Problem 1 [15]. Widgets are manufactured in three factories: A B and C. The proportion of defective widgets from each factory are as follows:

Factory A: .01
Factory B: .04
Factory C: .02

Factories A and B produce 30% of the widgets apiece, and the remaining 40% come from Factory C.

What is the likelihood that a given widget is defective?

Solve graphically *and* with Bayes'. Put your numeric answer in the box and show your work below.

$\boxed{2.3\%}$ or 0.023

Total Probabilities required here
NOT Bayes.

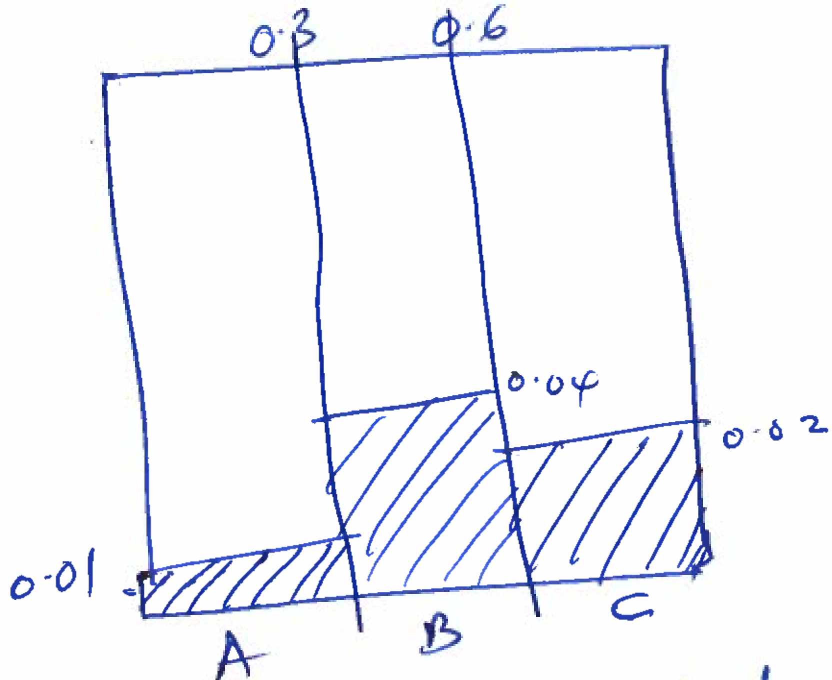
$$\sum_{\text{Factories}} P(\text{defective} \mid \text{factory}) * P(\text{factory})$$

$$= P(\text{defective} \mid \text{factory A}) * P(\text{factory A})$$
$$+ P(\text{defective} \mid \text{factor B}) * P(\text{factory B})$$
$$+ P(\text{defective} \mid \text{factory C}) * P(\text{factory C})$$

$$= (0.01 * 0.3) + (0.04 * 0.3) + (0.02 * 0.4)$$

$$= 0.023 \text{ or } 2.3\%$$

Question adapted from study.com.



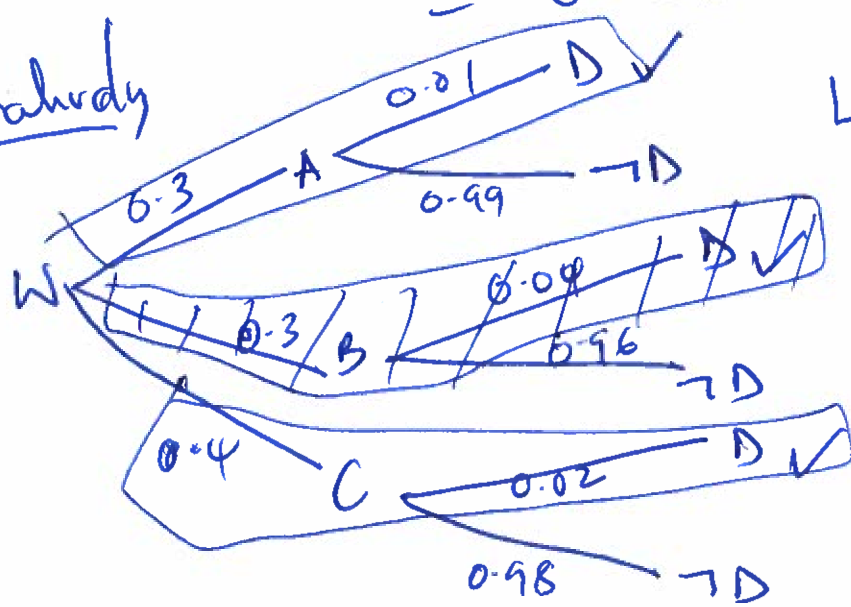
Shaded = Defective widgets

We needed to check the likelihood for a widget to be defective. This means adding up ratios of the shaded area.

$$= \overset{A}{(0.01 * 0.3)} + \overset{B}{(0.3 * 0.04)} + \overset{C}{(0.4 * 0.02)}$$

$$= 0.023$$

Alternatively



Look at the world where a widget is defective

Problem 2 [20]. Suppose there are two full bowls of cookies. Bowl #1 has 10 chocolate chip and 30 plain cookies, while bowl #2 has 20 of each. Our friend Stacy picks a bowl at random, and then picks a cookie at random. We may assume there is no reason to believe Stacy treats one bowl differently from another, likewise for the cookies. The cookie turns out to be a plain one. How probable is it that Stacy picked it out of Bowl #1?

