Prior Distribution = $P(X_0)$

Transition Model = $P(X_{t+1} | X_t)$

Sensor Model = $P(E_t | X_t)$

- Specify topology of the connections between successive slices & between state & evidence variables.

- Transition & sensor models are assumed to be stationary i.e same for all t
  - → Then just specify them for first slice

DBN : — a Bayesian Network that represents a temporal probability model.

- Each slice can have a number of state variables X_t & evidence variables, E_t.

Assume variables & their ~~links~~ links are ~~exted~~ exactly replicated from slice to slice & DBN represents $\boxed{\text{First-order Markov Process}}$

i.e. Each variable can have parents only in its own slice or immediately preceding slice

Particle Filtering

(1) A population of initial-state samples is created by sampling from the Prior distribution $P(X_0)$

Then update cycle is repeated for each step:

(a) Each Sample is propagated forward by sampling next-state value x_{t+1} given current value x_t for the sample based on transition model $P(X_{t+1} | X_t)$

(b) Each Sample is weighted by the likelihood it assigns to the new evidence $P(e_{t+1} | x_{t+1})$

(c) Population is re-sampled to generate new population of N samples. Each new sample is ~~selected~~ selected from the current population; the probability that a particular sample is selected is proportional to its weight. New samples are unweighted.

(4) Re-sampling

— each sample is replicated with probability proportional to its weight;

Thus number of samples in state x_{t+1} after resampling is proportional to the total weight in x_{t+1}

$$\frac{N(x_{t+1}|e_{1:t+1})}{N} = \alpha\, W(x_{t+1}|e_{1:t+1})$$

$$= \alpha\, P(e_{t+1}|x_{t+1})\, N(x_{t+1}|e_{1:t})$$

$$= \alpha P(e_{t+1}|x_{t+1}) \sum_{x_t} P(x_{t+1}|x_t) N(x_t|e_{1:t})$$

$$= \alpha' P(e_{t+1}|x_{t+1}) \sum_{x_t} P(x_{t+1}|x_t) P(x_t|e_{1:t})$$

This resolves to $P(x_{t+1}|e_{1:t+1})$

essentially filtering

Forward message at $\underline{\underline{t+1}}$

1) Sample population starts with correct representation of the forward message

$$f_{1:t} = P(X_t \mid e_{1:t}) \text{ at time } \underline{t}.$$

$N(x_t \mid e_{1:t})$

number of samples occupying state X_t after observations $e_{1:t}$ have been processed

Thus: $$\frac{N(x_t \mid e_{1:t})}{N} = P(x_t \mid e_{1:t})$$ ← i.e. consistent as $N \to \infty$ or for $\underline{\text{large } N}$

2) Propagate each sample forward by sampling state variables $t+1$ ⇐ given values for sample at t

$$N(x_{t+1} \mid e_{1:t}) = \sum_{x_t} P(x_{t+1} \mid x_t) N(x_t \mid e_{1:t})$$

i.e. number of samples reaching state x_{t+1} from each x_t is the transitional probabilities times the population of x_t.

3) Weigh each sample by its likelihood for evidence at $\underline{t+1}$

A sample at x_{t+1} receives weight $P(e_{t+1} \mid x_{t+1})$

Thus. $$W(t+1 \mid e_{1:t+1}) = P(e_{t+1} \mid x_{t+1}) N(x_{t+1} \mid e_{1:t})$$

This is the total weight of the samples in x_{t+1} after seeing e_{t+1}