Student: Phong Vo **Date:** 03/02/18	**Instructor:** Chuck Ormsby **Course:** Multi-Variable and Vector Calculus -- Calculus III Spring 2018 **Assignment:** Section 13.5 Homework

1. If w is a function of x, y, and z, which are all functions of t, explain how to find $\frac{dw}{dt}$.

Choose the correct answer below.

- **A.** $\frac{dw}{dt} = \frac{dw}{dx} + \frac{dw}{dy} + \frac{dw}{dz}$
- **B.** $\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dy} + \frac{\partial w}{\partial y}\frac{dy}{dz}$
- **C.** $\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt}\frac{\partial w}{\partial y}\frac{dy}{dt}\frac{\partial w}{\partial z}\frac{dz}{dt}$
- **D.** $\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$

2. Use the chain rule to find $\frac{dz}{dt}$, where $z = x \sin y$, $x = t^2$, and $y = 5t^5$. When feasible, express your answer in terms of the independent variable.

$\frac{dz}{dt} = 2t \sin\left(5t^5\right) + 25t^6 \cos\left(5t^5\right)$

3. Use the chain rule to find $\frac{dw}{dt}$, where $w = \cos 8x \sin 3y$, $x = \frac{t}{2}$, and $y = t^4$. When feasible, express your answer in terms of the independent variable.

$\frac{dw}{dt} = -4 \sin(4t) \sin\left(3t^4\right) + 12t^3 \cos\left(3t^4\right) \cos(4t)$

(Type an expression using t as the variable.)

4. Use the chain rule to find $\frac{dV}{dt}$, where $V = \frac{x+z}{z-y}$, $x = 4t$, $y = 3t$, and $z = 5t$. When feasible, express your answer in terms of the independent variable.

$\frac{dV}{dt} = 0$

5. The volume of a right circular cylinder of radius r and height h is $V = \pi r^2 h$.

(a) Assume that r and h are functions of t. Find $V'(t)$.

(b) Suppose that $r = e^{3t}$ and $h = e^{-3t}$. Use part (a) to find $V'(t)$.

(c) Does the volume of the cylinder of part (b) increase or decrease as t increases?

(a) Find $V'(t)$. Choose the correct answer below.

- **A.** $V'(t) = \pi(r(t))^2 h'(t)$
- **B.** $V'(t) = 2\pi r(t)h(t)r'(t)$
- **C.** $V'(t) = 2\pi r(t)h(t)r'(t) + \pi(r(t))^2 h'(t)$
- **D.** $V'(t) = 2\pi r(t)h(t)h'(t) + \pi(r(t))^2 r'(t)$

(b) $V'(t) = 3\pi e^{3t}$

(c) Does the volume of the cylinder of part (b) increase or decrease as t increases? Choose the correct answer below.

- **A.** The volume of the cylinder increases as t increases.
- **B.** The volume of the cylinder remains the same.
- **C.** The volume of the cylinder decreases as t increases.

6. Find the following derivatives. Express your answer in terms of the independent variables.

z_s and z_t, where $z = e^{2x + 2y}$, $x = st$, and $y = s + t$

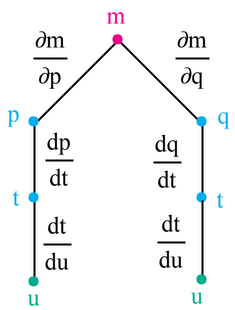
$z_s = 2e^{2(st + s + t)}(t + 1)$

$z_t = 2e^{2(st + s + t)}(s + 1)$

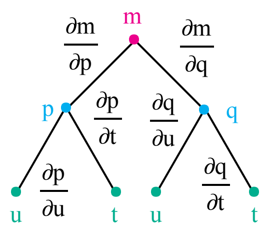
7. Use a tree diagram to write the Chain Rule formula for $\frac{dm}{du}$. m is a function of t, where t is a function of p and q, each of which is a function of u.

Choose the correct tree diagram below.

A.



B.



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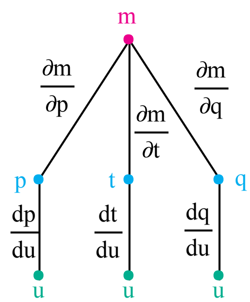
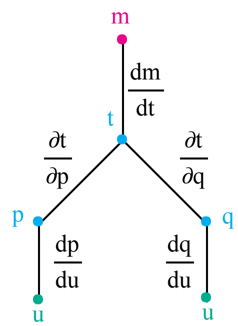


D.



What is the Chain Rule formula for $\frac{dm}{du}$?

