

CHAPTER 2

NUMBERS AND CODES

NA.

Numbers

11? Eleven?

Three?

Five?

Seventeen?

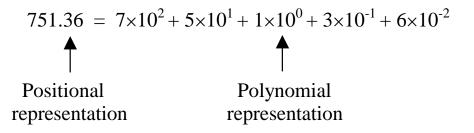
Nine?

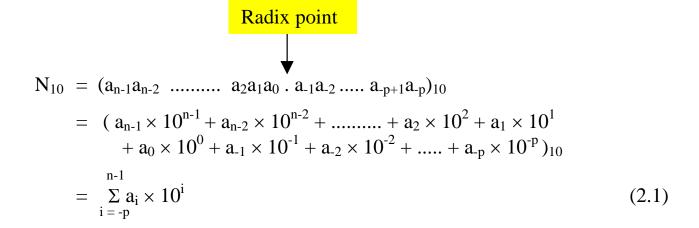
Radix, Base

Table 2.1 Example of five number systems.

System	Radix (base)	Digits
Binary	2	0, 1
Ternary	3	0, 1, 2
Octal	8	0, 1, 2, 3, 4, 5, 6, 7
Decimal	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Hexadecimal	16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Number representation





For N in base R and R \neq 10,

$$\begin{split} N_{R} &= (b_{m\text{-}1}b_{m\text{-}2} \ \ b_{2}b_{1}b_{0} \ . \ b_{-1}b_{-2} \ \ b_{-q+1}b_{-q})_{R} \\ &= (b_{m\text{-}1} \times 10^{m\text{-}1} + b_{m\text{-}2} \times 10^{m\text{-}2} + + b_{2} \times 10^{2} + b_{1} \times 10^{1} \\ &\quad + b_{0} \times 10^{0} + b_{-1} \times 10^{-1} + b_{-2} \times 10^{-2} + + b_{-q} \times 10^{-q})_{R} \\ &= (\sum_{i = -q}^{m\text{-}1} a_{i} \times 10^{i})_{R} \end{split}$$



Base	10	2	4	8	16
	0	0	0	0	0
	1	1	1	1	1
	2	10	2	2	2
	3	11	3	3	2 3
	4	100	10	4	4
	5	101	11	5	5
	6	110	12	6	6
	7	111	13	7	7
	8		20	10	8
	9		21	11	9
	10		22	12	A
	11		23	13	В
	12		30	14	C
	13		31	15	D
	14		32	16	E
	15		33	17	F
	16			20	10

$$0 \le b_i \le (R-1)$$

$$R_{10} = 10_R$$

Number Conversions

Conversion from Non-Decimal to Decimal

Polynomial substitution

$$(356.1)_8 = (?)_{10}$$

$$(356.1)_8 = (3 \times 10^2 + 5 \times 10^1 + 6 \times 10^0 + 1 \times 10^{-1})_8$$

$$(3 \times 8^2 + 5 \times 8^1 + 6 \times 8^0 + 1 \times 8^{-1})_{10} = (238.125)_{10}$$

$$N_R = (b_{m-1}b_{m-2} \dots b_2b_1b_0 \cdot b_{-1}b_{-2} \dots b_{-q+1}b_{-q})_R$$

$$\begin{array}{rcl}
\mathbf{N}_{R} &=& (b_{m-1}b_{m-2} & \dots & b_{2}b_{1}b_{0} & b_{-1}b_{-2} & \dots & b_{-q+1}b_{-q})_{R} \\
&=& (b_{m-1} \times R^{m-1} + b_{m-2} \times R^{m-2} + \dots + b_{2} \times R^{2} + b_{1} \times R^{1} \\
&& + b_{0} \times R^{0} + b_{-1} \times R^{-1} + b_{-2} \times R^{-2} + \dots + b_{-q} \times R^{-q})_{10}
\end{array} \tag{2.3}$$



$$(130.2)_4 = (1 \times 4^2 + 3 \times 4^1 + 0 \times 4^0 + 2 \times 4^{-1})_{10} = (28.5)_{10}$$

***** Example 2.3

$$(1101.01)_2 = (1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2})_{10} = (13.25)_{10}$$

***** Example 2.4

This example shows the conversion for R > 10.

$$(15C.F)_{16} = (1 \times 10^{2} + 5 \times 10^{1} + C \times 10^{0} + F \times 10^{-1})_{16}$$

$$= (1 \times 16^{2} + 5 \times 16^{1} + 12 \times 16^{0} + 15 \times 16^{-1})_{10} = (348.9375)_{10}$$

$$(120.2)_3 = (1 \times 3^2 + 2 \times 3^1 + 0 \times 3^0 + 2 \times 3^{-1})_{10} = (15.666...)_{10}$$

