Student: Phong Vo **Date:** 02/19/20	**Instructor:** Erica Yankowskas **Course:** Linear Algebra I (Spring 2020)	**Assignment:** Section 1.1 Homework

1. Solve the system by using elementary row operations on the equations. Follow the systematic elimination procedure.

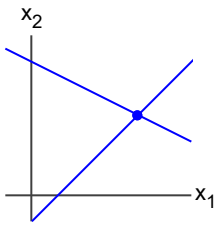
$$7x_1 + 14x_2 = 21$$
$$4x_1 + 5x_2 = 18$$

Find the solution to the system of equations.

$(7, -2)$

(Simplify your answer. Type an ordered pair.)

2. Find the point (x_1, x_2) that lies on the line $x_1 + 2x_2 = 10$ and on the line $x_1 - x_2 = 1$. See the figure.



The point (x_1, x_2) that lies on the line $x_1 + 2x_2 = 10$ and on the line $x_1 - x_2 = 1$ is $(4,3)$.

(Simplify your answer. Type an ordered pair.)

3. The augmented matrix of a linear system has been reduced by row operations to the form shown. Continue the appropriate row operations and describe the solution set of the original system.

$$\begin{bmatrix} 1 & 6 & 3 & -3 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & -1 & 3 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- **A.** The solution set has exactly one element, (, ,).
  (Type integers or simplified fractions.)
- **B.** The solution set has infintely many elements.
- **C.** The solution set is empty.

4. The augmented matrix of a linear system has been reduced by row operations to the form shown. Continue the appropriate row operations and describe the solution set of the original system.

$$\begin{bmatrix} 1 & -1 & 0 & 0 & -5 \\ 0 & 1 & -2 & 0 & -7 \\ 0 & 0 & 1 & -3 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- **A.** The solution set is empty.
- **B.** The solution has infinitely many elements.
- **C.** The solution set contains one solution:
  (18 , 23 , 15 , 4).
  (Type integers or simplified fractions.)

5. Solve the system.

$$\begin{aligned} x_2 + 3x_3 &= -6 \\ x_1 + 4x_2 + 4x_3 &= 5 \\ 2x_1 + 7x_2 + 5x_3 &= -2 \end{aligned}$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- **A.** The unique solution of the system is (, ,).
  (Type integers or simplified fractions.)
- **B.** The system has infinitely many solutions.
- **C.** The system has no solution.

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6. Determine if the given system is consistent. Do not completely solve the system.

$$\begin{aligned} 2x_1 \qquad\qquad - 4x_4 &= -16 \\ 3x_2 + 3x_3 \qquad &= 0 \\ x_3 + 4x_4 &= 2 \\ -3x_1 + 5x_2 + 3x_3 + x_4 &= 26 \end{aligned}$$

Choose the correct answer below.

- **A.** The system is inconsistent.
- **B.** The system is consistent.
- **C.** It is impossible to determine whether the system is consistent.

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7. Determine the value(s) of h such that the matrix is the augmented matrix of a consistent linear system.

$$\begin{bmatrix} 1 & h & 4 \\ 2 & 10 & 4 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The matrix is the augmented matrix of a consistent linear system if $h \neq$ 5 .
  (Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- **B.** The matrix is the augmented matrix of a consistent linear system if $h =$.
  (Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- **C.** The matrix is the augmented matrix of a consistent linear system for every value of h.
- **D.** The matrix is not the augmented matrix of a consistent linear system for any value of h.

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8. Determine the value(s) of h such that the matrix is the augmented matrix of a consistent linear system.

$$\begin{bmatrix} 1 & 2 & -2 \\ 3 & h & -6 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The matrix is the augmented matrix of a consistent linear system if h = _____.
  (Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- **B.** The matrix is the augmented matrix of a consistent linear system if h ≠ _____.
  (Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- **C.** The matrix is the augmented matrix of a consistent linear system for every value of h.
- **D.** The matrix is not the augmented matrix of a consistent linear system for any value of h.

9. Determine the value(s) of h such that the matrix is the augmented matrix of a consistent linear system.

$$\begin{bmatrix} 9 & 6 & h \\ -3 & -2 & 1 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The matrix is the augmented matrix of a consistent linear system if h ≠ _____.
  (Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- **B.** The matrix is the augmented matrix of a consistent linear system if h = −3.
  (Use a comma to separate answers as needed. Type an integer or a simplified fraction.)
- **C.** The matrix is the augmented matrix of a consistent linear system for every value of h.
- **D.** The matrix is not the augmented matrix of a consistent linear system for any value of h.

