CHAPTER 11					
DESIGN OF A	SIMPLE	SERIAL A	RITHMET	CIC PROC	ESSOR

### 11.1 Introduction

Register transfer design Data-path and control circuit.

# $\begin{array}{c} d_1 \\ + d_0 \\ \hline y_1 y_0 \\ \\ \text{carry} \end{array}$

### 11.2 Adder

Table 11.1 Truth table for a half adder.

$d_1 d_0$	<b>y</b> <sub>1</sub> <b>y</b> <sub>0</sub>
0 0	0 0
0 1	0 1
1 0	0 1
1 1	1 0

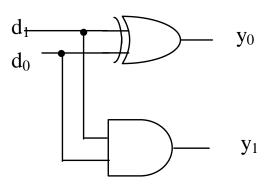


Figure 11.1 A half adder.

$$\begin{array}{c} c_i \\ a_i \\ + b_i \end{array}$$

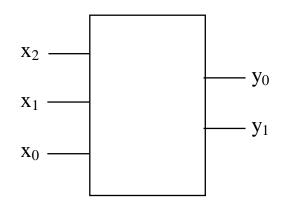


Figure 1.3 A 3-input, 2-output binary circuit.

Table 1.2 Truth table for the circuit in Figure 1.3.

X <sub>2</sub>	$\mathbf{x}_1$	$\mathbf{x}_0$		$\mathbf{y}_1$	$\mathbf{y}_0$
0	0	0		0	0
0	0	1		0	1
0	1	0		0	1
0	1	1		1	0
1	0	0		0	1
1	0	1		1	0
1	1	0		1	0
1	1	1		1	1
			I		

$$\begin{aligned} y_0 &= F(x_2, x_1, x_0) = \Sigma \ m(1, 2, 4, 7) \\ F(x_2, x_1, x_0) &= x_2' x_1' x_0 + x_2' x_1 x_0' + x_2 x_1' x_0' + x_2 x_1 x_0 \\ F(x_2, x_1, x_0) &= (m_1 + m_2) + (m_4 + m_7) \\ &= (x_2' x_1' x_0 + x_2' x_1 x_0') + (x_2 x_1' x_0' + x_2 x_1 x_0) \\ &= x_2' (x_1' x_0 + x_1 x_0') + x_2 (x_1' x_0' + x_1 x_0) \\ &= x_2' (x_1 \oplus x_0) + x_2 (x_1 \oplus x_0)' \end{aligned}$$

$X_2$	x <sub>1</sub> 00	01	11	10
$x_0$		1		1
1	1		1	

 $= \mathbf{x}_2 \oplus \mathbf{x}_1 \oplus \mathbf{x}_0$ 

Figure 5.27 K-map for  $y_0$  in Table 1.2.

$$S_i = a_i \oplus b_i \oplus c_i$$

The canonical sum-of-products expression for the carry bit  $c_{i+1}$  is

$$c_{i+1} = a_i' b_i c_i + a_i b_i' c_i + a_i b_i c_i' + a_i b_i c_i$$

The simplest sum-of-products expression is

$$c_{i+1} = a_i b_i + b_i c_i + a_i c_i$$
 (11.1)

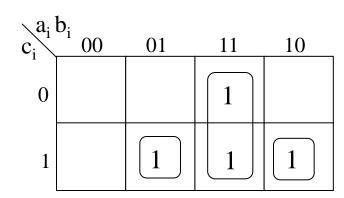
A different expression for the carry bit can be obtained as follows.

$$c_{i+1} = a_i' b_i c_i + a_i b_i' c_i + a_i b_i$$

$$= (a_i' b_i + a_i b_i')c_i + a_i b_i$$

$$= (a_i \oplus b_i) c_i + a_i b_i$$
(11.2)

$c_i$	b <sub>i</sub> 00	01	11	10
0				
1		1		1



$$\begin{split} S_i &= a_i \oplus b_i \oplus c_i \\ c_{i+1} &= (a_i \oplus b_i) \ c_i + a_i \ b_i \end{split}$$

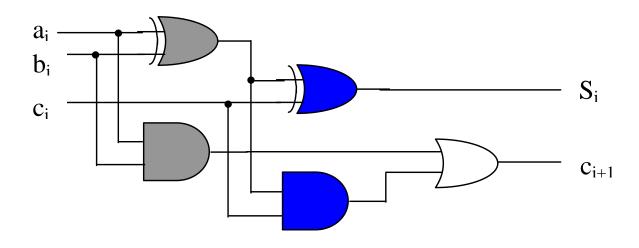


Figure 11.2 A full adder.

The addition of two n-bit numbers and an initial carry  $c_0$  is given below.

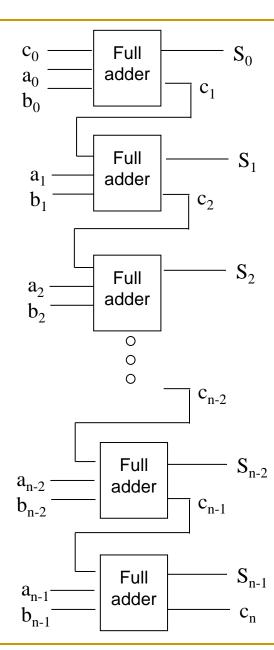


Figure 11.3 A ripple-carry adder.

