 Name SOLUTION

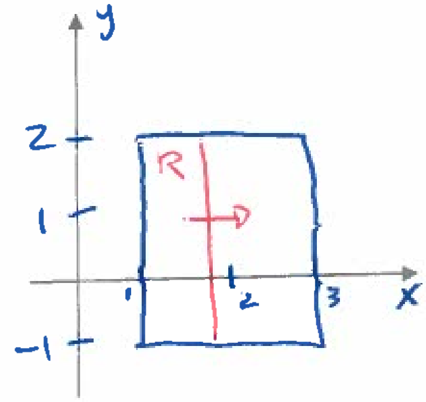
Instructions: No notes or electronic devices are allowed. Answers with little or no supporting work will get little or no credit. Work must be neat, organized and easily interpreted. **NOTE: Most of the credit for problems involving 2D and 3D integration will be awarded for correctly setting up the requested iterated integrals along with neat, well-labelled, and accurate diagrams of the regions of integrations showing the relevant windshield wipers.**

1. (15 Pts.) Find the volume under the surface $f(x,y)=4xy$ within the region R defined by $1\le x\le 3$ and $-1\le y\le 2$. Neatly draw your region of integration showing a "windshield wiper" that illustrates your order of integration.

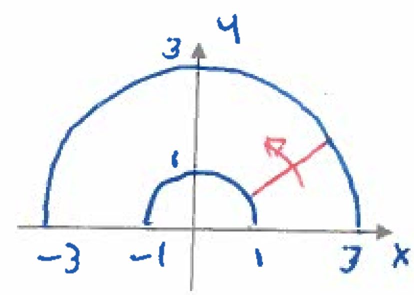
$$V=\iint_R 4xy\,dA=\int_1^3\int_{-1}^{2}4xy\,dy\,dx$$

$$=\int_1^3 2x\,dx\int_{-1}^{2}2y\,dy = x^2\Big|_1^3\, y^2\Big|_{-1}^{2}$$

$$=(9-1)(4-1)=24$$



2. (16 Pts) Determine the total quantity of gold having a density of $d(x,y,z)=6z^2/(x^2+y^2)^2$ gm/cm^3 in the volume D defined by $1\le x^2+y^2\le 9$, $y\ge 0$, and $0\le z\le 2$ where x, y and z are measured in cm. Be sure to draw a diagram of your region of integration in the xy-plane including a "windshield wiper" illustrating your intended method of integration.



$$Q=\iiint_D \frac{6z^2}{r^4}\,r\,dr\,d\theta\,dz$$

$$=\int_0^2 3z^2\,dz\int_0^{\pi}d\theta\int_1^3 2r^{-3}\,dr$$

$$=z^3\Big|_0^2\,(\pi)\,\left(-r^{-2}\right)\Big|_1^3$$

$$=8\pi\left(1-\frac{1}{9}\right)=64\pi/9$$

3. (16 Points) The density of electrons in an electron cloud is given by $G(x,y,z)=\dfrac{400}{(x^2+y^2+z^2)^2}\,electrons/cm^3$.

Determine the number of electrons in the first octant $(x, y, and\ z \geq 0)$ where $1 \leq x^2+y^2+z^2 \leq 4$ and where $x, y, and\ z\ are\ measured\ in\ cm$. Providing a figure is OK but is not required for this problem

$$\#\ electrons = \iiint_D \frac{400}{\rho^4}\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

$$= \int_0^{\pi/2} d\theta \int_0^{\pi/2} \sin\phi\, d\phi \int_1^2 400\rho^{-2}\, d\rho$$

$$= \left(\frac{\pi}{2}\right)(1)\left(-\frac{400}{\rho}\right)\Big|_1^2$$

$$= \frac{\pi}{2}(400-200) = 100\pi$$

~ 314 electrons

4. (6 Points each) If you were asked to perform integrations of the following functions over the specified regions in $\mathbb{R}^3$, what coordinate system would you use and what would you use for the relevant differential volume, dV ?

