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Homework 3

10/23/2018

1) a) Show $Y \sim N(\beta x, \sigma^2)$

$$P(y|x;\beta) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left(-\frac{1}{2}\frac{(y-\beta x)^2}{\sigma^2}\right)$$

$E[y] = \beta x$

$Var(y) = \sigma^2$

b) $$L(\beta) = \prod_{i=1}^{n} P(y^{(i)}|x^{(i)};\beta)$$

$$\log L(\beta) = \sum_{i=1}^{n} \log(P(y^{(i)}|x^{(i)};\beta))$$

$$= \sum_{i=1}^{n}\left[\log\left(\frac{1}{\sigma\sqrt{2\pi}}\right) + \log\left(\exp\left(-\frac{1}{2}\frac{(y^{(i)}-\beta x^{(i)})^2}{\sigma^2}\right)\right)\right]$$

$$\underset{\beta}{\operatorname{argmax}} \log L(\beta) = \underset{\beta}{\operatorname{argmax}} \sum_{i=1}^{n}\left[\cancel{\log\left(\frac{1}{\sigma\sqrt{2\pi}}\right)} - \frac{1}{\cancel{2\sigma^2}} \cdot (y^{(i)}-\beta x^{(i)})^2\right]$$

$$-1 \cdot \underset{\beta}{\operatorname{argmax}} \log L(\beta) = \underset{\beta}{\operatorname{argmax}} -\frac{1}{2}\sum_{i=1}^{n}(y^{(i)}-\beta x^{(i)})^2 \cdot -1$$

$$\boxed{= \underset{\beta}{\operatorname{argmin}} \frac{1}{2}\sum_{i=1}^{n}(y^{(i)}-\beta x^{(i)})^2}$$

$$\frac{\partial}{\partial\beta}\frac{1}{2}\sum_{i=1}^{n}(y^{(i)}-\beta x^{(i)})^2 \Longleftrightarrow \frac{\partial}{\partial\beta}\frac{1}{2}(y-\beta x)^2$$

$$\boxed{=\sum_{i=1}^{n}\frac{y^{(i)}x^{(i)}}{(x^{(i)})^2}}$$

$$=\cancel{\tfrac{1}{2}}\cdot 2(y-\beta x)\cdot -x$$

$$=(y-\beta x)-x$$

$$=-yx+\beta x^2$$

$$-yx+\beta x^2=0$$

$$+yx \qquad +yx$$

$$\frac{\beta \cancel{x^2}}{\cancel{x^2}}=\frac{yx}{x^2}$$

$$\beta=\frac{yx}{x^2}$$

This is scalar, but we can use it in the summation as shown in the left

2) a) $\frac{\partial}{\partial\beta}\frac{1}{2}\left(\sum_{i=1}^{n}(y^{(i)}-\beta x^{(i)})^2+\lambda\beta^2\right)$

$$\frac{\partial}{\partial\beta}\frac{1}{2}\sum_{i=1}^{n}(y^{(i)}-\beta x^{(i)})^2+\frac{\partial}{\partial\beta}\frac{1}{2}\sum_{i=1}^{n}\lambda\beta^2$$

$$\sum_{i=1}^{n}\left[\underbrace{-y^{(i)}x^{(i)}+\beta(x^{(i)})^2+\lambda\beta}\right]$$

$$-y^{(i)}x^{(i)}+\beta(x^{(i)})^2+\lambda\beta=0$$

$$\beta(x^{(i)})^2+\lambda\beta=y^{(i)}x^{(i)}$$

$$\beta((x^{(i)})^2+\lambda)=y^{(i)}x^{(i)}$$

$$\beta=\frac{y^{(i)}x^{(i)}}{(x^{(i)})^2+\lambda}\Rightarrow\boxed{\beta=\sum_{i=1}^{n}\frac{y^{(i)}x^{(i)}}{(x^{(i)})^2+\lambda}}$$

b)

After solving for $\hat{\beta}$, we can see that $\lambda$ is placed in the denomitor. We that idea, we can conclude that as $\lambda \to \infty$, $\hat{\beta}$ will equal 0.

3) a)

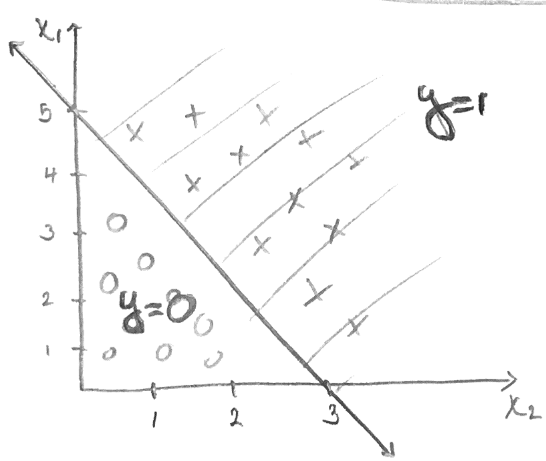
$$h(x) = 3x_1 + 5x_2 - 15$$

$$3x_1 + 5x_2 - 15 \geq 0$$

$$3x_1 + 5x_2 \geq 15$$

$$\cancel{\tfrac{3x_1}{3} + 5(0)} \geq \tfrac{15}{3} \qquad \cancel{3(0)} + \tfrac{5x_2}{5} \geq \tfrac{15}{5}$$

$$\boxed{x_1 \geq 5 \qquad x_2 \geq 3}$$




b) $P(y=1 \mid x_1, x_2) = \sigma(h(x)) = \dfrac{1}{1 - \exp(-h(x))}$

$$= \boxed{\frac{1}{1 - \exp(-3x_1 - 5x_2 + 15)}}$$

a) Higher

b) The same

c) Model 2