Prob. & Stat. Exam 2 - Review | Phong Vo

1. $F_X(x) = \ln x \qquad 1 < x < e$

(a) $f_X(x) = \frac{dF_Y(y)}{dy} = \boxed{1/x}$

(b) $P(X<2) = \int_1^2 f_X(x)dx = F_X(x)\Big|_1^2 = \ln x\Big|_1^2 = \ln 2 - \ln 1 = \boxed{\ln 2}$

(c) $P(2 < X \leq 2.5) = \int_2^{2.5} f_X(x)dx = F_X(x)\Big|_2^{2.5} = \ln x\Big|_2^{2.5} = \ln(2.5) - \ln(2)$
$= \ln\left(\frac{2.5}{2}\right) = \ln(1.25)$

(d) $P(2<X<2.5) = \int_2^{2.5} f_X(x)dx = \text{----} = \ln(1.25)$

2. Variance $= Var(x) = E(x^2) - [E(X)]^2 \qquad 0<x<1$

(a) $E(x^2) = \int_0^1 x^2 f_X(x)\,dx = \int_0^1 x^2 3\underbrace{(1-x)^2}_{x^2-2x+1} dx$

$= \int_0^1 (3x^4 - 6x^3 + 3x^2)dx = \left(\frac{3}{5}x^5 - \frac{3}{2}x^4 + x^3\right)\Big|_0^1 = \frac{3}{5} - \frac{3}{2} + 1$
$= \frac{6-15+10}{10} = \boxed{1/10}$

$E(x) = \int_0^1 x f_X(x)\,dx = \int_0^1 x\,3(1-x)^2 dx$

$= \int_0^1 (3x^3 - 6x^2 + 3x)\,dx = \left(\frac{3}{4}x^4 - 2x^3 + \frac{3}{2}x^2\right)\Big|_0^1 = \frac{3}{4} - 2 + \frac{3}{2}$
$= \frac{3-8+6}{4} = \boxed{1/4}$

$Var = E(x^2) - [E(x)]^2$

$= \frac{1}{10} - \left(\frac{1}{4}\right)^2 = \frac{1}{10} - \frac{1}{16} = \frac{8-5}{80} = \boxed{3/80}$

(3) $f_X(x) = \frac{1}{9}x^2 \quad 0 < x < 3$

a) $F_X(x) = \int f(x)\,dx = \int_0^3 \frac{1}{9}x^2\,dx = \frac{1}{27}x^3 \Big|_0^3 = 1$

$E(x) = \int x f_X(x)\,dx = \int_0^3 x \cdot \frac{1}{9}x^2\,dx = \int_0^3 \frac{1}{9}x^3\,dx = \frac{1}{36}x^4 \Big|_0^3 = \boxed{\frac{81}{36}}$

$E(x^2) = \int x^2 f_X(x)\,dx = \int_0^3 \frac{1}{9}x^4\,dx = \frac{1}{45}x^5 \Big|_0^3 = \frac{243}{45} = \frac{27}{5}$

(b) m = the median value

$P(X<m) = \int_0^m f(x)\,dx = \int_0^m \frac{1}{9}x^2\,dx = \frac{1}{27}x^3 \Big|_0^m = \frac{m^3}{27} = 0.5$

$\Rightarrow m^3 = 27 \cdot (.5) = 13.5 \Rightarrow m = \sqrt[3]{13.5}$

(4) $n = 6000 \quad p = \frac{1}{3250} \quad 1-p = 1 - \frac{1}{3250} = \frac{3249}{3250}$

$p(x=0) = \binom{n}{0}(p)^0(1-p)^{n-0} = \binom{6000}{0}\left(\frac{1}{3250}\right)^0\left(\frac{3249}{3250}\right)^{6000}$

$= (1) \times (1)(.1578) = \boxed{.1578}$

(5) $p(x=0) = 1/3, n=9 \Rightarrow \binom{9}{0} p^0 (1-p)^9 = \frac{1}{3} \Rightarrow (1)(1)(1-p)^9 = \frac{1}{3} \Rightarrow 1-p = \sqrt[9]{\frac{1}{3}} = .8851$

$\Rightarrow P = .1149$

$p(x=1) = \binom{9}{1}(.1149)^1(.8851)^8 = (9)(.1149)(.3767) = .3895$

The chances of getting two or more hit $= 1 - p(x=0) - p(x=1)$

$= 1 - \frac{1}{3} - .3895 = \boxed{.2772}$

(6)

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(6) 2 Blk + 6 Brown + 2 Blue

Black: $\frac{2}{10} \cdot \frac{1}{9} = \frac{1}{45}$ | Brown: $\frac{6}{10} \cdot \frac{5}{9} = \frac{1}{3}$

Blue: $\frac{2}{10} \cdot \frac{1}{9} = \frac{1}{45}$

$\Rightarrow$ The prob. that he picked a matched pair $= \frac{1}{45} + \frac{1}{3} + \frac{1}{45}$

$= \boxed{.378}$

(7) there are 4 aces out of 52-card deck.

$$P(X=2 \mid Y=2) = \frac{P(X=2 \text{ and } Y=2)}{P(Y=2)} = \frac{\binom{4}{2}\binom{4}{2}\binom{44}{1}}{\binom{4}{2}\binom{48}{3}}$$

$$= \frac{\left(\frac{4\cdot3\cdot2\cdot1}{2\cdot1\cdot2\cdot1}\right)\cdot\left(\frac{4\cdot3\cdot2\cdot1}{2\cdot1\cdot2\cdot1}\right)\cdot(44)}{\left(\frac{4\cdot3\cdot2\cdot1}{2\cdot1\cdot2\cdot1}\right)\frac{48\cdot47\cdot46}{3\cdot2\cdot1}} = \frac{(\cancel{6})(6)(44)}{(\cancel{6})(8\cdot47\cdot46)} = \boxed{.0153}$$