Analyzing Algorithms

Text

Chapters 2

Analyzing Algorithms

- goal: predicting resources that an algorithm requires
  - memory, communication bandwidth, hardware, computational time
  - compare several candidate algorithms, identify most efficient one

Analyzing Algorithms

- one-processor, random-access machine (RAM)
  - instructions executed one after another, no concurrent operations
  - common instructions:
    - arithmetic (+, -, *, /, %, $\lfloor\ \rfloor$, $\lceil\ \rceil$)
    - data movement (load, store, copy)
    - control (branch, subroutine call and return)
  - do not consider memory hierarchy

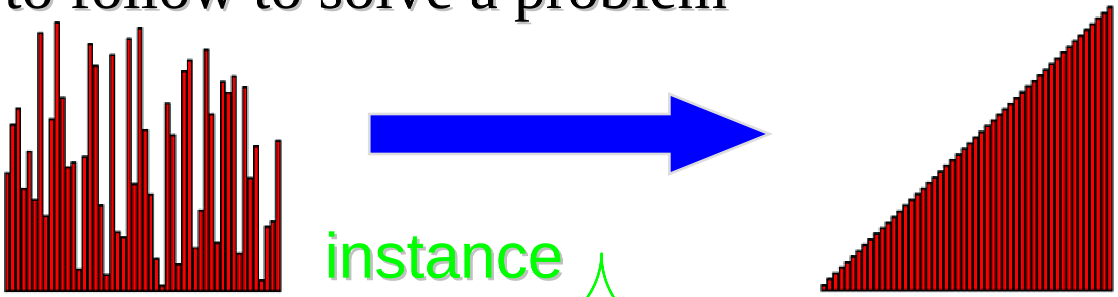
- elementary (primitive) operation: execution time can be bounded above by a constant depending only on the particular implementation—the machine, the programming language, etc.

Efficiency of an algorithm

- Efficiency
  - **Time**, space, energy
  - Measured as a function of the size of the instances considered
- Input Size
  - The ***size*** of an instance/input
    - corresponds formally the number of the bits needed to represent the instance on a computer
    - A less formal definition: any integer that in some way measures the number of components in an instance
      - For example, sorting, graphs
    - For problems involving integers, we use ***value*** rather than size
- Running time
  - The number of primitive operations executed in terms of input size.
- mathematic tools include combinatorics, probability theory, identify most significant terms in a formula

Sorting as Example

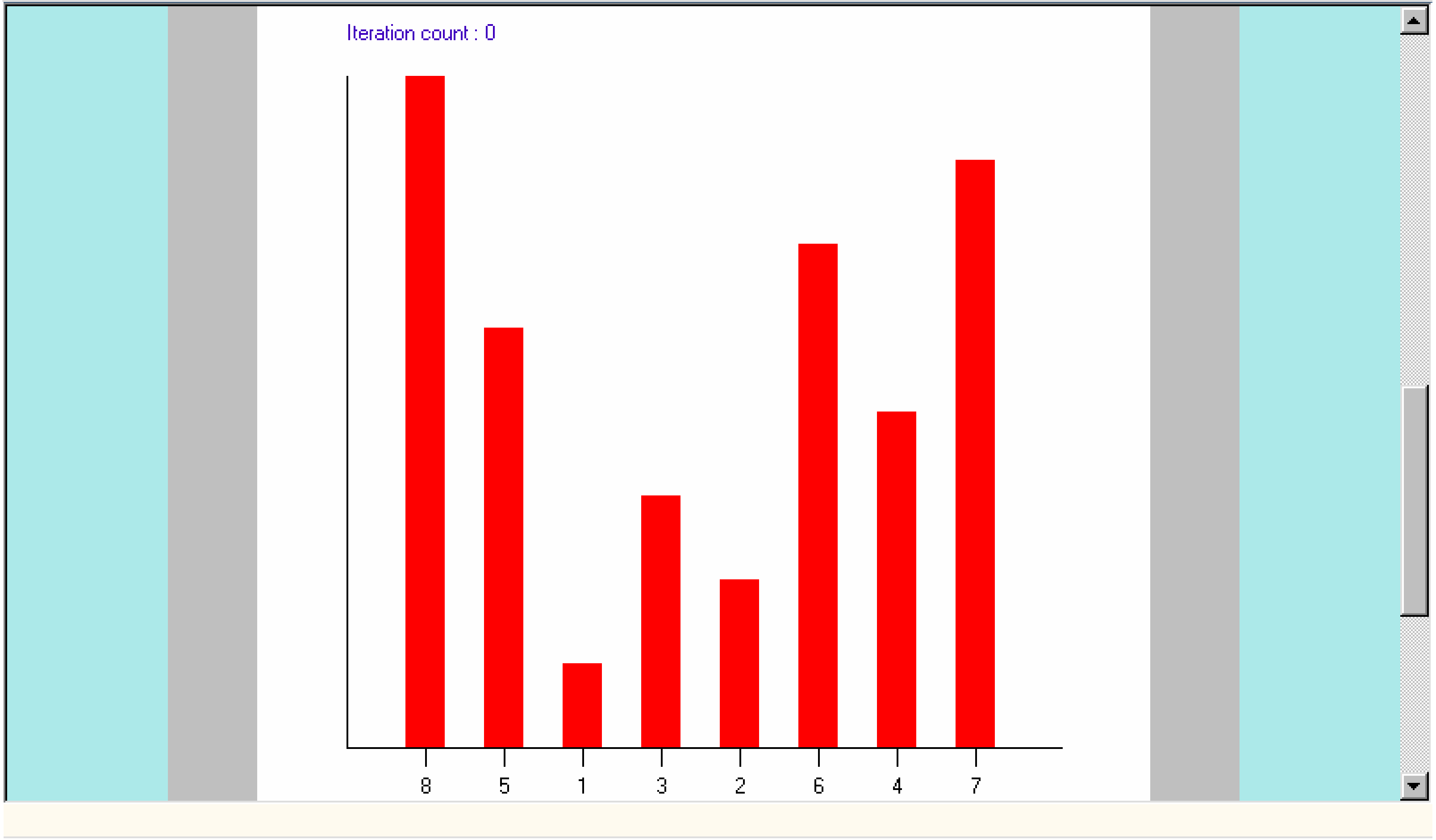
- Algorithm:
  - *well-defined computational procedure* that transforms input into output
  - steps for the computer to follow to solve a problem



- Sorting Problem:
  - Input: A sequence of *n* numbers $< a_1 , a_2 , \cdots , a_n >$
  - Output: A permutation (reordering) $< a'_1 , a'_2 , \cdots , a'_n >$ of the input sequence such that: $a'_1 \le a'_2 \le \cdots \le a'_n$

Insertion Sort Animation

Finding a place for item with value 5 in position 1: Swap item in position 0 with item in position 1.

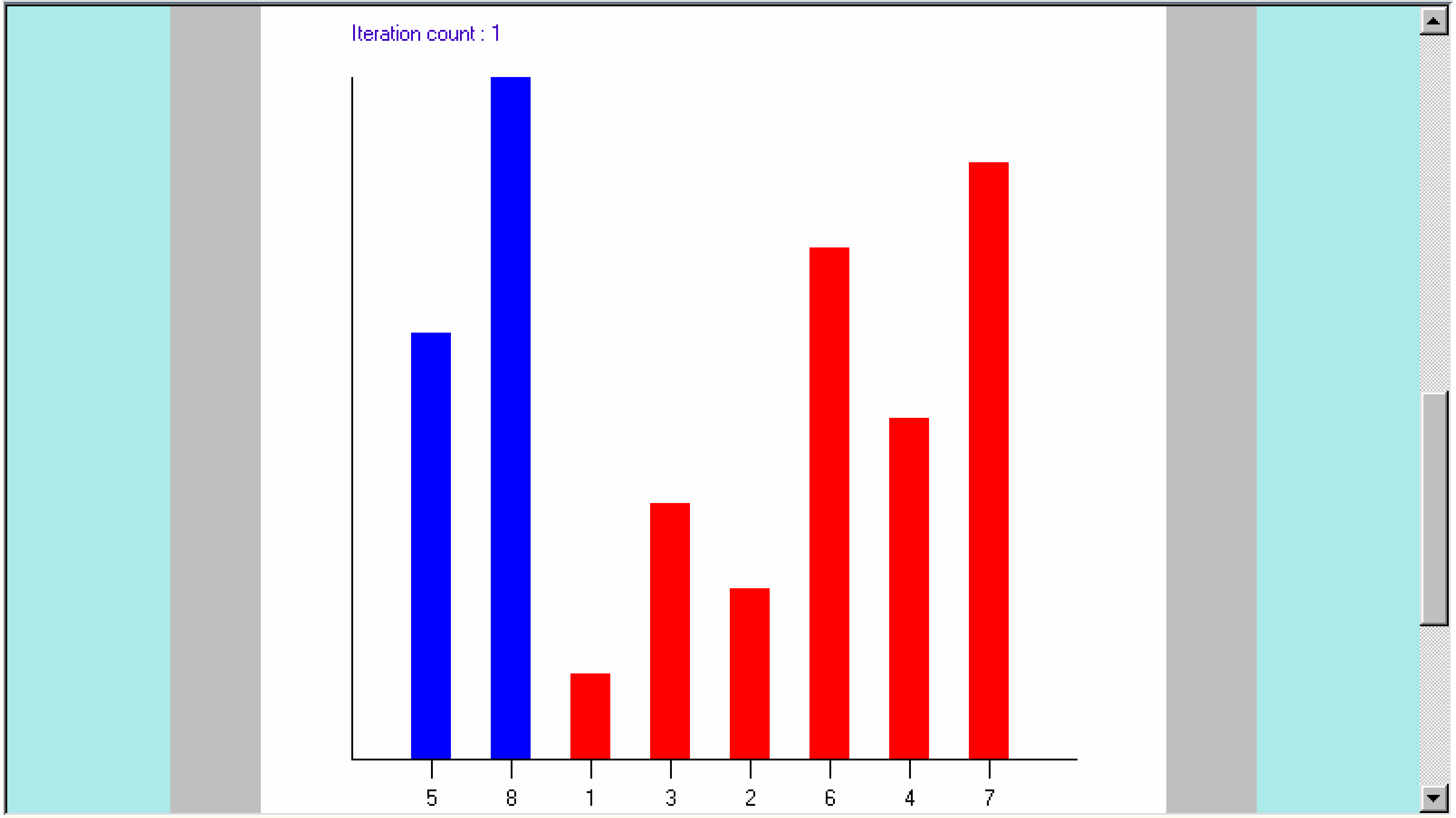


Iteration count : 0

http://www.cs.brockport.edu/cs/java/apps/sorters/insertsortaniminp.html

Insertion Sort Animation

Positions 0 through 1 are now in non-decreasing order.

