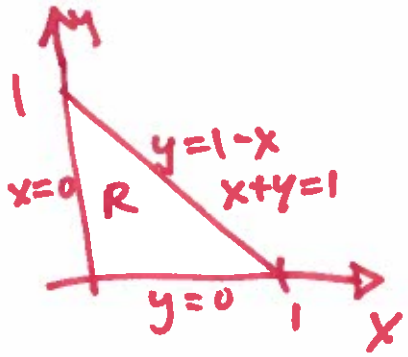
JACOBIAN EXAMPLE

$$Q=\int_0^1\int_0^{1-x}\sqrt{x+y}\,(y-2x)^2\,dy\,dx=?$$



Try $u=x+y$, $v=y-2x$ — Solve for x & y

$u-v=3x \Rightarrow x=\frac{1}{3}(u-v)$

$2u+v=3y \Rightarrow y=\frac{1}{3}(2u+v)$

COMPUTE JACOBIAN

$$J=\begin{vmatrix} 1/3 & -1/3 \\ 2/3 & 1/3 \end{vmatrix}=\left(\frac{1}{9}+\frac{2}{9}\right)=\frac{3}{9}=\frac{1}{3}$$

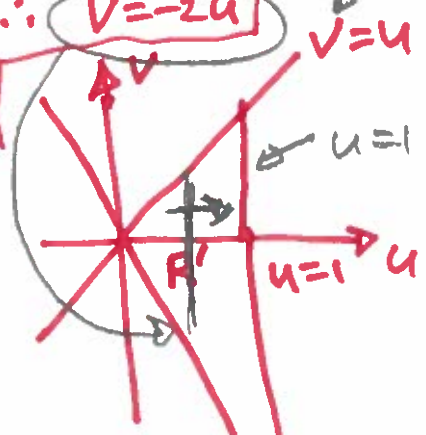
Determine corresponding region in uv space

Boundary of R

$x=0 \Rightarrow u=y$ & $v=y$ $\therefore$ $u=v$

$y=0 \Rightarrow u=x$ & $v=-2x$ $\therefore$ $v=-2u$

$x+y=1 \Rightarrow u=1$



Jacobian

$$\Rightarrow Q=\int_0^1\int_{-2u}^{u} u^{1/2}v^2\,\frac{1}{3}\,dv\,du$$

$$=\int_0^1 u^{1/2}\frac{1}{9}v^3\Big|_{-2u}^{u}du=\int_0^1\frac{1}{9}\left(u^{7/2}+8u^{7/2}\right)du$$

$$=\int_0^1 u^{7/2}du=\frac{2}{9}u^{9/2}\Big|_0^1=2/9$$