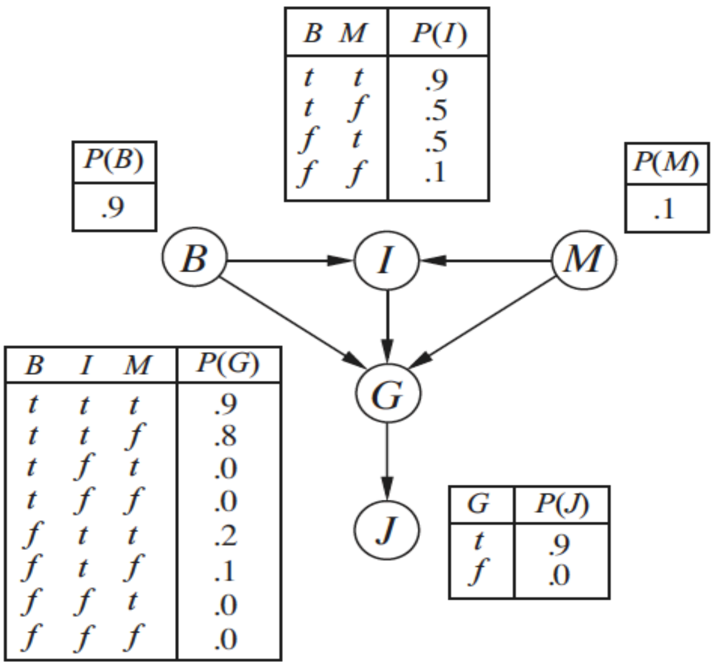
NAME:
DATE:

1. Consider the belief network shown below:



P(B)
.9

B	M	P(I)
t	t	.9
t	f	.5
f	t	.5
f	f	.1

P(M)
.1

B	I	M	P(G)
t	t	t	.9
t	t	f	.8
t	f	t	.0
t	f	f	.0
f	t	t	.2
f	t	f	.1
f	f	t	.0
f	f	f	.0

G	P(J)
t	.9
f	.0

Fig 1: A simple Bayes net with Boolean variables B =BrokeElectionLaw, I =Indicted M=PoliticallyMotivatedProsecutor , G=FoundGuilty , J =Jailed

a) (4 points) Which of the following are asserted by the network structure?
   (i) $P(B, I, M) = P(B)P(I)P(M)$.

   (ii) $P(J \mid G) = P(J \mid G, I)$.

   (iii) $P(M \mid G, B, I) = P(M \mid G, B, I, J)$.

b) (5 points) Calculate the value of $P(b, i, \neg m, g, j)$.

c) (6 points) Calculate the probability that someone goes to jail given that they broke the law, have been indicted, and face a politically motivated prosecutor.

d) (5 points) Suppose we want to add the variable P =PresidentialPardon to the network; draw the new network and briefly explain any links you add.

SOLUTION FOR Q1

a. The network asserts (ii) and (iii). (For (iii), consider the Markov blanket of M.)

b. $P(b, i, \neg m, g, j) = P(b)P(\neg m)P(i|b, \neg m)P(g|b, i, \neg m)P(j|g)$
$= 0.9 * 0.9 * 0.5 * 0.8 * 0.9 = 0.2196$

c. Since B, I,M are fixed true in the evidence, we can treat G as having a prior of 0.9 and just look at the sub-model with G, J:

$$P(J|b, i,m) = \alpha\, \Sigma_g\, P(J, g) = \alpha[P(J, g) + P(J,\neg g)]$$

$$= \alpha[\langle P(j, g), P(\neg j, g)\rangle + \langle P(j,\neg g), P(\neg j,\neg g)\rangle$$