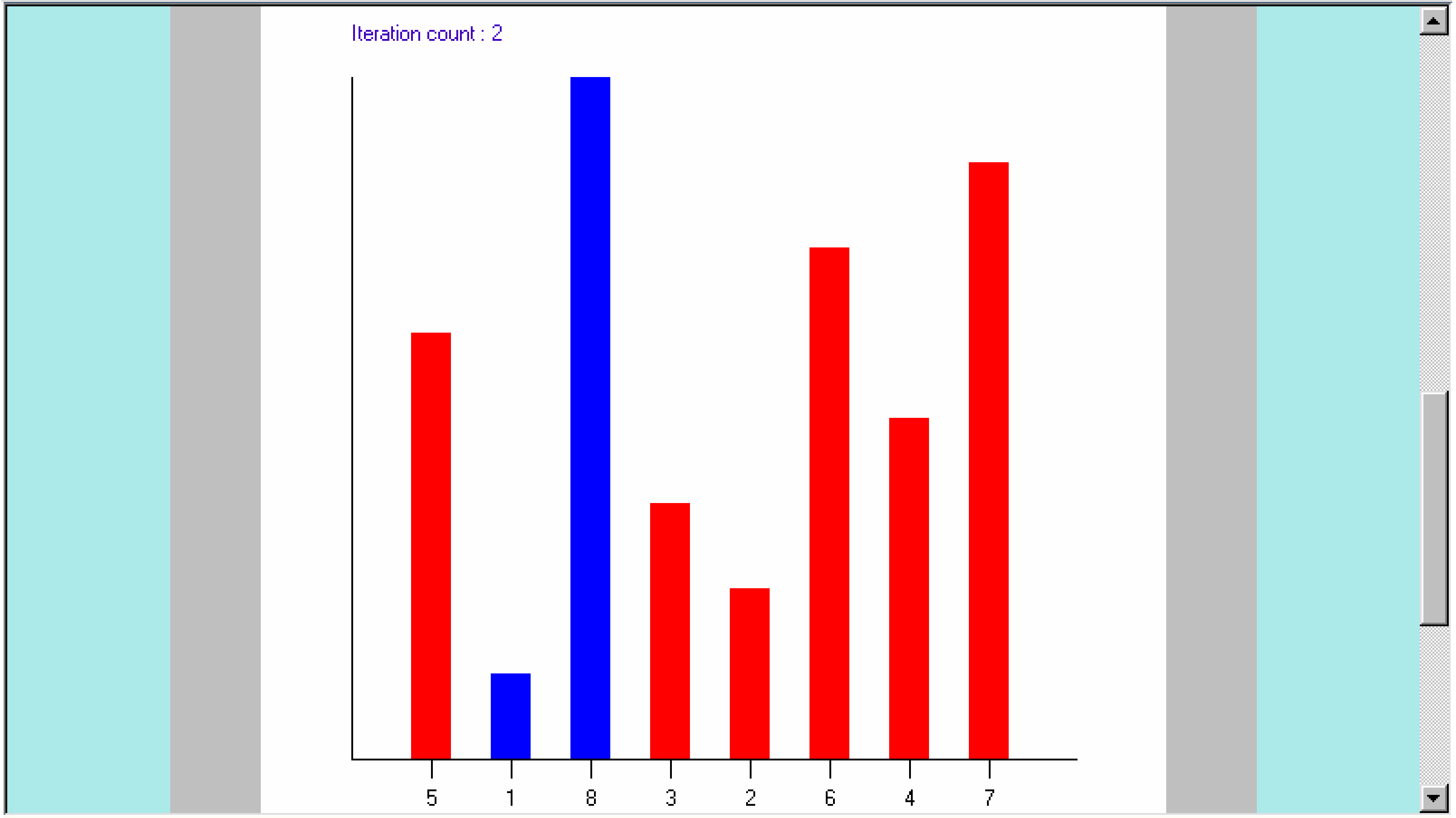


Iteration count : 1

http://www.cs.brockport.edu/cs/java/apps/sorters/insertsortaniminp.html

Insertion Sort Animation

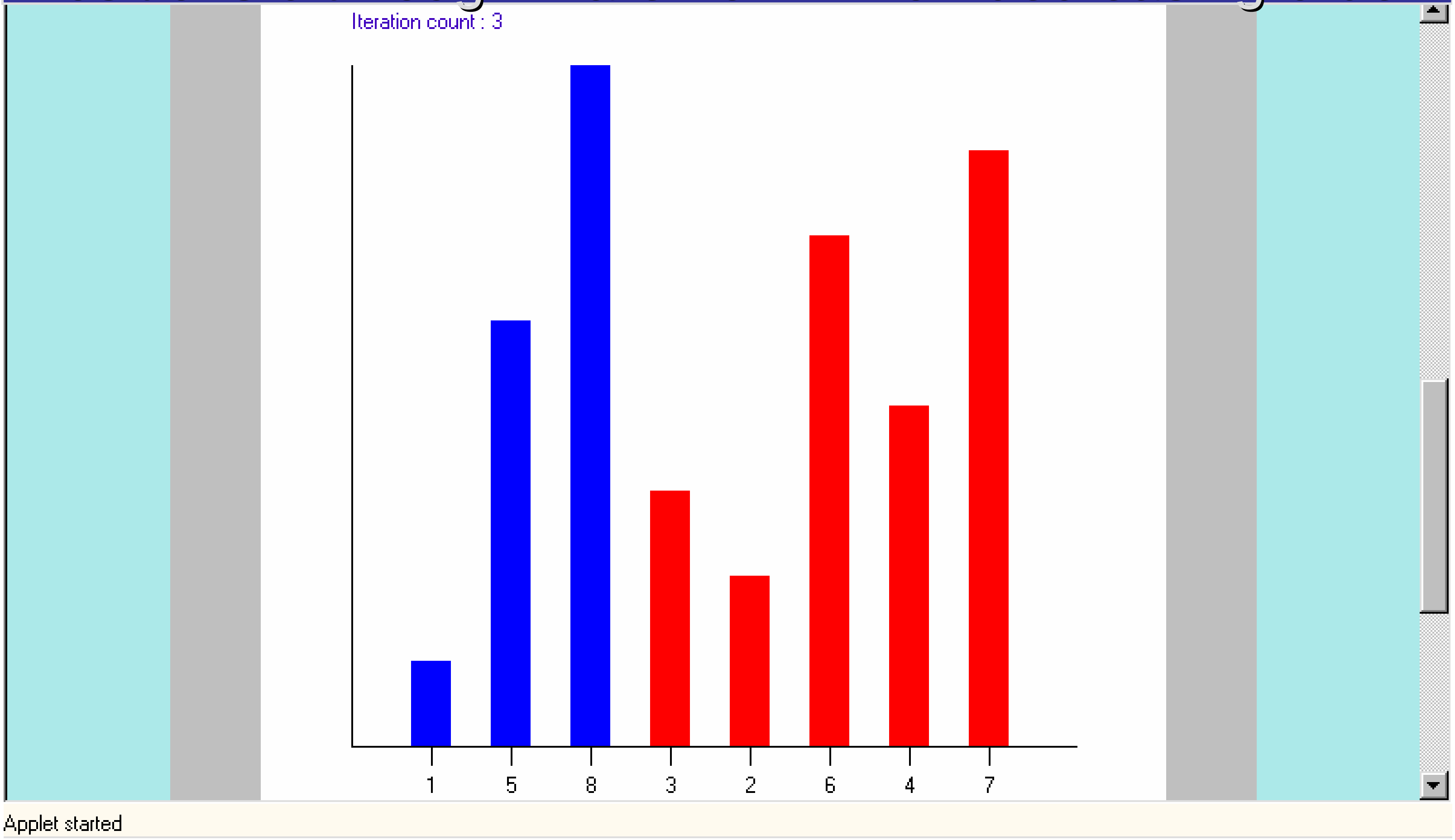
Finding a place for item with value 1 in position 2: Swap item in position 1 with item in position 2.



http://www.cs.brockport.edu/cs/java/apps/sorters/insertsortaniminp.html

Insertion Sort Animation

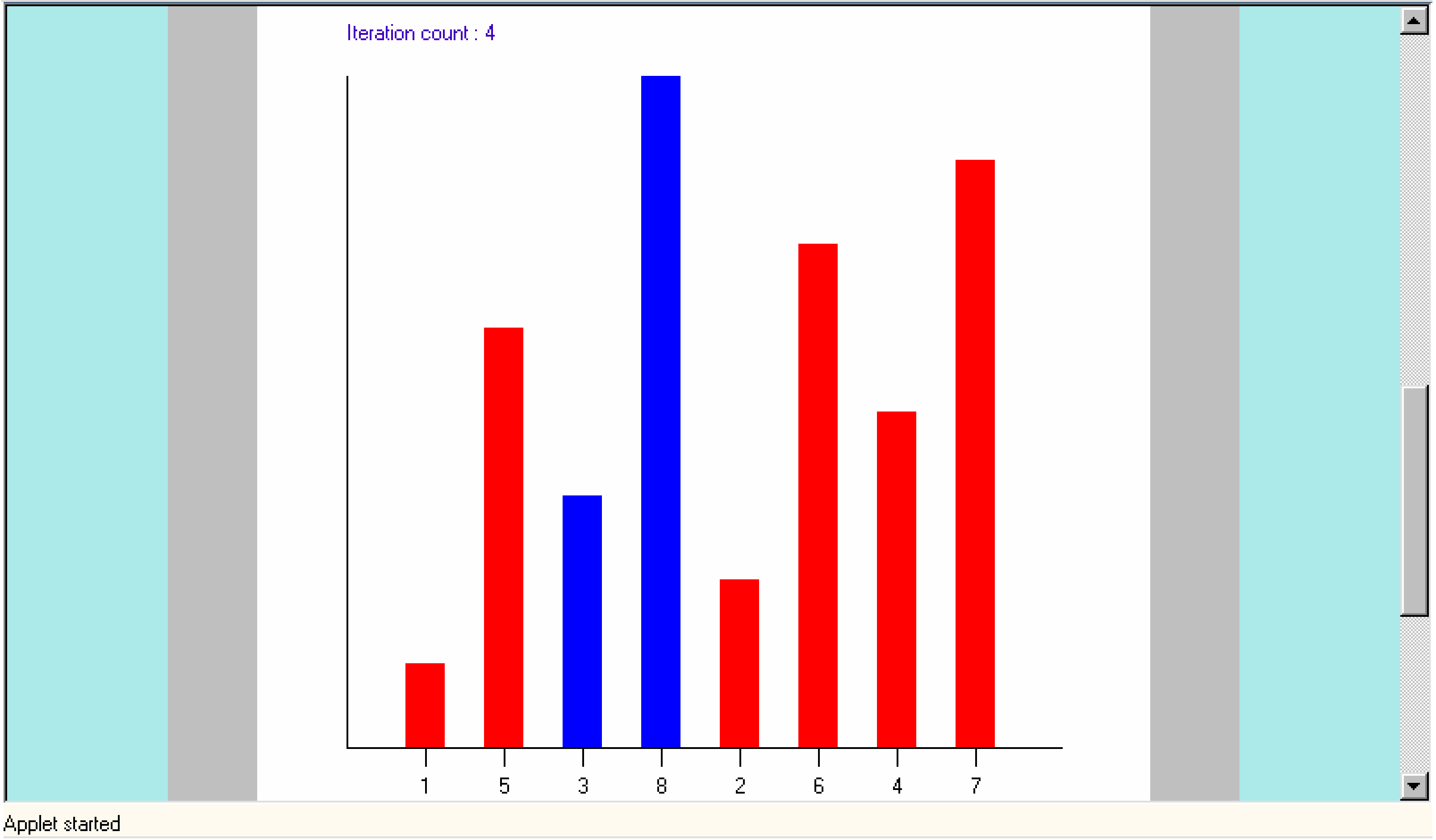
Finding a place for item with value 1:
Swap item in position 0 with item in position 1.
Positions 0 through 2 are now in non-decreasing order.



http://www.cs.brockport.edu/cs/java/apps/sorters/insertsortaniminp.html

Insertion Sort Animation

Finding a place for item with value 3 in position 3: Swap item in position 2 with item in position 3.



