Name:

Linear Algebra I: Exam 3 (Summer 2019)

<u>Show ALL work, as unjustified answers may receive no credit</u>. Calculators are not allowed on any quiz or test paper. *<u>Make sure to exhibit skills discussed in class</u>*. Box all answers and simplify answers as much as possible.

Good Luck! ☺

1. <u>Linearly Independent Sets; Bases</u>

Let $\vec{v}_1 = (1, 1, 1)$, $\vec{v}_2 = (1, 2, 3)$, $\vec{v}_3 = (1, 1, 2)$.

(a) [5 pts] Show that the vectors are Linearly Independent.

(b) [5 pts] Find the unique weights(scalars) c_1, c_2, c_3 such that $\vec{v} = (2, 1, 3)$ can be written as $\vec{v} = c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3$

2. *Null Spaces, Column Spaces, and Linear Transformations*

Define the Linear Transformation $\mathrm{T} : \mathbb{R}^4 \to \mathbb{R}^3$ *by* $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ x_2 - x_3 \\ x_1 + x_4 \end{bmatrix}.$

(a) [5 pts] Find the column space of T.
(b) [5 pts] Find the null space of T.
(c) [2 pts] Find a basis for the column space of T.
(d) [2 pts] Find the basis for the null space of T.

Hint: The column space of T is Col(A) and the null space of T is Nul(A), where A is the standard matrix of T ☺

3. Vector Spaces and Subspaces

Let H and K be subspaces of a vector space V. Let $H + K = \{ \vec{w} : \ \vec{w} = \vec{u} + \vec{v} \ , \ \vec{u} \in H \ \ and \ \ \vec{v} \in K \}$.

[9 pts] Show that $H + K$ is a subspace of V.

4. ***Null Spaces, Column Spaces, and Linear Transformations***

Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation and $\mathcal{B} = \{ \vec{v}_1, \vec{v}_2, \vec{v}_3 \}$ be a basis for $\mathbb{R}^3$. Suppose that $T(\vec{v}_1) = (-2, 1, 1)$, $T(\vec{v}_2) = (0, 1, -1)$, $T(\vec{v}_3) = (-2, 2, 0)$.

(a) [5 pts] Determine whether $\vec{w} = (-6, 5, 0)$ is in the range of T.

(b) [5 pts] Find a basis for the kernel of T.

5. ***Coordinate Systems***

The set $B = \{\, 1 + t^2\,,\ 2t - t^2\,, 1 - t + t^2 \,\}$ be a basis for $\mathbb{P}_2$.

[5pts] Find the coordinate vector $p(t) = 1 + 16t - 6t^2$ relative to B.

Bonus Question: ***<u>Coordinate Systems</u>***

Let $B = \left\{ \begin{bmatrix} 1 \\ -4 \end{bmatrix}, \begin{bmatrix} -2 \\ 9 \end{bmatrix} \right\}$. Since the coordinate mapping determined by B is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$, this mapping must be implemented by some 2 x 2 matrix A.

[5pts] Find it.

Scratch Work (Not Graded)