Student: Phong Vo **Date:** 02/21/18	**Instructor:** Chuck Ormsby **Course:** Multi-Variable and Vector Calculus -- Calculus III Spring 2018	**Assignment:** Section 12.9 Homework

1. Find the unit tangent vector **T** and the curvature κ for the following parameterized curve.

$\mathbf{r}(t) = \langle 2t + 2,\ 4t - 4, 6t + 14\rangle$

$\mathbf{T} = \left\langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle$

(Type exact answers, using radicals as needed.)

$\kappa = 0$

(Type exact answers, using radicals as needed.)

2. Find the unit tangent vector **T** and the curvature κ for the following parameterized curve.

$\mathbf{r}(t) = \langle 4\cos t, 4\sin t, \sqrt{2}\,t\rangle$

Choose the correct answer for the unit tangent vector of **r**(t).

- **A.** $\mathbf{T}(t) = \left\langle \frac{2\sqrt{2}}{3}\sin t, \frac{2\sqrt{2}}{3}\cos t, \frac{1}{3}\right\rangle$
- **B.** $\mathbf{T}(t) = \left\langle \frac{2\sqrt{2}}{3}\sin t, -\frac{2\sqrt{2}}{3}\cos t, \frac{1}{3}\right\rangle$
- **C.** $\mathbf{T}(t) = \left\langle -\frac{2\sqrt{2}}{3}\sin t, -\frac{2\sqrt{2}}{3}\cos t, \frac{1}{3}\right\rangle$
- ✔ **D.** $\mathbf{T}(t) = \left\langle -\frac{2\sqrt{2}}{3}\sin t, \frac{2\sqrt{2}}{3}\cos t, \frac{1}{3}\right\rangle$

$\kappa = \frac{2}{3\sqrt{3}}$

3. Find the unit tangent vector **T** and the curvature κ for the following parameterized curve.

$\mathbf{r}(t) = \langle \sqrt{7}\sin t, 3\sin t, 4\cos t\rangle$

The unit tangent vector is $\mathbf{T} = \left\langle \frac{\sqrt{7}}{4}\cos t, \frac{3}{4}\cos t, -\sin t \right\rangle$.

(Type exact answers, using radicals as needed.)

The curvature is $\kappa = \frac{1}{4}$.

4. Use the alternative curvature formula $\kappa = \frac{|\mathbf{a}\times\mathbf{v}|}{|\mathbf{v}|^3}$ to find the curvature of the following parameterized curve.

$\mathbf{r}(t) = \langle 6 + 4t^2, t, 0\rangle$

$\kappa = \frac{8}{(64t^2+1)\sqrt{64t^2+1}}$

5. Find the unit tangent vector **T** and the principal unit normal vector **N** for the following parameterized curve. Verify that $|\mathbf{T}| = |\mathbf{N}| = 1$ and $\mathbf{T} \cdot \mathbf{N} = 0$.

$$\mathbf{r}(t) = \left\langle \frac{t^2}{2}, 3 - 2t, 8 \right\rangle$$

$$\mathbf{T} = \left\langle \frac{t}{\sqrt{t^2+4}}, -\frac{2}{\sqrt{t^2+4}}, 0 \right\rangle$$

$$\mathbf{N} = \langle 0, 0, 1 \rangle$$

6. Find the unit tangent vector **T** and the principal unit normal vector **N** for the following parameterized curve. Verify that $|\mathbf{T}| = |\mathbf{N}| = 1$ and $\mathbf{T} \cdot \mathbf{N} = 0$.

$$\mathbf{r}(t) = \langle 10 \cos t^2, 10 \sin t^2 \rangle \text{ for } 0 \le t \le 2\pi$$

$$\mathbf{T} = \langle -\sin t^2, \cos t^2 \rangle$$

$$\mathbf{N} = \langle -\cos t^2, -\sin t^2 \rangle$$