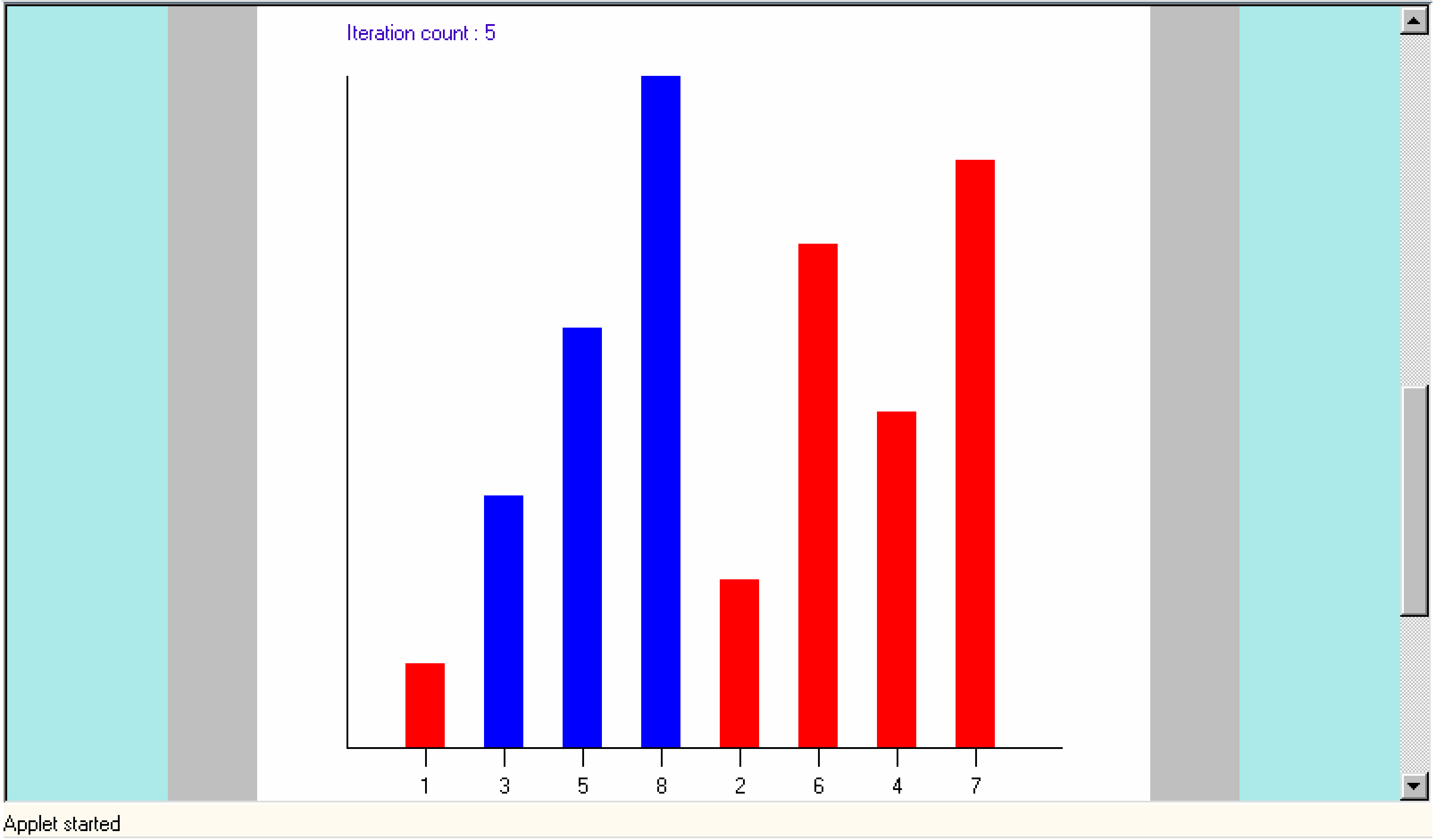
Iteration count : 4

Applet started

http://www.cs.brockport.edu/cs/java/apps/sorters/insertsortaniminp.html

Insertion Sort Animation

Finding a place for item with value 3:
Swap item in position 1 with item in position 2.



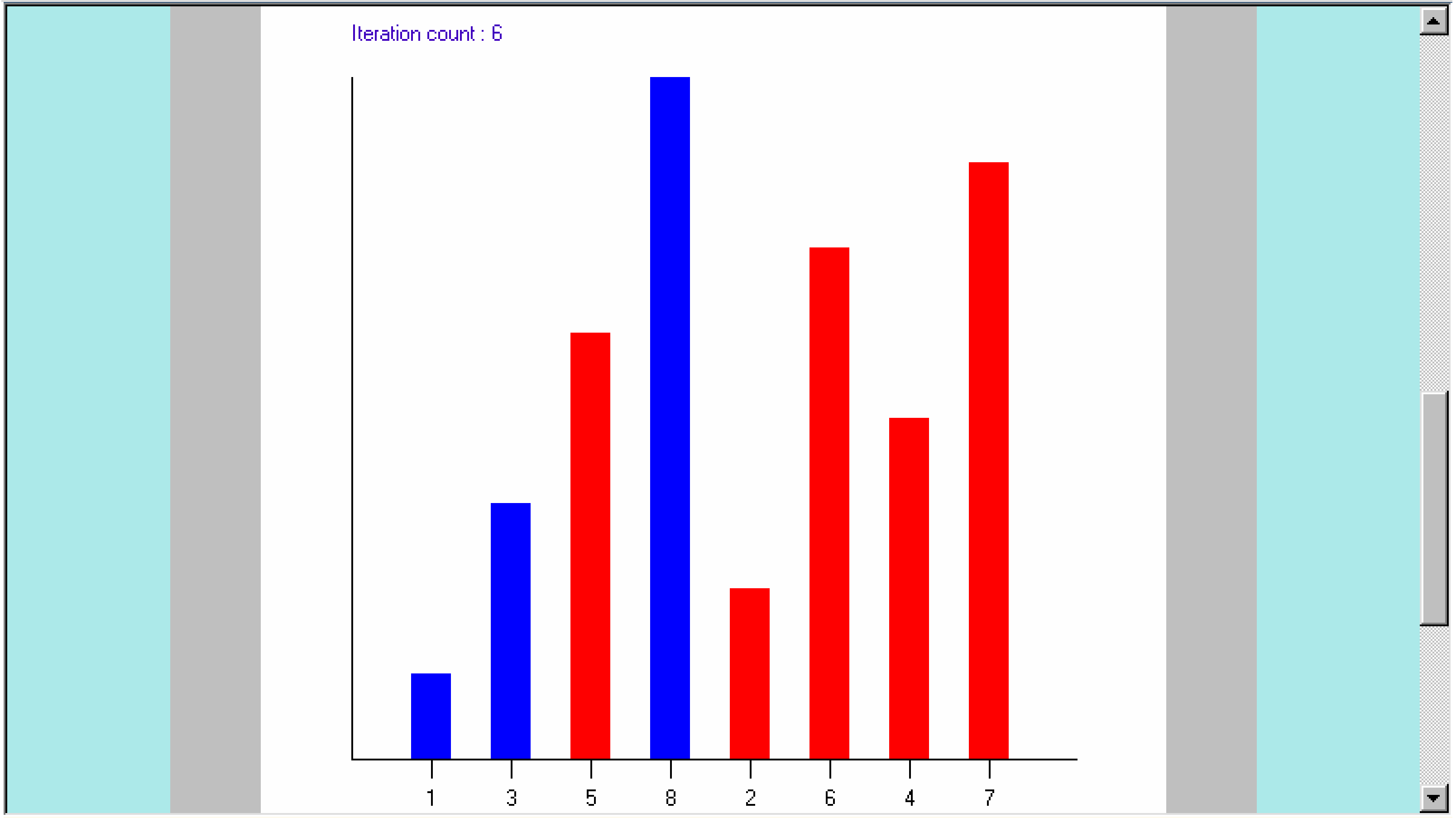
Iteration count : 5

Applet started

http://www.cs.brockport.edu/cs/java/apps/sorters/insertsortaniminp.html

Insertion Sort Animation

Positions 0 through 3 are now in non-decreasing order.

