Exam 1 Review Sheet / Discrete Structures II / Fall 2018

Exam 1 is on Tuesday, October 9 and will cover all sections from chapter 6 (excluding 6.4), section 8.5, and the "Counting Using the Principle of Inclusion-Exclusion" notes. No notes, calculator, book, etc.

Extra office hours: Monday, October 8 from noon to 3 pm in 428L Olney, or by appointment.

Review Problems

1. A group of eight people consists of four husband and wife couples.
   (a) In how many different ways can the eight people be seated in a row if
      i. no two men are seated next to each other, and no two women are seated next to each other?
      ii. each husband sits next to his wife?
   (b) In how many different ways can a group of four people be selected
      i. if there are no restrictions?
      ii. if one member of a couple is selected, the person's partner must also be selected?
      iii. if the group contains either the husband or the wife from a couple, but not both?
      iv. if exactly one couple is included in the group?
2. A password for a certain website consists of 5 digits chosen from $\{0, 1, \ldots, 9\}$. How many passwords
   (a) are there in total?
   (b) contain the digit 1?
   (c) consist of five different digits?
   (d) consist of five different digits and contain 145 as a consecutive substring?
   (e) have digits that are alternating? (For example, 34343 or 51515.)
   (f) contain a string of four or more consecutive 5's?
   (g) consist of three 2's and two 4's? (For example, 34433.)
   (h) contain exactly three 2's?
3. There are four different types of cookies available from a bakery (chocolate chip, oatmeal, peanut butter, and lemon drop). Assume that cookies of the same type are considered to be identical.
   (a) How many ways are there to pick select 6 cookies if ...
      i. there's no restrictions?
      ii. at least two chocolate chip cookies and one lemon drop cookie are selected?
      iii. at least two chocolate chip cookies or at least one lemon drop cookie are selected?
      iv. the selection doesn't include at least one of the types of cookies?
      v. at most three chocolate chip cookies are picked?
   (b) Bob buys three chocolate chip cookies, two oatmeal cookies, and five peanut butter cookies. He eats exactly one cookie per day for ten days. In how many different orders can he consume the cookies?
   (c) How many ways can 12 chocolate chip cookies be distributed among five people if every person gets at least one cookie?
   (d) How many ways can 5 oatmeal cookies and 6 peanut butter cookies be distributed among four people?
   (e) Alice has one of each of the four types of cookies. In how many ways can she distribute the cookies among four friends if ...
      i. each person gets one cookie?
      ii. there's no restrictions? (In particular, this means that someone could get no cookies or more than one cookie.)

4. There are 6 seniors and 5 juniors in a class.

   (a) Every student in the class will receive an A or B. How many ways are there to assign grades if exactly three juniors and two seniors will get A's?

   (b) The students are competing for four scholarships. How many ways are there to pick the scholarship winners if ...

      i. the scholarships are identical $1000 prizes?

      ii. the scholarships are identical, and at least one junior receives a scholarship?

      iii. the scholarships are all different amounts?

5. Let $S = \{1, 2, 3, 4, 5, 6, 7\}$.

   (a) Consider permutations of S. Using lexicographic order, what's the smallest permutation that begins with 624? What's the largest permutation that begins with 624?

   (b) Consider permutations of S. What are the next three permutations after 1457632 in lexicographic order?

   (c) Consider 5-combinations of S under lexicographic order. What are the smallest 5-combination and the largest 5-combination whose smallest elements are 1 and 3?

   (d) Consider 5-combinations of S. What are the next two 5-combinations after $\{1, 2, 4, 5, 7\}$ in lexicographic order?

6. State the generalized pigeonhole principle.

7. For each statement below, determine if it's true or false. If it's true, use the generalized pigeonhole principle to explain why. If it's false, explain why.

   (a) If 25 marbles are selected at random from an urn which contains red, blue, green, and yellow marbles, then there must be at least six red marbles among the selected marbles.

   (b) If there are seven different types of bagels available at a bakery, and 20 bagels are selected at random, then there must be at least three bagels of the same type among the selected bagels.

   (c) Suppose a class of 25 students contains only freshmen, sophomores, and juniors. Then there must be at least 8 freshmen, at least 8 sophomores, and at least 8 juniors.

8. Complete the following statement of the Principle of Inclusion-Exclusion for four sets A, B, C, D:
   $|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |B \cap C| - |C \cap D| + |A \cap B \cap C| + |B \cap C \cap D| +$ _____
   (There's some terms missing from the right hand side – put the rest of the terms in the blank.)

9. Review the assigned homework problems from Weeks 1-4 on the course website, particularly Section 8.5.

Answers:

1. (a) i. 1152
   ii. 384
   (b) i. 70
   ii. 6
   iii. 16
   iv. 96

2. (a) 100,000
   (b) 40,951
   (c) 30,240
   (d) 126 252
   (e) 90
   (f) 19
   (g) 10
   (h) 810

3. (a) i. 84
   ii. 20
   iii. 71
   iv. 74
   v. 74
   (b) 2520
   (c) 330
   (d) 4704
   (e) i. 24
   ii. 256

4. (a) 150
   (b) i. 330
   ii. 315
   iii. 7920

5. (a) 6241357; 6247531
   (b) 1462357, 1462375, 1462537
   (c) $\{1, 3, 4, 5, 6\}$ and $\{1, 3, 5, 6, 7\}$
   (d) $\{1, 2, 4, 6, 7\}$, $\{1, 2, 5, 6, 7\}$, $\{1, 3, 4, 5, 6\}$

6. See book!

7. (a) False. (Find a counterexample.)
   (b) True.
   (c) False. (Find a counterexample.)

8. $-|A \cap C| - |A \cap D| - |B \cap D| + |A \cap B \cap D| + |A \cap C \cap D| - |A \cap B \cap C \cap D|$