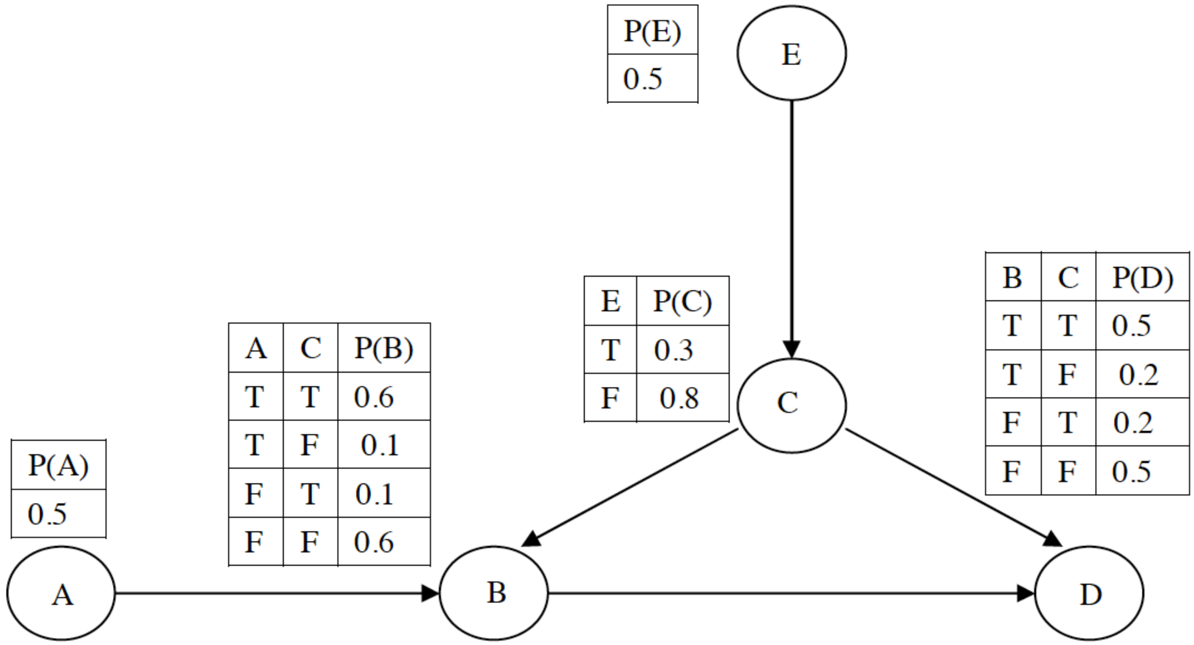
$$= \alpha[\langle .81, .09\rangle + \langle 0, 0.1\rangle] = \langle .81, .19\rangle$$

That is, the probability of going to jail is 0.81.

d. A pardon is unnecessary if the person is not indicted or not found guilty; so I and G are parents of P. One could also add B and M as parents of P, since a pardon is more likely if the person is actually innocent and if the prosecutor is politically motivated.

(There are other causes of *Pardon*, such as *LargeDonationToPresidentsParty*, but such variables are not currently in the model.) The pardon (presumably) is a get out-of-jail-free card, so P is a parent of J.*)*

2. (10 points) Given the Bayes net shown in the Figure below; A, B, C, D, and E are all Boolean variables. P(A="T") is simply denoted as P(A).



P(E)
0.5

E	P(C)
T	0.3
F	0.8

P(A)
0.5

A	C	P(B)
T	T	0.6
T	F	0.1
F	T	0.1
F	F	0.6

B	C	P(D)
T	T	0.5
T	F	0.2
F	T	0.2
F	F	0.5

Fig 2: Bayes Net

Note: In the *following, the notations:* $A \perp B$ *means A is independent of B;* $A \perp B \mid C$ *means A is conditionally independent of B given C.*

a. Please judge if the following independence assumptions are correct or not:
   i. (1 point) $B \perp E \mid C$ ----TRUE
   ii. (1 point) $A \perp D$ -- FALSE
   iii. (2 point) $A \perp D \mid B$ --FALSE
   iv. (2 point) $A \perp D \mid B, C$ --TRUE

b. (2 point) Compute the value of P(C)

