

# Solution

Name:

## Linear Algebra: Quiz 3

Show ALL work, as unjustified answers may receive no credit. Calculators are not allowed on any quiz or test paper. Make sure to exhibit skills discussed in class. Box all answers and clean up answers as much as possible.

1. Consider the following Linear System:

$$\begin{cases} 4x_1 + 2x_2 + 6x_3 = 2 \\ 2x_1 + 1x_2 + 3x_3 = 1 \\ -6x_1 - 3x_2 - 9x_3 = -3 \end{cases}$$

(a) [6pts] Find the solution set of the given nonhomogeneous system in the parametric-vector form.

(b) [2pts] Using your solution for part (a), find the solution set of the corresponding homogeneous system in the parametric-vector form.

(c) [2pts] Without sketching a graph, use a geometric interpretation to describe the two solutions sets.

(a) Row-reduce  $[A | \vec{b}]$  to rref:

$$\begin{bmatrix} 4 & 2 & 6 & | & 2 \\ 2 & 1 & 3 & | & 1 \\ -6 & -3 & -9 & | & -3 \end{bmatrix} \xrightarrow[\sim]{\begin{matrix} \frac{1}{2}R_1 \\ -\frac{1}{3}R_3 \end{matrix}} \begin{bmatrix} 2 & 1 & 3 & | & 1 \\ 2 & 1 & 3 & | & 1 \\ 2 & 1 & 3 & | & 1 \end{bmatrix} \xrightarrow[\sim]{\begin{matrix} -R_1 \\ -R_2 \\ -R_3 \end{matrix}} \begin{bmatrix} 2 & 1 & 3 & | & 1 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1} \begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} & | & \frac{1}{2} \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$$\Rightarrow \begin{cases} x_1 = \frac{1}{2} - \frac{1}{2}x_2 - \frac{3}{2}x_3 \\ x_2 \text{ is free} \\ x_3 \text{ is free} \end{cases}$$

$$\therefore \vec{x} = \begin{bmatrix} \frac{1}{2} \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -\frac{3}{2} \\ 0 \\ 1 \end{bmatrix}$$

st  $x_2, x_3 \in \mathbb{R}$

Ans

(b) Find  $\vec{x}$  for  $A\vec{x} = \vec{0}$ : Removing  $\begin{bmatrix} \frac{1}{2} \\ 0 \\ 0 \end{bmatrix}$  we get,

$$\vec{x} = x_2 \begin{bmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -\frac{3}{2} \\ 0 \\ 1 \end{bmatrix}, \quad x_2, x_3 \in \mathbb{R}$$

Ans

(c) Describe the solution sets:

The Solution Sets are parallel planes in  $\mathbb{R}^3$ ; Both contain the vectors  $\langle -\frac{1}{2}, 1, 0 \rangle$  &  $\langle -\frac{3}{2}, 0, 1 \rangle$ , but  $A\vec{x} = \vec{b}$ 's solution passes through the pt.  $(\frac{1}{2}, 0, 0)$

Valentine's Day Bonus (+1):

Fill in the blank with a specific, mathematical-example ☺

Love is like  $\pi$ , it is real irrational and neverending

\*Any irrational number works!

Ans