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| **Student:** Phong Vo<br>**Date:** 05/03/18 | **Instructor:** Chuck Ormsby<br>**Course:** Multi-Variable and Vector Calculus -- Calculus III Spring 2018 | **Assignment:** Section 15.7 Homework |

1. Verify that the line integral and the surface integral of Stokes' Theorem are equal for the following vector field, surface S, and closed curve C. Assume that C has counterclockwise orientation and S has a consistent orientation.

$\mathbf{F} = \langle y, -x, 6\rangle$; S is the upper half of the sphere $x^2 + y^2 + z^2 = 16$ and C is the circle $x^2 + y^2 = 16$ in the xy-plane.

Construct the line integral of Stokes' Theorem using the parameterization $\mathbf{r}(t) = \langle 4\cos t, 4\sin t, 0\rangle$, for $0 \le t \le 2\pi$ for the curve C. Choose the correct answer below.

A. $\int_0^{2\pi} 16\,dt$

B. $\int_0^{2\pi} -16\,dt$

C. $\int_0^{2\pi} 32\,dt$

D. $\int_0^{2\pi} -32\,dt$

Construct the surface integral of Stokes' Theorem using $R = \{(x,y): x^2 + y^2 \le 16\}$ as the region of integration. Choose the correct answer below.

A. $-2\iint_R dA$

B. $-4\iint_R dA$

C. $2\iint_R dA$

D. $4\iint_R dA$

Evaluate both integrals to verify that they are equal. What is the result?

$-32\pi$ (Type an exact answer, using $\pi$ as needed.)

2. Evaluate the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ by evaluating the surface integral in Stokes' Theorem with an appropriate choice of S.

Assume that C has a counterclockwise orientation when viewed from above and will spin clockwise when viewed from below.

**F** = $\langle -9y, -z, x\rangle$
C is the circle $x^2 + y^2 = 11$ in the plane $z = 0$.

$\oint_C \mathbf{F} \cdot d\mathbf{r} =$ $99\pi$ (Type an exact answer, using $\pi$ as needed.)

3. Evaluate the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ by evaluating the surface integral in Stokes' Theorem with an appropriate choice of S.

Assume that C has a counterclockwise orientation.

**F** = $\left\langle 6xy \sin z, 3x^2 \sin z, 3x^2 y \cos z \right\rangle$; C is the boundary of the plane $z = 10 - 5x - 2y$ in the first octant.

$\oint_C \mathbf{F} \cdot d\mathbf{r} =$ 0 (Type an exact answer, using $\pi$ as needed.)

4. Evaluate the line integral in Stokes' Theorem to find the surface integral $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$. Assume that **n** is in the positive z-direction.

**F** = $\langle x, y, z\rangle$; S is the upper half of the ellipsoid $\frac{x^2}{25} + \frac{y^2}{36} + z^2 = 1$.

$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS =$ 0

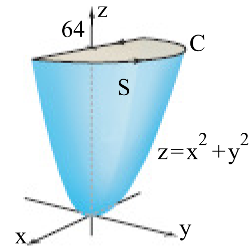
5. Evaluate the line integral in Stokes' Theorem to evaluate the surface integral $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$. Assume that **n** is in the positive z-direction.

**F** = $\langle x + y, y + z, z + x\rangle$; S is the tilted disk enclosed by $\mathbf{r}(t) = \left\langle \cos t, 4 \sin t, \sqrt{7} \cos t \right\rangle$.

$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS =$ $2\left(2\sqrt{7} - 2\right)\pi$

(Type an exact answer, using $\pi$ as needed.)

6. Begin with the paraboloid $z = x^2 + y^2$, for $0 \le z \le 64$, and slice it with the plane $y = 0$. Let S be the surface that remains for $y \ge 0$ (including the planar surface in the xz-plane) (see figure). Let C be the semicircle and line segment that bound the cap of S in the plane $z = 64$ with counterclockwise orientation. Let $\mathbf{F} = \langle 4z + 3y, 4x + 3z, 4y + 3x \rangle$. Complete parts (a) through (c) below.




**a.** Describe the direction of the vectors normal to the surface. Choose the correct answer below.

- **A.** The normal vectors point away from the z-axis on the curved surface of S and in the direction of $\langle 0, -1, 0 \rangle$ on the flat surface of S.
- **B.** The normal vectors point away from the z-axis on the curved surface of S and in the direction of $\langle 0, 1, 0 \rangle$ on the flat surface of S.
- **C.** The normal vectors point toward the z-axis on the curved surface of S and in the direction of $\langle 0, 1, 0 \rangle$ on the flat surface of S.
- **D.** The normal vectors point toward the z-axis on the curved surface of S and in the direction of $\langle 0, -1, 0 \rangle$ on the flat surface of S.

**b.** Evaluate $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dS$.

$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dS =$ $32\pi$

(Type an exact answer, using $\pi$ as needed.)

**c.** Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ and check for agreement with part (b).

$\oint_C \mathbf{F} \cdot d\mathbf{r} =$ 0

(Type an exact answer, using $\pi$ as needed.)

YOU ANSWERED: $-4096 + 32\pi$

7. The goal is to evaluate $A = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dS$, where $\mathbf{F} = \langle 3yz, -3xz, 3xy \rangle$ and S is the surface of the upper half of the ellipsoid $x^2 + y^2 + 6z^2 = 1$ $(z \ge 0)$.

**a.** Evaluate a surface integral over a more convenient surface to find the value of A.

**b.** Evaluate A using a line integral.

**a.** A = 0

**b.** A = 0