   0.55

c. (2 point) Compute the value of P(B|A)

   0.375

3. Consider the Bayesian network below

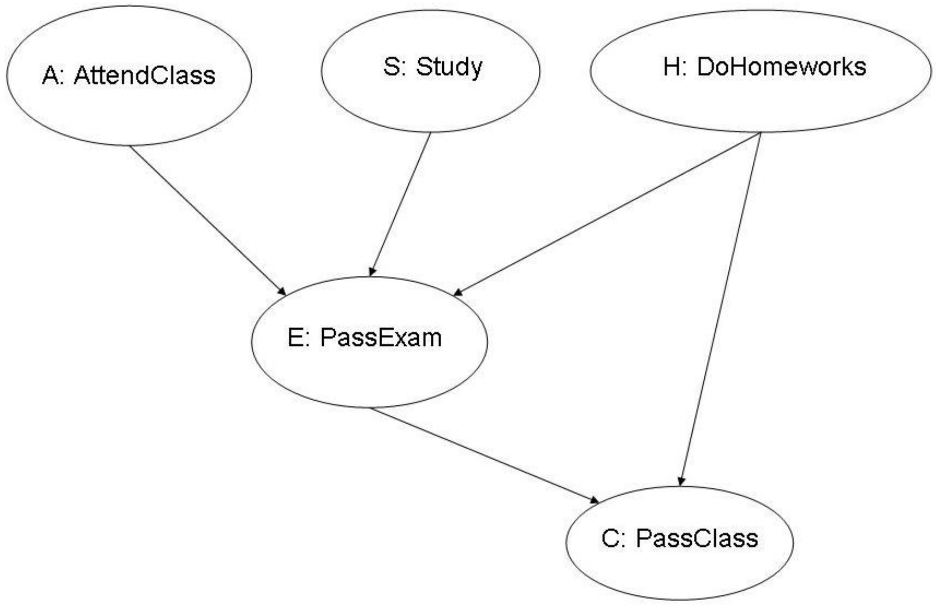


Fig 3: Bayes net for studying and passing

a) (5points) Write down the joint distribution as it factorizes according to the graph below.

b) (5 points) Use variable elimination and your result from (a) above to write down the expression for the probability of passing the class, given that you attend class and study, but don't do the home works.

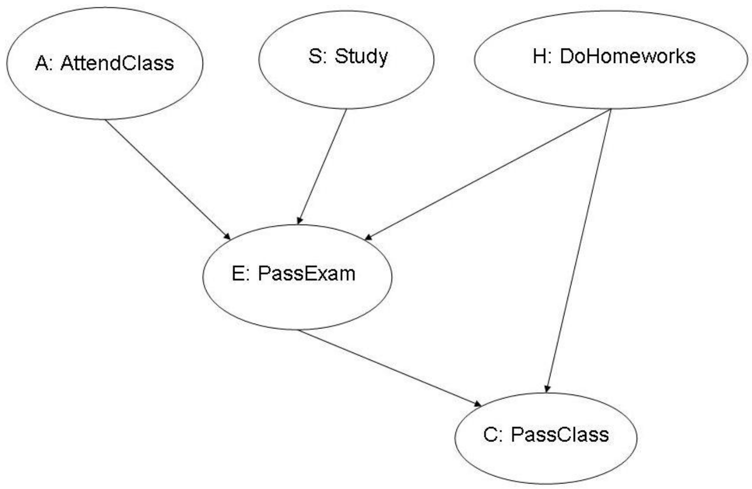
c) (10 points) Use the following CPTs for the graph of question 1 to compute P (A|C,H).

