Q1

$$\vec{b}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \quad \vec{b}_2 = \begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix} \quad \vec{b}_3 = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$$

It is linearly Dependent when

$c_1\vec{b}_1 + c_2\vec{b}_2 + c_3\vec{b}_3 = \vec{0}$ has only trivial solution

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 2 & 0 & 3 & 0 \\ 0 & 6 & 4 & 0 \end{array}\right] \xrightarrow[=nR_2]{2R_1 - R_2} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & 6 & -1 & 0 \\ 0 & 6 & 4 & 0 \end{array}\right]$$

$$\xrightarrow[=nR_3]{-R_2 + R_3} \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 6 & -1 & 0 \\ 0 & 0 & 5 & 0 \end{bmatrix}$$

$R_3/5$

Since the matrix has 3 pivot positions $\Rightarrow$

$\therefore$ The columns of A are <u>NOT</u> linearly Dependent

Q2

a) Dimension of the Col (A): $\vec{a}_1$, $\vec{a}_3$ and $\vec{a}_6$ are pivot columns $\Rightarrow$ Since A as h3 pivot colums

$\Rightarrow \therefore \dim[\text{col}(A)] = 3$

Basis for A: $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ -2 \\ 1 \\ 0 \end{bmatrix}$

b) Dimension of the Null Space of A: $\vec{a}_2$, $\vec{a}_4$, $\vec{a}_5$ and $\vec{a}_7$ are free variables $\therefore$ Since A has 4 free variables

$\therefore$ $\dim[\text{Nul}(A)] = 4$

Basis for A: $\begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -6 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 3 \\ 0 \end{bmatrix}$