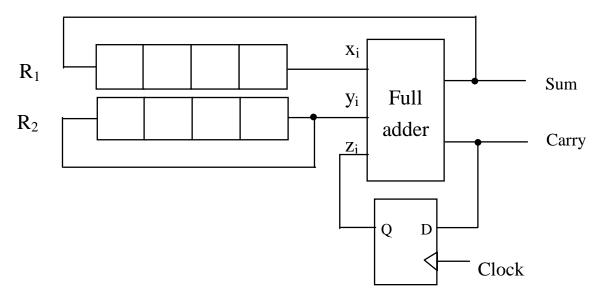
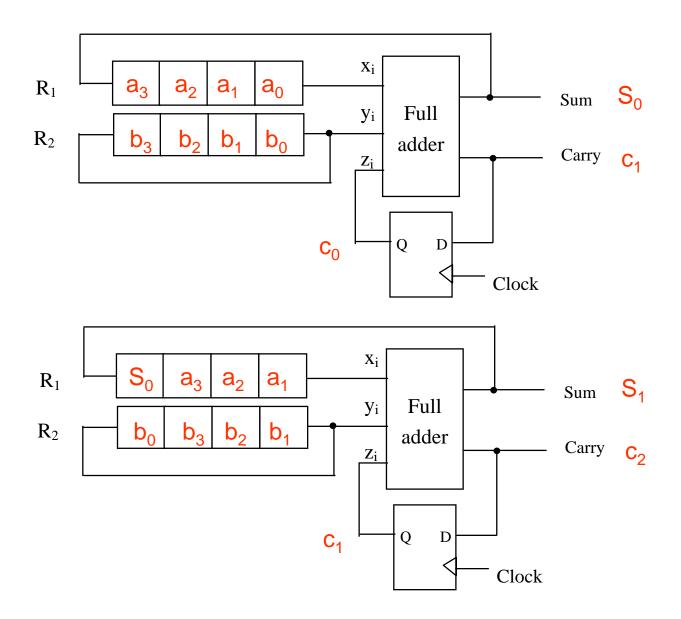
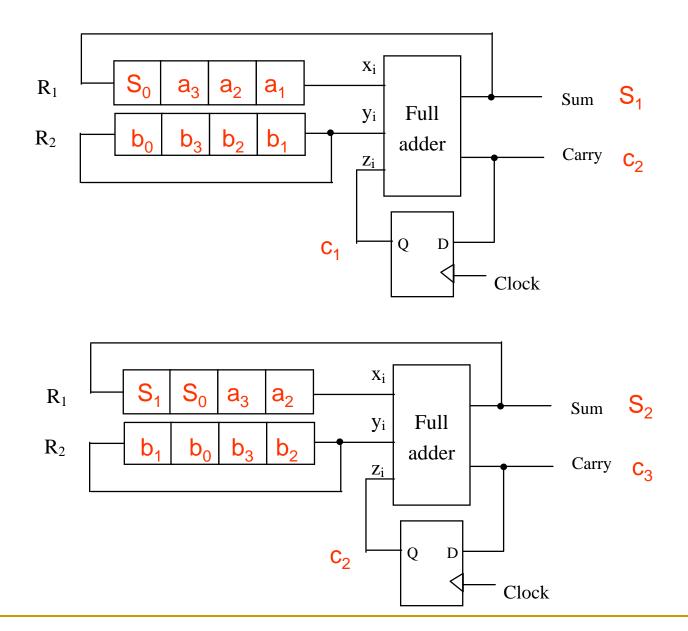


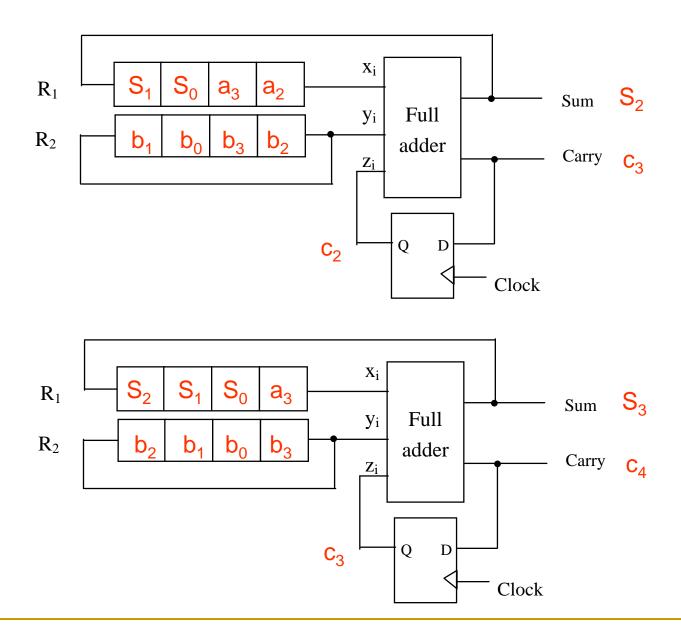
Figure 11.4 A serial adder.

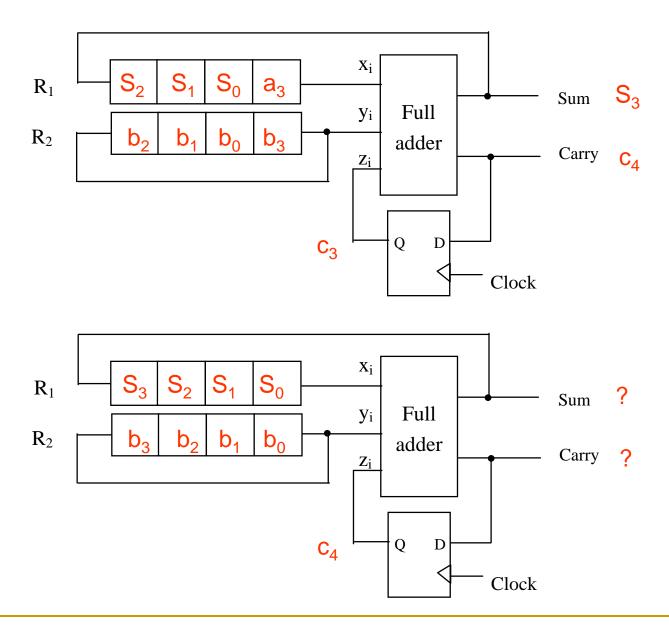
Table 11.2 Contents of the serial adder.

Clock cycle	$R_1$	$R_2$	Xi	y <sub>i</sub>	$z_{i}(Q)$	Sum	Carry (D)
0	$a_3 a_2 a_1 a_0$	b <sub>3</sub> b <sub>2</sub> b <sub>1</sub> b <sub>0</sub>	$a_0$	$b_0$	$c_0$	$S_0$	$c_1$
1	$S_0 a_3 a_2 a_1$	$b_0 \ b_3 \ b_2 \ b_1$	$a_1$	$b_1$	$c_1$	$S_1$	$c_2$
2	$S_1 S_0 a_3 a_2$	$b_1  b_0  b_3  b_2$	$a_2$	$b_2$	$c_2$	$S_2$	$c_3$
3	$S_2 S_1 S_0 a_3$	$b_2  b_1  b_0  b_3$	$a_3$	$b_3$	$c_3$	$S_3$	$c_4$
4	$S_3 S_2 S_1 S_0$	$b_3  b_2  b_1  b_0$	$S_0$	$b_0$	c <sub>4</sub>	N/A	N/A









### 11.3 Signed Numbers

### Sign-magnitude representation

$$-(2^{n-1}-1) \le N \le (2^{n-1}-1)$$

$$0 1 0 0 1 0 1 1 + 75$$

$$0 1 1 1 1 1 1 1 1 + 127$$

$$1 1 1 1 1 1 1 1 1 - 127$$

$$1 0 0 0 0 0 0 0 1 - 1$$

$$0 0 0 0 0 0 0 0 0 + 0$$

$$1 0 0 0 0 0 0 0 0 - 0$$

### 2's complement representation

$$-2^{n-1} \le N \le (2^{n-1}-1)$$

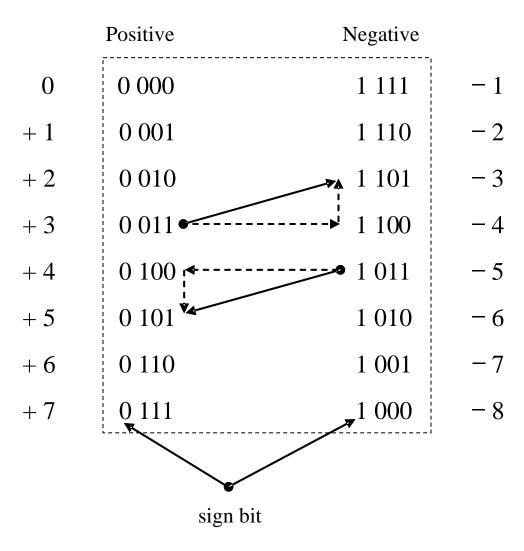


Figure 11.5 Two's complement representation for 4-bit signed numbers.

