

Due Date: Dec. 12, 2018 (W), BEFORE the class starts

This assignment covers textbook Chapter 12&13.

1. BST (30 points)

Using the definitions on p. 1177 of our textbook for *depth* of a tree node and *height* of a tree, consider the set of keys $K = \{10, 4, 2, 8, 7\}$ and the different possible insertion orders for the keys in K . Based on the different possible insertion orders and their resulting Binary Search Trees, answer the following questions.

- a) What is the minimum height of a Binary Search Tree constructed from K ? Show an insertion order for the keys in K that generates a Binary Search Tree of minimum height. Draw the corresponding Binary Search Tree.
- b) Are there any other insertion orders (beyond what you found in (a) above) for the keys in K that produce a Binary Search Tree of minimum height? If so, provide one such sample insertion order and its accompanying Binary Search Tree.
- c) What is the maximum height of a Binary Search Tree constructed from K ? Show an insertion order for the keys in K that generates a Binary Search Tree of maximum height. Draw the corresponding Binary Search Tree.

2. Red-Black Tree (20 points)

For the set of keys given above in Problem 1, show the Binary Search Tree resulting from inserting the keys in the order given: $K = \langle 10, 4, 2, 8, 7 \rangle$. Is it possible to color the resulting Binary Search Tree so that it becomes a red-black tree? Either prove that it is not possible or show a legal coloring that makes it a red-black tree.

3. Red-Black Tree (40 points)

Show the sequence of red-black trees that result after successively inserting the keys into an initially empty red-black tree T in the order given: $K = \langle 41, 38, 31, 12, 19, 28 \rangle$. You should use the algorithm(s) provided in the textbook/notes. Show at least one tree resulting from each insertion (you may draw trees in the middle to get partial credits). Be sure to state which case from the textbook applies when applicable. Assume that the root is always colored black. Please mark black node and red node clearly.

4. Red-Black Tree (10 points)

In a red-black tree, the length of the path from the root to the farthest leaf is defined as lf , and the length of the path from the root to the nearest leaf is defined as ln . What is the value of lf in terms of ln ? Briefly explain your answer.

Algorithms -- COMP.4040 Honor Statement
(Courtesy of Prof. Tom Costello and Karen Daniels with modifications)

Must be attached to each submission

Academic achievement is ordinarily evaluated on the basis of work that a student produces independently. Infringement of this Code of Honor entails penalties ranging from reprimand to suspension, dismissal or expulsion from the University.

Your name on any exercise is regarded as assurance and certification that what you are submitting for that exercise is the result of your own thoughts and study. Where collaboration is authorized, you should state very clearly which parts of any assignment were performed with collaboration and name your collaborators.

In writing examinations and quizzes, you are expected and required to respond entirely on the basis of your own memory and capacity, without any assistance whatsoever except such as what is specifically authorized by the instructor.

I certify that the work submitted with this assignment is mine and was generated in a manner consistent with this document, the course academic policy on the course website on Blackboard, and the UMass Lowell academic code.

Date: _____

Name (please print): _____

Signature: _____