10. Indicate whether the statements given in parts (a) through (d) are true or false and justify the answer.

a. Is the statement "Every elementary row operation is reversible" true or false? Explain.

- **A.** False, because only scaling and interchanging are reversible row operations.
- **B.** True, because interchanging can be reversed by scaling, and scaling can be reversed by replacement.
- **C.** False, because only interchanging is a reversible row operation.
- **D.** True, because replacement, interchanging, and scaling are all reversible. ✓

b. Is the statement "A 5×6 matrix has six rows" true or false? Explain.

- **A.** True, because a 5×6 matrix has five columns and six rows.
- **B.** False, because a 5×6 matrix has five rows and five columns.
- **C.** True, because a 5×6 matrix has six columns and six rows.
- **D.** False, because a 5×6 matrix has five rows and six columns. ✓

c. Is the statement "The solution set of a linear system involving variables $x_1, ..., x_n$ is a list of numbers $(s_1, ..., s_n)$ that makes each equation in the system a true statement when the values $s_1, ..., s_n$ are substituted for $x_1, ..., x_n$, respectively" true or false? Explain.

- **A.** False, because the list of numbers $(s_1, ..., s_n)$ is the solution set for a linear system involving the variables $x_1, ..., x_{n-1}$.
- **B.** True, because the solution set of a linear system will have the same number of elements as the list of the variables in the system.
- **C.** False, because the description applies to a single solution. The solution set consists of all possible solutions. ✓
- **D.** True, because the list of variables $(x_1, ..., x_n)$ and the list of numbers $(s_1, ..., s_n)$ have a one-to-one correspondence.

d. Is the statement "Two fundamental questions about a linear system involve existence and uniqueness" true or false? Explain.

- **A.** True, because two fundamental questions address whether the solution exists and whether there is only one solution. ✓
- **B.** False, because two fundamental questions address whether it is possible to solve the system with row operations or whether a computer is necessary.
- **C.** False, because two fundamental questions address the type of row operations that can be used on the system and whether the linear operations fundamentally change the system.
- **D.** True, because two fundamental questions address whether the equations of the linear system exist in n-dimensional space and whether they can exist in more than one instance of n-dimensional space.

11. Indicate whether the statements given in parts (a) through (d) are true or false and justify the answer.

a. Is the statement "Two matrices are row equivalent if they have the same number of rows" true or false? Explain.

- **A.** False, because if two matrices are row equivalent it means that there exists a sequence of row operations that transforms one matrix to the other.
- **B.** False, because if two matrices are row equivalent it means that they have the same number of row solutions.
- **C.** True, because two matrices are row equivalent if they have the same number of rows and column equivalent if they have the same number of columns.
- **D.** True, because two matrices that are row equivalent have the same number of solutions, which means that they have the same number of rows.

b. Is the statement "Elementary row operations on an augmented matrix never change the solution set of the associated linear system" true or false? Explain.

- **A.** False, because the elementary row operations make a system inconsistent.
- **B.** True, because the elementary row operations replace a system with an equivalent system.
- **C.** True, because elementary row operations are always applied to an augmented matrix after the solution has been found.
- **D.** False, because the elementary row operations augment the number of rows and columns of a matrix.

c. Is the statement "Two equivalent linear systems can have different solution sets" true or false? Explain.

- **A.** False, because two systems are called equivalent if they have the same solution set.
- **B.** False, because two systems are called equivalent only if they both have no solution.
- **C.** True, because equivalent linear systems are systems that have the same number of rows and columns when they are written as augmented matrices, which means that they can have different solution sets.
- **D.** True, because equivalent linear systems are systems with the same number of variables, which means that they can have different solution sets.

d. Is the statement "A consistent system of linear equations has one or more solutions" true or false? Explain.

- **A.** False, because a consistent system has infinitely many solutions.
- **B.** True, because a consistent system is made up of equations for planes in three-dimensional space.
- **C.** True, a consistent system is defined as a system that has at least one solution.
- **D.** False, because a consistent system has only one unique solution.

12. Find an equation involving g, h, and k that makes this augmented matrix correspond to a consistent system.

$$\begin{bmatrix} 1 & -5 & 6 & g \\ 0 & 9 & -9 & h \\ -3 & 6 & -9 & k \end{bmatrix}$$

What equation must be satisfied for the augmented matrix to correspond to a consistent system? Select the correct choice below and fill in the answer box to complete your choice.
(Simplify your answer.)

- **A.** $3g + h - k =$ ____
- **B.** $3g - h - k =$ ____
- **C.** $3g - h + k =$ ____
- **D.** $3g + h + k =$ 0