The bit inversion of an n-bit number  $Y = y_{n-1} y_{n-2} \dots y_2 y_1 y_0$  is equivalent to the subtraction of an n-bit number  $2^n - 1$  by Y, which is shown below.

<sup>2</sup>Y, the 2's complement of Y, is defined as

$$^{2}Y = ^{1}Y + 1$$

Thus the 2's complement of Y can also be obtained by subtracting Y from 2<sup>n</sup> because

$$^{2}Y = ^{1}Y + 1 = (2^{n} - 1) - Y + 1 = 2^{n} - Y$$

Subtraction of two n-bit signed numbers: A - B

$$A - B = A + (-B)$$
  $\longrightarrow$   $A + {}^{2}B = A + (2^{n} - B) = (A - B) + 2^{n}$ 

Decimal: 
$$2-5=2+(-5)=-3$$
  
Binary:  $0010-0101=0010+(-0101)$ 

The result is a negative number. The 2's complement of this number is 0011.

$$5 + 4 = +9$$

Binary:

$$0101 + 0100 = 1001$$

Positive Positive

Negative

0101

+ 1110

1001

$$-5-6 = -5 + (-6) = -11$$

Binary:

$$-(0101) + (-0110)$$

$$(1011) + (1010) = 1(0101)$$

Negative

Negative

Positive

1011

+ 1010

10101

### Addition and Subtraction Methods for 2's Complement Signed Number Conversion

The conversions for positive signed numbers between decimal and binary are no difference from what have been done in Chapter 2.

## The conversions for negative signed numbers between decimal and .

Note: The weight of signed bit is negative.

8-bit negative signed number 10011010

Binary number 1 0 0 1 1 0 1 0

Weight -128 + 64 + 32 + 16 + 8 + 4 + 2 + 1

(Add weights of all 1-bits)

Decimal number -128 +16 +8 +2 = -102

### Convert –79 to a 2's complement 8-bit signed number using the subtraction method

Weight 
$$-2^7 = -128$$
  $-(-128)$   $-(-128)$   $-(-128)$   $-(-128)$  difference  $2^5 = 32$   $-32$   $-32$  difference  $2^4 = 16$   $-16$   $-16$  difference  $2^0 = 1$   $-1$  difference  $-1$  difference  $-1$  difference  $-1$  difference  $-1$  difference  $-1$  difference  $-1$ 

$$a_7 = a_5 = a_4 = a_0 = 1$$
,  $a_6 = a_3 = a_2 = a_1 = 0$   
 $(-79)_{10} = (a_7 a_6 a_5 a_4 a_3 a_2 a_1 a_0)_2 = (10110001)_2$ 

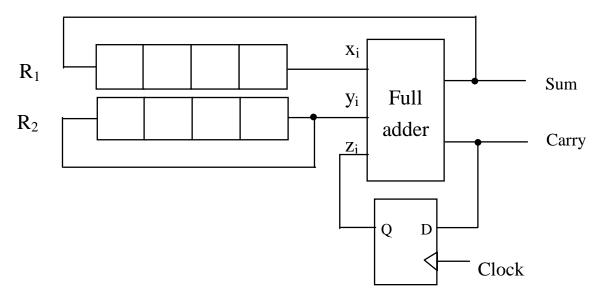


Figure 11.4 A serial adder.

Table 11.2 Contents of the serial adder.

Clock cycle	$R_1$	$R_2$	Xi	y <sub>i</sub>	$z_{i}(Q)$	Sum	Carry (D)
0	$a_3 a_2 a_1 a_0$	b <sub>3</sub> b <sub>2</sub> b <sub>1</sub> b <sub>0</sub>	$a_0$	$b_0$	$c_0$	$S_0$	$c_1$
1	$S_0 a_3 a_2 a_1$	$b_0 \ b_3 \ b_2 \ b_1$	$a_1$	$b_1$	$c_1$	$S_1$	$c_2$
2	$S_1 S_0 a_3 a_2$	$b_1  b_0  b_3  b_2$	$a_2$	$b_2$	$c_2$	$S_2$	$c_3$
3	$S_2 S_1 S_0 a_3$	$b_2  b_1  b_0  b_3$	$a_3$	$b_3$	$c_3$	$S_3$	$c_4$
4	$S_3 S_2 S_1 S_0$	$b_3  b_2  b_1  b_0$	$S_0$	$b_0$	c <sub>4</sub>	N/A	N/A

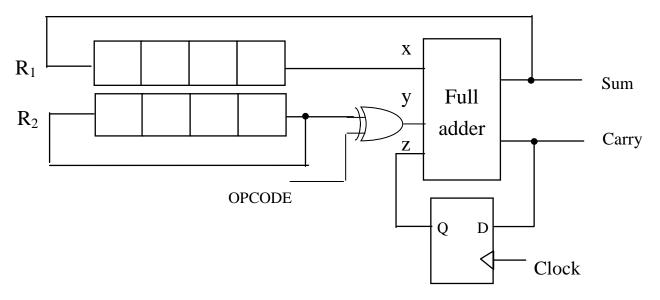


Figure 11.4 A serial adder.

#### 11.4 Algorithmic State Machine (ASM) Chart

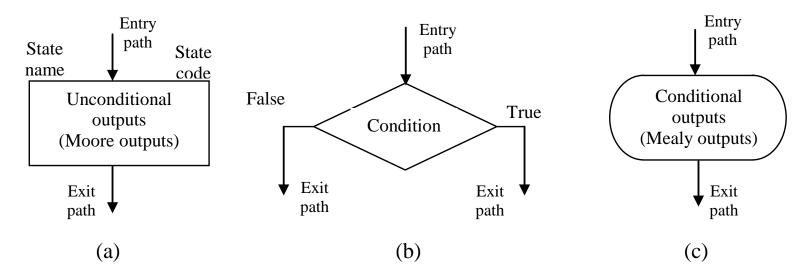
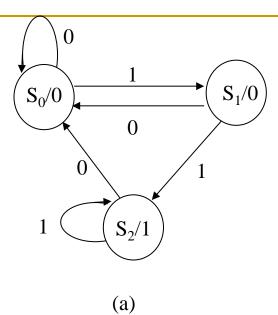
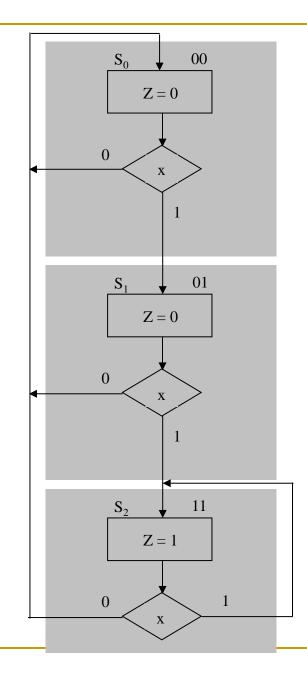


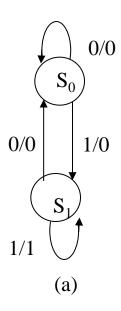
Figure 11.6 Basic elements of ASM charts. (a) State box. (b) Decision box. (c) Conditional output box.



