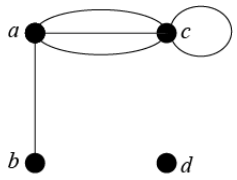
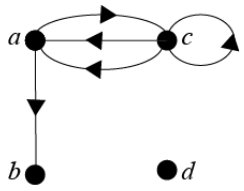
Section 10.3 Homework

1. Find the adjacency matrix for the graph below using the vertex order a, b, c, d.



2. Find the adjacency matrix for the directed graph below using the vertex order a, b, c, d.

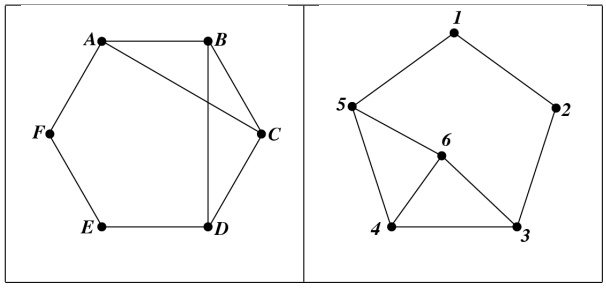


3. Suppose that the adjacency matrix of a graph G is $A = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ using the vertex order $V = \{1, 2, 3\}$. Draw the corresponding graph.

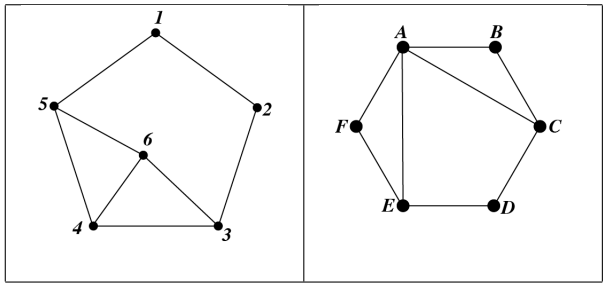
4. Suppose that the adjacency matrix of a directed graph G is $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ using the vertex order $V = \{1, 2, 3\}$. Draw the corresponding directed graph.

5. Consider the pair of graphs in each part. If the graphs are isomorphic, then find a graph isomorphism from the first graph to the second graph. If the graphs are not isomorphic, explain why no isomorphism exists.

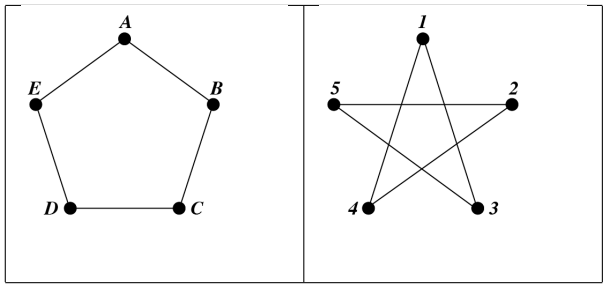
(a)



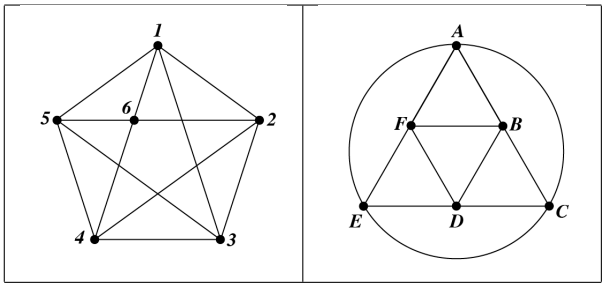
(b)



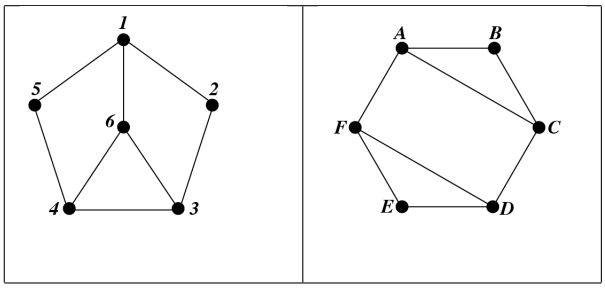
(c)



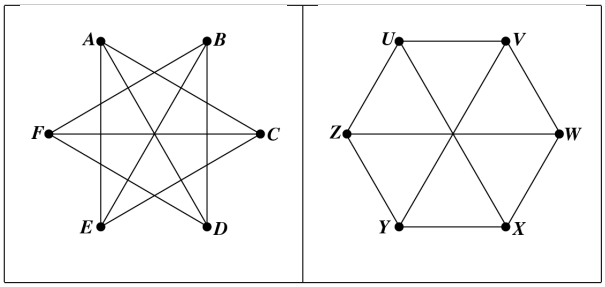
(d)



(e)



(f)



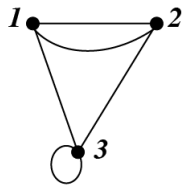
6. (*Optional*) Suppose that G and H are two simple graphs with n vertices, each of which have degree $n-2$. Prove that G and H must be isomorphic. (*Note:* n must be even – why?)

Answers:

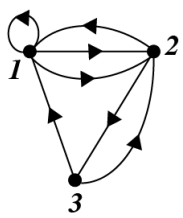
1. $A = \begin{bmatrix} 0 & 1 & 3 & 0 \\ 1 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

2. $A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

3.



4.



5. (a) isomorphic. An isomorphism is $f(A) = 3$, $f(B) = 6$, $f(C) = 4$, $f(D) = 5$, $f(E) = 1$, and $f(F) = 2$.

   (b) not isomorphic

   (c) isomorphic. An isomorphism is $f(A) = 1$, $f(B) = 3$, $f(C) = 5$, $f(D) = 2$, and $f(E) = 4$.

   Note: There are other isomorphisms! For example, $f(A) = 3$, $f(B) = 5$, $f(C) = 2$, $f(D) = 4$, and $f(E) = 1$ is also a valid isomorphism. How many different isomorphisms are there?

   (d) isomorphic

   (e) not isomorphic

   (f) not isomorphic