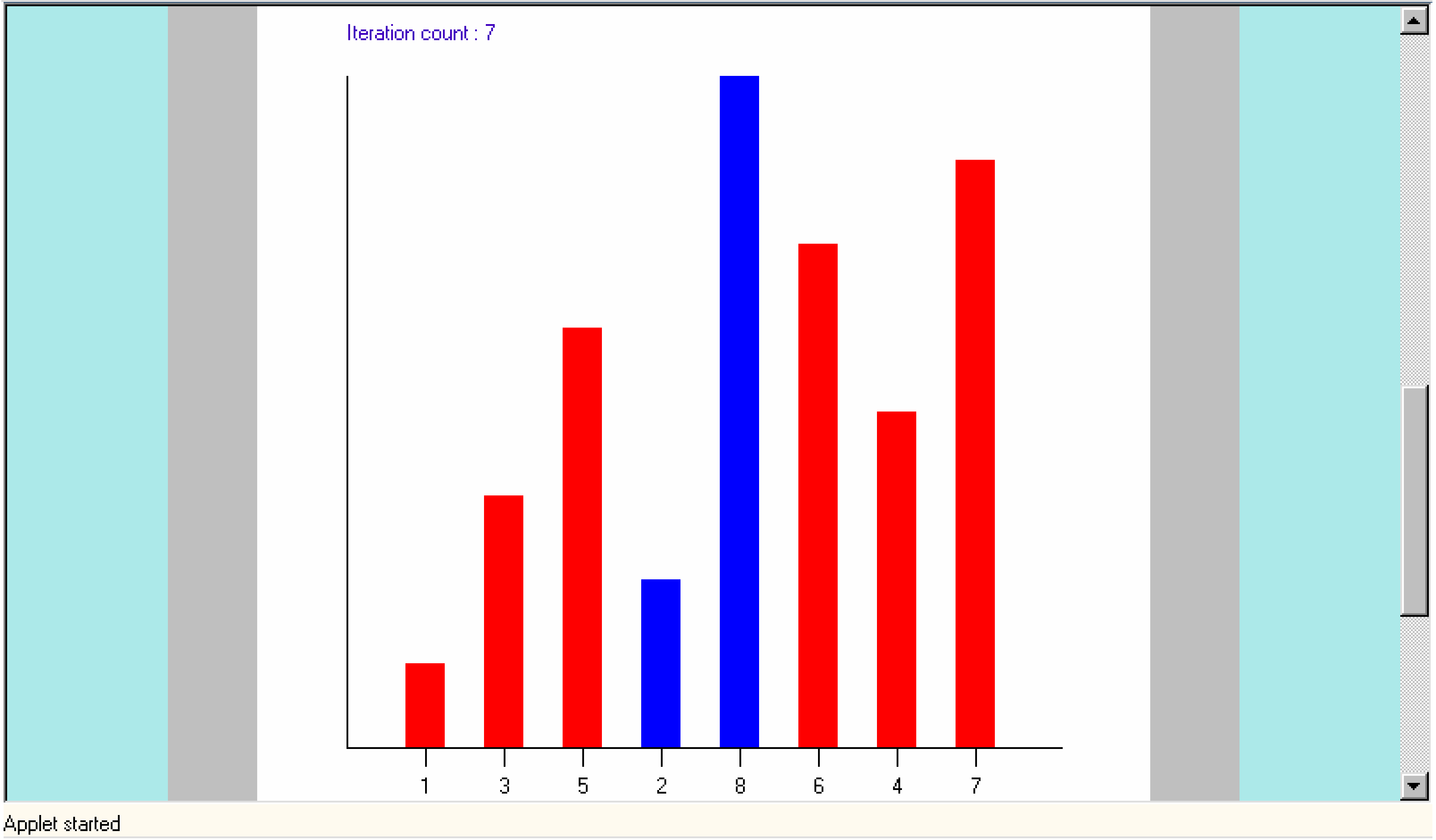


Iteration count : 6

http://www.cs.brockport.edu/cs/java/apps/sorters/insertsortaniminp.html

Insertion Sort Animation

Finding a place for item with value 2 in position 4: Swap item in position 3 with item in position 4.

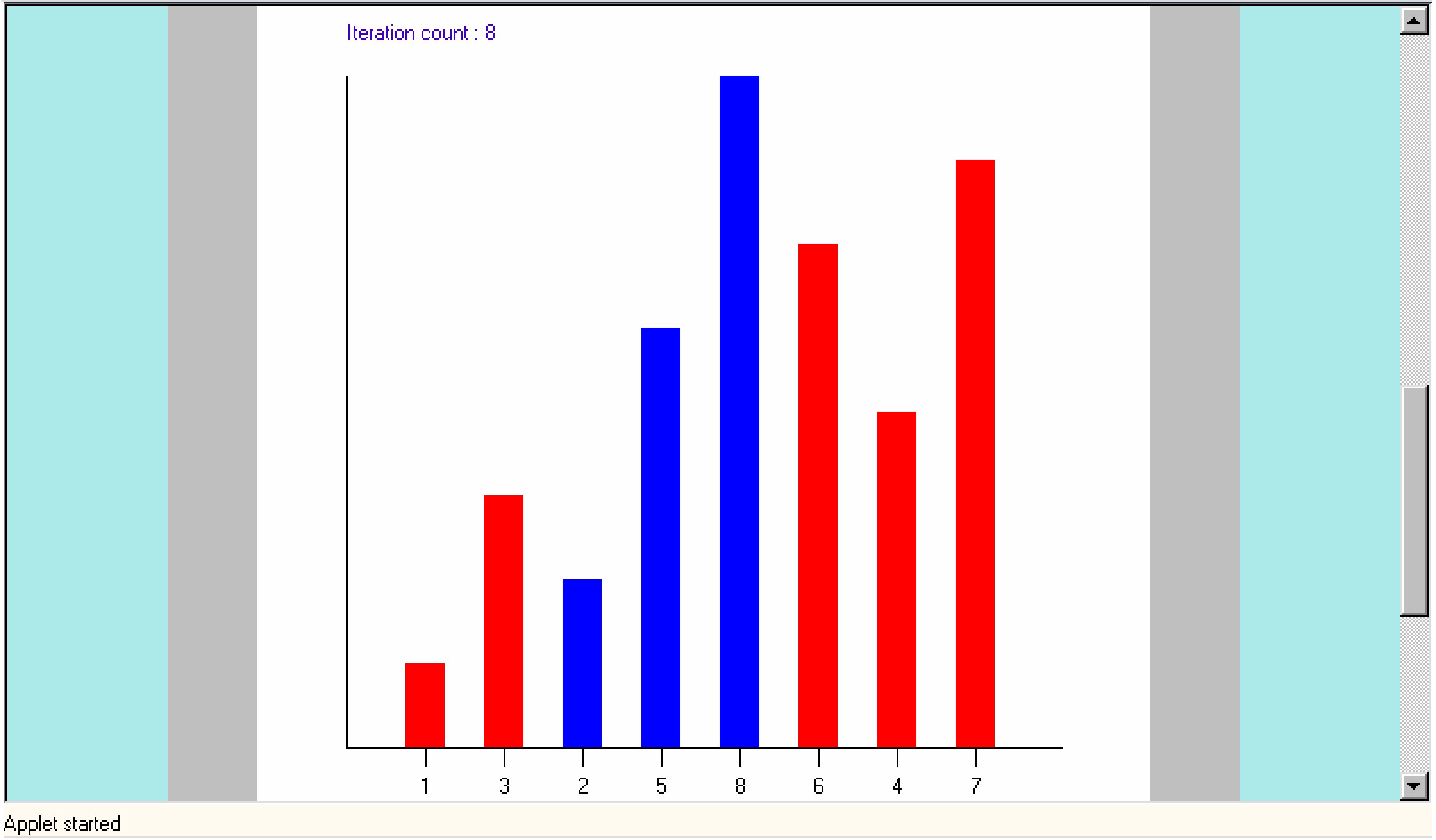


Applet started

http://www.cs.brockport.edu/cs/java/apps/sorters/insertsortaniminp.html

Insertion Sort Animation

Finding a place for item with value 2:
Swap item in position 2 with item in position 3.

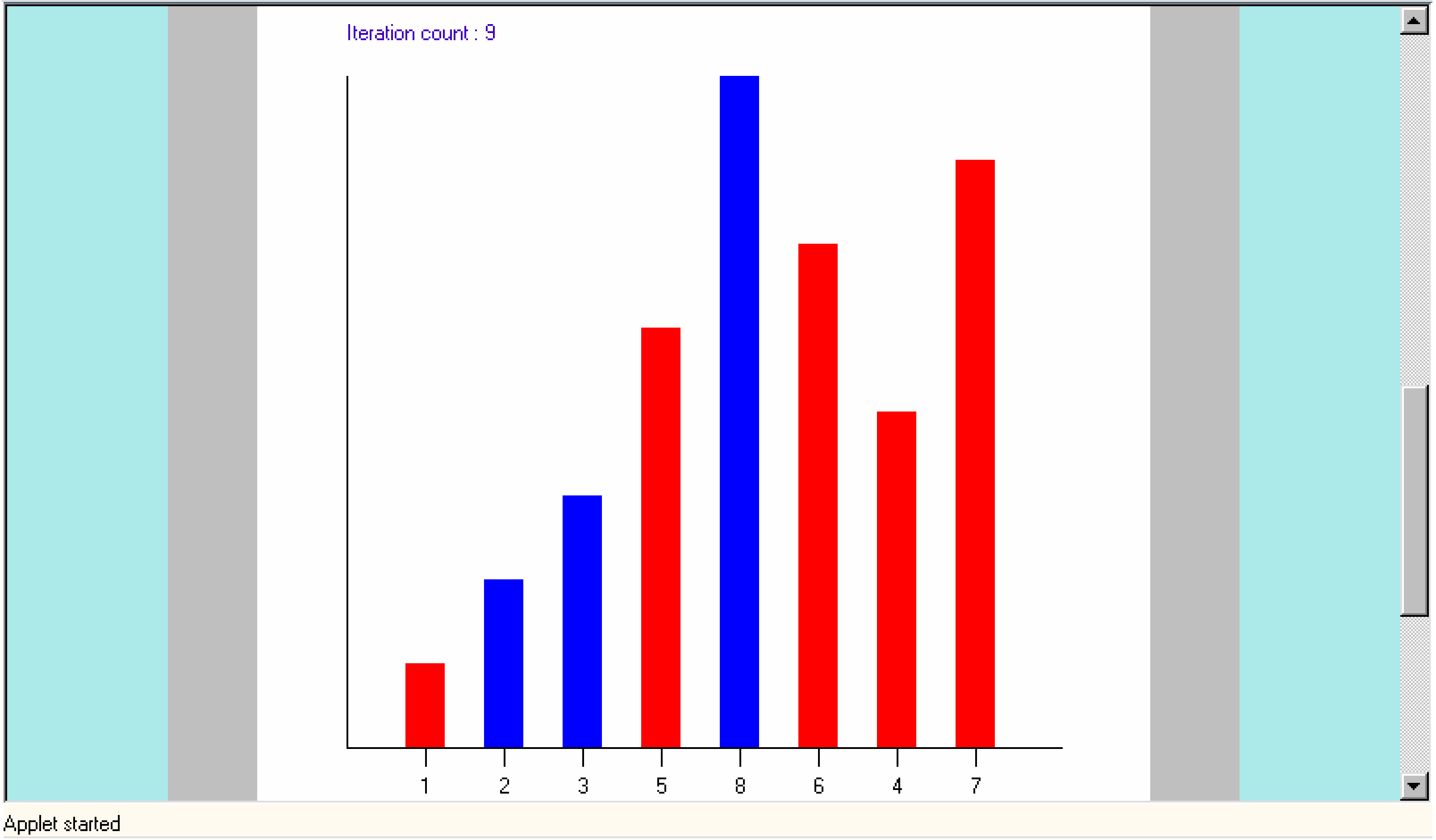


Iteration count : 8

http://www.cs.brockport.edu/cs/java/apps/sorters/insertsortaniminp.html

Insertion Sort Animation

Finding a place for item with value 2:
Swap item in position 1 with item in position 2.



