Student: Phong Vo **Date:** 02/19/20	**Instructor:** Erica Yankowskas **Course:** Linear Algebra I (Spring 2020)	**Assignment:** Section 1.9 Homework

1. Assume that T is a linear transformation. Find the standard matrix of T.

   T: $\mathbb{R}^2 \to \mathbb{R}^4$, $T(\mathbf{e}_1) = (8, 1, 8, 1)$, and $T(\mathbf{e}_2) = (-5, 9, 0, 0)$, where $\mathbf{e}_1 = (1,0)$ and $\mathbf{e}_2 = (0,1)$.

   $$A = \begin{bmatrix} 8 & -5 \\ 1 & 9 \\ 8 & 0 \\ 1 & 0 \end{bmatrix}$$ (Type an integer or decimal for each matrix element.)

2. Assume that T is a linear transformation. Find the standard matrix of T.

   T: $\mathbb{R}^3 \to \mathbb{R}^2$, $T(\mathbf{e}_1) = (1,3)$, and $T(\mathbf{e}_2) = (-5,7)$, and $T(\mathbf{e}_3) = (3,-8)$, where $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ are the columns of the 3×3 identity matrix.

   $$A = \begin{bmatrix} 1 & -5 & 3 \\ 3 & 7 & -8 \end{bmatrix}$$ (Type an integer or decimal for each matrix element.)

3. Assume that T is a linear transformation. Find the standard matrix of T.

   T: $\mathbb{R}^2 \to \mathbb{R}^2$, rotates points (about the origin) through $\frac{5\pi}{3}$ radians.

   $$A = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

   (Type an integer or simplified fraction for each matrix element. Type exact answers, using radicals as needed.)

4. Assume that T is a linear transformation. Find the standard matrix of T.

   T: $\mathbb{R}^2 \to \mathbb{R}^2$, first performs a horizontal shear that transforms $\mathbf{e}_2$ into $\mathbf{e}_2 + 15\mathbf{e}_1$ (leaving $\mathbf{e}_1$ unchanged) and then reflects points through the line $x_2 = -x_1$.

   $$A = \begin{bmatrix} 0 & -1 \\ -1 & -15 \end{bmatrix}$$

   (Type an integer or simplified fraction for each matrix element.)

5. Show that the transformation T: $\mathbb{R}^2 \to \mathbb{R}^2$ that reflects points through the horizontal x_1-axis and then reflects points through the line $x_2 = x_1$ is merely a rotation about the origin. What is the angle of rotation?

If T: $\mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation, then there exists a unique matrix A such that the following equation is true.

$$T(\mathbf{x}) = A\mathbf{x} \text{ for all } \mathbf{x} \text{ in } \mathbb{R}^n$$

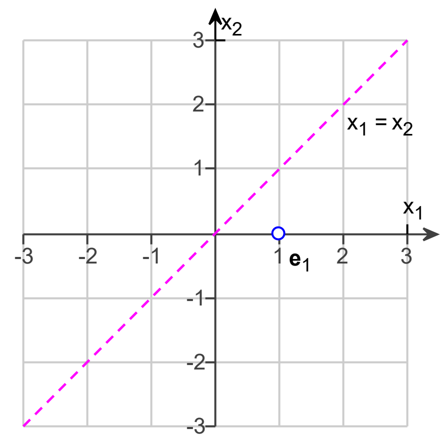
In fact, A is the $m \times n$ matrix whose jth column is the vector $T(\mathbf{e}_j)$, where $\mathbf{e}_j$ is the jth column of the identity matrix in $\mathbb{R}^n$, as shown in the following equation.

$$A = \begin{bmatrix} T(\mathbf{e}_1) & \cdots & T(\mathbf{e}_n) \end{bmatrix}$$

Find A by transforming the columns of the identity matrix, $\mathbf{e}_1$ and $\mathbf{e}_2$.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Reflect $\mathbf{e}_1$ through the horizontal x_1-axis and then through the line $x_2 = x_1$.

