(a) $\underset{C \leftarrow B}{P}$

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Quiz 9

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right]$$

$R_1 - R_2 = nR_1$

$R_2 - R_3 = nR_2$

$$\longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right]$$

$$\therefore \underset{C \leftarrow B}{P} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(b) $\underset{B \leftarrow C}{P} = \left(\underset{C \leftarrow B}{P}\right)^{-1} = \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}^{-1}}_{[A]}$

$$\det(A) = 1\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1 \neq 0$$

$C_{11} = 0$	$C_{12} = 0$	$C_{13} = 1\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$
$C_{21} = 0$	$C_{22} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$	$C_{23} = 0$
$C_{31} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$	$C_{32} = 0$	$C_{33} = 0$

$$\Rightarrow C = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \Rightarrow \text{adj}(A) = C^T = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A}\,\text{adj}\,A = \frac{1}{-1}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\therefore \underset{B \leftarrow C}{P} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(c)

$$[\vec{x}]_B = \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}}_{3\times 3}\underbrace{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}_{3\times 1} = \begin{bmatrix} 6 \\ 3 \\ 1 \end{bmatrix}$$

$$\therefore \vec{x} = \begin{bmatrix} 6 \\ 3 \\ 1 \end{bmatrix}$$

$$[\vec{x}]_C = \underset{C \leftarrow B}{[P]}[\vec{x}_B] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$\therefore \vec{x}_C = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$