$$P(A) = 0.5, P(S) = 0.7, P(H) = 0.9$$

A	S	H	P (E|A, S,
0	0	0	0.2
0	0	1	0.5
0	1	0	0.4
0	1	1	0.8
1	0	0	0.3
1	0	1	0.7
1	1	0	0.6
1	1	1	0.9

E	H	P (C|E, H)
0	0	0.1
0	1	0.4
1	0	0.3
1	1	0.9

SOLUTION FOR Q3



$$P(A,S,H,E,C) = P(A) * P(S) * P(H) * P(E|A,S,H) * P(C|E,H)$$

b. (5 points) Use variable elimination and your result from the previous question to write down the expression for the probability of passing the class, given that you attend class and study, but don't do the homeworks.

$$\begin{aligned}
P(C|A,S,\neg H) &= \frac{P(C,A,S,\neg H)}{P(A,S,\neg H)} \\
&= \frac{\Sigma_e P(A,S,\neg H, E=e, C))}{\Sigma_{e,c} P(A,S,\neg H, E=e, C=c)} \\
&= \frac{\Sigma_e P(A) * P(S) * P(\neg H) * P(E=e|A,S,\neg H) * P(C|E=e,\neg H)}{\Sigma_{e,c} P(A) * P(S) * P(\neg H) * P(E=e|A,S,\neg H) * P(C=c|E=e,\neg H)} \\
&= \frac{P(A) * P(S) * P(\neg H) * \Sigma_e P(E=e|A,S,\neg H) * P(C|E=e,\neg H)}{P(A) * P(S) * P(\neg H) * \Sigma_e P(E=e|A,S,\neg H) * \Sigma_c P(C=c|E=e,\neg H)} \\
&= \frac{P(A) * P(S) * P(\neg H) * \Sigma_e P(E=e|A,S,\neg H) * P(C|E=e,\neg H)}{P(A) * P(S) * P(\neg H)} \\
&= \Sigma_e P(E=e|A,S,\neg H) * P(C|E=e,\neg H)
\end{aligned}$$

c. (10 points) Use the following CPTs for the graph of question 1 to compute $P(A|C, H)$.

$P(A) = 0.5, P(S) = 0.7, P(H) = 0.9$

A	S	H	$P(E\|A, S, H)$
0	0	0	0.2
0	0	1	0.5
0	1	0	0.4
0	1	1	0.8
1	0	0	0.3
1	0	1	0.7
1	1	0	0.6
1	1	1	0.9

E	H	$P(C\|E, H)$
0	0	0.1
0	1	0.4
1	0	0.3
1	1	0.9

$$
\begin{aligned}
P(A|C, H) &= \frac{P(A, C, H)}{P(C, H)} \\
&= \frac{\Sigma_{e,s} P(A, S = s, H, E = e, C)}{\Sigma_{a,e,s} P(A = a, S = s, H, E = e, C)} \\
&= \frac{\Sigma_{e,s} P(A) * P(S = s) * P(H) * P(E = e|A, S = s, H) * P(C|E = e, H)}{\Sigma_{a,e,s} P(A = a) * P(S = s) * P(H) * P(E = e|A = a, S = s, H) * P(C|E = e, H)} \\
&= \frac{P(A) * P(H) * \Sigma_s P(S = s) * \Sigma_e P(E = e|A, S = s, H) * P(C|E = e, H)}{P(H) * \Sigma_a P(A = a) * \Sigma_s P(S = s) * \Sigma_e P(E = e|A = a, S = s, H) * P(C|E = e, H)} \\
&= \frac{P(A) * \Sigma_s P(S = s) * \Sigma_e P(E = e|A, S = s, H) * P(C|E = e, H)}{\Sigma_a P(A = a) * \Sigma_s P(S = s) * \Sigma_e P(E = e|A = a, S = s, H) * P(C|E = e, H)} \\
&= \frac{.5(.7(.9 * .9 + .1 * .4) + .3(.7 * .9 + .3 * .4))}{(.5(.7(.9 * .9 + .1 * .4) + .3(.7 * .9 + .3 * .4)) + .5(.7(.8 * .9 + .2 * .4) + .3(.5 * .9 + .5 * .4)))} \\
&= \frac{0.41}{0.7875} \\
&= 0.5206
\end{aligned}
$$