

$$\begin{array}{c|c|c|c}
 \begin{array}{cccc}
 -1 & 3 & 6 & 0 \\
 3 & 3 & 5 & 0 \\
 7 & 6 & 8 & 6 \\
 5 & 3 & 5 & 3
 \end{array}
 &
 \begin{array}{l}
 3R_1 + R_2 = nR_2 \\
 7R_1 + R_3 = nR_3 \\
 \hline
 5R_1 + R_4 = nR_4
 \end{array}
 &
 \begin{array}{cccc}
 -1 & 3 & 6 & 0 \\
 0 & 12 & 23 & 0 \\
 0 & 27 & 50 & 6 \\
 0 & 18 & 35 & 3
 \end{array}
 &
 \begin{array}{l}
 2R_4 \\
 -R_3 \\
 \hline
 \rightarrow \\
 = nR_3
 \end{array}
 \end{array}
 \begin{array}{c|c|c|c}
 \begin{array}{cccc}
 -1 & 3 & 6 & 0 \\
 0 & 12 & 23 & 0 \\
 0 & 9 & 20 & 0 \\
 0 & 18 & 35 & 3
 \end{array}
 &
 \begin{array}{l}
 \\
 \\
 \\
 \end{array}
 &
 \begin{array}{cccc}
 -1 & 3 & 6 & 0 \\
 0 & 12 & 23 & 0 \\
 0 & 9 & 20 & 0 \\
 0 & 18 & 35 & 3
 \end{array}
 &
 \begin{array}{l}
 \\
 \\
 \\
 \end{array}
 \end{array}$$

$$= (3) \begin{vmatrix} -1 & 3 & 6 \\ 0 & 12 & 23 \\ 0 & 9 & 20 \end{vmatrix} = (-3)(-1)[(12)(20) - (9)(23)] = \\
 = +3(240 - 207) = +3(33) = \boxed{99}$$

3.2.21

$$\begin{vmatrix} 5 & 0 & -1 \\ 1 & -3 & -2 \\ 0 & -5 & -3 \end{vmatrix} = 5(-1) + (-1)(-5) \\
 = -5 + 5 = 0 \Rightarrow \boxed{\text{NOT invertible}}$$

3.2.23

$$\begin{array}{c|c|c|c}
 \begin{array}{cccc}
 1 & -1 & -3 & 0 \\
 0 & 1 & 5 & 4 \\
 3 & -1 & -2 & 4 \\
 -1 & 2 & 8 & 5
 \end{array}
 &
 \begin{array}{l}
 3R_1 - R_3 \\
 = nR_3 \\
 \hline
 R_1 + R_4 \\
 = nR_4
 \end{array}
 &
 \begin{array}{cccc}
 1 & -1 & -3 & 0 \\
 0 & 1 & 5 & 4 \\
 0 & -2 & -7 & 4 \\
 0 & 1 & 5 & 5
 \end{array}
 &
 \begin{array}{l}
 \\
 \\
 \\
 \end{array}
 \end{array}$$

$$\begin{array}{l}
 2R_2 + R_3 \\
 = nR_3 \\
 \hline
 R_2 - R_4 \\
 = nR_4
 \end{array}
 \begin{array}{c|c|c|c}
 \begin{array}{cccc}
 1 & -1 & -3 & 0 \\
 0 & 1 & 5 & 4 \\
 0 & 0 & 3 & 12 \\
 0 & 0 & 0 & -1
 \end{array}
 &
 \begin{array}{l}
 \\
 \\
 \\
 \end{array}
 &
 \begin{array}{cccc}
 1 & -1 & -3 & 0 \\
 0 & 1 & 5 & 4 \\
 0 & 0 & 3 & 12 \\
 0 & 0 & 0 & -1
 \end{array}
 &
 \begin{array}{l}
 \\
 \\
 \\
 \end{array}
 \end{array}
 \Rightarrow \det(A) = (1)(1)(3)(-1) \\
 = \boxed{-3}$$

3.2.26

$$\begin{array}{c|cccc|l} 3 & 2 & -2 & 0 & 5R_1 - 3R_2 \\ & & & & = nR_2 \\ 5 & -6 & -1 & 0 & 2R_1 + R_3 = nR_3 \\ & & & & \xrightarrow{\quad} \\ -6 & 0 & 3 & 0 & R_1 - R_4 \\ & & & & = nR_4 \\ 3 & 7 & 0 & -2 & \end{array} \quad \begin{array}{c|cccc|} 3 & 2 & -2 & 0 & \\ 0 & 28 & -7 & 0 & \\ 0 & 4 & -1 & 0 & \\ 0 & -5 & -2 & 2 & \end{array}$$

$$= 3 \begin{vmatrix} 28 & -7 & 0 \\ 4 & -1 & 0 \\ -5 & -2 & 2 \end{vmatrix} = (3)(2) \begin{vmatrix} 28 & -7 \\ 4 & -1 \end{vmatrix} = 6(-28 + 28) \\ = \boxed{0} \Rightarrow \boxed{\text{linearly dependent}}$$

3.2.29

$$\det B = \begin{array}{c|ccc|l} 1 & 0 & 2 & 2R_1 - R_2 \\ & & & = nR_2 \\ 2 & 2 & 3 & \xrightarrow{\quad} \\ 1 & 2 & 2 & R_1 - R_3 \\ & & & = nR_3 \end{array} \quad \begin{array}{c|ccc|l} 1 & 0 & 2 & R_2 - R_3 \\ & & & = nR_3 \\ 0 & -2 & 1 & \xrightarrow{\quad} \\ 0 & -2 & 0 & \end{array} \quad \begin{array}{c|ccc|} 1 & 0 & 2 & \\ 0 & -2 & 1 & \\ 0 & 0 & 1 & \end{array}$$

$$= (1)(-2)(1) = -2$$

$$\Rightarrow \det(B^4) = [\det(B)]^4 = (-2)^4 = \boxed{16}$$