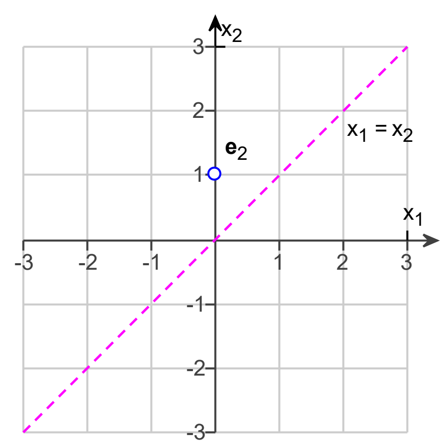


Use this plotted point to construct $T(\mathbf{e}_1)$.

$$T(\mathbf{e}_1) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

(Type an integer or simplified fraction for each matrix element.)

Reflect $\mathbf{e}_2$ through the horizontal x_1-axis and then through the line $x_2 = x_1$.



Use this plotted point to construct $T(\mathbf{e}_2)$.

$T(\mathbf{e}_2) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ (Type an integer or simplified fraction for each matrix element.)

Use the transformed columns to construct A.

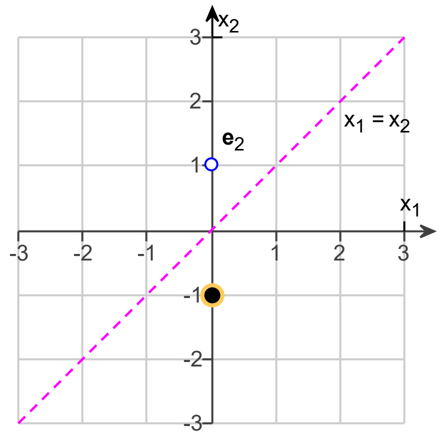
$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (Type an integer or simplified fraction for each matrix element.)

Compare this matrix to the $\mathbb{R}^2 \to \mathbb{R}^2$ rotation matrix, $\begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix}$, to determine the angle of roation, φ.

$\varphi = \frac{\pi}{2}$

(Simplify your answer. Type your answer in radians. Use angle measures greater than or equal to 0 and less than 2π.)

YOU ANSWERED:



6. Fill in the missing entries of the matrix, assuming that the equation holds for all values of the variables.

$$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4x_1 - 2x_2 \\ 4x_1 - x_3 \\ -x_2 + 4x_3 \end{bmatrix}$$

Fill in the missing entries of the matrix below.

$$\begin{bmatrix} 4 & -2 & 0 \\ 4 & 0 & -1 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4x_1 - 2x_2 \\ 4x_1 - x_3 \\ -x_2 + 4x_3 \end{bmatrix}$$

7. Show that T is a linear transformation by finding a matrix that implements the mapping. Note that x_1, x_2, ... are not vectors but are entries in vectors.

$$T(x_1,x_2,x_3,x_4) = (x_1 + 9x_2,\ 0,\ 7x_2 + x_4,\ x_2 - x_4)$$

$$A = \begin{bmatrix} 1 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 7 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$ (Type an integer or decimal for each matrix element.)

8. Show that T is a linear transformation by finding a matrix that implements the mapping. Note that x_1, x_2, ... are not vectors but are entries in vectors.

$$T(x_1,x_2,x_3) = (x_1 - 8x_2 + 3x_3,\ x_2 - 6x_3)$$

$$A = \begin{bmatrix} 1 & -8 & 3 \\ 0 & 1 & -6 \end{bmatrix}$$ (Type an integer or decimal for each matrix element.)

9. Let T: $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation such that $T(x_1,x_2) = (x_1 + x_2,\ 4x_1 + 2x_2)$. Find **x** such that T(**x**) = (1,14).

$$\mathbf{x} = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$$

10. Mark each statement True or False and justify each answer for parts **a** through **e**.

a. If A is a 4×3 matrix, then the transformation $\mathbf{x} \mapsto A\mathbf{x}$ maps $\mathbb{R}^3$ onto $\mathbb{R}^4$. Choose the correct answer below.

- **A.** False. The columns of A are not linearly independent.
- **B.** False. The columns of A do not span $\mathbb{R}^4$.
- **C.** True. The the columns of A are linearly independent.
- **D.** True. The columns of A span $\mathbb{R}^4$.

b. Every linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ is a matrix transformation. Choose the correct answer below.

- **A.** False. Not every vector $\mathbf{x}$ in $\mathbb{R}^n$ can be assigned to a vector $T(\mathbf{x})$ in $\mathbb{R}^m$.
- **B.** True. Every matrix transformation spans $\mathbb{R}^n$.
- **C.** True. There exists a unique matrix A such that $T(\mathbf{x}) = A\mathbf{x}$ for all $\mathbf{x}$ in $\mathbb{R}^n$.
- **D.** False. Not every image $T(\mathbf{x})$ is of the form $A\mathbf{x}$.

c. The columns of the standard matrix for a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ are the images of the columns of the $n \times n$ identity matrix under T. Choose the correct answer below.

