$\boxed{4.5.1}$

$$\left\{\begin{bmatrix} s-6t \\ s+t \\ 5t \end{bmatrix}\right\} = s\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -6 \\ 1 \\ 5 \end{bmatrix} = s\vec{v}_1 + t\vec{v}_2$$

$$\{\vec{v}_1, \vec{v}_2\} = \boxed{\left\{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 1 \\ 5 \end{bmatrix}\right\}}$$

$\therefore$ dimension $= 2$ b/c there are 2 vectors.

$\boxed{4.5.3}$

$$\begin{bmatrix} p-4q \\ 4p+3r \\ -4q+9r \\ -4p+8r \end{bmatrix} = p\begin{bmatrix} 1 \\ 4 \\ 0 \\ -4 \end{bmatrix} + q\begin{bmatrix} -4 \\ 0 \\ -4 \\ 0 \end{bmatrix} + r\begin{bmatrix} 0 \\ 3 \\ 9 \\ 8 \end{bmatrix}$$

$$\Rightarrow \{\vec{v}_1, \vec{v}_2, \vec{v}_3\} = \left\{\begin{bmatrix} 1 \\ 4 \\ 0 \\ -4 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ -4 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 9 \\ 8 \end{bmatrix}\right\}$$

$\therefore$ dimension $= 3$ b/c there are 3 vectors $\exists$

$\boxed{4.5.8}$

$$\left\{\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : a - 5b + 3c = 0\right\}$$

$a - 5b + 3c = 0 \Rightarrow \begin{cases} a = 5b - 3c \\ b, c, d: \text{free} \end{cases}$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 5b - 3c \\ b \\ c \\ d \end{bmatrix} = b \begin{bmatrix} 5 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + d \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore$ Basis of the subspace $= \left\{ \begin{bmatrix} 5 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

$\therefore$ Dimension $= 3$ b/c there are 3 vectors.

$\boxed{4.5.10}$

$$\left\{ \underset{\vec{v}_1}{\begin{bmatrix} 1 \\ 5 \end{bmatrix}}, \underset{\vec{v}_2}{\begin{bmatrix} -2 \\ -10 \end{bmatrix}}, \underset{\vec{v}_3}{\begin{bmatrix} -4 \\ -20 \end{bmatrix}} \right\} \sim$$

$v_2 = -2\vec{v}_1$
$v_3 = -4\vec{v}_1$ $\Rightarrow$ dimension $= 1$ b/c there is 1 vector $\vec{v}_1$

$\boxed{4.5.13}$

$$\begin{bmatrix} 1 & 6 & -2 & 7 & 6 \\ 0 & 0 & -8 & -8 & -4 \\ 0 & 0 & 0 & -4 & -2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{4 \times 5}$$

$\therefore$ 4 pivot positions

$\Rightarrow \dim \text{Col}(A) = \boxed{4}$

$\therefore \dim \text{Nul}(A) = n - 4$

$= 5 - 4 = \boxed{1}$

$\boxed{4.5.14}$

$$A = \begin{bmatrix} \underline{1} & 3 & -6 & 2 & -3 & 6 & -1 \\ 0 & 0 & 0 & 0 & \underline{1} & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & \underline{1} & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & \underline{1} \end{bmatrix}_{4\times 7}$$

$\therefore$ 4 pivot positions $\Rightarrow$ dim Col $(A) = \boxed{4}$

$\Rightarrow$ dim Nul $(A) = 7-4 = \boxed{3}$

$\boxed{4.5.17}$

$$A = \begin{bmatrix} \underline{1} & 1 & 0 & 0 \\ 0 & \underline{1} & -6 & 0 \\ 0 & 0 & \underline{1} & -3 \\ 0 & 0 & 0 & \underline{1} \end{bmatrix}_{4\times 4}$$

$\therefore$ 4 pivot positions

$\Rightarrow$ Dim Col $(A) = 4$

$\Rightarrow$ dim Nul $(A) = 4-4=0$

$\boxed{4.5.23}$

$$\{1,\ 2t,\ -2+4t^2,\ -12t+8t^3\}$$

$$p(t) = 6 + 4t^2 + 8t^3$$

$$[\vec{p}_B] = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} \Rightarrow c_1(1) + c_2(2t) + c_3(-2+4t^2) + c_4(-12t+8t^3)$$

$$= (c_1 - 2c_3) + (2c_2 - 12c_4)t + 4c_3 t^2 + 8c_4 t^3$$

$$\Rightarrow \begin{cases} C_1 - 2C_3 = 6 \\ 2C_2 - 12C_4 = 0 \\ 4C_3 = 4 \\ 8C_4 = 8 \end{cases} \Longleftrightarrow \begin{cases} C_1 = 8 \\ C_2 = 6 \\ C_3 = 1 \\ C_4 = 1 \end{cases} \Rightarrow \boxed{\begin{bmatrix} 8 \\ 6 \\ 1 \\ 1 \end{bmatrix}}$$