Solve graphically *and* with Bayes'. Put your numeric answer in the box and show your work below.

$\boxed{60\%}$ or 0.6

$$P(B_1 | Plain) = \frac{P(Plain | B_1) * P(B_1)}{P(Plain)}$$

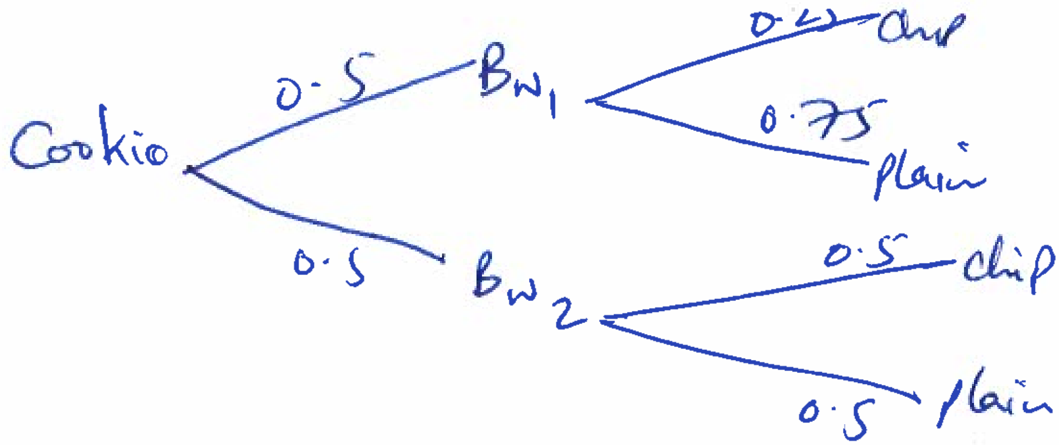
Note $P(Plain | B_1) \Rightarrow$ means area or means in the KB where the selection is a plain cookie & is in Bowl 1 $= 0.75$

$$\therefore = \frac{0.75 * 0.5}{P(Plain)} = \frac{0.75 * 0.5}{(0.75 * 0.5) + (0.5 * 0.5)}$$

Entire Interpretation where a cookie is plain

$$= \frac{0.375}{0.375 + 0.25}$$

$$= \underline{0.6}$$

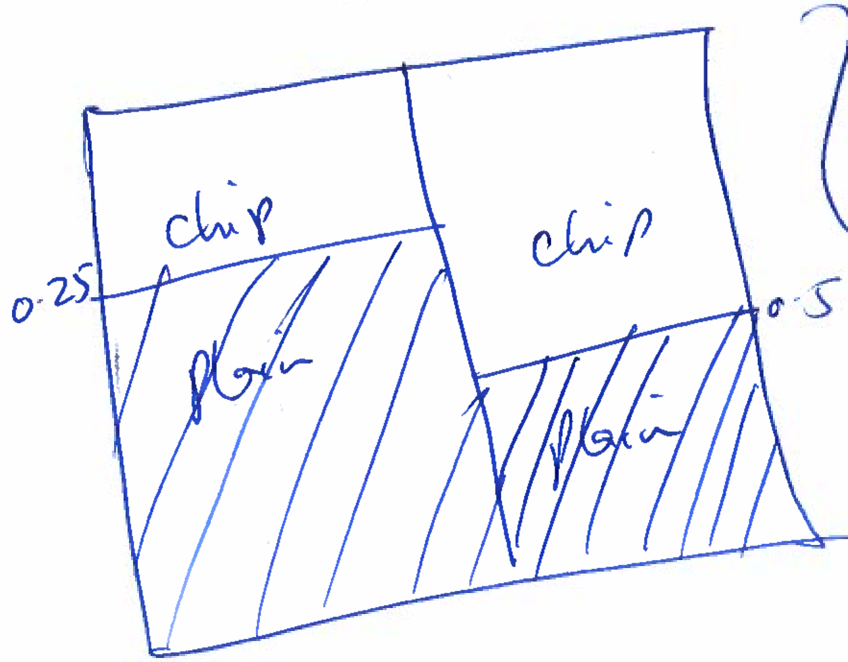


check the world where we have a plain chip.

