Section 6.3 HW / Discrete Structures II / Fall 2018

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1. Let $S = \{1, 2, 3, 4, 5\}$.
   (a) List all 3-permutations of S which begin with 4. 12 sets: 412, 413, 415, 421, 423, 425, 431, 432, 435, 451, 452, 453
   (b) List all 3-combinations of S. 10 sets: 123, 124, 125, 134, 135, 145, 234, 235, 245, 345

2. There are 11 members in a club which consists of 6 men and 5 women. Total 11 people
   (a) How many different ways are there to select a president, vice-president, and secretary for the club? =P(11,3)=11!/8!=11*10*9=990
   (b) How many different ways are there to select a group consisting of three club members? =C(11,3)=11!/(8!3!)=165
   (c) How many different ways are there to select a group consisting of four club members with two men and two women? picking 2/6 men * picking 2/5 women = C(6,2)*C(5,2)=(6!/(2!4!))*(5!/(2!3!)) = 15*10 = 150

3. How many bit strings of length 8 contain ...
   (a) ... exactly three 1s? = C(8,3) = P(8,3)/3! = 8*7*6/3*2*1 = 56
   (b) ... at most three 1s? None 1: C(8,0) + one 1: C(8,1) + two 1's: C(8,2) + three 1's: C(8,3) = 1 + 8 + 28 + 56 = 93
   (c) ... at least five 0s? At least five 0s equals to at most three 1s = 93

4. A coin is flipped ten times. Note there are $2^{10} = 1024$ possible sequences of flips. How many sequences of coin flips contain ... 10-time coin flipping is same format as a 10-bit string
   (a) ... exactly three heads? = C(10,3) = 120
   (b) ... at most one head? = Zero head: C(10,0) + one head: C(10,1) = 1 + 10/1! = 1 + 10 = 11
   (c) ... at least two heads? (*Hint:* Use part (b) and "counting the complement.") = total - zero head - one head = total - at most one head = 1024 - 11 = 1013

5. Twelve tickets, numbered 1, 2, ..., 12, are sold to 12 different people for a drawing. Four different prizes are awarded, including a grand prize. How many ways are there to award the prizes if ...
   (a) ... there are no restrictions? pick randomly 4 tickets to award differently = P(12,4) = 12*11*10*9 = 11,880
   (b) ... the person holding ticket 7 wins the grand prize? pick randomly 3 out of 11 tickets = P(11,3) = 11*10*9 = 990
   (c) ... the person holding ticket 7 wins one of the prizes? There are 4 chances for that person to win 4 prizes => 4*990 = 3,960
   (d) ... the people holding tickets 7 and 11 both win prizes?

6. Suppose all phone numbers for a certain area code can consist of any sequence of seven digits (e.g. 011-5690). How many phone numbers ...
   (a) ... contain four 1s and three 2s? (e.g. 122-1211)
   (b) ... contain exactly four 1s? (e.g. 411-5101)
   (c) ... contain exactly four digits that are the same?
   (d) ... are a sequence of decreasing numbers? (For example, 976-4321.)

Answers:

1. (a) 412, 413, 415, 421, 423, 425, 431, 432, 435, 451, 452, 453
   (b) $\{1,2,3\}$, $\{1,2,4\}$, $\{1,2,5\}$, $\{1,3,4\}$, $\{1,3,5\}$, $\{1,4,5\}$, $\{2,3,4\}$, $\{2,3,5\}$, $\{2,4,5\}$, $\{3,4,5\}$
   Note: There's an algorithm for methodically listing r-combinations, which we will go over in section 6.6.

2. (a) 990
   (b) 165
   (c) 150

3. (a) 56
   (b) 93
   (c) 93
   (How are the bit strings in part (b) related to the ones in part (c)?)

4. (a) 120
   (b) 11
   (c) 1013

5. (a) 11880
   (b) 990
   (c) 3960
   (d) 1080

6. (a) 35
   (b) 25,515
   (c) 255,150
   (d) 120