

Q1.

Credit : Navaneeth Chandrasekaran

Compare Function:

- Comparing insertion sort and Merge sort.

n - input of size - n.

Insertion sort running time steps - $8n^2$.

Merge sort running in steps - $64n \lg n$.

Value of n, where insertion sort beat Merge sort,

$$(i) \quad 8n^2 < 64n \lg n$$

$$\Rightarrow n^2 < 8n \lg n.$$

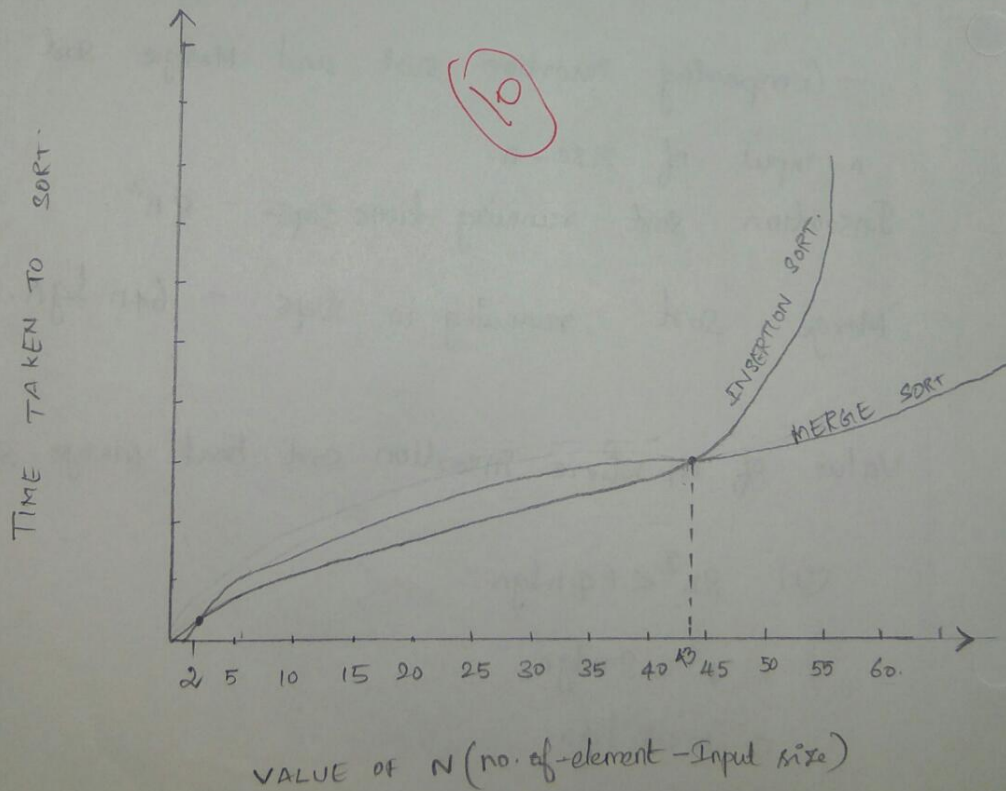
$$\Rightarrow n < 8 \lg n.$$

$$2^{n/8} < n$$

$$\left[\begin{array}{l} \because \lg_{10} x = y \\ 10^y = x \end{array} \right]$$

After doing trial and error method in the above function by substituting value of n from $1 \rightarrow 44$, it is finally proved that,

$$2^{n/8} < n \text{ is true for } \boxed{2 \leq n \leq 43.}$$



Q2.

Credit: Christopher Primes

Question 2

2.1-3

Consider the *searching problem*:**Input:** A sequence of n numbers $A = \langle a_1, a_2, \dots, a_n \rangle$ and a value v .**Output:** An index i such that $v = A[i]$ or the special value NIL if v does not appear in A .Write pseudocode for *linear search*, which scans through the sequence, looking for v . Using a loop invariant, prove that your algorithm is correct. Make sure that your loop invariant fulfills the three necessary properties.

```

Linear_Search(A, v)
1   for i = 0 to A.length
2       if A[i] == v
3           return i
4   return NIL

```

Loop invariant:	At the start of each iteration of the for loop, $A[i] \neq v$.
Initialization:	The function holds true if the function has not run yet.
Maintenance:	The loop invariant holds true for every iteration. If a match is found, the function will return.
Termination:	When the function stops running, the function will either return an index, or NIL, which are both valid values.