A. $\frac{dm}{du} = \frac{\partial m}{\partial p}\frac{dp}{du} + \frac{\partial m}{\partial t}\frac{dt}{du} + \frac{\partial m}{\partial q}\frac{dq}{du}$

B. $\frac{dm}{du} = \frac{\partial m}{\partial p}\frac{dp}{dt}\frac{dt}{du} + \frac{\partial m}{\partial q}\frac{dq}{dt}\frac{dt}{du}$

C. $\frac{dm}{du} = \frac{\partial m}{\partial p}\frac{\partial p}{\partial u} + \frac{\partial m}{\partial q}\frac{\partial q}{\partial u}$

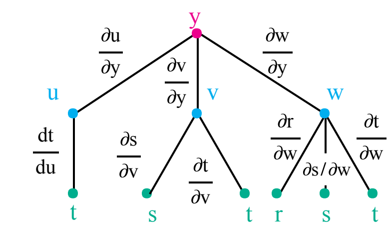
D. $\frac{dm}{du} = \frac{dm}{dt}\frac{\partial t}{\partial p}\frac{dp}{du} + \frac{dm}{dt}\frac{\partial t}{\partial q}\frac{dq}{du}$

8.

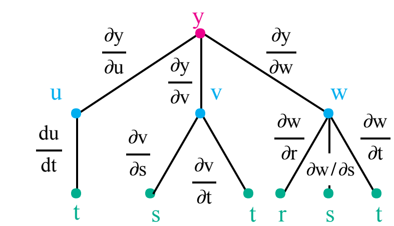
Use a tree diagram to write the Chain Rule formula for $\frac{\partial y}{\partial t}$, where $y = f(u,v,w)$, $u = g(t)$, $v = h(s,t)$, and $w = p(r,s,t)$.

Choose the correct tree diagram below.

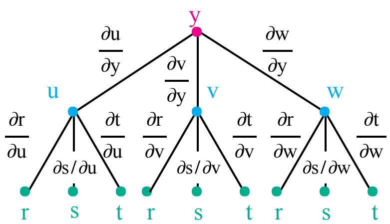
A.



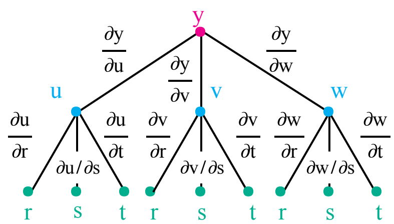
B.



C.



D.



Choose the correct Chain Rule below.

A. $\frac{\partial y}{\partial t} = \frac{\partial y}{\partial u} \frac{dy}{dv} \frac{\partial y}{\partial w} + \frac{\partial u}{\partial t} \frac{\partial v}{\partial t} \frac{\partial w}{\partial t}$

B. $\frac{\partial y}{\partial t} = \frac{\partial y}{\partial u}\left(\frac{\partial u}{\partial r} + \frac{\partial u}{\partial s} + \frac{\partial u}{\partial t}\right) + \frac{\partial y}{\partial v}\left(\frac{\partial v}{\partial r} + \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}\right) + \frac{\partial y}{\partial w}\left(\frac{\partial w}{\partial r} + \frac{\partial w}{\partial s} + \frac{\partial w}{\partial t}\right)$

C. $\frac{\partial y}{\partial t} = \frac{\partial u}{\partial y} \frac{dt}{du} + \frac{\partial v}{\partial y} \frac{\partial t}{\partial v} + \frac{\partial w}{\partial y} \frac{\partial t}{\partial w}$

D. $\frac{\partial y}{\partial t} = \frac{\partial y}{\partial u} \frac{du}{dt} + \frac{\partial y}{\partial v} \frac{\partial v}{\partial t} + \frac{\partial y}{\partial w} \frac{\partial w}{\partial t}$

9.

Given the equation $y \ln\left(x^3 + y^3 + 7\right) = 5$, evaluate $\frac{dy}{dx}$. Assume that the equation implicitly defines y as a differentiable function of x.

Choose the correct answer below.

A. $\frac{dy}{dx} = \frac{5y \ln\left(x^3 + y^3 + 7\right)}{5\left(x^3 + y^3 + 7\right) + 3y^4}$

B. $\frac{dy}{dx} = -\frac{x^2}{y^2}$

C. $\frac{dy}{dx} = -\frac{3yx^2}{\left(x^3 + y^3 + 7\right) \ln\left(x^3 + y^3 + 7\right) + 3y^3}$

D. $\frac{dy}{dx} = \frac{-3x^2y^2 - 3y^4}{5 \ln\left(x^3 + y^3 + 7\right)}$