Section 8.2 Homework / Discrete Structures II / Fall 2018

1. State the degree and characteristic equation for the recurrence relation $a_n = 3a_{n-1} - 7a_{n-3} + 5a_{n-4}$.

2. Suppose that the characteristic equation of a degree 6 recurrence relation can be partially factored as

$$(x^2 - 3x - 10)(x^2 - 6x + 5)^2$$

   State the general form for solutions to this recurrence relation.

3. Find the general solution for each recurrence relation.

   (a) $a_n = 5a_{n-1}$

   (b) $a_n = 8a_{n-1} - 12a_{n-2}$

   (c) $a_n = -6a_{n-1} - 9a_{n-2}$

4. Solve the following recurrence relations with initial conditions.

   (a) $a_n = 4a_{n-1}$, $a_0 = 16$

   (b) $a_n = 4a_{n-1} + 5a_{n-2}$, $a_0 = 2$, $a_1 = -1$

   (c) $a_n = 4a_{n-2}$, $a_0 = 0$, $a_1 = 8$

   (d) $a_n = 4_{n-1} - 4a_{n-2}$, $a_0 = 1$, $a_1 = 1$

5. Consider the recurrence relation $a_n = 4a_{n-1} - 6n + 5$.

   (a) Prove that $a_n = 2n + 1$ is a particular solution for this recurrence relation using the method of section 2.4.

   (b) Find the general solution for the recurrence relation.

6. Consider the recurrence relation $a_n = 7a_{n-1} - 10a_{n-2} - 2 \cdot 3^n$.

   (a) Prove that $a_n = 3^{n+2}$ is a particular solution for this recurrence relation using the method of section 2.4.

   (b) Find the general solution for the recurrence relation.

7. (Optional) Suppose that it's known that $a_n = bn + c$ is a solution for the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2} + 2n$ for some real numbers b, c. Determine what the values of b, c must be. Then find general solution for the recurrence relation.

Answers

1. Degree 4; characteristic equation is $x^4 - 3x^3 + 7x - 5 = 0$

   (*Note:* You can use r for the variable as the book does, if you wish.)

2. $a_n = (\alpha_0 + \alpha_1 n + \alpha_2 n^2)5^n + \beta(-2)^n + (\gamma_0 + \gamma_1 n)$

   (*Hint:* Factor the characteristic equation completely before proceeding.)

3. (a) $a_n = \alpha \cdot 5^n$

   (b) $a_n = \alpha \cdot 2^n + \beta \cdot 6^n$

   (c) $a_n = (\alpha + \beta n)(-3)^n$

4. (a) $a_n = 4^{n+2}$

   (b) $a_n = \dfrac{5^n + 11(-1)^n}{6}$

   (c) $a_n = 2^{n+1} + (-2)^{n+1}$

   (d) $a_n = (1 - \frac{n}{2})2^n$

5. (b) $a_n = \alpha 4^n + 2n + 1$

6. (b) $a_n = 3^{n+2} + \alpha \cdot 2^n + \beta \cdot 5^n$

7. $b = 1, c = 7/2$

   General solution for recurrence: $a_n = n + \frac{7}{2} + \alpha \cdot 3^n + \beta \cdot 2^n$