Section 2.4 Homework / Discrete Structures II / Fall 2018

1. In each part below, a_n is a sequence defined by a recurrence relation with initial conditions. Find a_n for all n satisfying $1 \leq n \leq 4$.

   (a) $a_n = na_{n-1} + 3n$, $a_0 = 2$

   (b) $a_n = 3a_{n-1} + 2a_{n-2} - n + 2, \quad a_0 = -1, \quad a_1 = 2$

2. Consider the recurrence relation $a_n = 2a_{n-1} + 8a_{n-2}$. Which of the following sequences are solutions? If a sequence is a solution, prove it. If it's not a solution, find specific terms in the sequence that don't satisfy the recurrence.

   (a) $a_n = 4^n$

   (b) $a_n = 5 \cdot 4^n$

   (c) $a_n = 4n$

   (d) $a_n = 2^n$

   (e) $a_n = (-2)^n$

   (f) $a_n = 4^n + (-2)^n$.

3. Consider the recurrence relation $a_n = 7a_{n-1} - 10a_{n-2}$.

   (a) Prove that $a_n = 5^n$ and $a_n = 2^n$ are both solutions for the recurrence relation.

   (b) Let α, β be any real numbers. Prove that $a_n = \alpha \cdot 5^n + \beta \cdot 2^n$ is a solution.

4. Consider the recurrence relation $a_n = 3a_{n-1} + 2^n$. Prove that there's a solution of the form $a_n = 3^n + c \cdot 2^n$ for some real number c. Also, determine the value of c.

Answers:

1. (a) $a_1 = 5$, $a_2 = 16$, $a_3 = 57$, $a_4 = 240$

   (b) $a_1 = 2$, $a_2 = 4$, $a_3 = 15$, $a_4 = 51$

2. (a), (b), (e), and (f) are solutions; the other sequences are not.

4. $c = -2$