}dV$ where D is the region defined by $0 \le x^2+y^2 \le 4$, $0 \le z \le \pi$ and $y \ge 0$. Provide an appropriate figure that illustrates either D or a relevant region, R, in the xy-plane.

same as above but replace the 3 with a 4

so last two lines become

$$= \pi\,\frac{1}{2}\ln(4+r^2)\Big|_0^{2}\left(-\cos(z)\Big|_0^{\pi}\right)$$

$$= \pi\,\frac{1}{2}\left[\ln(8)-\ln(4)\right]2$$

$$= \pi\ln(8/4) = \pi\ln(2)$$