Name:

<u>Show ALL work, as unjustified answers may receive no credit</u>. Calculators are not allowed on any quiz or test paper. *<u>Make sure to exhibit skills discussed in class</u>*. Box all answers and simplify answers as much as possible.

Good Luck! ☺

1. <u>Row-Reduction and Echelon Form</u>

[6pts] Determine when the augmented matrix below represents a consistent linear system:

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & a \\ 2 & 1 & 5 & b \\ 1 & -1 & 1 & c \end{array}\right]$$

2. ***Vector Equations***

(a) [9pts] Determine if $\vec{b}$ is a linear combination of the vectors $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$ where:

$$\vec{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} -2 \\ -1 \\ -1 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 3 \\ -1 \\ -3 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}$$

(b) [2pts] If $\vec{b}$ is a linear combination of the vectors $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$, then express $\vec{b}$ as a linear combination of the vectors $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$.

3. ***The Matrix Equation, $A\vec{x} = \vec{b}$***

(a) [9pts] Solve the matrix equation $A\vec{x} = \vec{b}$ where:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 0 \\ 1 & 1 & 2 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(b) [2pts] Is it possible to solve $A\vec{x} = \vec{b}$ for any vector $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$, where A is the matrix given above?

Explain.

4. ***Solution Sets of Linear Systems***

Consider the linear system $A\vec{x} = \vec{b}$, where:

$$A = \begin{bmatrix} 1 & -1 & -2 & -2 & -2 \\ 3 & -2 & -2 & -2 & -2 \\ -3 & 2 & 1 & 1 & 1 \end{bmatrix}, \ \vec{b} = \begin{bmatrix} 3 \\ -1 \\ -1 \end{bmatrix}$$

(a) [9pts] Solve the linear system. Write the general solution in parametric-vector form.

(b) [2pts] Using your answer from (a), write the solution set for the homogeneous equation $A\vec{x} = \vec{0}$.

5. ***Linear Independence***

Determine if the following sets of vectors are linearly independent. Explain.

(a) [2pts] $\left\{\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\}$

(b) [2pts] $\left\{\begin{bmatrix} -5 \\ 10 \end{bmatrix}, \begin{bmatrix} -4 \\ -2 \end{bmatrix}, \begin{bmatrix} 36 \\ 12 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \end{bmatrix}\right\}$

(c) [2pts] $\left\{\begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \end{bmatrix}\right\}$

(d) [2pts] $\left\{\begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ -6 \end{bmatrix}\right\}$

(e) [3pts] $\left\{\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -1 \end{bmatrix}\right\}$

Bonus Question [5pts]:

Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by:

$$T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3 \,, \, 2x_1 + x_3 \,, -x_1 - 2x_2 + 2x_3)$$

Find the standard matrix of T.

Scratch Work (Not Graded)