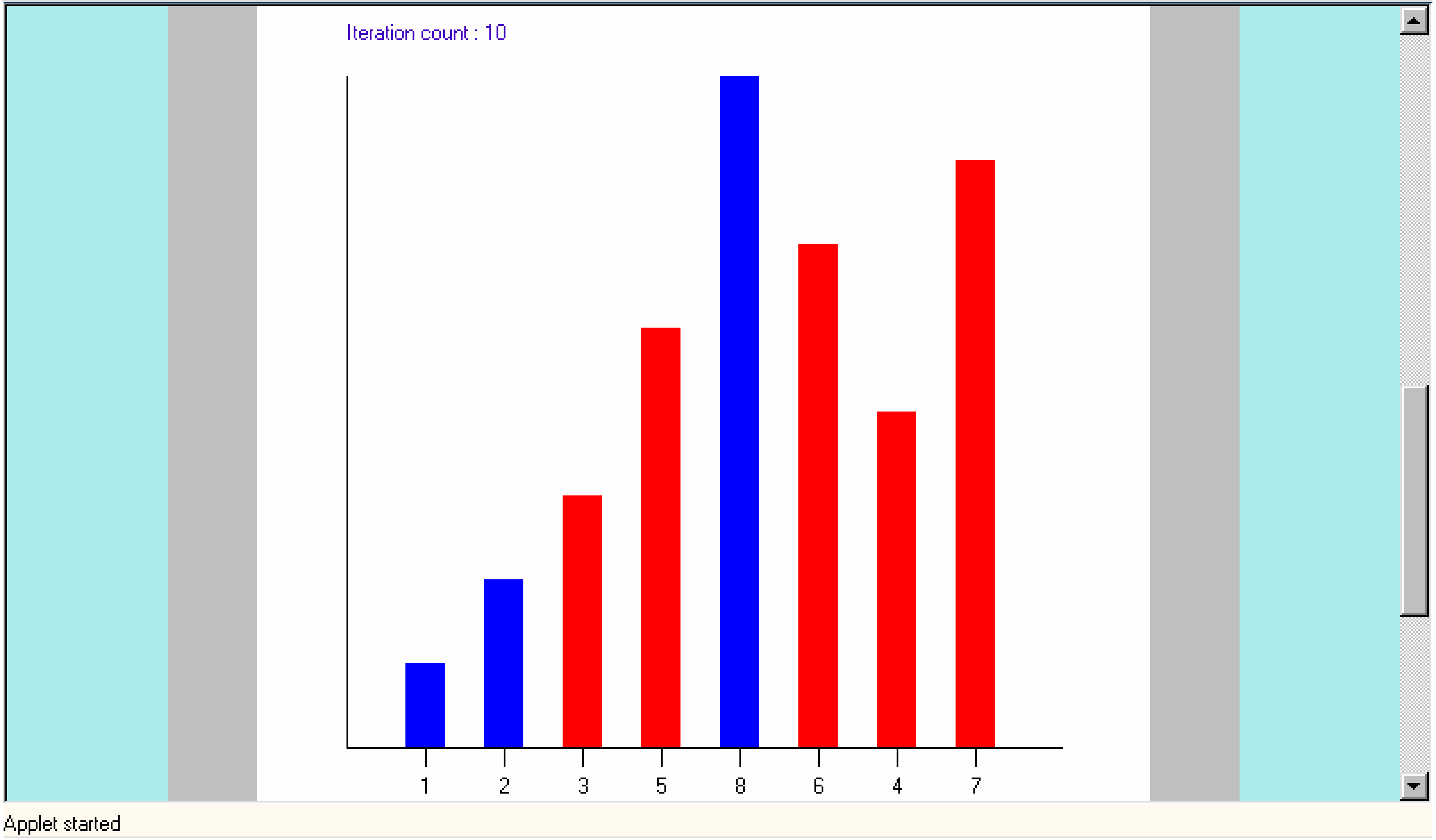
Iteration count : 9

Applet started

http://www.cs.brockport.edu/cs/java/apps/sorters/insertsortaniminp.html

Insertion Sort Animation

Positions 0 through 4 are now in non-decreasing order.

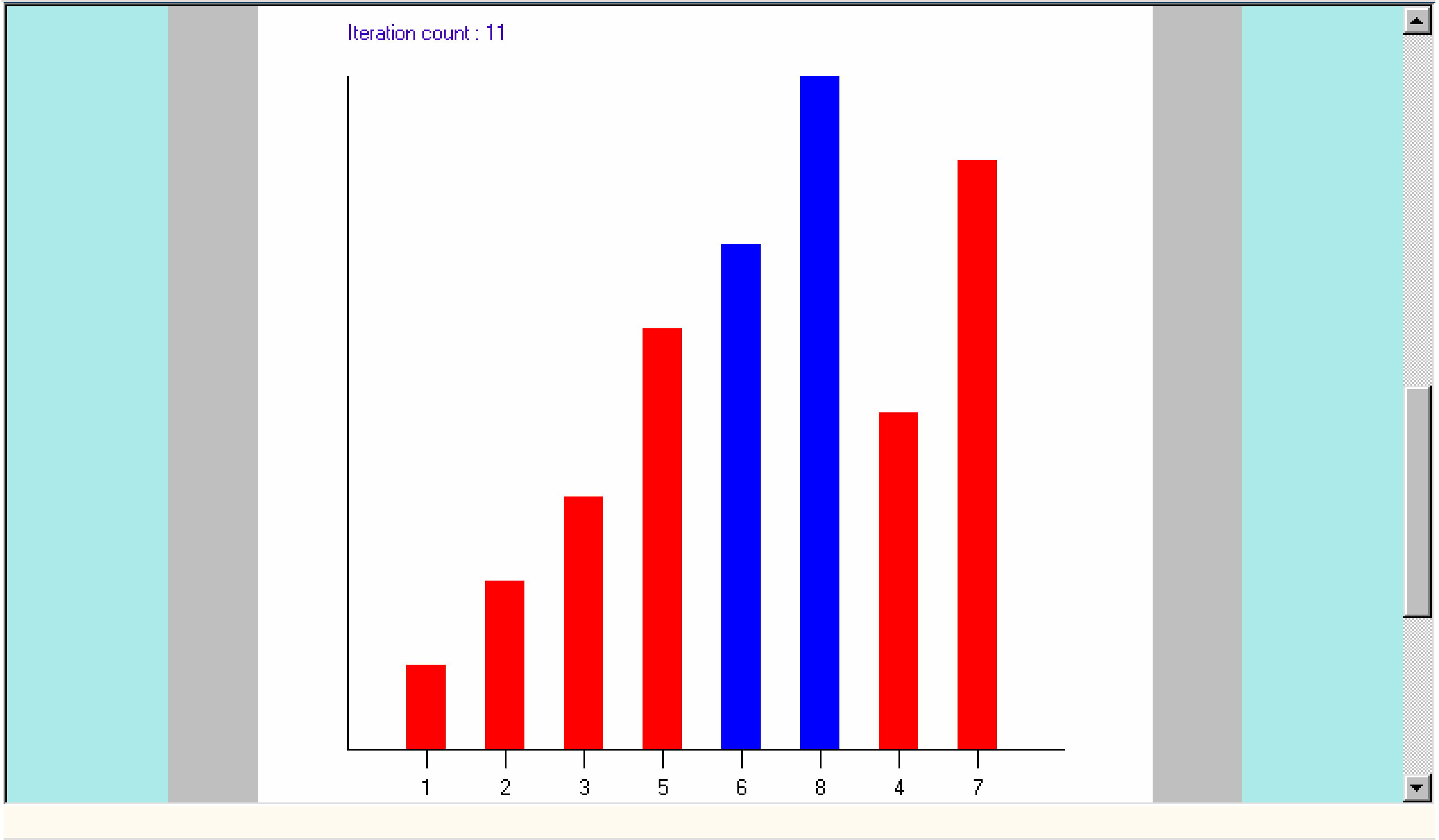

Iteration count : 10

Applet started

http://www.cs.brockport.edu/cs/java/apps/sorters/insertsortaniminp.html

Insertion Sort Animation

Finding a place for item with value 6 in position 5: Swap item in position 4 with item in position 5.