$$P(B_1 \mid Plain) = \frac{P(Plain \mid B_1) * P(B_1)}{P(Plain)}$$

$$\approx \frac{Bw_1 \wedge \text{plain}}{\text{KB where Cookie is plain}} = \frac{0.5 * 0.75}{(0.5 * 0.75) + (0.5 * 0.5)}$$

$$= \frac{0.375}{0.625} = 0.6$$



Our Investigation should be in Shaded area.

$$\frac{\text{Area of } B_1 \text{ \& plain}}{\text{Total area of Plain}}$$

Problem 3[15]. The blue M&M was introduced in 1995. Before then, the color mix in a bag of plain M&Ms was (30% Brown, 20% Yellow, 20% Red, 10% Green, 10% Orange, 10% Tan). Afterward it was (24% Blue, 20% Green, 16% Orange, 14% Yellow, 13% Red, 13% Brown).

A friend has two bags of M&Ms, and tells me that one is from 1994 and one from 1996. My friend won't tell me which is which, but gives me one M&M from each bag. One is yellow and one is green. What is the probability that the yellow M&M came from the 1994 bag?

Solve graphically *and* with Bayes'. Put your numeric answer in the box and show your work below.

$\boxed{0.7407}$

We shall consider only colour Green & Yellow

→ We are Interested in all world (KB) where we have Green & Yellow

Hypothesis = Yellow from 94 (i.e Y94)

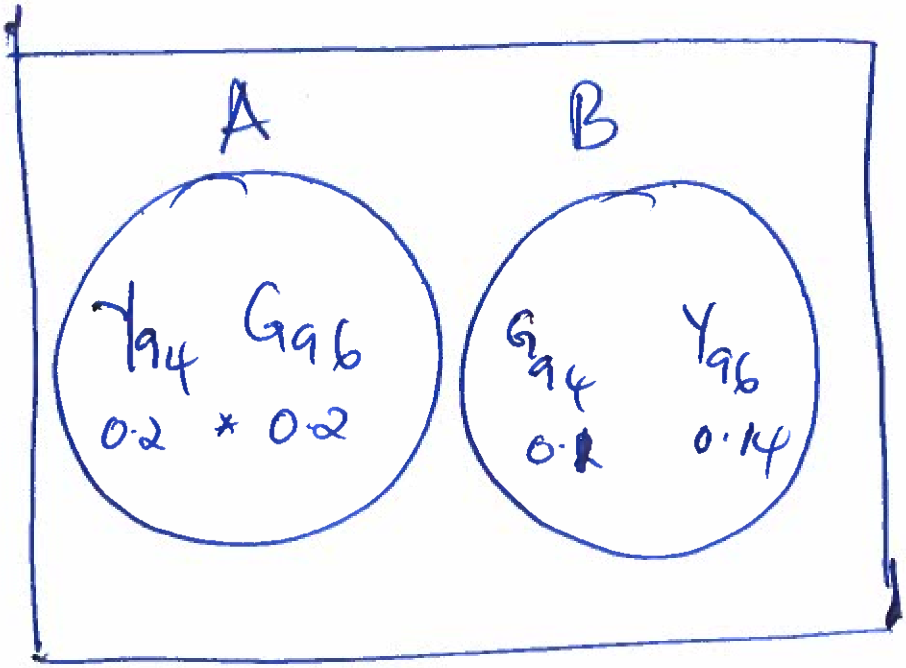
$$P(Y_{94} | Y \& G) = \frac{P(Y \& G | Y_{94}) * P(Y_{94})}{P(Y \& G)}$$ ← entire KB

NB : $P(Y \& G | Y_{94})$ i.e The ~~[illegible]~~ Interpretation in the KB where $Y \& G$ is True Given Y_{94}. This is only when $G96 = 0.2$

$$\therefore P(Y_{94} | Y \& G) = \frac{0.2 \times 0.2}{(0.2 * 0.2) + (0.14 * 0.1)} = \frac{0.04}{0.054}$$

Thanks Allen Downey for these two, who also points out that these are "urn problems."

$\simeq 0.7407$



Only Two events are possible in the KB where we have Y & G

Y & G

Event A: Y_{94} & $G_{96} = 0.04$

Event B: G_{94} & $Y_{96} = 0.1 * 0.14 = 0.014$

Y & G = A + B

$$P(A) = \frac{0.04}{0.04 + 0.014} = \frac{0.04}{0.054} \simeq 0.7407$$

PS a student might also use the probability of selecting an event. This wont change the answer, i.e

$$\frac{\frac{1}{2} * 0.04}{(\frac{1}{2} * 0.04) + \frac{1}{2}(0.014)} = \frac{0.02}{0.027} \simeq 0.740$$