State name	State code $Q_1Q_0$				
$S_0$	0 0				
$S_1$	0 1				
$S_2$	1 1				
(b)					

Figure 11.7 Conversion of a Moore state diagram to ASM chart. (a)
State diagram. (b) State assignment. (c) ASM chart.





State name	State code Q			
$S_0$	0			
$S_1$	1			
(b)				

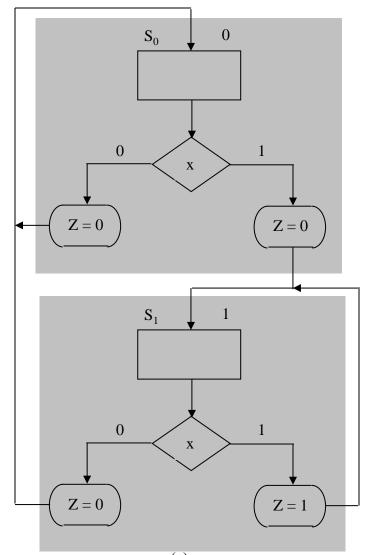
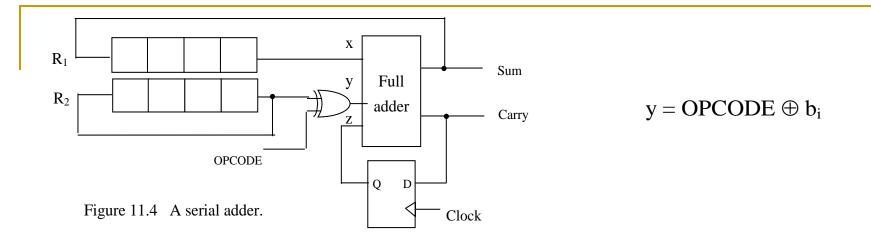
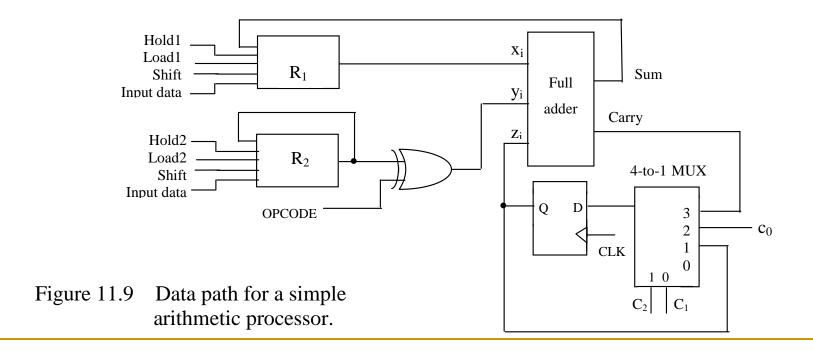
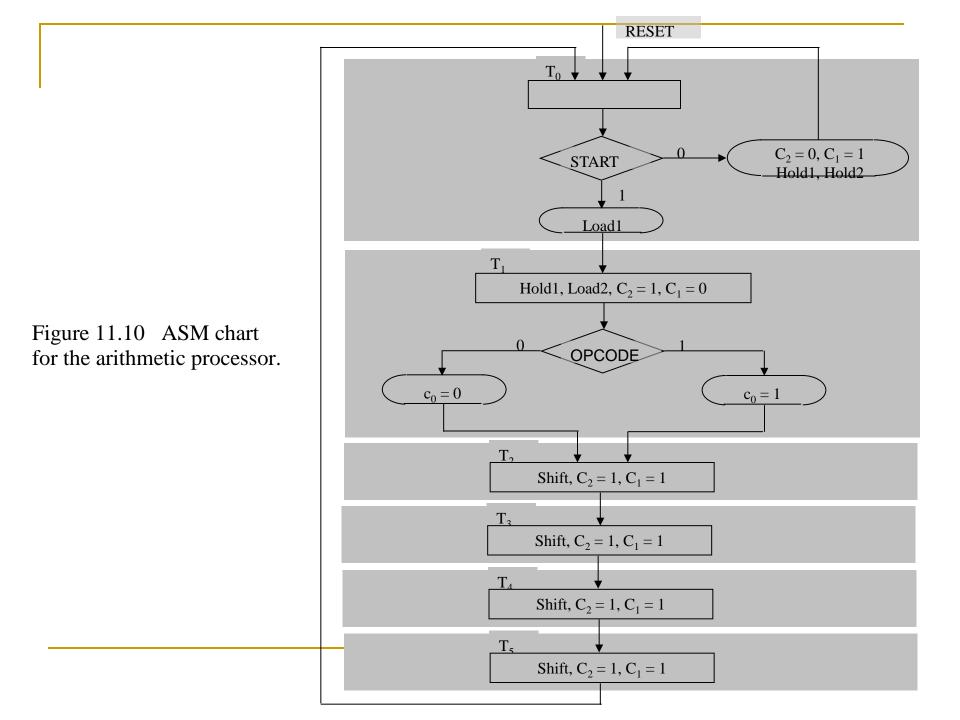


Figure 11.8 Conversion of a Mealy state diagram to ASM chart. (a9) State diagram. (b) State assignment. (c) ASM chart.







### **Ring Counter**

Table 10.2 State assignment table for a 4-state ring counter

State	$Q_0 Q_1 Q_2 Q_3$			
$T_0$	1 0 0 0			
$T_1$	0 1 0 0			
$T_2$	0 0 1 0			
$T_3$	0 0 0 1			

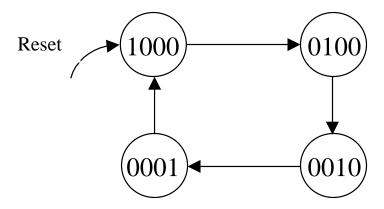


Figure 10.6 State diagram for a 4-state ring counter.

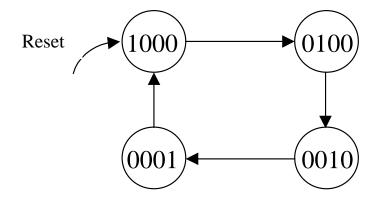


Figure 10.6 State diagram for a 4-state ring counter.