Conversion from Decimal to Non-Decimal

***** Example 2.6

$$(39)_{10} = (?)_4$$

 $(39)_{10} = (3 \times 10^1 + 9 \times 10^0)_{10}$ (2.4)

$$(39)_{10} = (3 \times 10^{1} + 9 \times 10^{0})_{10} = (3 \times 22^{1} + 21 \times 22^{0})_{4} = (3 \times 22 + 21)_{4}$$

Verification:

$$(213)_4 = (2 \times 4^2 + 1 \times 4^1 + 3 \times 4^0)_{10} = (39)_{10}$$

Division method for converting an integer

$$(a_{n-1}a_{n-2} \dots a_2a_1a_0 ./10)_{10} = (a_{n-1}a_{n-2} \dots a_2a_1 . a_0)_{10}$$

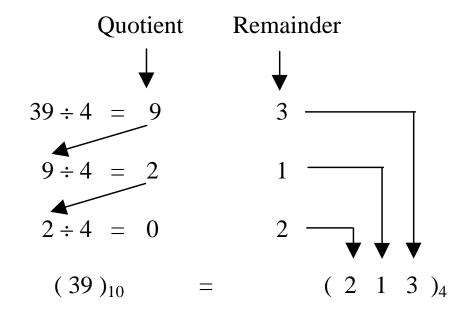
$$Quotient = (a_{n-1}a_{n-2} \dots a_2a_1)_R \qquad Remainder = (a_0)_R$$

In general

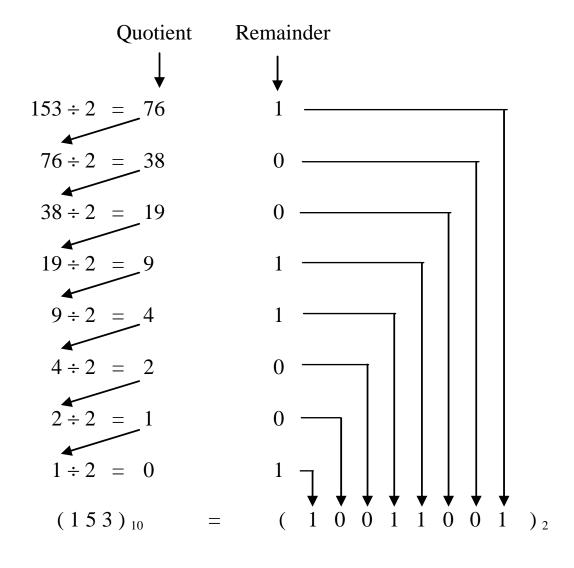
Division method for conversion of decimal integer to non-decimal integer

***** Example 2.6 (revisit)

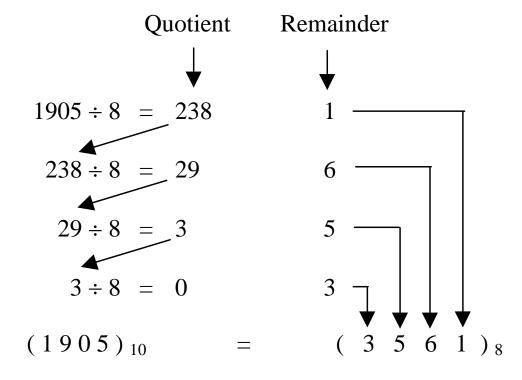
$$(39)_{10} = (?)_4$$



Find the binary equivalent of (153) $_{10}$.

