$\frac{11}{10}$

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Linear Algebra: Quiz 3

Show ALL work, as unjustified answers may receive no credit. Calculators are not allowed on any quiz or test paper. Make sure to exhibit skills discussed in class. Box all answers and clean up answers as much as possible.

1. Consider the following Linear System:

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$$\begin{cases} 4x_1 + 2x_2 + 6x_3 = 2 \\ 2x_1 + 1x_2 + 3x_3 = 1 \\ -6x_1 - 3x_2 - 9x_3 = -3 \end{cases}$$

(a) [6pts] Find the solution set of the given nonhomogeneous system in the parametric-vector form.

(b) [2pts] Using your solution for part (a), find the solution set of the corresponding homogeneous system in the parametric-vector form.

(c) [2pts] Use a geometric interpretation to describe the two solutions sets *DO NOT NEED TO SKETCH*

(a) $A\vec{x} = \vec{b} \Rightarrow \left[\begin{array}{ccc|c} 4 & 2 & 6 & 2 \\ 2 & 1 & 3 & 1 \\ -6 & -3 & -9 & -3 \end{array}\right] \xrightarrow[-\frac{1}{3}R_3]{\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 2 & 1 & 3 & 1 \\ 2 & 1 & 3 & 1 \end{array}\right] \xrightarrow[R_1 - R_3 = nR_3]{R_1 - R_2 = nR_2}$

$\rightarrow \left[\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ Since x_2, x_3 are free variables $\Rightarrow x_1 = \frac{1}{2} - \frac{x_2}{2} - \frac{3x_3}{2}$

x_1 is a Basic variable

$\left[\begin{array}{ccc|c} 1 & 1/2 & 3/2 & 1/2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\Leftrightarrow \vec{x} = \begin{bmatrix} \vec{x}_1 \\ \vec{x}_2 \\ \vec{x}_3 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -1/2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3/2 \\ 0 \\ 1 \end{bmatrix}$ (S_a)

(b) $A\vec{x} = \vec{0} \Rightarrow \left[\begin{array}{ccc|c} 4 & 2 & 6 & 0 \\ 2 & 1 & 3 & 0 \\ -6 & -3 & -9 & 0 \end{array}\right] \xrightarrow[-\frac{1}{3}R_3]{\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 2 & 1 & 3 & 0 \\ 2 & 1 & 3 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\Rightarrow$

$\Rightarrow \begin{cases} x_2, x_3 \text{ are free vars.} \\ x_1 \text{ is basic var.} \end{cases} \Rightarrow x_1 = -x_2 - 3x_3$

$\Rightarrow \vec{x} = \begin{bmatrix} \vec{x}_1 \\ \vec{x}_2 \\ \vec{x}_3 \end{bmatrix} = x_2 \begin{bmatrix} -1/2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3/2 \\ 0 \\ 1 \end{bmatrix}$ (S_b)

(c) Since Solutions S_a and S_b have the same scalars of x_2 and x_3

So those two solution sets S_a is the solution S_b shifting by $\begin{bmatrix} 1/2 \\ 0 \\ 0 \end{bmatrix}$

+ parallel planes ...

Valentine's Day Bonus (+1): Fill in the blank with a specific, mathematical-example ☺

Love is like pie (π) *, it is real irrational and neverending*