a. $f(x,y,z)=\cos(z)e^{x^2+y^2}$ over D: $1 \leq x^2+y^2 \leq \pi$ and $0 \leq z \leq \pi/3$

Your selected coordinate system: Cylindrical $dV =$ $r\,dr\,d\theta\,dz$

b. $f(x,y,z)=x+2y-z$ over D: $0 \leq x^2+y^2+z^2 \leq 5$, $x \leq 0$ and $y \geq 0$

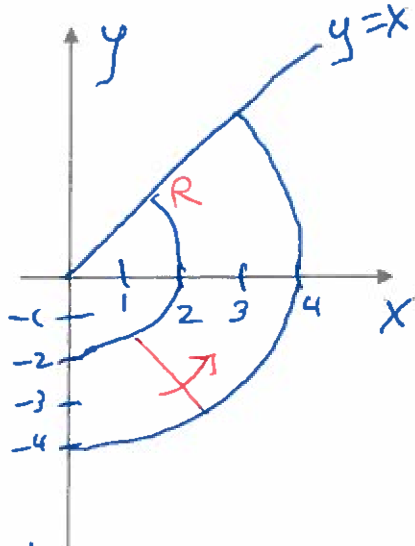
Your selected coordinate system: spherical $dV =$ $\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

c. $f(x,y,z=\pi x^2y^2z$ over D: $0 \leq z \leq 9-3x-2y$, $x \geq 0$ and $y \geq 0$

Your selected coordinate system: Cartesian $dV =$ $dx\,dy\,dz$

5. (16 Points) Evaluate the integral $\iint_R \frac{4xy}{(x^2+y^2)}dA$ using polar coordinates, where R is the region $4 \le x^2+y^2 \le 16$, $y \le x$, and $x \ge 0$. You <u>must</u> draw a neat diagram of the region including your windshield wiper.

$$\iint_R \frac{4xy\,dA}{(x^2+y^2)} = \int_{-\pi/2}^{\pi} \int_2^4 \frac{4r\cos\theta\, r\sin\theta\, r\,dr\,d\theta}{r^2}$$



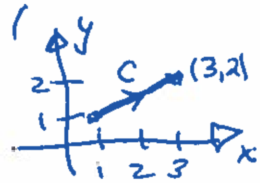
$$= \int_{-\pi/2}^{\pi/4} 2\sin\theta\cos\theta\,d\theta \int_2^4 2r\,dr$$

$$= \sin^2(\theta)\Big|_{-\pi/2}^{\pi/4} r^2\Big|_2^4 = \left(\tfrac{1}{2}-1\right)(16-4)$$

$$= -\tfrac{1}{2}(12) = -6$$

6. (15 Points) Evaluate the work integral $\int_C \vec{F}(x,y)\cdot\vec{dr}$ where C is the straight line segment from (1,1) to (3,2) and $\vec{F}(x,y) = \langle x, 2y \rangle$.

C: $\vec{r}(t) = \langle 1+2t, 1+t \rangle \quad 0 \le t \le 1$



$\vec{r}\,'(t) = \langle 2, 1 \rangle$

$$\int_C \vec{F}\cdot\vec{dr} = \int_0^1 \langle 1+2t, 2+2t \rangle \cdot \langle 2, 1 \rangle\, dt$$

$\frac{d\vec{F}}{dt}dt$

$$= \int_0^1 (2+4t+2+2t)\,dt = \int_0^1 (4+6t)\,dt$$

$$= (4t+3t^2)\Big|_0^1 = 7$$

7. (9 Points) If you are given the coordinates of a point in spherical coordinates, (ρ, ϕ, θ), provide the equations you would use to convert to Cartesian coordinates:

