MATH.3860

Prob and Stats – Test 2 Spring 18   Name: SOLNS PRASAD

1. (20 points) Let X be a continuous random variable on the interval $[1, e]$ with cumulative distribution function (cdf)

$$F_X(x) = \ln x, \quad 1 < x < e.$$

   (a) Find a formula for the pdf $f_X(x)$. $4x$ for $1<x<e$
   (b) Compute the probability $P(X < 2)$. $\ln 2$
   (c) Find $P(2 < X \leq 2.5)$ $\ln 2.5 - \ln 2 = \ln(1.25)$
   (d) Find $P(2 < X < 2.5)$.   "

2. (20 points) (a) Find the variance of the random variable X, where the pdf of X is

$$f_X(x) = 3(1 - x)^2, \quad 0 < x < 1$$

$3/80$

   (b) If Y denotes the temperature recorded in degrees Fahrenheit, then $\frac{5}{9}(Y - 32)$ is the corresponding temperature in degrees Celsius. If the standard deviation for a set of temperatures is 15.7 degrees F, what is the standard deviation of the equivalent Celsius temperatures? (Hint: first consider the relation between the variances.)
   $(5/9)(15.7)$

3. (20 points) On planet Alpha, the prison sentence X (in years) of persons convicted of cheating on probability exams has the pdf

$$f_X(x) = \frac{1}{9}x^2 \quad 0 < x < 3$$

   (a) What is the *average* length of time these cheaters spend in jail? $81/36$
   (b) What is the *median* time in jail (the number m so that $P(X < m) = P(X > m)$)
   $(18)^{1/4}$

4. (10 points) If a typist averages one misspelling in every 3250 words, what are the chances a 6000 word report is free of all such errors? Answer the question two ways — first by using an exact binomial analysis, and second by using a Poisson approximation. not covered

Bin: $P(X=0) = \binom{6000}{0}\left(\frac{1}{3250}\right)^0\left(\frac{3249}{3250}\right)^{6000} \approx .158$

5. (10 points) Assume that the number of hits, X, that a baseball team makes in a nine-inning game has a Poisson distribution. If the probability that a team makes 0 hits is 1/3, what are the chances of getting two or more hits? $1-P(X=0)-P(X=1)$ Poisson not covered

6. (10 points) A bleary eyed student awakens one morning late for an 8:00 class, and pulls out two socks out of a drawer that contains two black, six brown and two blue socks, all randomly arranged. Compute the probability that the two he draws are a matched pair.
   $\frac{1+15+1}{45} \cong .378$

7. (10 points) Five cards are dealt from a standard poker deck. Let X be the number of aces received, and Y the number of kings in the hand. Compute the conditional probability $P(X = 2|Y = 2)$. $\frac{P(X=2,Y=2)}{P(Y=2)} = \frac{\binom{4}{2}\binom{4}{2}\binom{44}{1}}{\binom{4}{2}\binom{48}{3}} = .015$