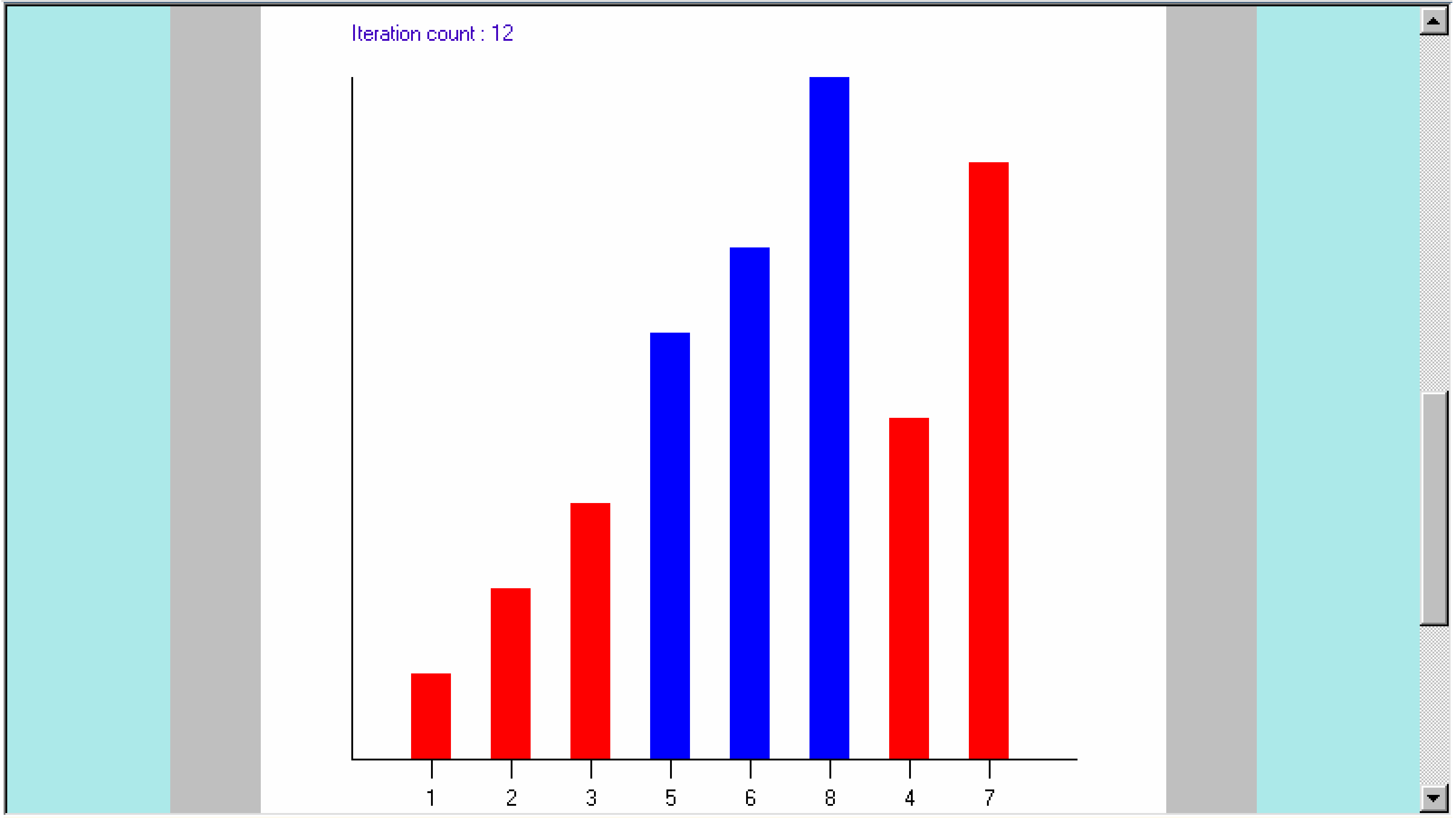


Iteration count : 11

http://www.cs.brockport.edu/cs/java/apps/sorters/insertsortaniminp.html

Insertion Sort Animation

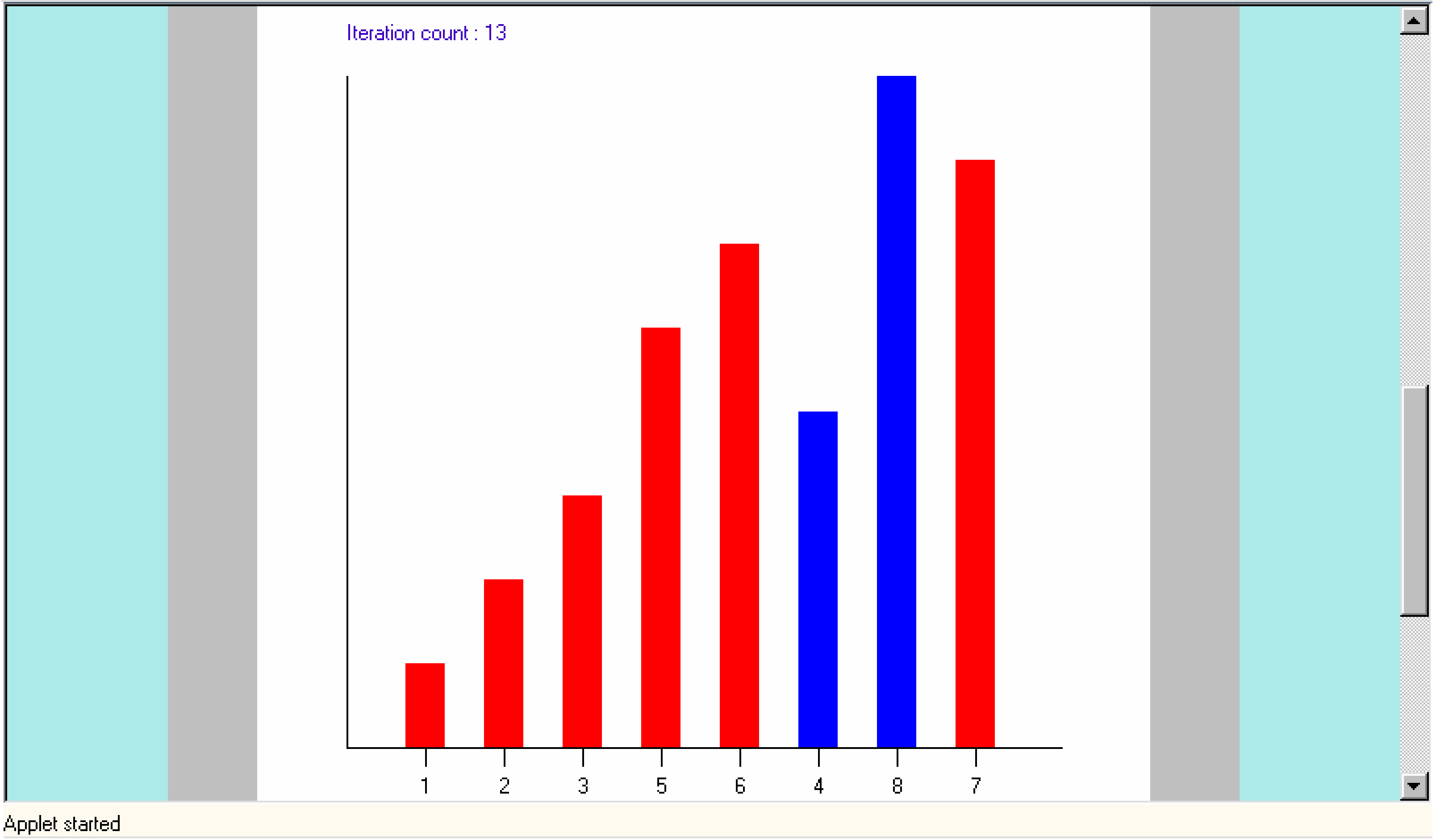
Positions 0 through 5 are now in non-decreasing order.



http://www.cs.brockport.edu/cs/java/apps/sorters/insertsortaniminp.html

Insertion Sort Animation

Finding a place for item with value 4 in position 6:
Swap item in position 5 with item in position 6.



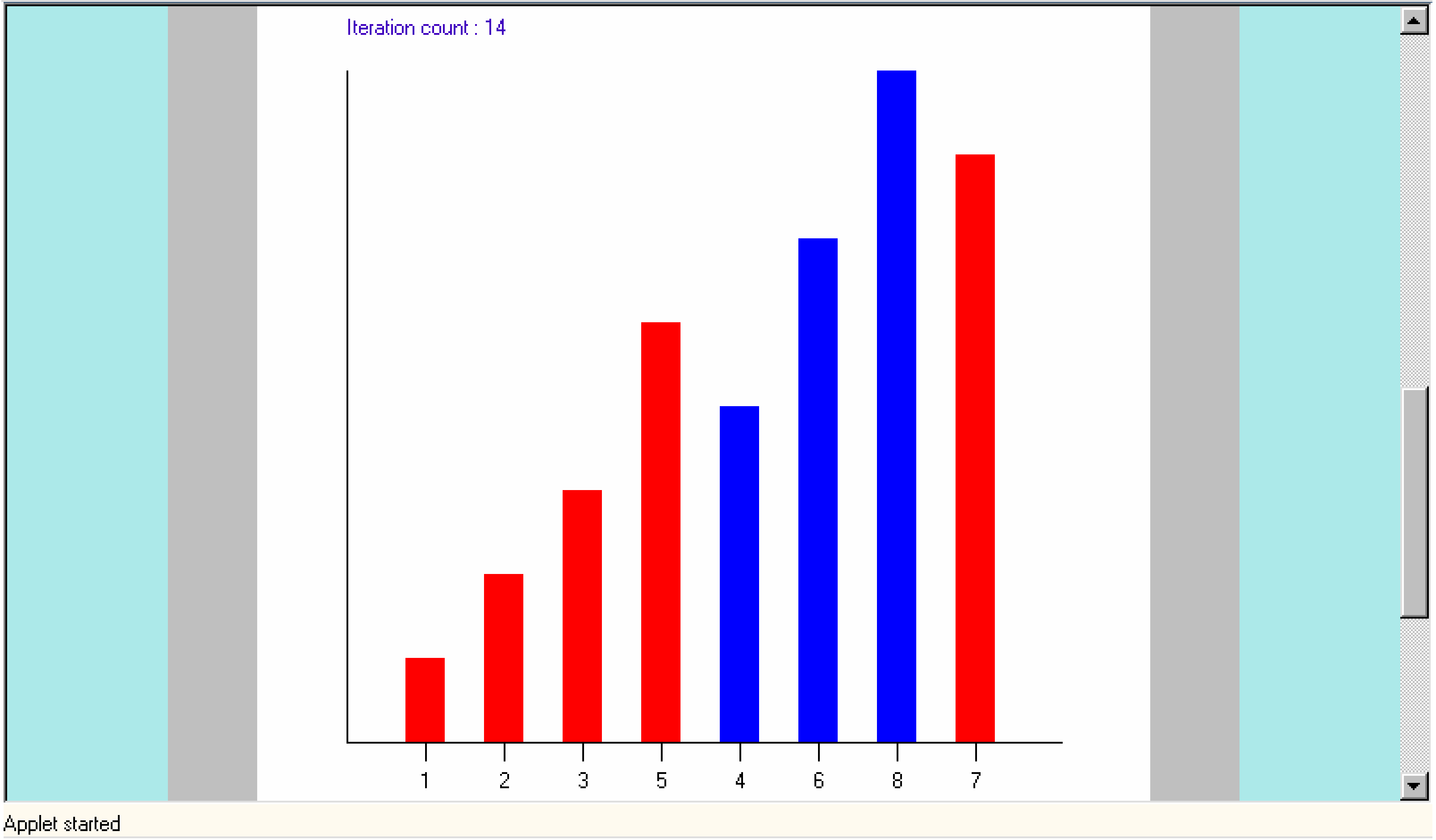
Iteration count : 13

Applet started

http://www.cs.brockport.edu/cs/java/apps/sorters/insertsortaniminp.html

Insertion Sort Animation

Finding a place for item with value 4:
Swap item in position 4 with item in position 5.