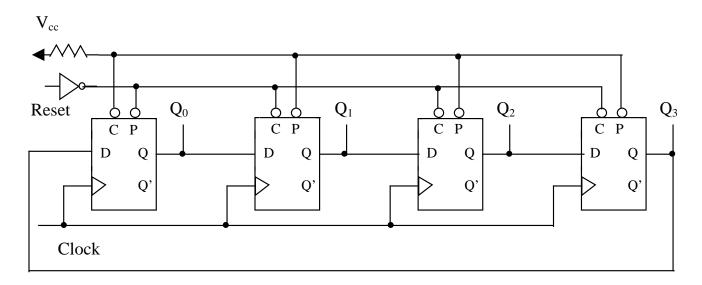


Figure 10.7 Circuit diagram for a 4-bit ring counter.

RESET Figure 11.10 ASM chart  $T_0$ for the arithmetic processor. START = 0 $C_2 = 0, C_1 = 1$ START RESET Hold1, Hold2  $T_0$ Load1  $T_1$ Hold1, Load2,  $C_2 = 1$ ,  $C_1 = 0$ OPCODE  $T_2$  $c_0 = 0$  $c_0 = 1$  $T_3$ Shift,  $C_2 = 1$ ,  $C_1 = 1$  $T_4$ Shift,  $C_2 = 1$ ,  $C_1 = 1$ Shift,  $C_2 = 1$ ,  $C_1 = 1$  $T_5$ Shift,  $C_2 = 1$ ,  $C_1 = 1$ 

#### **Design of State Generator**

Table 11.3 State assignment.

State	$Q_0$	$Q_1$	$Q_2$	$Q_3$	$Q_4$	$Q_5$
$T_0$	1	0	0	0	0	0
$T_1$	0	1	0	0	0	0
$T_2$	0	0	1	0	0	0
$T_3$	0	0	0	1	0	0
$T_4$	0	0	0	0	1	0
$T_5$	0	0	0	0	0	1

Table 11.4 State table for state generator.

Present state	START	Next state
$T_0$	0	$T_0$
$\mathrm{T}_{\mathrm{0}}$	1	$T_1$
$T_1$	d	$T_2$
$T_2$	d	$T_3$
$T_3$	d	$T_4$
$\mathrm{T}_4$	d	$T_5$
$T_5$	d	$\mathrm{T}_{\mathrm{0}}$

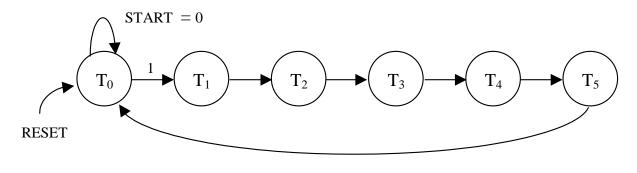


Figure 11.11 State diagram for state generator.

$$Q_0^+ = Q_0 \cdot START' + Q_5$$
 $Q_1^+ = Q_0 \cdot START$ 
 $Q_2^+ = Q_1$ 
 $Q_3^+ = Q_2$ 
 $Q_4^+ = Q_3$ 
 $Q_5^+ = Q_4$ 

$$D_0 = Q_0^+ = Q_0 \cdot START' + Q_5$$
 $D_1 = Q_1^+ = Q_0 \cdot START$ 
 $D_2 = Q_2^+ = Q_1$ 
 $D_3 = Q_3^+ = Q_2$ 
 $D_4 = Q_4^+ = Q_3$ 
 $D_5 = Q_5^+ = Q_4$ 

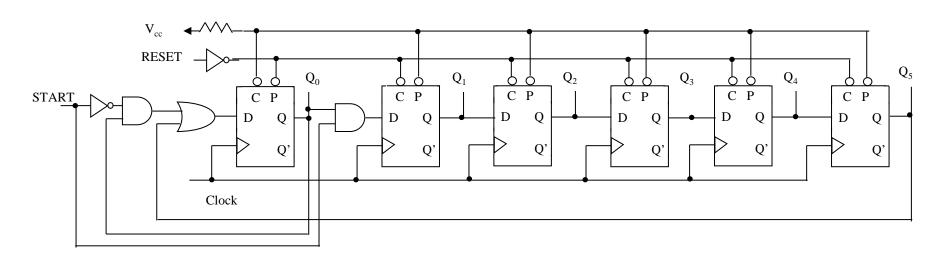
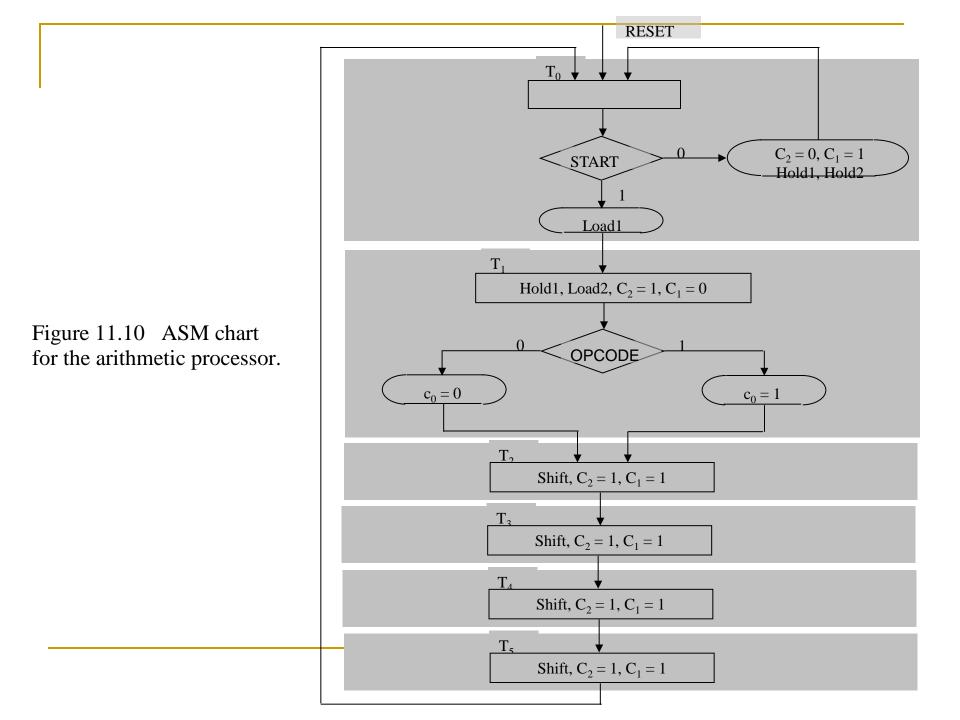
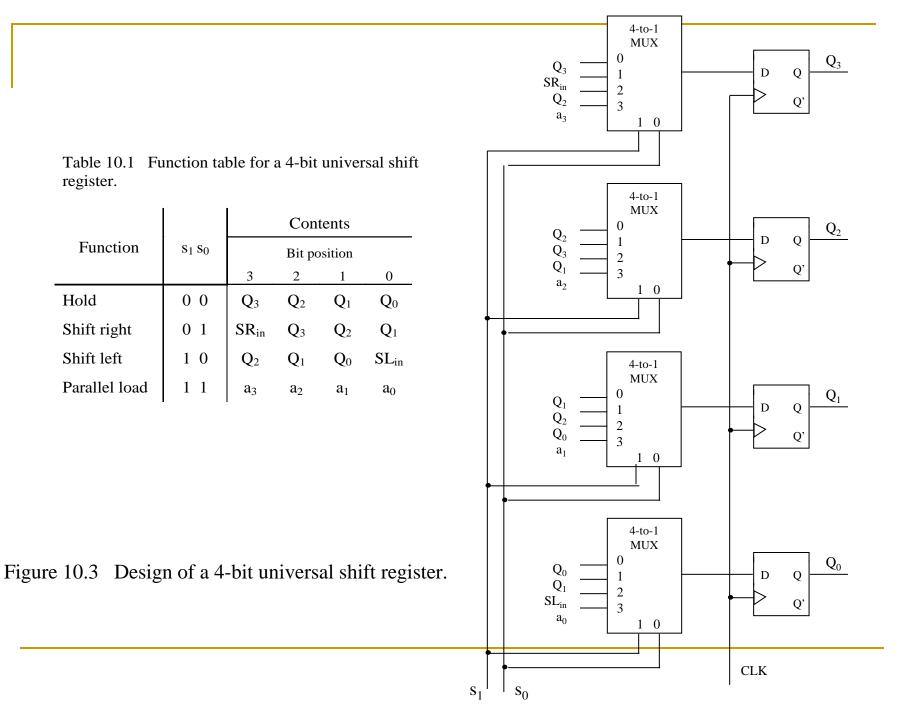


