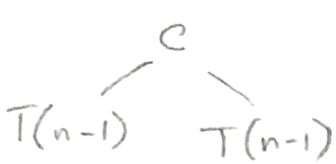
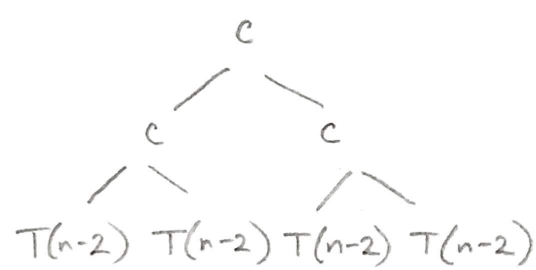
Name: (Print) PHONG VO

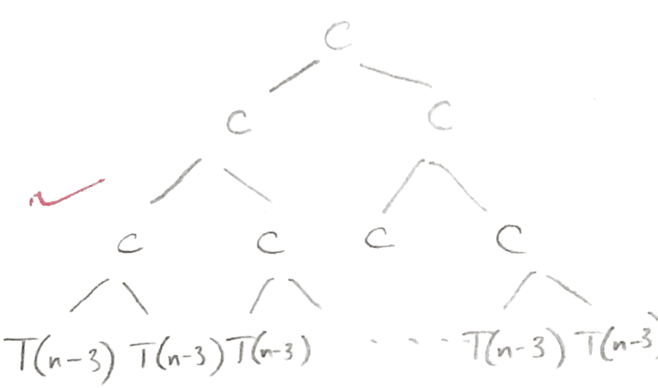
2. **Recurrence** (15 points) Derive a recurrence for the running time of the algorithm below and then solve the recurrence with one of the three methods we have learned. The solution should be a tight upper and lower bound solution. You must show the detailed work of how to solve the recurrence. You do NOT need to write the algorithm.

An algorithm solves a problem of size n by recursively solving **two** sub-problems of size (n – 1), and then combining the solutions in constant time.

$$T(n) = 2T(n-1) + \theta(1)$$
$$= 2T(n-1) + c \quad (c: \text{pos. const})$$

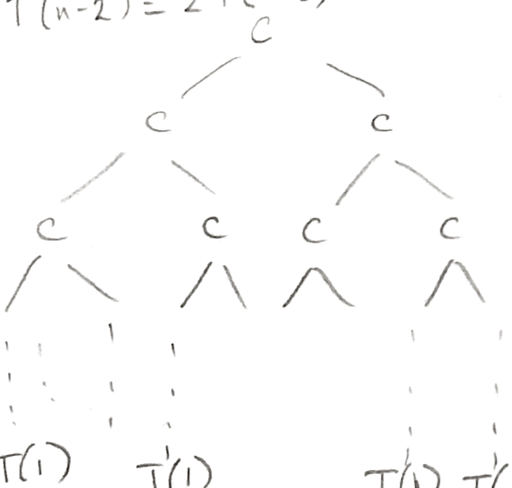






$T(n-1) = 2T(n-2) + c$

$T(n-2) = 2T(n-3) + c$



Level	Total cost
1	c
2	2c
3	4c
4	8c
i	$2^{i-1}c$
n	$2^n c$

$$T(n) = c + 2c + 4c + 8c + \dots + 2^{n-1}c$$
$$= c(1 + 2^1 + 2^2 + 2^3 + \dots + 2^{n-1})$$
$$= c\frac{2^n}{2-1} = c \cdot 2^n = \theta(2^n)$$

$T(1): \quad 2^{i-1} = n \Rightarrow i-1 = \lg n$ −4

Substitution:
$T(n) = \theta(2^n)$
$T(n-1) = \theta(2^{n-1})$
$c\,2^{n-1}$

Upper bound:
$T(n) = O(2^n)$
$T(n-1) \le d2^{n-1}$
Substitution: $T(n) \le 2d\,2^{n-1} + c$
$\le d2^n + c$
$\le d2^n$ if $c \le 0$
not possible

New guess
$T(n) \le d2^n - d'$
$T(n-1) \le d2^{n-1} - d'$
$T(n) \le 2(d2^{n-1} - d') + c$
$\le d2^n - 2d' + c$
$\le d2^n - d' - d' + c$
$\le d2^n - d'$ if
$-d' + c \le 0$
$0 < c \le d'$

0 4. **Analysis of an Algorithm (20 points)**

You are given an array A, which stores *n* distinct numbers (n>=2). There is a mystery function called Mystery(A, n) that works on the array. The pseudocode of the algorithm is shown as below.

Please analyze the worst-case asymptotic execution time of this algorithm. (1) List the cost for executing each line of code and the number of executions for each line; and then derive a recurrence of the running time; (2) solve the recurrence by using one of the methods we have learned; must show your work clearly.

```
Mystery(A, n)
    return Mystery_helper(A, 1, n)


Mystery_helper (A, p, r)
  if p==r-1
      if A[p] > A[r]
          return p
      else return r
  q = ⌊ (p+r)/2 ⌋
  if A[q] < A[q+1]
      return Mystery_helper (A, q+1, r)
  else
      return Mystery_helper (A, p, q)
```

	c_1	1
	c_2	2
	c_3	3
	c_4	4
	c_5	5
	c_6	6
$T(\frac{n}{2})$	c_7	7
	c_8	8
$T(\frac{n}{2})$	c_9	9

$$T(n) = c_1 + c_2 + c_3\,(\text{or } c_4) + \cancel{c_5 + c_6} + T(\tfrac{n}{2})$$

-20

$$= T(\tfrac{n}{2}) + c$$

$a = 1 \quad b = 2 \qquad n^{\log_b a} = n^{\log_2 1} = n^0 = 1$

n^0 vs c

case 2 $\Rightarrow \Theta(\lg n)$

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