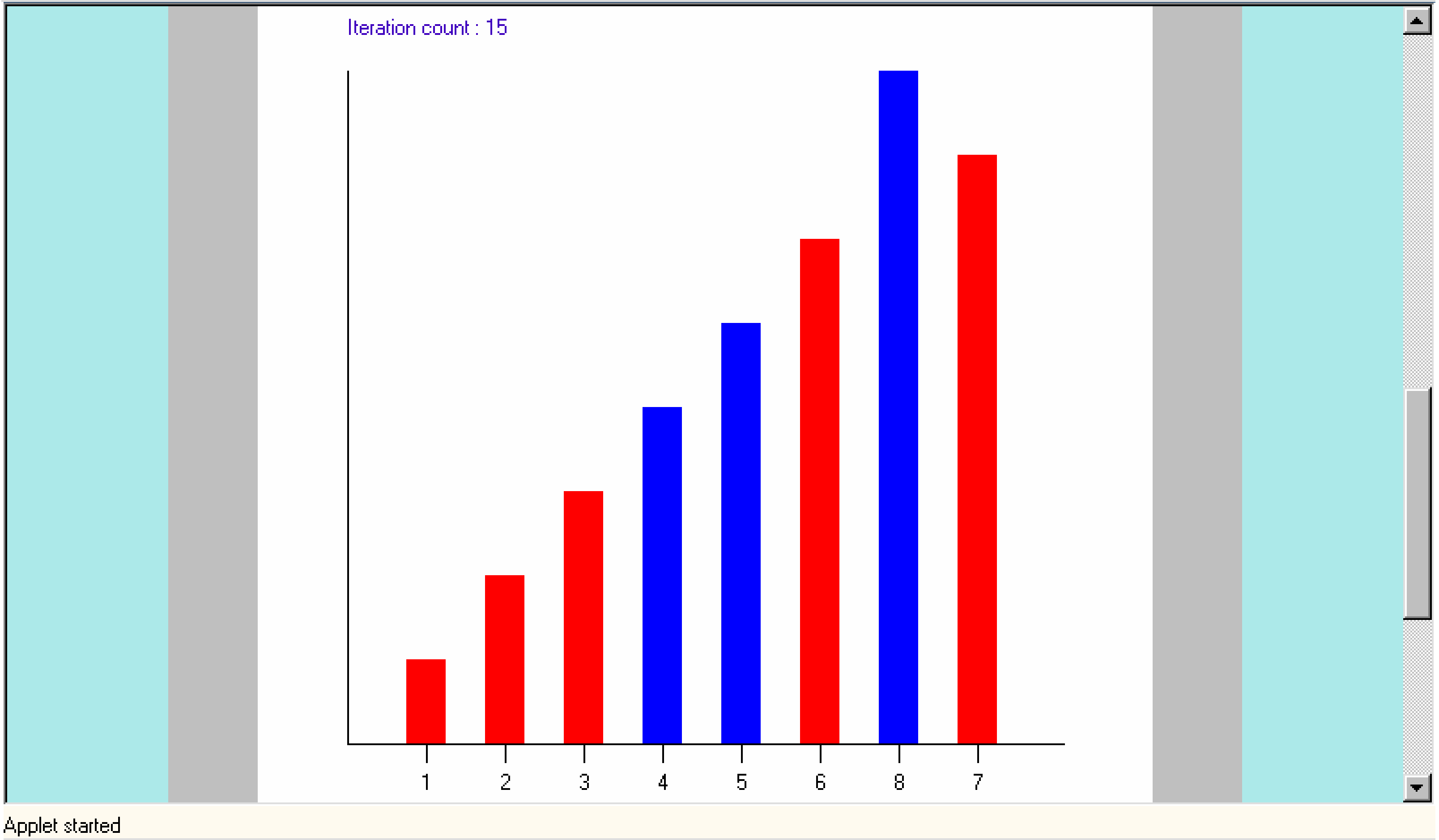


Applet started

http://www.cs.brockport.edu/cs/java/apps/sorters/insertsortaniminp.html

Insertion Sort Animation

Positions 0 through 6 are now in non-decreasing order.



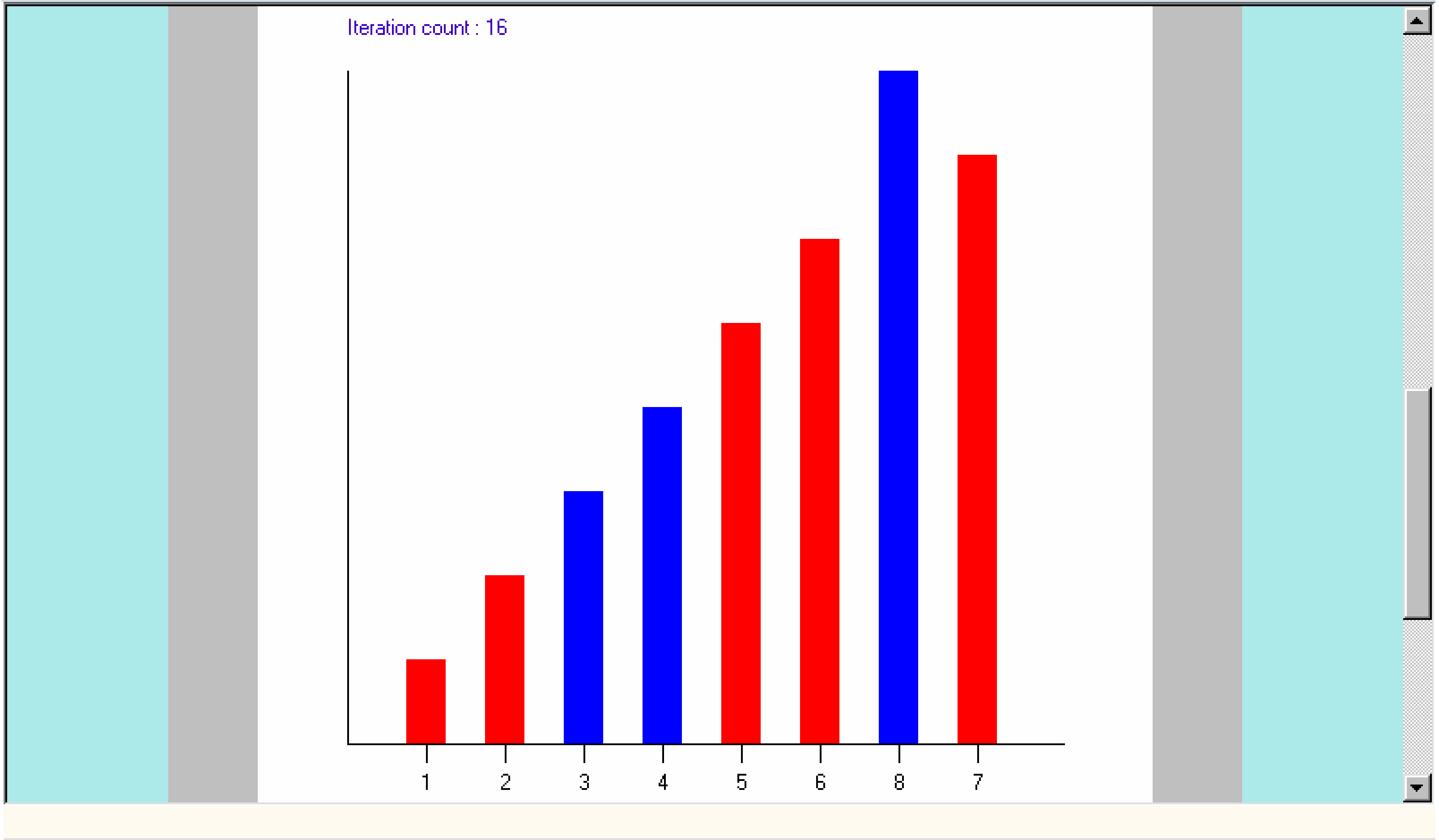
Iteration count : 15

Applet started

http://www.cs.brockport.edu/cs/java/apps/sorters/insertsortaniminp.html

Insertion Sort Animation

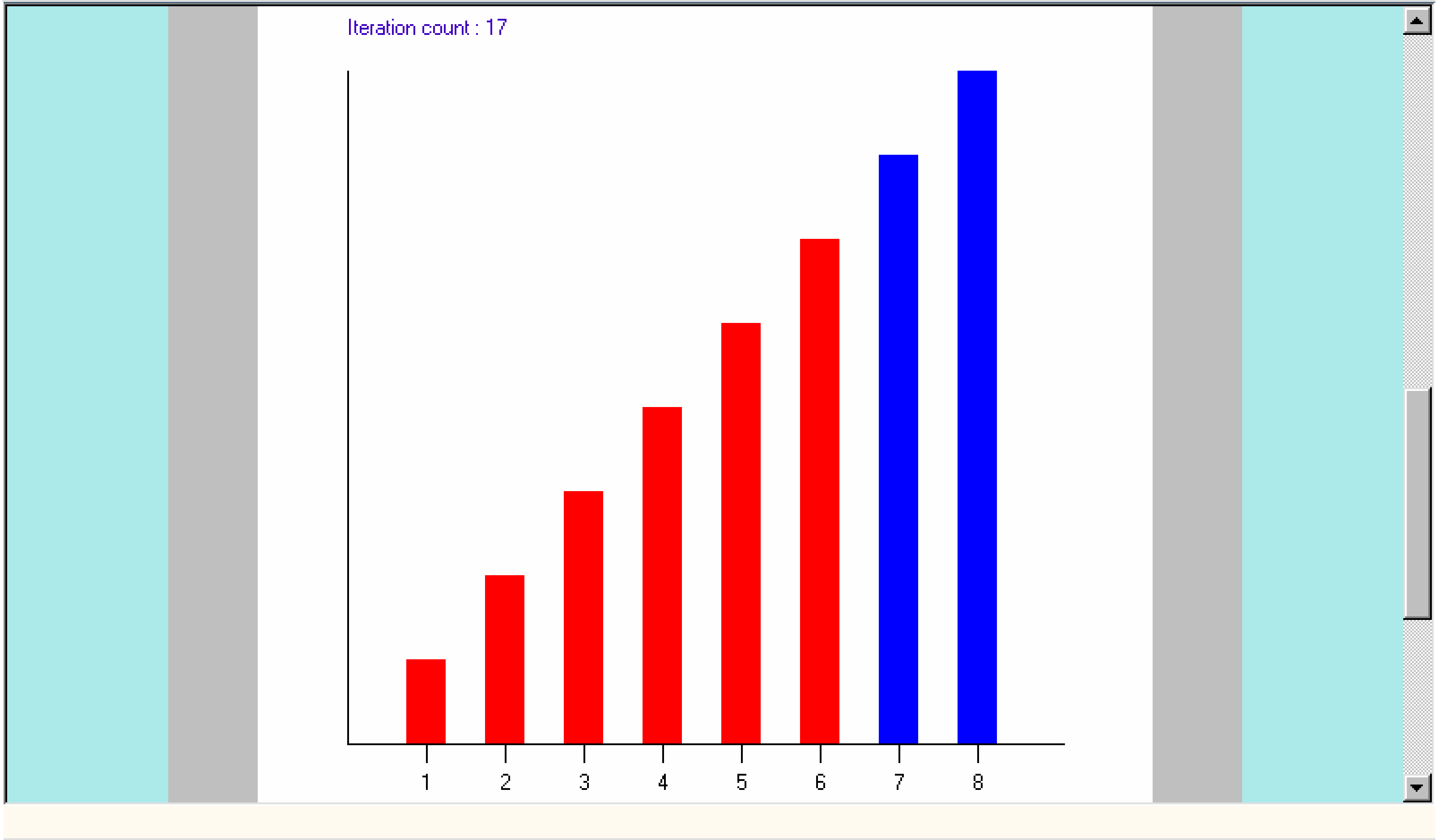
Finding a place for item with value 7 in position 7: Swap item in position 6 with item in position 7.



http://www.cs.brockport.edu/cs/java/apps/sorters/insertsortaniminp.html

Insertion Sort Animation

Positions 0 through 7 are now in non-decreasing order.