Figure 11.12 State generator.





## Design of Control Circuit

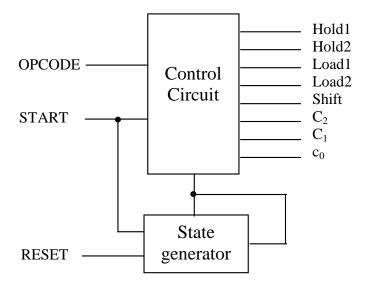
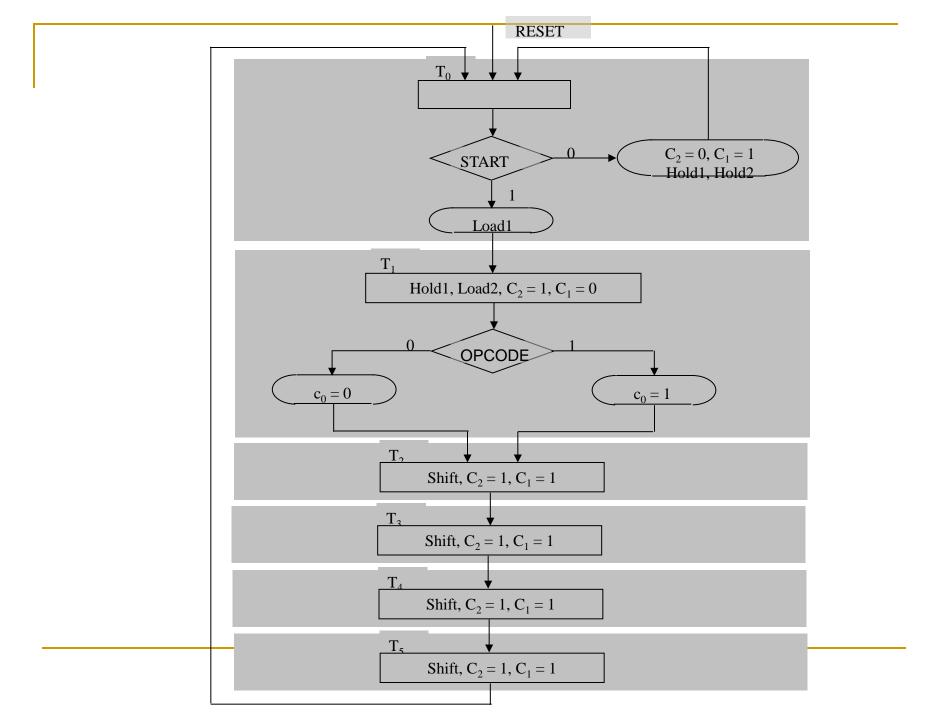


Figure 11.13 Block diagram for control circuit.

Table 11.5 Conversion of asserted signals to selection signals for shift register.

Asserted signal	$(s_1s_0)_{R1}$	$(s_1s_0)_{R2}$
Hold1	0 0	N/A
Hold2	N/A	0 0
Load1	1 1	N/A
Load2	N/A	1 1
Shift	0 1	0 1



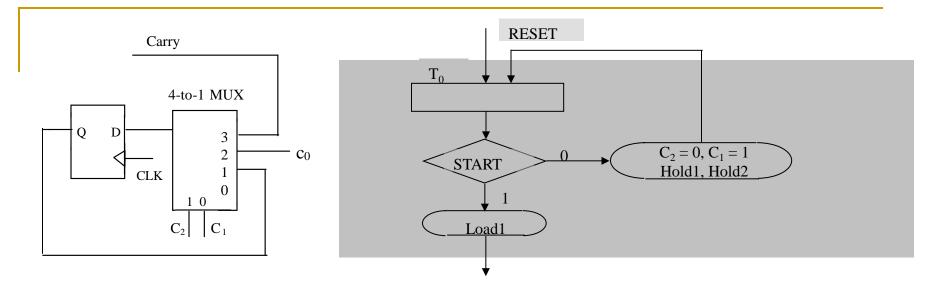


Table 11.6 Truth table for the control circuit.

State	START	$(s_1s_0)_{R1}$	$(s_1s_0)_{R2}$	$C_2$	$C_1$	$c_0$	
$T_0$	0	0 0	0 0	0	1	d	
$T_0$	1	1 1	d d	d	d	d	

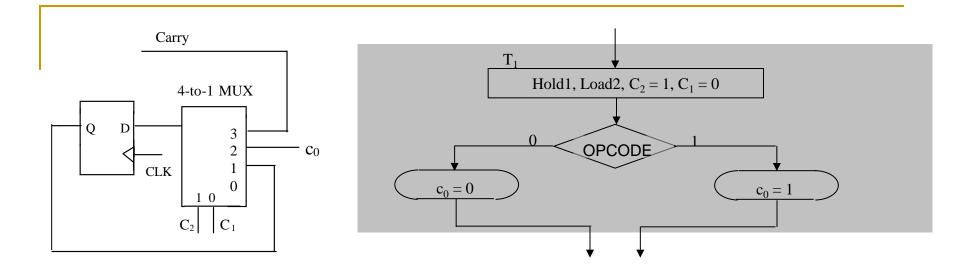


Table 11.6 Truth table for the control circuit.

