# Section 7.2 Homework / Discrete Structures II / Fall 2018

1. An unfair die has the following properties:

   - The probability of rolling a 2 is twice as likely as rolling a 1.
   - The probability of rolling a 3 is three times as likely as rolling a 2.
   - Rolling the numbers 1, 4, 5, 6 are all equally likely.

   *Questions:*

   (a) Find the probability of each outcome (i.e. find $p(k)$ for $k = 1, 2, \ldots, 6$).

   (b) The die is rolled. What's the probability of getting an even number?

   (c) Suppose a pair of the unfair dice is rolled.

   i. What's the probability that 4 occurs on the first die and 5 occurs on the second die?

   ii. What's the probability that 4 occurs on the first die or 5 occurs on the second die?

   iii. What's the probability that the sum is equal to 4?

2. Suppose that a number is picked from the set $S = \{0, 1, 2, 3, \ldots\}$, and that the probability that the number $n$ is picked is $p(n) = 0.3(0.7)^n$.

   (a) Find the probability that

   i. a number less than 3 is picked.

   ii. a number greater than 2 is picked.

   (b) Find the probability that an even number is picked given that it's known that a number less than 3 is picked.

3. A pair of dice is rolled. What is the probability that the sum of the faces is greater than 7, given that

   (a) the first roll was a 4?

   (b) the first roll was greater than 4?

4. Three cards are drawn from a deck of cards, one at a time, without replacement. Use the "multiplication rule" to compute the probability that

   (a) the cards are all hearts?

   (b) the first card is a jack, the second card is a queen, and the third card is a king?

   (c) the cards are a jack, queen, king, in some order?

5. A coin is flipped seven times.

   (a) What's the probability that heads occurs exactly three times?

   (b) What's the probability that heads occurs exactly three times, given that the first flip was heads?

6. A class contains 12 students who are taking calculus, 17 students who are taking physics, and 5 students who are taking both. Every student in the class is taking calculus or physics. Suppose a student is selected at random from the class.

   (a) What's the probability that the student is taking calculus but not physics?

   (b) What's the probability that the student is taking calculus given that it's known they're taking physics?

7. Let $E, F$ be events so that $p(E \cap F) = 0.2$, $p(E \cup F) = 0.9$, and $p(E|F) = 0.5$. Compute the following probabilities:

   (a) $p(F)$
   (*Hint:* Use the definition of $p(E|F)$.)

   (b) $p(E)$
   (*Hint:* Inclusion-exclusion.)

   (c) $p(F|E)$

   (d) $p(E \cap \overline{F})$
   (*Hint:* How are the sets $E$, $E \cap F$, and $E \cap \overline{F}$ related? Sketch a Venn diagram if you're not sure.)

8. It's known that 10% of the widgets produced by a certain factory are defective. Suppose that five widgets are selected at random from the factory. What's the probability that

   (a) all the widgets are defective?

   (b) exactly two are defective?

   (c) at least three are defective?

   NOTE: Assume that picking one widget is a Bernoulli trial with 10% of being defective, and 90% of not being defective. (This is a reasonable assumption if the total number of widgets produced by the factory is large, and the number selected is small.)

9. A true-false quiz has ten questions. (Note: You can use a calculator on this problem, but you should be able to set up an expression for what you need to compute.)

   (a) Assume that the student randomly guesses on the entire quiz. What's the probability that the student gets at least 80% on the quiz?

   (b) Suppose that the student believes he has a 70% probability of getting each question right. Supposing this assumption is correct, what's the probability that the student gets at least 80% on the quiz?

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## Answers

1. (a) $p(1) = p(4) = p(5) = p(6) = 1/12$,
   $p(2) = 1/6$,
   $p(3) = 1/2$.
   (b) $1/3$
   (c) i. $1/144$
   ii. $23/144$
   iii. $1/9$

2. (a) i. 0.657
   ii. 0.343
   (*Hint:* Use the complement of the event.)
   (b) $0.447/0.657 \approx 0.6804$

3. (a) $1/2$
   (b) $3/4$

4. (a) $\frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \approx 0.012941$
   (b) $\frac{4}{52} \cdot \frac{4}{51} \cdot \frac{4}{50} \approx 0.00048265$
   (c) $6 \cdot \frac{4}{52} \cdot \frac{4}{51} \cdot \frac{4}{50} \approx 0.0028959$

5. (a) $35/128$
   (b) $15/64$

6. (a) $7/24$
   (b) $5/17$

7. (a) 0.4
   (b) 0.7
   (c) $2/7$
   (d) 0.5

8. (a) 0.00001
   (b) 0.0729
   (c) 0.00856

9. (a) $C(10,8) \cdot (0.5)^{10} + C(10,9) \cdot (0.5)^{10} + C(10,10) \cdot (0.5)^{10} \approx 0.0547$
   (b) $C(10,8) \cdot (0.7)^8(0.3)^2 + C(10,9) \cdot (0.7)^9(0.3) + C(10,10) \cdot (0.7)^{10} \approx 0.3828$

   *Hint:* Use the binomial distribution.