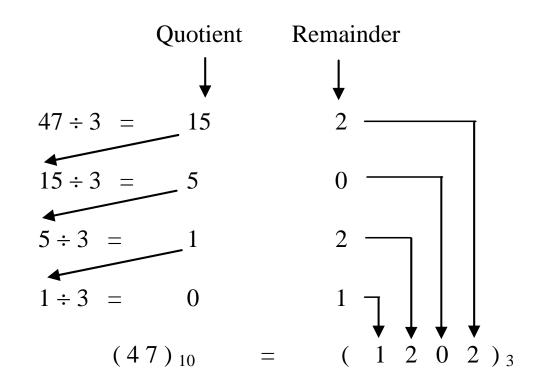


$$(1905)_{10} = (?)_8$$



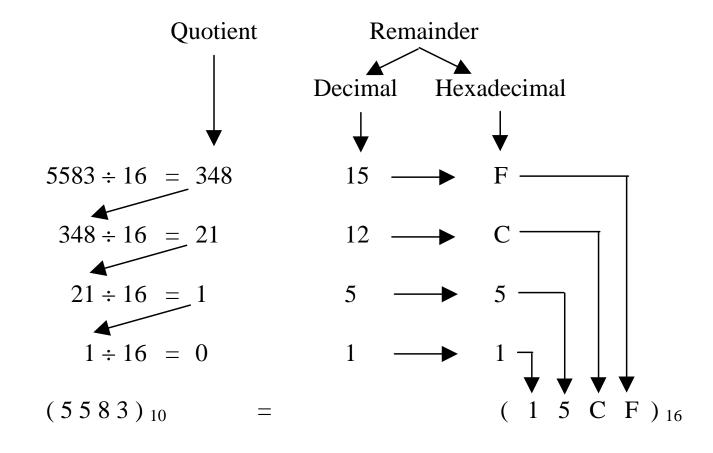
M

$$(47)_{10} = (?)_3$$



M

$$(5583)_{10} = (?)_{16}$$



Multiplication method for converting a fraction

$$(a_{p-1}a_{p-2} \dots a_{p-m+1}a_{p-m} \times 10)_{10} = (a_{p-1} \cdot a_{p-2} \dots a_{p-m+1}a_{p-m})_{10}$$

In general,

$$(.b_{q-1}b_{q-2} b_{q-m+1}b_{q-m} \times 10)_R = (b_{q-1}.b_{q-2} b_{q-m+1}b_{q-m})_R$$

Given Unknown
$$(.a_{p-1}a_{p-2} a_{p-m+1}a_{p-m})_{10} = (.b_{q-1}b_{q-2} b_{q-m+1}b_{q-m})_R$$

$$(.a_{p-1}a_{p-2} a_{p-m+1}a_{p-m} \times R)_{10} = (.b_{q-1}b_{q-2} b_{q-m+1}b_{q-m} \times 10)_R$$

= $(b_{q-1} . b_{q-2} b_{q-m+1}b_{q-m})_R$

M

\Delta Example 2.11

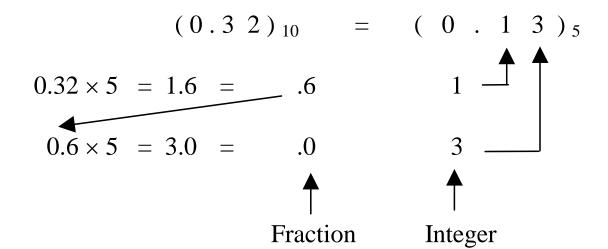
$$(0.4375)_{10} = (?)_8$$

$$(0.4375)_{10} = (0.34)_{8}$$
 $0.4375 \times 8 = 3.5 = .5$
 $0.5 \times 8 = 4.0 = .0$
Fraction Integer

$$(0.625)_{10} = (?)_2$$

$$(0.625)_{10} = (0.101)_{2}$$
 $0.625 \times 2 = 1.25 = .25$
 $0.25 \times 2 = 0.5 = .5$
 $0.5 \times 2 = 1.0 = .0$
Fraction Integer

$$(0.32)_{10} = (?)_5$$



M

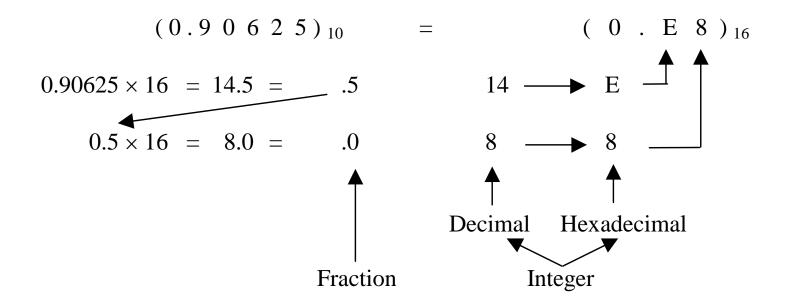
$$(0.75)_{10} = (?)_5$$

$$(0.75)_{10} = (0.333333...)_5$$

$$0.75 \times 5 = 3.75 = .75$$

$$0.75 \times 5 = 3.75 = .75$$
Fraction Integer

$$(0.90625)_{10} = (?)_{16}$$



Addition method for conversion of binary integer to decimal integer

$$(a_{n-1}a_{n-2} \dots a_2a_1a_0)_2$$

$$= (a_{n-1} \times 2^{n-1} + a_{n-2} \times 2^{n-2} + \dots + a_2 \times 2^2 + a_1 \times 2^1 + a_0 \times 2^0)_{10}$$