State	START	$(s_1s_0)_{R1}$	$(s_1s_0)_{R2}$	$C_2$	$C_1$	$c_0$	
$T_0$	0	0 0	0 0	0	1	d	
$T_0$	1	1 1	d d	d	d	d	
$\mathbf{T}_1$	d	0 0	1 1	1	0	OPCODE	

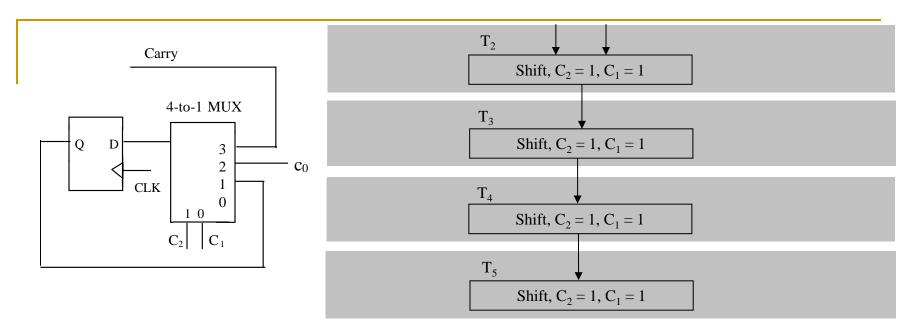


Table 11.6 Truth table for the control circuit.

State	START	$(s_1s_0)_{R1}$	$(s_1s_0)_{R2}$	$C_2$	$\mathbf{C}_1$	$c_0$
$T_0$	0	0 0	0 0	0	1	d
$T_0$	1	1 1	d d	d	d	d
$T_1$	d	0 0	1 1	1	0	OPCODE
$T_2$	d	0 1	0 1	1	1	d
$T_3$	d	0 1	0 1	1	1	d
$T_4$	d	0 1	0 1	1	1	d
$T_5$	d	0 1	0 1	1	1	d

Table 11.6 Truth table for the control circuit.

TART	$(s_1s_0)_{R1}$	$(s_1s_0)_{R2}$	$C_2$	$C_1$	$c_0$
0	0 0	0 0	0	1	d
1	1 1	d d	d	d	d
d	0 0	1 1	1	0	OPCODE
d	0 1	0 1	1	1	d
d	0 1	0 1	1	1	d
d	0 1	0 1	1	1	d
d	0 1	0 1	1	1	d
	1 d d d d	0 0 0 1 1 1 d 0 0 d 0 1 d 0 1 d 0 1	0       0       0       0       0         1       1       1       d       d         d       0       0       1       1         d       0       1       0       1         d       0       1       0       1         d       0       1       0       1         d       0       1       0       1	0     0     0     0     0       1     1     1     1     d       d     0     0     1     1     1       d     0     1     0     1     1       d     0     1     0     1     1       d     0     1     0     1     1       d     0     1     0     1     1	0       0       0       0       0       1         1       1       1       1       0       d       d         d       0       0       1       1       1       0       0       d       0       1

$$(s_1)_{R1} = T_0 \bullet START$$

$$(s_0)_{R1} = T_0 \bullet START + T_2 + T_3 + T_4 + T_5$$

$$(s_1)_{R2} = T_1$$

$$(s_0)_{R2} \; = \; T_1 + T_2 + T_3 + T_4 + T_5$$

$$C_2 = T_1 + T_2 + T_3 + T_4 + T_5$$

$$C_1 \; = \; T_0 + T_2 + T_3 + T_4 + T_5$$

$$c_0 = OPCODE$$

Table 11.7 Truth table for  $s_0$ ',  $C_2$ ', and  $C_1$ '

State	START	$(s_0')_{R1}$	(s <sub>0</sub> ') <sub>R2</sub>	$C_2$	$C_1$
$T_0$	0	1	1	1	0
$T_0$	1	0	d	d	d
$\mathrm{T}_1$	d	1	0	0	1
$T_2$	d	0	0	0	0
$T_3$	d	0	0	0	0
$T_4$	d	0	0	0	0
$T_5$	d	0	0	0	0

$$(s_1)_{R1} = T_0 \cdot START$$
  
 $(s_0)_{R1} = T_0 \cdot START + T_2 + T_3 + T_4 + T_5$   
 $(s_1)_{R2} = T_1$   
 $(s_0)_{R2} = C_2 = T_1 + T_2 + T_3 + T_4 + T_5$   
 $C_1 = T_0 + T_2 + T_3 + T_4 + T_5$   
 $c_0 = OPCODE$ 

From Table 11.7,  

$$(s_0')_{R1} = T_0 \cdot START' + T_1$$
  
 $(s_0')_{R2} = C_2' = T_0$   
 $C_1' = T_1$   
 $(s_0)_{R1} = (T_0 \cdot START' + T_1)'$   
 $(s_0)_{R2} = C_2 = T_0'$   
 $C_1 = T_1'$ 

#### 11.6 Revisit of Arithmetic Processor

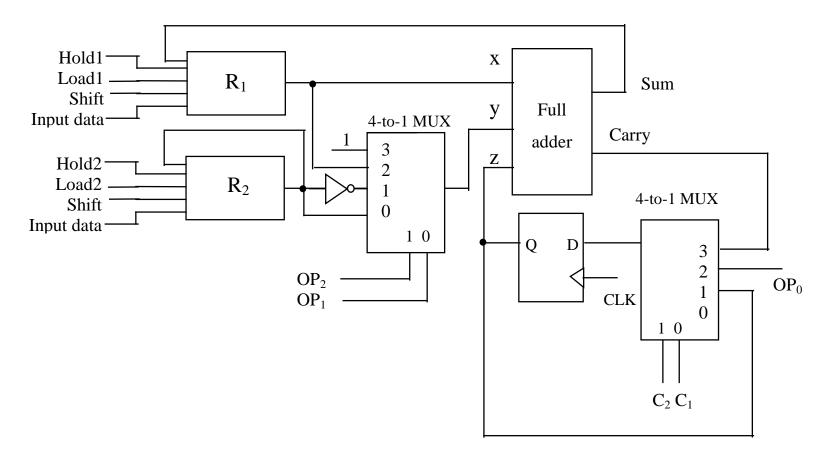


Figure 11.14 A processor for eight arithmetic functions.

