Section 8.5: Principle of Inclusion-Exclusion Problems

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1. A class of 20 students contains only students majoring in math or physics (or both). There are 12 students majoring in math and 15 majoring in physics.

   (a) How many students are majoring in both areas?

   (b) How many students are majoring in only math?

2. In a survey of 60 students, it's found that

   - 20 like asparagus,
   - 30 like broccoli,
   - 25 like cauliflower,
   - 8 like both asparagus and broccoli,
   - 6 like both asparagus and cauliflower,
   - 10 like both broccoli and cauliflower, and
   - 3 like all three vegetables.

   Question: How many of the survey's participants like at least one of the vegetables?

3. Let A_1, A_2, A_3, A_4 be sets, and let $A = A_1 \cup A_2 \cup A_3 \cup A_4$. Suppose the sets A_i satisfy the following properties:

   - $|A_i| = 20$ for all i;
   - $|A_i \cap A_j| = 3$ for all $i \neq j$;
   - the intersection of any three sets is $\varnothing$.

   Question: What is $|A|$?

Counting Using Inclusion-Exclusion

1. Suppose that there are 5 freshmen, 6 sophomores, and 3 juniors in a class. How many groups of five students ...

   (a) ... contain exactly two freshmen?

   (b) ... contain exactly two freshmen or two sophomores?

2. Suppose that a bakery sells five types of muffins (apple, blueberry, carrot, pineapple, and strawberry). How many ways can ten muffins be selected if ...

   (a) ... at least three apple and four blueberry muffins are picked?

   (b) ... at least three apple muffins or at least four blueberry muffins are picked?

3. Alice has ten identical marbles she wants to distribute between four friends (Bob, Candy, Darren, and Ernest). How many ways can she do this if ...

   (a) ... Bob gets at least two marbles and Candy gets at least four marbles?

   (b) ... Bob gets at least two marbles or Candy gets at least four marbles?