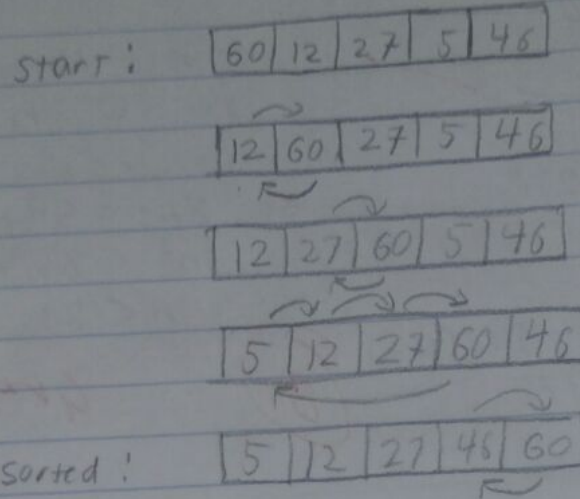
(15)

the array index should start from 1, not 0

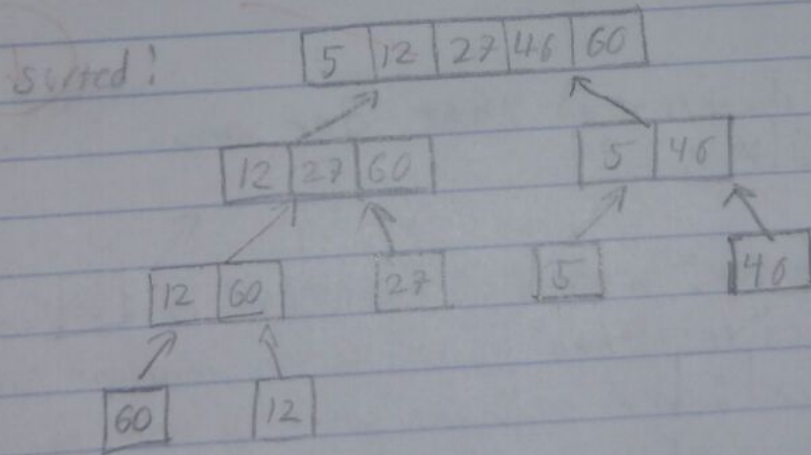
Q3.

Credit: Wesley Wuzzo

3.) insertion sort:



merge sort:



Q4.

Credit: Wesley Wuzzo

4.) $T(n) = 3T(\frac{n}{3}) + 3cn^2$

\nearrow recursive calls \nearrow nested for loops

recursion tree:

level	$T(?)$	leaf cost	# leaves	total cost
1	$T(n)$	$3cn^2$	1	$3cn^2$
2	$T(\frac{n}{3})$	$\frac{1}{3}cn^2$	3	cn^2
3	$T(\frac{n}{9})$	$\frac{1}{27}cn^2$	9	$\frac{1}{3}cn^2$
...
i	$T(\frac{n}{3^{i-1}})$	$3^{2-i}cn^2$	3^{i-1}	$3^{2-i}cn^2$

$T(\frac{n}{3^{i-1}}) = c$ where $\frac{n}{3^{i-1}} = 1$
 $n = 3^{i-1}$
 $\log_3 n = i-1$
 $i = \log_3 n + 1$

$T(n) = \sum_{i=1}^{\log_3 n + 1} 3^{2-i} cn^2$

(4 cont.)

$$T(n) = \sum_{i=1}^{\log_3 n + 1} 3^{2-i} c n^2$$

$$= 3^2 c n^2 \sum_{i=1}^{\log_3 n + 1} \frac{1}{3^i} = 9 c n^2 \left(\sum_{i=1}^{\log_3 n + 1} \left(\frac{1}{3}\right)^i - 1 \right)$$

$$= 9 c n^2 \left(\left(\frac{\left(\frac{1}{3}\right)^{\log_3 n + 2} - 1}{\frac{1}{3} - 1} \right) - 1 \right)$$

$$= 9 c n^2 \left(\frac{-3}{2} \left(\frac{1}{3^{\log_3 n + 2}} - 1 \right) - 1 \right)$$

