

10-1

| | UnSorted single linked | Sorted single linked | Unsorted double linked | Sorted double linked |
|-------------------|------------------------------|----------------------------|------------------------------|----------------------------|
| Search(L, k) | $\theta(L)$ | $\theta(L)$ | $\theta(L)$ | $\theta(L)$ |
| Insert(L, x) | $\theta(1)$ | $\theta(L)$ | $\theta(1)$ | $\theta(1)$ |
| Delete(L, x) | $\theta(L)$ | $\theta(L)$ | $\theta(L)$ | $\theta(L)$ |
| Successor(L, x) | $\theta(L)$ | $\theta(L)$ | $\theta(L)$ | $\theta(L)$ |
| Predecessor(L, x) | $\theta(L)$ | $\theta(1)$ | $\theta(L)$ | $\theta(1)$ |
| Minimum(L) | $\theta(L)$ | $\theta(1)$ | $\theta(1)$ | $\theta(1)$ |
| Maximum(L) | $\theta(L)$ | $\theta(L)$ | $\theta(1)$ | $\theta(1)$ |

* Assuming the linked list accepts duplicated elements.

12.2-2

Tree-Minimum(x)

if left[x] = NIL

return x

else return Tree-Minimum(left[x])

Tree-Maximum(x)

if right[x] = NIL

return x

else return Tree-Maximum(right[x])

12.2-3

Tree-Predecessor(x)

if left[x] \neq NIL

then return

Tree-Maximum(left[x])

y \leftarrow p[x]

while y \neq NIL and x = left[y]

do x \leftarrow y

y \leftarrow p[y]

return y

2. (a) Open addressing with linear probing:

| K | $h(k) = k^2 \bmod m$ | $h(k, i) = (h'(k) + i) \bmod m$ | Final Position |
|---|----------------------|---|----------------|
| 3 | $3^2 \bmod 5 = 4$ | $(4 + 0) \bmod 5 = 4$ | 4 |
| 4 | $4^2 \bmod 5 = 1$ | $(1 + 0) \bmod 5 = 1$ | 1 |
| 2 | $2^2 \bmod 5 = 4$ | $(4 + 0) \bmod 5 = 4, (4 + 1) \bmod 5 = 0$ | 0 |
| 5 | $5^2 \bmod 5 = 0$ | $(0 + 0) \bmod 5 = 0, (0 + 1) \bmod 5 = 1, (0 + 2) \bmod 5 = 2$ | 2 |
| 1 | $1^2 \bmod 5 = 1$ | $(1 + 0) \bmod 5 = 1, (1 + 1) \bmod 5 = 2, (1 + 2) \bmod 5 = 3$ | 3 |

(b) Open addressing with quadratic probing, where $c_1 = 1$ and $c_2 = 2$:

| K | $h'(k) = k^2 \bmod m$ | $h(k, i) = (h'(k) + i * c_1 + i^2 * c_2) \bmod m$ | Final Position |
|---|-----------------------|---|----------------|
| 3 | $3^2 \bmod 5 = 4$ | $(4 + 0 * 1 + 0^2 * 2) \bmod 5 = 4$ | 4 |
| 4 | $4^2 \bmod 5 = 1$ | $(1 + 0 * 1 + 0^2 * 2) \bmod 5 = 1$ | 1 |
| 2 | $2^2 \bmod 5 = 4$ | $(4 + 0 * 1 + 0^2 * 2) \bmod 5 = 4,$ $(4 + 1 * 1 + 1^2 * 2) \bmod 5 = 2$ | 2 |
| 5 | $5^2 \bmod 5 = 0$ | $(0 + 0 * 1 + 0^2 * 2) \bmod 5 = 0$ | 0 |
| 1 | $1^2 \bmod 5 = 1$ | $(1 + 0 * 1 + 0^2 * 2) \bmod 5 = 1, (1 + 1 * 1 + 1^2 * 2) \bmod 5 = 4,$ $(1 + 2 * 1 + 4 * 2) \bmod 5 = 1, (1 + 3 * 1 + 9 * 2) \bmod 5 = 2,$ $(1 + 4 * 1 + 16 * 2) \bmod 5 = 2.$ No place for 1 in the hash table. | |

