MATH.3860

Prob and Stats – Test 1 Spring19 Name: Phong Vo

1. (20 points) Events A and B have respective probabilities $P(A) = \frac{1}{3}$ and $P(B) = \frac{2}{5}$, while $P(B|A) = \frac{3}{5}$.
   For each of the following, give an exact numerical answer as a **reduced fraction.**
   (a) What is $P(A \cap B)$?
   (b) What is $P(A \cup B)$?
   (c) What is $P(A - B)$?
   (d) What is $P(A|B)$?
   (e) What is $P(A|B^c)$?

2. (20 points) Two sections of a senior probability class are being taught. From what she has heard about the two instructors listed, Francesca estimates that her chances of passing course are 0.85 if she gets Professor A, and 0.60 if she gets Professor F. The section into which she is put is determined by the registrar. Suppose the chances of being assigned to Professor A are 4 out of 10. Fifteen weeks later we learn that Francesca passed the course. What is the probability she was enrolled in Professor A's section?

3. (20 points) At UML, 30% of the students are majoring in humanities, 50% in engineering, and 20% in science. Moreover, according to figures released by the registrar, the percentages of women majoring in humanities, engineering, and science are 75%, 45%, and 30% respectively. Suppose Jason meets Anna at a UML frat party. What is the probability that Anna is an engineering major?

4. (12 points) (a) If $P(A) = 1/4$ and $P(B) = 1/8$, determine $P(A \cup B)$ if A and B are mutually exclusive.
   (b) If $P(A) = 1/4$ and $P(B) = 1/8$, determine $P(A \cup B)$ if A and B are independent.
   (c) If $P(A) = 1/4$ and $P(B) = 1/8$, determine $P(A|B)$ if A and B are mutually exclusive.
   (d) If $P(A) = 1/4$ and $P(B) = 1/8$, determine $P(A|B)$ if A and B are independent.

5. (18 points) Consider a set of 10 urns, nine of which contain three white chips and three red chips each. The tenth urn contains five white chips and one red chip. An urn is picked at random. Then a sample of size three is drawn without replacement from that urn. If all three chips drawn are white, what is the probability that the urn being sampled is the one with five white chips?

6. (10 points) To qualify as a "three-of-a-kind" hand in a five card poker hand, the five cards must include three of the same denomination and two "single" cards — cards whose denominations match neither the triple nor each other. Compute the probability that a random poker hand draws a three-of-a-kind hand.

G.S.:
converges to $\frac{a}{1-r} = \dfrac{\frac{1}{1+\lambda}}{\left(\frac{1+\lambda}{1+\lambda}\right) 1 - \frac{\lambda}{1+\lambda}} = \dfrac{1/1+\lambda}{\frac{1+\lambda-\lambda}{1+\lambda}}$

Exam 1:
$= 1$

2

B : event she passes class

A : enrolls in prof A class $p(A) = 4/10$

prof F class $p(F) = 6/10$

$p(B|A) = 0.85 \qquad p(B|F) = 0.6$

Want $p(A|B) = \dfrac{p(A \cap B)}{p(B)} = \dfrac{p(A)\, p(B|A)}{p(B \cap A) + p(B \cap F)}$

$= \dfrac{\frac{4}{10}(0.85)}{\left(\frac{4}{10}\right)(.85) + \left(\frac{6}{10}\right)(0.6)} = \dfrac{.340}{.200} \approx .49$

3 W : event student chosen is women.

$p(W|H) = .75 \qquad p(W|E) = .45 \quad p(W|S) = .3$

H .. enrolled in humanities $p(H) = .3$

E engineering $p(E) = .5$

S science $p(S) = .2$

$p(\text{Anna in engineering}) = p(E|W) =$

$= \dfrac{p(E \cap W)}{p(W)} = \dfrac{p(W|E)\, p(E)}{p(W \cap H) + p(W \cap E) + p(W \cap S)}$

$$= \frac{(.5)(.45)}{(.30)(.75)+(.5)(.45)+(.20)(.30)} = \frac{.225}{.510}$$

$= 44\%$

④ A, B are mutual exclusive

$\Rightarrow p(A \cap B)$ ~~p~~ $p(\emptyset) = 0$

a) $p(A \cup B) = p(A) + p(B) = 3/8$

b) $p(A \cup B) = p(A) + p(B) - p(A \cap B)$ b/c independent.

c) $p(A|B) = \frac{p(A \cap B)\ (=0)}{p(B)} = 0$

d) $p(A|B) = \frac{p(A \cap B)}{p(B)} = \frac{p(A)\,p(B)}{p(B)} = p(A)$

⑤ A : event 10^{th} urn is chosen.

B : event all three chips in sample are white

A^c : urns $1^{st} \rightarrow 9^{th}$ is chosen

$$p(B|A) = \frac{\binom{5}{3}\binom{1}{0}}{\binom{6}{3}} = 1/2$$

$p(A) = 1/10$

$$p(A|B) = \frac{p(A \cap B)}{p(B)} =$$

$$= \frac{p(A)\,p(B|A)}{p(B \cap A) + p(B \cap A^c)} =$$

$p(B|A^c)\,p(A^c)$

$$p(B|A^c) = \frac{\binom{3}{3}\binom{3}{0}}{\binom{6}{3}} = \frac{1}{\frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1}} = 1/20$$

$$\Rightarrow \quad p(A|B) = \frac{\left(\frac{1}{10}\right)\left(\frac{1}{2}\right)}{\frac{1}{10}\cdot\frac{1}{2}+\frac{9}{10}\cdot\frac{1}{20}} = 10/19$$

(6)

$$p(3 \text{ of kind}) = \frac{\#(3 \text{ of kinds hands})}{\#(\text{poker hands})} = \frac{\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}}{\binom{52}{5}}$$

$$\approx .0211$$