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**Course:** Multi-Variable and Vector  
 Calculus -- Calculus III Spring 2018

**Assignment:** Section 15.6 Homework

1. Explain how to compute the surface integral of a scalar-valued function  $f$  over a sphere using an explicit description of the sphere.

Choose the correct answer below.

- ☐ A. Compute  $\int_0^{2\pi} \int_0^h f(a \cos u, a \sin u, v) \, dv \, du$ .
- ☒ B. Compute  $\int_0^\pi \int_0^{2\pi} f(a \sin u \cos v, a \sin u \sin v, a \cos u) a^2 \sin u \, dv \, du$ .
- ☐ C. Compute  $\int_0^\pi \int_0^{2\pi} f(a \sin u \cos v, a \sin u \sin v, a \cos u) \, dv \, du$ .
- ☐ D. Compute  $\int_0^{2\pi} \int_0^h f(a \cos u, a \sin u, v) a^2 \sin u \, dv \, du$ .

2. Explain how to compute a surface integral  $\iint_S \mathbf{F} \cdot \mathbf{n} \, dS$  over a hemisphere using a parametric description of a hemisphere of radius  $a$  and a given orientation.

Choose the correct answer below.

- ☒ A. Compute  $\int_0^{\frac{\pi}{2}} \int_0^{2\pi} a^2 \sin u (f \tan u \cos v + g \tan u \sin v + h) \, dv \, du$ .
- ☐ B. Compute  $\int_0^{2\pi} \int_0^h a^2 \sin u (f \cos u + g \sin u + hv) \, dv \, du$ .
- ☐ C. Compute  $\int_0^{2\pi} \int_0^h (f \cos u + g \sin u + hv) \, dv \, du$ .
- ☐ D. Compute  $\int_0^{\frac{\pi}{2}} \int_0^{2\pi} (f \tan u \cos v + g \tan u \sin v + h) \, dv \, du$ .

3. Give a parametric description of the form  $\mathbf{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$  for the frustum of the cone  $z^2 = x^2 + y^2$ , for  $5 \leq z \leq 7$ .

Select the correct choice below.

- ☒ A.  $\mathbf{r}(u,v) = \langle v \cos u, v \sin u, v \rangle$ ,  $0 \leq u \leq 2\pi$ ,  $5 \leq v \leq 7$
- ☐ B.  $\mathbf{r}(u,v) = \langle 7v \cos u, 5v \sin u, v \rangle$ ,  $0 \leq u \leq 2\pi$ ,  $5 \leq v \leq 7$
- ☐ C.  $\mathbf{r}(u,v) = \langle 7v \cos u, 5v \sin u, v \rangle$ ,  $0 \leq u \leq 2\pi$ ,  $5 \leq v \leq 7$
- ☐ D.  $\mathbf{r}(u,v) = \left\langle \frac{7v}{5} \cos u, \frac{7v}{5} \sin u, v \right\rangle$ ,  $0 \leq u \leq 2\pi$ ,  $5 \leq v \leq 7$
- ☐ E.  $\mathbf{r}(u,v) = \left\langle \frac{5v}{7} \cos u, \frac{5v}{7} \sin u, v \right\rangle$ ,  $0 \leq u \leq 2\pi$ ,  $5 \leq v \leq 7$

4. Describe the surface with the parametric representation shown below.

$$\mathbf{r}(u,v) = \langle v \cos u, v \sin u, 5v \rangle, \text{ for } 0 \leq u \leq 2\pi, 0 \leq v \leq 1$$

Select the correct choice below and fill in the answer boxes within your choice.

- ☐ A. The surface is a cylinder with a height of \_\_\_\_\_ and a radius of \_\_\_\_\_.
- ☐ B. The surface is a sphere with its center at ( \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_ ) and a radius of \_\_\_\_\_.
- ☒ C. The surface is a cone with height of \_\_\_\_\_ 5 \_\_\_\_\_ and radius of \_\_\_\_\_ 1 \_\_\_\_\_ at the widest point.

5. Find the area of the surface of the half cylinder  $\{(r,\theta,z): r=4, 0 \leq \theta \leq \pi, 0 \leq z \leq 6\}$  using a parametric description of the surface.

The surface area is \_\_\_\_\_  $24\pi$  \_\_\_\_\_.

(Type an exact answer, using  $\pi$  as needed.)

6. Evaluate the surface integral  $\iint_S f(x,y,z) \, dS$  using a parametric description of the surface.

$$f(x,y,z) = 4x^2 + 4y^2, \text{ where } S \text{ is the hemisphere } x^2 + y^2 + z^2 = 9, \text{ for } z \geq 0$$

The value of the surface integral is \_\_\_\_\_.

(Type an exact answer, using  $\pi$  as needed.)

7. Find the area of the following cone using the given explicit description of the surface.

$$z^2 = 9(x^2 + y^2), \text{ for } 0 \leq z \leq 9$$

The surface area is \_\_\_\_\_  $9\pi\sqrt{10}$  \_\_\_\_\_.

(Type an exact answer, using  $\pi$  as needed.)

8. Evaluate the surface integral  $\iint_S f(x,y,z) \, dS$  using an explicit representation of the surface.

$f(x,y,z) = e^z$ ,  $S$  is the plane  $z = 18 - 2x - 6y$  in the first octant.

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The value of the surface integral is \_\_\_\_\_ .  
(Type an exact answer, using radicals as needed.)

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9. Find the average temperature on that part of the plane  $x + 3y + z = 7$  over the square  $|x| \leq 1$ ,  $|y| \leq 1$ , where the temperature is given by  $T(x,y,z) = e^{-z}$ .
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The average value is \_\_\_\_\_ .  
(Type an exact answer, using radicals as needed.)

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10. Find the flux of the vector field  $\mathbf{F} = \langle 0, 0, 3 \rangle$  across the slanted face of the tetrahedron  $z = 1 - x - y$  in the first octant with the normal vectors pointing in the positive  $z$ -direction. You may use either an explicit or parametric description of the surface.
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The flux is \_\_\_\_\_ .  
(Simplify your answer.)