Section 7.3 Homework / Discrete Structures II / Fall 2018

Note: You can use a calculator on problems 2(j) and 3(fg).

- 1. Suppose that E and F are events in a sample space so that $p(E)=1/3, \quad p(F)=1/2, \text{ and } p(E\,|\,F)=2/5.$ Compute $p(E\cap F)$ and $p(F\,|\,E)$.
- 2. Suppose that 2% of a certain population has disease X. There's a test for the disease which satisfies the property that 99% of the people with the disease test positive, and 3% of the people who don't have the disease test positive. A person is selected at random from the population. Let D be the event that the random person has the disease, and let P be the event that the person tests positive for the disease. Compute the following:
 - (a) p(D)
 - (b) $p(\overline{D})$
 - (c) $p(P \mid D)$
 - (d) $p(\overline{P} \mid D)$
 - (e) $p(P \mid \overline{D})$
 - (f) $p(\overline{P} | \overline{D})$
 - (g) $p(P \cap D)$
 - (h) $p(P \cap \overline{D})$
 - (i) p(P)
 - (j) p(D | P)
 - (k) $p(P \cup D)$
- 3. Suppose a certain test for opium use has a 2% false positive rate and a 4% false negative rate. That is, 2% of people who do not use opium test positive for opium, and 4% of people who do use opium test negative. Furthermore, suppose that 1% of people actually use opium. A person is selected at random. Let D be the event that the random person uses opium, and let N be the event that the person tests negative. Compute the following:
 - (a) $p(\overline{D})$
 - (b) $p(N \mid \overline{D})$
 - (c) The probability that the person doesn't use opium and tests negative.
 - (d) The probability that the person uses opium and tests negative.
 - (e) p(N)
 - (f) The probability that the person uses opium given that they test negative.
 - (g) $p(\overline{D} | N)$

Answers

- 1. (a) 1/5
 - (b) 3/5
- 2. (a) 0.02
 - (b) 0.98
 - (c) 0.99
 - (d) 0.01
 - (e) 0.03
 - (f) 0.97
 - (g) 0.0198
 - (h) 0.0294
 - (i) 0.0492

(How is this related to the previous two parts?)

- (j) ≈ 0.4024
- (k) 0.0494

(Hint: Use inclusion-exclusion.)

- 3. (a) 0.99
 - (b) 0.96
 - (c) 0.9504
 - (d) 0.0004
 - (e) 0.9508
 - (f) ≈ 0.0004
 - (g) ≈ 0.9996