- **A.** False. The standard matrix only has the trivial solution.
- **B.** True. The standard matrix is the identity matrix in $\mathbb{R}^n$.
- **C.** False. The standard matrix is the $m \times n$ matrix whose jth column is the vector $T\left(\mathbf{e}_j\right)$, where $\mathbf{e}_j$ is the jth column wh
- **D.** True. The standard matrix is the $m \times n$ matrix whose jth column is the vector $T\left(\mathbf{e}_j\right)$, where $\mathbf{e}_j$ is the jth column of th

d. A mapping $T: \mathbb{R}^n \mapsto \mathbb{R}^m$ is one-to-one if each vector in $\mathbb{R}^n$ maps onto a unique vector in $\mathbb{R}^m$. Choose the correct answer below.

- **A.** False. A mapping T is said to be one-to-one if each $\mathbf{b}$ in $\mathbb{R}^m$ is the image of at least one $\mathbf{x}$ in $\mathbb{R}^n$.
- **B.** True. A mapping T is said to be one-to-one if each $\mathbf{x}$ in $\mathbb{R}^n$ has at least one image for $\mathbf{b}$ in $\mathbb{R}^m$.
- **C.** False. A mapping T is said to be one-to-one if each $\mathbf{b}$ in $\mathbb{R}^m$ is the image of at most one $\mathbf{x}$ in $\mathbb{R}^n$.
- **D.** True. A mapping T is said to be one-to-one if each $\mathbf{b}$ in $\mathbb{R}^m$ is the image of exactly one $\mathbf{x}$ in $\mathbb{R}^n$.

e. The standard matrix of a horizontal shear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ has the form $\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$. Choose the correct answer below.

- **A.** False. The standard matrix has the form $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.
- **B.** False. The standard matrix has the form $\begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$.
- **C.** False. The standard matrix has the form $\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$.
- **D.** True.

11. Determine if the specified linear transformation is (**a**) one-to-one and (**b**) onto. Justify each answer.

$$T(x_1, x_2, x_3) = (x_1 - 3x_2 + 4x_3,\ x_2 - 9x_3)$$

(**a**) Is the linear transformation one-to-one?

- **A.** T is not one-to-one because the columns of the standard matrix A are linearly independent.
- **B.** T is not one-to-one because the columns of the standard matrix A are linearly dependent.
- **C.** T is one-to-one because T(x) = 0 has only the trivial solution.
- **D.** T is one-to-one because the column vectors are not scalar multiples of each other.

(**b**) Is the linear transformation onto?

- **A.** T is not onto because the standard matrix A does not have a pivot position for every row.
- **B.** T is onto because the columns of the standard matrix A span $\mathbb{R}^2$.
- **C.** T is not onto because the columns of the standard matrix A span $\mathbb{R}^2$.
- **D.** T is onto because the standard matrix A does not have a pivot position for every row.

12. Describe the possible echelon forms of the standard matrix for a linear transformation T where T: $\mathbb{R}^4 \to \mathbb{R}^3$ is onto.

Give some examples of the echelon forms. The leading entries, denoted ■, may have any nonzero value; the starred entries, denoted $*$, may have any value (including zero). Select all that apply.

A. $\begin{bmatrix} ■ & * & * & * \\ 0 & 0 & ■ & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$

B. $\begin{bmatrix} ■ & * & * & * \\ 0 & 0 & ■ & * \\ 0 & 0 & 0 & ■ \end{bmatrix}$

D. $\begin{bmatrix} ■ & * & * & * \\ 0 & ■ & * & * \\ 0 & 0 & ■ & * \end{bmatrix}$

E. $\begin{bmatrix} 0 & ■ & * & * \\ 0 & ■ & * & * \\ 0 & ■ & * & * \end{bmatrix}$

G. $\begin{bmatrix} 0 & 0 & ■ & * \\ 0 & 0 & 0 & ■ \\ 0 & 0 & 0 & 0 \end{bmatrix}$

H. $\begin{bmatrix} 0 & ■ & * & * \\ 0 & 0 & ■ & * \\ 0 & 0 & 0 & ■ \end{bmatrix}$