RESET  $T_0$ ,  $C_2 = 0, C_1 = 1$ Hold1, Hold2 0 START Figure 11.15 ASM chart for the arithmetic Load1 processor in Figure 11.14. Hold1, Load2,  $C_2 = 1$ ,  $C_1 = 0$ Shift,  $C_2 = 1$ ,  $C_1 = 1$ 

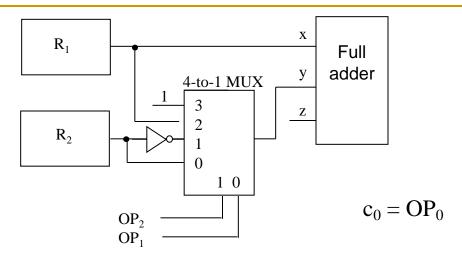


Table 11.8 Arithmetic functions for the processor in Figure 11.14.

$OP_2$	$OP_1$	$OP_0$	$x_3x_2x_1x_0 + y_3y_2y_1y_0 + c_0$	Arithmetic function
0	0	0	$a_3a_2a_1a_0 + b_3b_2b_1b_0 + 0$	A + B
0	0	1	$a_3a_2a_1a_0 + b_3b_2b_1b_0 + 1$	A + B + 1
0	1	0	$a_3a_2a_1a_0 + b_3'b_2'b_1'b_0' + 0$	A - B - 1
0	1	1	$a_3a_2a_1a_0 + b_3'b_2'b_1'b_0' + 1$	A - B
1	0	0	$a_3 a_2 a_1 a_0 + a_3 a_2 a_1 a_0 + 0$	2A
1	0	1	$a_3a_2a_1a_0 + a_3a_2a_1a_0 + 1$	2A + 1
1	1	0	$a_3a_2a_1a_0 + 1111 + 0$	A – 1
1	1	1	$a_3a_2a_1a_0 + 1111 + 1$	A

## Experiment 5 Arithmetic Processor

#### 1. Sequence assignment

Op <sub>2</sub> Op <sub>1</sub>	Arithmetic Function
0 0	
0 1	
1 0	
1 1	

# 2. Processor Design Construct the truth table for the input processor. (Use a<sub>i</sub>, b<sub>i</sub>, 0, 1)

Op <sub>2</sub> Op <sub>1</sub>	X <sub>i</sub>	y <sub>i</sub>	$\mathbf{c}_0$
00			
01			
10			
11			

## Experiment 5 Arithmetic Processor

#### 1. Sequence assignment

Op <sub>2</sub> Op <sub>1</sub>	Arithmetic Function
0 0	2B + 1
0 1	A - B
1 0	A + B
1 1	A - 1

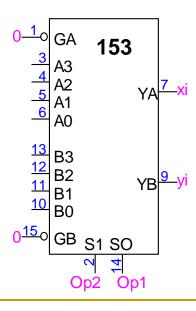
# 2. Processor Design Construct the truth table for the input processor. (Use a<sub>i</sub>, b<sub>i</sub>, 0, 1)

Op <sub>2</sub> Op <sub>1</sub>	X <sub>i</sub>	y <sub>i</sub>	$c_0$
00	b <sub>i</sub>	b <sub>i</sub>	1
01	a <sub>i</sub>	b <sub>i</sub> ,	1
10	a <sub>i</sub>	b <sub>i</sub>	0
11	a <sub>i</sub>	1	0

# Processor Design Construct the truth table for the input processor. (Use a<sub>i</sub>, b<sub>i</sub>, 0, 1)

Op <sub>2</sub> Op <sub>1</sub>	X <sub>i</sub>	y <sub>i</sub>	C <sub>0</sub>
00	b <sub>i</sub>	b <sub>i</sub>	1
01	a <sub>i</sub>	b <sub>i</sub>	1
10	a <sub>i</sub>	b <sub>i</sub>	0
11	a <sub>i</sub>	1	0

Label the data inputs of the two 4-to-1 multiplexers given below for the realization of x<sub>i</sub>, y<sub>i</sub>.



Express the initial carry  $c_0$  as a function of  $Op_2$  and  $Op_1$ .

$$c_0 =$$

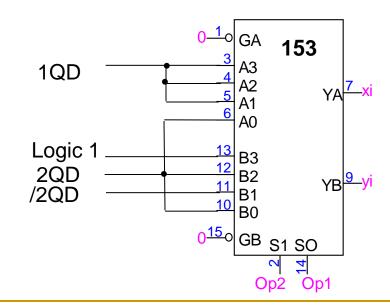
## Processor Design Construct the truth table for the input processor. (Use a<sub>i</sub>, b<sub>i</sub>, 0, 1)

Op <sub>2</sub> Op <sub>1</sub>	X <sub>i</sub>	y <sub>i</sub>	c <sub>0</sub>
00	b <sub>i</sub>	bi	1
01	a <sub>i</sub>	, bi	1
10	a <sub>i</sub>	b <sub>i</sub>	0
11	a <sub>i</sub>	1	0

Label the data inputs of the two 4-to-1 multiplexers given below for the realization of  $x_i$ ,  $y_i$ .

Use the following signal names for input processor 1QD for ai, 2QD for bi, Op2, Op1, xi, yi, zi

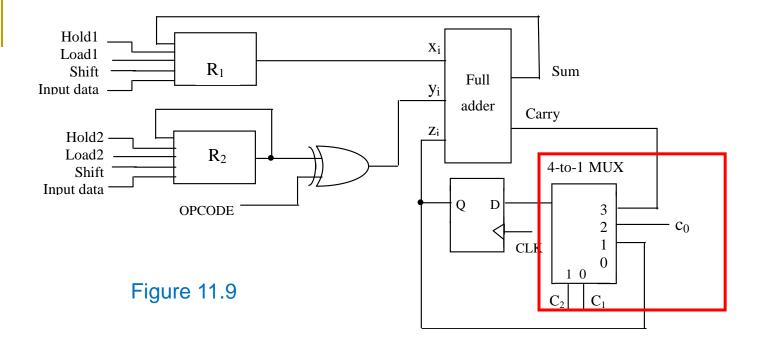
Use the following signal names for control signals 1s1, 1s0, 2s1, 2s0, c0, C2, C1



Express the initial carry  $c_0$  as a function of  $Op_2$  and  $Op_1$ .  $c_0 = Op_2$ ' Construct the truth table for the control circuit.

State	START	(s <sub>1</sub> ) <sub>R1</sub>	(s <sub>0</sub> ) <sub>R1</sub>	(s <sub>1</sub> ) <sub>R2</sub>	$(s_0)_{R2}$	C <sub>2</sub>	C <sub>1</sub>
T <sub>0</sub>	0						
T <sub>0</sub>	1						
T <sub>1</sub>	d						
T <sub>2</sub>	d						
T <sub>3</sub>	d						
T <sub>4</sub>	d						
T <sub>5</sub>	d						

$$(s_1)_{R1} = (s_0)_{R1} = (s_1)_{R2} = (s_0)_{R2} = C_2 = C_1 =$$



Note the difference in the data inputs of the two multiplexers.

Lab 5 schematic diagram

