Due Date: 09-29-2016(Th)   BEFORE the class starts

1. **Function Order of Growth**: (20 points)
   List the 4 functions below in nondecreasing asymptotic order of growth.

   $$(\lg n)^2 \qquad\qquad n^{-2} \qquad\qquad \lg(2^{\lg (n^2)}) \qquad\qquad n^2$$

   (1)   (2)   (3)   (4)
      smallest   largest

   Justify your answer mathematically by showing values of c and n_0 for each pair of functions that are adjacent in your ordering.

2. **O, Ω, Θ Notation Practice**: (30 points, 6 points for each)
   Given (for large n):
   (1) $f_1(n) \in \Omega((\lg n)^2)$ (2) $f_2(n) \in O(n^2 - \frac{1}{n})$
   (3) $f_3(n) \in \Omega\left(\frac{1}{n^2}\right)$ (4) $f_4(n) \in \Theta\left(\lg(2^{\lg (n^2)})\right)$

   Draw the arrow diagram associated with the 4 statements above and use the diagram to help answer the following questions (a) ~(e)

   (a) ~ (e) For each statement below, state if it is TRUE (if the statement must always be true, given the assumptions) or FALSE otherwise. In the TRUE case, provide a proof. In the FALSE case, give a counter-example.

   (a) $f_1(n) \in \Omega(f_3(n))$
   (b) $f_4(n) \in O(f_1(n))$
   (c) $f_2(n) \in \Omega(f_3(n))$
   (d) $f_1(n) \in O(f_2(n))$
   (e) $f_4(n) \in \Theta(\lg^3 n)$

3. **O, Ω, Θ Notation Practice**: (15 points)
   Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Using the basic definition of Θ-notation to prove that $\max(f(n), g(n)) = \Theta(f(n) + g(n))$.
   Hint: $\max(f(n), g(n)) = \begin{cases} f(n), if\ f(n) \geq g(n) \\ g(n), if\ f(n) < g(n) \end{cases}$

4. **Analysis**: (10 points)
   Your client is developing two new algorithms. $f_1(n)$ and $f_2(n)$ *are* the worst-case running time for these two algorithms: $f_1(n) = nlgn$, and $f_2(n) = 256n$. As a consultant, which algorithm will you recommend to your client? Justify your answer.   (Hint: Please consider the asymptotical growth of the functions and also consider the reality.)

5. **Pseudocode Analysis** (25 points)
   For the pseudocode below for procedure Mystery(n), derive tight upper and lower bounds on its asymptotic _worst-case_ running time $f(n)$. That is, for the set of inputs including those that force Mystery to work its hardest, find $g(n)$ such that $f(n) \in \Theta(g(n))$. Assume that the input n is a positive integer.   Justify your answer.

      Mystery (n)
   1. if n is an even number

   2.   for $i = 1$ to n

   3.   for $j = n$ downto $n/2$

   4.   print "even number"

   5. else

   6.   for $k = 1$ to $n/4$

   7.   for $m = 1$ to n

   8.   print "odd number"

Algorithms -- COMP.4040 Honor Statement
(Courtesy of Prof. Tom Costello and Karen Daniels with modifications)

Must be attached to each submission

Academic achievement is ordinarily evaluated on the basis of work that a student produces independently. Infringement of this Code of Honor entails penalties ranging from reprimand to suspension, dismissal or expulsion from the University.

Your name on any exercise is regarded as assurance and certification that what you are submitting for that exercise is the result of your own thoughts and study. Where collaboration is authorized, you should state very clearly which parts of any assignment were performed with collaboration and name your collaborators.

In writing examinations and quizzes, you are expected and required to respond entirely on the basis of your own memory and capacity, without any assistance whatsoever except such as what is specifically authorized by the instructor.

I certify that the work submitted with this assignment is mine and was generated in a manner consistent with this document, the course academic policy on the course website on Blackboard, and the UMass Lowell academic code.

Date: _____

Name (please print): _____

Signature: _____