$$= 9 c n^2 \left(\frac{-3}{2} \left(\frac{1}{9n} - 1 \right) - 1 \right)$$

$$= 9 c n^2 \left(\frac{-3}{2} \left(\frac{1 - 9n}{9n} \right) - 1 \right)$$

$$= 9 c n^2 \left(\frac{-(1 - 9n)}{6n} - 1 \right)$$

$$= 9 c n^2 \left(\frac{9n - 1 - 6n}{6n} \right)$$

$$= 9 c n^2 \left(\frac{3n - 1}{6n} \right)$$

$$= 3cn(3n-1) = 9cn^2 - 3cn$$

$$\Rightarrow O(n^2)$$

The range of i is not very accurate. I will review it in the class.

Q5.

Credit: Sai Krishnan Kiran

5. Algorithm Design

(a) Array with the elements in the reverse order
 i.e., $n, n-1, n-2, \dots, 3, 2, 1$ from the set $\{1, 2, 3, \dots, n\}$
 has the most number of inversions.

Number of inversions = Sum of all the terms till $(n-1)$

$$= 1+2+3+\dots+(n-1)$$

$$= \frac{n(n+1)}{2} - n$$

$$= \frac{n^2+n-2n}{2}$$

$$= \frac{n(n-1)}{2}$$

(b) Number of inversions:

INVERSIONSCOUNT(A, P, Q, R)

1. $n_1 = Q - P + 1$
2. $n_2 = R - Q$
3. Let $L[1 \dots n_1+1]$ and $R[1 \dots n_2+1]$ be new arrays.
4. for $i = 1$ to n_1
5. $L[i] = A[P+i-1]$
6. for $j = 1$ to n_2
7. $R[j] = A[Q+j]$
8. $L[n_1+1] = \infty$
9. $R[n_2+1] = \infty$
10. $i = 1$
11. $j = 1$

HW1

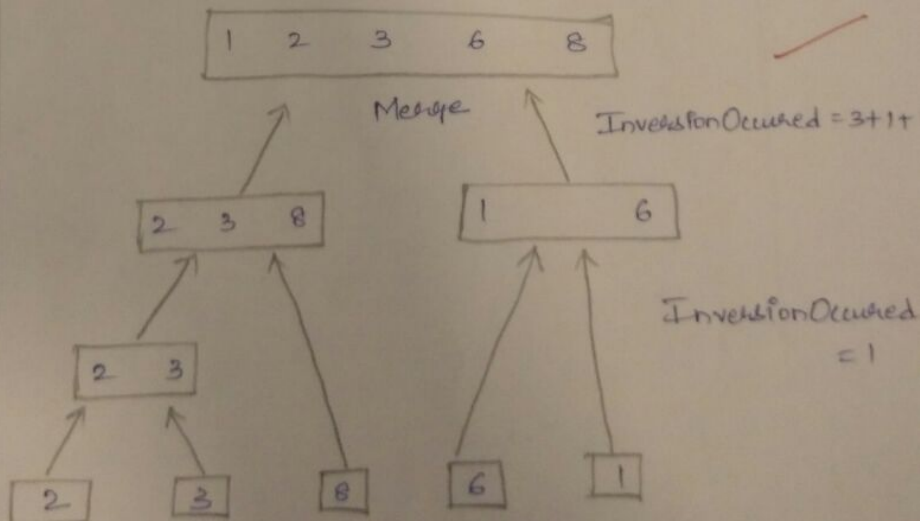
design

```

12. InversionOccurred = 0
13. count = FALSE
14. for K = p to r
15.     if count == FALSE and  $R[i] < L[i]$ 
16.         InversionOccurred +=  $n - i + 1$ 
17.         count = TRUE
18.     if  $L[i] \leq R[j]$ 
19.          $A[k] = L[i]$ 
20.          $i = i + 1$ 
21.     else  $A[k] = R[j]$ 
22.          $j = j + 1$ 
23.     count = FALSE
24. return InversionOccurred
    
```

c. Working of the above algorithm on the array

$\langle 2, 3, 8, 6, 1 \rangle$



(P.T.O)

Total number of inversion in the
above array is

$$\text{inversion Occured} = 3 + 1 + 1 \\ = 5.$$