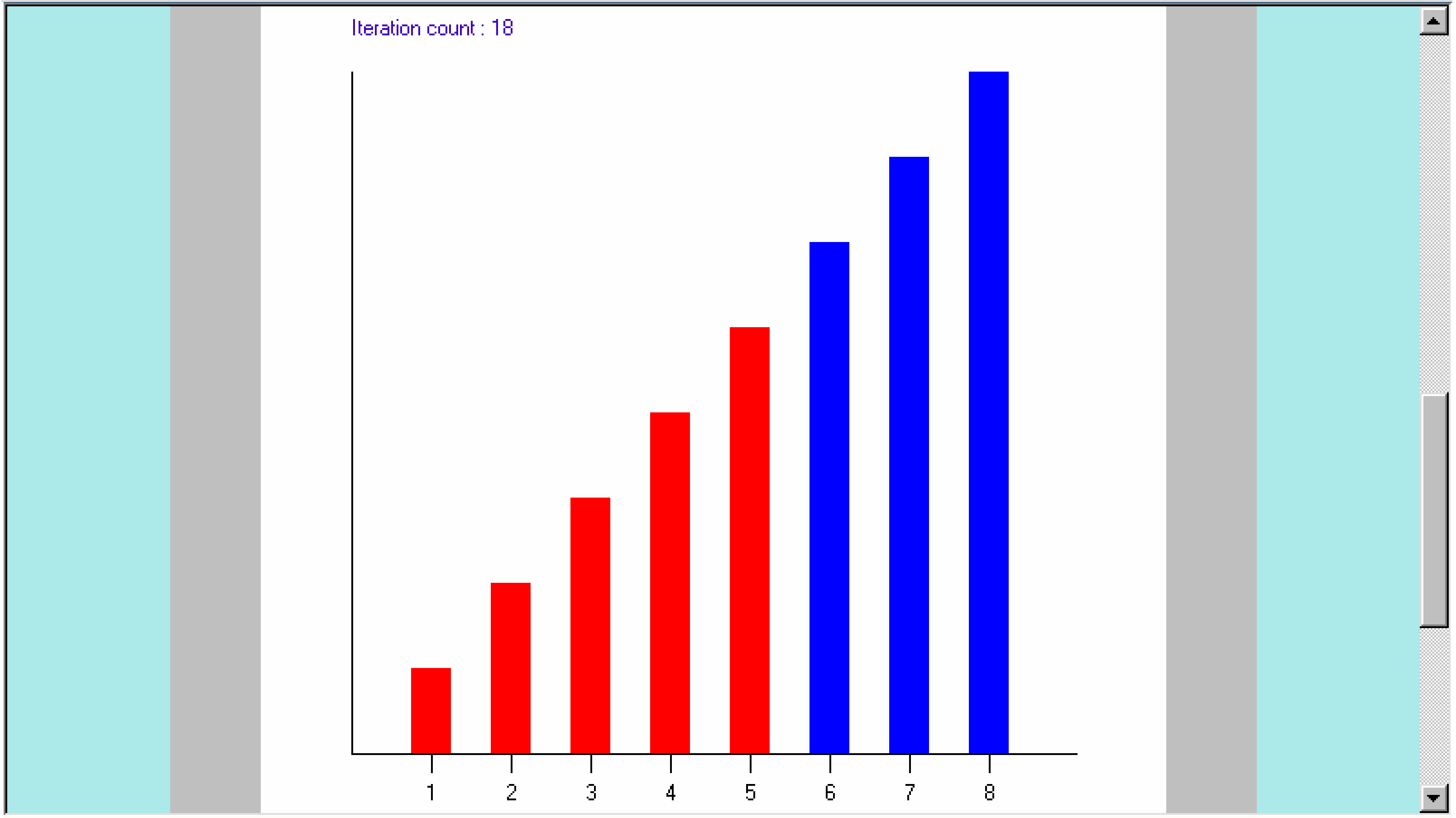


Iteration count : 17

http://www.cs.brockport.edu/cs/java/apps/sorters/insertsortaniminp.html

Insertion Sort Animation

Positions 0 through 7 are now in non-decreasing order.

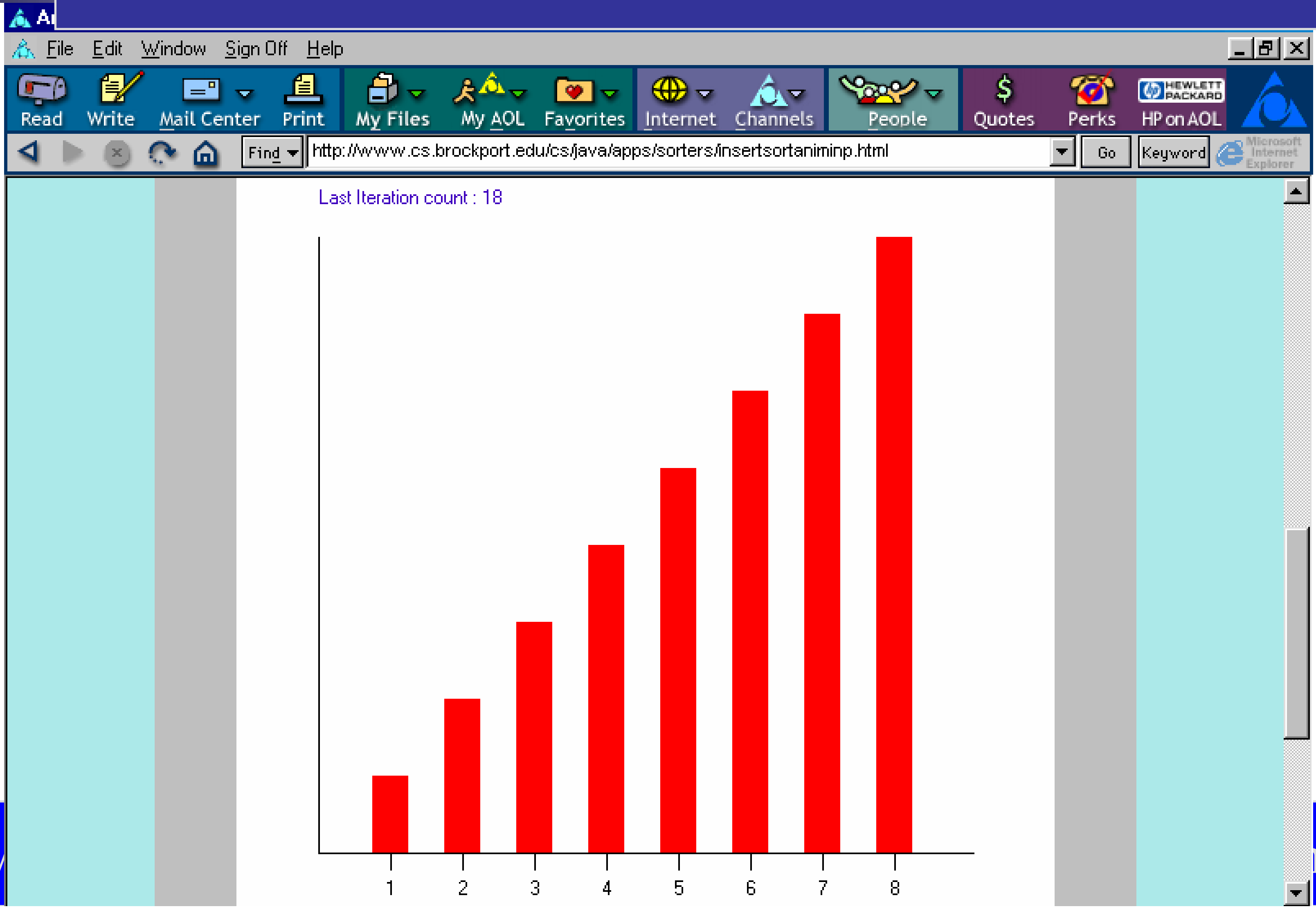


Iteration count : 18

http://www.cs.brockport.edu/cs/java/apps/sorters/insertsortaniminp.html

Insertion Sort Animation

Positions 0 through 7 are now in non-decreasing order.



http://www.cs.brockport.edu/cs/java/apps/sorters/insertsortaniminp.html

Last Iteration count : 18

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Insertion sort analysis

	cost	times
`void insertionSort(int A[], int n)`		
`{`		
`  int i, j, tmp;`		
`   for (i=1; i<n; i++) {`	c_1	n
`      tmp=A[i];`	c_2	n-1
`      j = i-1;`	c_4	n-1
`      while (j>=0 && tmp<A[j]) {`	c_5	$\sum_{i=1}^{n-1} t_i$
`        A[j+1] = A[j];`	c_6	$\sum_{i=1}^{n-1}(t_i - 1)$
`        j--;`	c_7	$\sum_{i=1}^{n-1}(t_i - 1)$
`      }`		
`      A[j+1] = tmp;`	c_8	n-1
`   }`		
`}`		

best case $t_i = 1$

worst case $t_i = i + 1$

Insert sort cost

$$T(n) = c_1 n + c_2(n-1) + c_4(n-1) + c_5 \sum_{i=1}^{n-1} t_i + c_6 \sum_{i=1}^{n-1} (t_i - 1) + c_7 \sum_{i=1}^{n-1} (t_i - 1) + c_8(n-1)$$

best case $t_i = 1$

$$T(n) = c_1 n + c_2(n-1) + c_4(n-1) + c_5 \sum_{i=1}^{n-1} t_i + c_6 \sum_{i=1}^{n-1} (t_i - 1) + c_7 \sum_{i=1}^{n-1} (t_i - 1) + c_8(n-1)$$

$$= c_1 n + c_2(n-1) + c_4(n-1) + c_5(n-1) + c_8(n-1)$$

$$= (c_1 + c_2 + c_4 + c_5 + c_8)n - (c_2 + c_4 + c_5 + c_8)$$

worst case $t_i = i + 1$

$$T(n) = c_1 n + c_2(n-1) + c_4(n-1) + c_5 \sum_{i=1}^{n-1} t_i + c_6 \sum_{i=1}^{n-1} (t_i - 1) + c_7 \sum_{i=1}^{n-1} (t_i - 1) + c_8(n-1)$$

$$= c_1 n + c_2(n-1) + c_4(n-1) + c_5 \sum_{i=1}^{n-1} (i+1) + c_6 \sum_{i=1}^{n-1} i + c_7 \sum_{i=1}^{n-1} i + c_8(n-1)$$

$$= \frac{c_5 + c_6 + c_7}{2} n^2 + (c_1 + c_2 + c_4 + \frac{c_5}{2} - \frac{c_6 + c_7}{2} + c_8)n - (c_2 + c_4 + c_5 + c_8)$$