(c)

| K | $h(k) = k^2 \bmod m, m = 5$ | Final Position |
|---|-----------------------------|----------------|
| 3 | $3^2 \bmod 5 = 4$ | 4 |
| 4 | $4^2 \bmod 5 = 1$ | 1 |
| 2 | $2^2 \bmod 5 = 4$ | 4 |
| 5 | $5^2 \bmod 5 = 0$ | 0 |
| 1 | $1^2 \bmod 5 = 1$ | 1 |

The load factor is $\alpha = n/m = 5/5 = 1$.

(d)

| K | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|-----------------------|---|---|---|---|---|---|---|---|---|---|----|
| $h(k) = k^2 \bmod 11$ | 0 | 1 | 4 | 9 | 5 | 3 | 3 | 5 | 9 | 4 | 1 |

For $k \geq 11$, we can represent $k = 11 * a + b$, where a is the quotient of k divided by 11 and b is the remainder, $0 \leq b \leq 10$. Then we have:

$$h(k) = k^2 \bmod 11 = (11a + b)^2 \bmod 11 = (11^2 a^2 + b^2 + 2 * 11 * ab) \bmod 11 \\ = b^2 \bmod 11, 0 \leq b \leq 10$$

This goes back to the above table. Thus, for any $k \geq 0$, all the possible values of $h(k) = k^2 \bmod 11$ are: 0, 1, 3, 4, 5, and 9

Solutions for HW#9.

1. a) minimum height = 2.

$K = \langle 35, 14, 76, 2, 27, 43, 89 \rangle$

OR

$K = \langle 35, 14, 2, 27, 76, 43, 89 \rangle$

There are also other insertion orders for the keys in K that can generate the same Binary Search Tree of minimum height above.

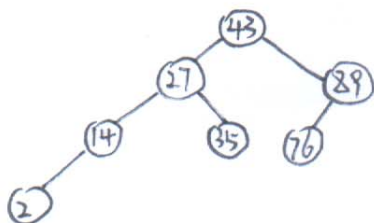
b) maximum height = 6.

$K = \langle 2, 14, 27, 35, 43, 76, 89 \rangle$ OR $K = \langle 89, 76, 43, 35, 27, 14, 2 \rangle$



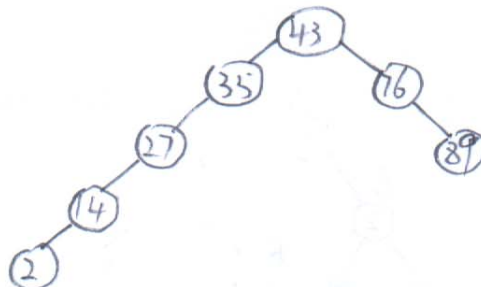
c) height = 3

$K = \langle 43, 27, 14, 2, 35, 89, 76 \rangle$



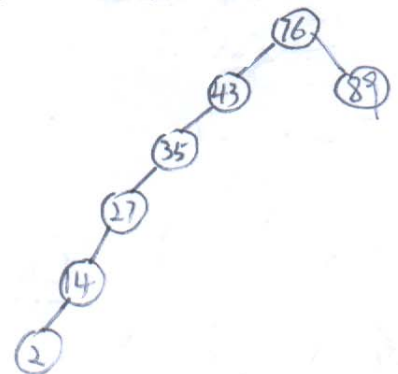
height = 4

$K = \langle 43, 35, 27, 14, 2, 76, 89 \rangle$



height = 5

$K = \langle 76, 43, 35, 27, 14, 2, 89 \rangle$



Note: There are also other solutions.