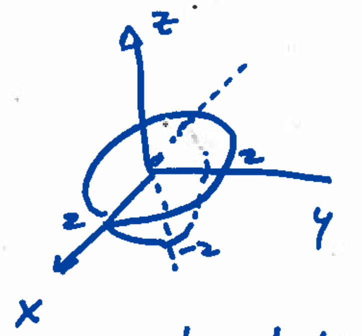
Calculus III Quiz #7 Spring 2018 NAME: SOLUTION

1. (6 Pts) Evaluate the triple integral $\iiint\limits_D 9(4+(x^2+y^2+z^2)^{3/2})^{1/2}\,dV$ where D is the interior of the hemisphere having radius 2 centered at the origin where $z \le 0$.



Hemisphere below xy-plane

spherical coordinates.

$$\int_0^{2\pi}\int_{\pi/2}^{\pi}\int_0^{2} 9(4+\rho^3)^{1/2}\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$= \int_0^{2\pi} d\theta \int_{\pi/2}^{\pi} \sin\phi\,d\phi \int_0^2 9(4+\rho^3)^{1/2}\rho^2\,d\rho$$

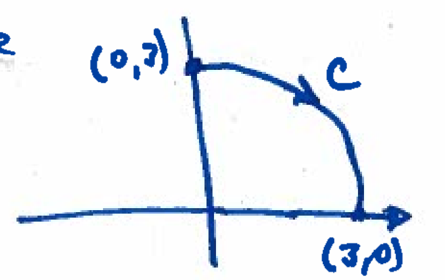
$$= 2\pi(-\cos(\phi))\Big|_{\pi/2}^{\pi}\ \frac{3\cdot 2}{3}(4+\rho^3)^{3/2}\Big|_0^2 = 2\pi(1-0)\,2\left[12^{3/2}-4^{3/2}\right]$$

$$= 4\pi[8\cdot 3\sqrt{3} - 8]$$

SEE SECOND PROBLEM (over)

$$= 32\pi(3\sqrt{3}-1)$$

2. (6 Pts) Evaluate the scalar line integral $\int\limits_C xy\,ds$ where C is the quarter circle centered at the origin from (0,3) to (3,0).



For scalar line integrals we can trace the path in either direction, so I will go from (3,0) to (0,3) $\vec{r}(t) = \langle 3\cos(t), 3\sin(t)\rangle$

$0 \le t \le \pi/2$

$\vec{r}\,'(t) = \langle -3\sin(t), 3\cos(t)\rangle$

$|\vec{r}\,'(t)| = \text{speed} = 3$

$$\int_C xy\,ds = \int_0^{\pi/2} \underbrace{3\cos(t)}_{x}\,\underbrace{3\sin(t)}_{y}\cdot\underbrace{3}_{\text{speed}}\,dt = \frac{27}{2}\sin^2(t)\Big|_0^{\pi/2} = \frac{27}{2}$$

2nd Version of problem had radius = 2

$$= \int_0^{\pi/2} \underbrace{2\cos(t)}_{x}\,\underbrace{2\sin(t)}_{y}\,\underbrace{2}_{\text{speed}}\,dt = \frac{8}{2}\sin^2(t)\Big|_0^{\pi/2} = 4$$