Name: Solution

Spring 2020

Linear Algebra: Quiz 7

Show ALL work, as unjustified answers may receive no credit. Calculators are not allowed on any quiz or test paper. Make sure to exhibit skills discussed in class. Box all answers and clean up answers as much as possible.

1. ***Vector Spaces & Subspaces (4.1)***

[2pts] Find all values of h such that $\vec{y}$ will be in the subspace spanned by $\vec{v_1}$, $\vec{v_2}$, $\vec{v_3}$ if:

$$\vec{v_1} = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}, \quad \vec{v_2} = \begin{bmatrix} 3 \\ 4 \\ -8 \end{bmatrix}, \quad \vec{v_1} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \quad \vec{y} = \begin{bmatrix} 4 \\ 2 \\ h \end{bmatrix}$$

* Row-reduce $[\vec{v}_1\ \vec{v}_2\ \vec{v}_3 \,\vdots\, \vec{y}]$ to Echelon Form & solve for h:

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 4 \\ 2 & 4 & 0 & 2 \\ -4 & -8 & 0 & h \end{array}\right] \begin{array}{c} \frac{1}{2}R_2 \\ \sim \\ -\frac{1}{4}R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 3 & -1 & 4 \\ 1 & 2 & 0 & 1 \\ 1 & 2 & 0 & -\frac{h}{4} \end{array}\right] \begin{array}{c} -R_1+R_2 \to N.R_2 \\ \sim \\ -R_1+R_3 \to N.R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 4 \\ 0 & -1 & 1 & -3 \\ 0 & -1 & 1 & -4-\frac{h}{4} \end{array}\right] \begin{array}{c} -R_2+R_3 \to N.R_3 \\ \sim \\ -R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 3 & -1 & 4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & -1-\frac{h}{4} \end{array}\right]$$

* $\vec{y}$ will be in the subspace spanned by $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ IFF

$$-1 - \frac{h}{4} = 0 \to -4 - h = 0 \to \boxed{h = -4}$$

Ans

2. ***Null Space, Column Space, & Linear Transformations (4.2) & Basis (4.3)***

Define a Linear Transformation $T: \mathbb{R}^4 \to \mathbb{R}^3$ by $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ x_2 - x_3 \\ x_1 + x_4 \end{bmatrix}$.

(a) [2pts] Find the Null Space of T

(b) [2pts] Find the Column Space of T

(c) [2pts] Find the Basis for the Null Space of T

(d) [2pts] Find the Basis for the Column Space of T

** The Column Space of T is $Col(A)$ & the Null Space of T is the $Nul(A)$, where A is the Standard Matrix of T **

* Given: $T: \mathbb{R}^4 \to \mathbb{R}^3$ ST $T(\vec{x}) = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \to \therefore A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

Row-Reduce $[A \,\vdots\, \vec{0}]$ to RREF:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \underset{-R_1 + R_3 \to N.R_3}{\sim} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \underset{\substack{-R_2 + R_1 \to N.R_1 \\ R_2 + R_3 \to N.R_3}}{\sim} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \underset{\substack{R_3 + R_1 \to N.R_1 \\ -R_3 + R_2 \to N.R_2 \\ -R_3}}{\sim}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \Leftrightarrow \begin{cases} x_1 = -x_4 \\ x_2 = x_4 \\ x_3 = x_4 \\ x_4 \text{ is free} \end{cases} \Rightarrow \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_4 \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \; x_4 \in \mathbb{R}$$

(a) $\boxed{Nul(A) = \left\{ x_4 \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \; x_4 \in \mathbb{R} \right\}}$

(c) Basis for $Nul(A)$:

$$\mathcal{B}_N = \left\{ \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

(b) $Col(A) = \left\{ c_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} + c_4 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

ST $c_1, c_2, c_3, c_4 \to$ weights

(d) Basis for $Col(A)$:

$$\mathcal{B}_C = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \right\}$$