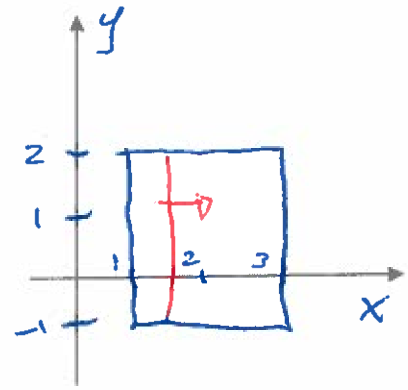
$x = \rho\sin\phi\cos\theta$ $\quad y = \rho\sin\phi\sin\theta$ $\quad z = \rho\cos\phi$

Calculus III Exam III B Pg1 Spring 2018 COrmsby Name SOLUTION

Instructions: No notes or electronic devices are allowed. Answers with little or no supporting work will get little or no credit. Work must be neat, organized and easily interpreted. **NOTE: Most of the credit for problems involving 2D and 3D integration will be awarded for correctly setting up the requested iterated integrals along with neat, well-labelled, and accurate diagrams of the regions of integrations showing the relevant windshield wipers.**

1. (15 Pts.) Find the volume under the surface $f(x,y)=6x^2y$ within the region R defined by $1\le x\le 3$ and $-1\le y\le 2$. Neatly draw your region of integration showing a "windshield wiper" that illustrates your order of integration.

$$V=\iint_R 6x^2y\,dA=\int_1^3 3x^2dx\int_{-1}^2 2y\,dy$$

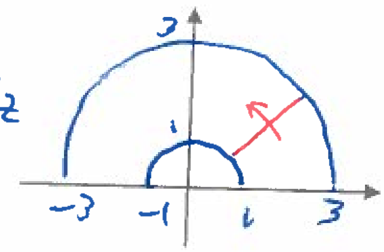


$$=x^3\Big|_1^3\; y^2\Big|_{-1}^2$$

$$=(27-1)(4-1)=78$$

2. (16 Pts) Determine the total quantity of gold having a density of $d(x,y,z)=8z/(x^2+y^2)^2$ gm/cm^3 in the volume D defined by $1\le x^2+y^2\le 9$, $y\ge 0$, and $0\le z\le 2$ where x, y and z are measured in cm. Be sure to draw a diagram of your region of integration in the xy-plane including a "windshield wiper" illustrating your intended method of integration.

$$Q=\iiint_D \frac{8z}{(x^2+y^2)^2}dV=\iiint_D \frac{8z}{r^4}\,r\,dr\,d\theta\,dz$$



$$=\int_0^2 2z\,dz\int_0^\pi d\theta\int_1^3 4r^{-3}dr$$

$$=z^2\Big|_0^2\;\pi\;(-2r^{-2})\Big|_1^3$$

$$=-8\pi\left(\tfrac{1}{9}-1\right)=64\pi/9$$

3. (16 Points) The density of electrons in an electron cloud is given by $G(x,y,z)=\dfrac{400}{(x^2+y^2+z^2)^2}\,electrons/cm^3$.

Determine the number of electrons in the first octant $(x, y,\ and\ z \geq 0)$ where $1 \leq x^2+y^2+z^2 \leq 4$ and where $x, y, and\ z\ are\ measured\ in\ cm$. Providing a figure is OK but is not required for this problem

$$\#\ \text{electrons} = \iiint_D \frac{400}{\rho^4}\,\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

$$= \int_0^{\pi/2} d\theta \int_0^{\pi/2} \sin\phi\, d\phi \int_1^2 400\,\rho^{-2}\, d\rho$$

$$= \left(\frac{\pi}{2}\right)(1)\left(-\frac{400}{\rho}\right)\Big|_1^2$$

$$= \frac{\pi}{2}(400-200) = 100\pi$$

~ 314 electrons

4. (6 Points each) If you were to perform integrations of the following functions over the specified regions in $\mathbb{R}^3$, what coordinate system would you use and what would you use for the relevant differential volume, dV ?

a. $f(x,y,z)=x+2y-z$ over D: $0 \leq x^2+y^2+z^2 \leq 5$, $x \leq 0$ and $y \geq 0$

