Vector Calculus Note Sheet for Final Exam

(May be modified/updated before Final Exam)

Line integrals of scalar functions: $\int_C f(x,y,z)ds = \int_a^b f(x(t),y(t),z(t))\left|\vec{r}'(t)\right|dt$ where $\vec{r}(t) = < x(t), y(t), z(t) >$ is a parameterization of C for $a \le t \le b$

Line integrals of vector fields: $\int_C \vec{F} \bullet \vec{T} ds = \int_C \vec{F} \bullet d\vec{r} = \int_a^b \vec{F}(t) \bullet \vec{r}'(t) dt$ where the final expression assumes the parameterization detailed above. If C is a closed curve, these integrals represent circulation.

Flux integrals of vector fields: $\int_C \vec{F} \bullet \vec{n}\, ds = \int_C f dy - g dx = \int_a^b \vec{F}(t) \bullet < y'(t), -x'(t) > dt$ where $\vec{F} = < f(x,y), g(x,y) >$, $\vec{r}(t) = < x(t), y(t) >$ is the parameterization of C detailed above, and $\vec{n}$ in a unit vector pointed outward if C is a closed curve or pointed to the right (viewed from $z > 0$) if C is not closed.

Green's Theorem (Circulation form): $\oint_C \vec{F} \bullet d\vec{r} = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dA$ where R is the region enclosed by the curve C

Green's Theorem (Flux form): $\oint_C \vec{F} \bullet \vec{n}\, ds = \iint_R \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} \right) dA$ where R is the region enclosed by the curve C

Surface integrals of scalar-valued functions on explicitly defined surfaces S given by $z = g(x,y)$ for $(x,y) \in R$:

$$\iint_S f(x,y,z) dS = \iint_R f(x,y,g(x,y)) \sqrt{g_x^2 + g_y^2 + 1}\, dA$$

Flux of vector Fields through explicitly defined surfaces S given by $z = g(x,y)$ for $(x,y) \in R$:

$$\iint_S \vec{F} \bullet \vec{n}\, dS = \iint_R \vec{F} \bullet < -g_x, -g_y, 1 > dA$$ where $\vec{F}(x,y,z)$ is a vector field.

Stokes' Theorem: $\oint_C \vec{F} \bullet d\vec{r} = \iint_S (\vec{\nabla} \times \vec{F}) \bullet \vec{n}\, dS$ where the direction of travel for C, the orientation of S, and the direction of $\vec{n}$ are consistent.

Divergence Theorem: $\oiint_S \vec{F} \bullet \vec{n}\, dS = \iiint_D \vec{\nabla} \bullet \vec{F}\, dV$ where $\vec{n}$ is the unit outward normal vector on S .