Your selected coordinate system: spherical $dV =$ $\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

b. $f(x,y,z)=\cos(z)e^{x^2+y^2}$ over D: $1 \leq x^2+y^2 \leq \pi$ and $0 \leq z \leq \pi/3$

Your selected coordinate system: cylindrical $dV =$ $r\,dr\,d\theta\,dz$

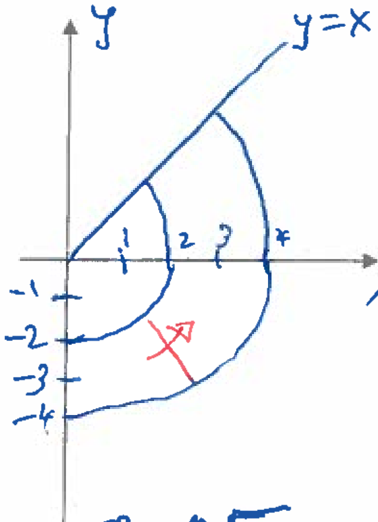
c. $f(x,y,z=\pi x^2y^2z$ over D: $0 \leq z \leq 9-3x-2y$, $x \geq 0$ and $y \geq 0$

Your selected coordinate system: carterian $dV =$ $dx\,dy\,dz$

Calculus III Exam III B Pg3 Spring 2018 COrmsby Name SOLUTION

5. (16 Points) Evaluate the integral $\iint_R \frac{4x}{(x^2+y^2)} dA$ using polar coordinates, where R is the region $4 \le x^2+y^2 \le 16$, $y \le x$, and $x \ge 0$. You must draw a neat diagram of the region including your windshield wiper.

$$\iint_R \frac{4x \, dA}{(x^2+y^2)} = \int_{-\pi/2}^{\pi/4} \int_2^4 \frac{4r\cos\theta}{r^2} r\,dr\,d\theta$$



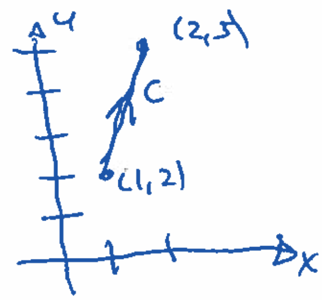
$$= \int_{-\pi/2}^{\pi/4} \cos\theta \, d\theta \int_2^4 4\,dr$$

$$= \sin\theta \Big|_{-\pi/2}^{\pi/4} \; 4r \Big|_2^4 = \left(\frac{1}{\sqrt{2}} + 1\right) 8 = 8 + 4\sqrt{2}$$

6. (15 Points) Evaluate the work integral $\int_C \vec{F}(x,y) \cdot \vec{dr}$ where C is the straight line segment from (1,2) to (2,5) and $\vec{F}(x,y) = \langle 2x, y \rangle$.

$$C: \vec{r}(t) = \langle 1+t, 2+3t \rangle \quad 0 \le t \le 1$$

$$\vec{r}\,'(t) = \langle 1, 3 \rangle$$



$$\int_C \vec{F} \cdot \vec{dr} = \int_0^1 \vec{F} \cdot \frac{\vec{dr}}{dt} dt = \int_0^1 \langle 2+2t, 2+3t \rangle \cdot \langle 1, 3 \rangle dt$$

$$= \int_0^1 (2+2t+6+9t)\,dt = \int_0^1 (8+11t)\,dt = \left(8t + \frac{11}{2}t^2\right)\Big|_0^1$$

$$= 8 + \frac{11}{2} = 27/2$$

7. (9 Points) If you are given the coordinates of a point in spherical coordinates, (ρ, ϕ, θ), provide the equations you would use to convert to Cartesian coordinates:

$x = \rho \sin\phi \cos\theta$ $\quad$ $y = \rho \sin\phi \sin\theta$ $\quad$ $z = \rho\cos\phi$

DO NOT TURN OVER BEFORE TOLD TO DO SO

YOU MAY USE THIS PAGE FOR SCRATCH WORK

IT WILL NOT BE GRADED