$$a_{n-1} \quad a_{n-2} \quad a_{n-3} \quad \dots \quad a_3 \quad a_2 \quad a_1 \quad a_0$$
Weight $2^{n-1} \quad 2^{n-2} \quad 2^{n-3} \quad \dots \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$

$$2^{n-1} \quad 2^{n-2} \quad 2^{n-3} \quad \dots \quad 8 \quad 4 \quad 2 \quad 1$$

$$(10011010)_2 = (?)_{10}$$

Binary number 1 0 0 1 1 0 0 1 0 Weight 128 64 32 16 8 4 2 1 Decimal number 128 +
$$16 + 8 + 2 = 154$$

Convert $(11111111111)_2$ to decimal.

$$1111111111 = 10000000000 - 1$$

$$(10000000000)_2$$
 $2^9 = 512.$

Range of an n-bit binary number N is

$$0 \le N \le (2^n - 1)$$

$$(1658)_{10} = (?)_2$$

Weight
$$2^{10} = 1024$$
 -1024 634 $2^9 = 512$ -512 122 $2^6 = 64$ -64 58 $2^5 = 32$ -32 26 $2^4 = 16$ -16 10 $2^3 = 8$ -8 2 $2^1 = 2$ -2 2

decimal number N

difference

difference

difference

difference

difference

difference

stop

$$a_{10} = a_9 = a_6 = a_5 = a_4 = a_3 = a_1 = 1$$

$$a_8 = a_7 = a_2 = a_0 = 0$$

 $(1658)_{10} = (a_{10} a_9 a_8 a_7 a_6 a_5 a_4 a_3 a_2 a_1 a_0)_2 = (11001111010)_2$

Extension to Fraction

Conversion without Computations

$$R_1 = (R_2)^n \qquad n \ge 2$$

One digit in base $R_1 \iff n$ digits in R_2

Table 2.2 (a) Conversion between binary and octal. (b) Conversion between binary and hexadecimal.

(a	ı)	(1)	o)
R = 2	R = 8	R =2	R = 16
000	0	0000	0
001	1	0001	1
010	2	0010	2
011	3	0011	3
100	4	0100	4
101	5	0101	5
110	6	0110	6
111	7	0111	7
	•	1000	8
		1001	9
		1010	A
		1011	В
		1100	C
		1101	D
		1110	E

1111

(a)
$$(F8.A7)_{16}$$

$$= (1111 1 1000.1010 01111)_2$$

$$= (011 111 000.101 001 110)_2$$

$$= (370.516)_8$$
(b) $(20000)_{16}$

$$= (0010 0000 000 000 000 000)_2$$

$$= (100 000 000 000 000 000 000)_2$$

$$= (400000)_8$$
(c) $(320.71)_8$

$$= (011 010 000.1111 001)_2$$

$$= (1101 0000.1111 0100)_2$$

 $= (D0.E4)_{16}$

Binary Codes

Table 2.3 Weighted codes for decimal digits.

Decimal digit	BCD code (8, 4, 2, 1) weighted code	(6, 3, 1, 1) weighted code	(6, 3, 1, 1) weighted code
0	0000	0000	0000
1	0001	0001	0010
2	0010	0011	0011
3	0 0 1 1	0100	0100
4	0100	0101	0 1 1 0
5	0101	0 1 1 1	0 1 1 1
6	0110	1000	$1\ 0\ 0\ 0$
7	0 1 1 1	1001	1010
8	1000	1011	1011
9	1001	1 1 0 0	1100
	1010	0010	0001
Unused	1011	0110	0101
(invalid)	1100	1010	1001
codes	1 1 0 1	1 1 0 1	1 1 0 1
	1110	1110	1110
	1111	1111	1111

Table 2.4 Other binary codes.

Decimal digit	Excess-3 code	Reflected (Gray) code	2-out-of-5 code
0	0 0 1 1	0000	00011
1	0100	0 0 0 1	00101
2	0101	0011	$0\ 0\ 1\ 1\ 0$
3	0110	0010	$0\ 1\ 0\ 0\ 1$
4	0111	0110	01010
5	1000	1110	01100
6	1001	1010	$1\ 0\ 0\ 0\ 1$
7	1010	1011	10010
8	1011	1001	10100
9	1100	1000	1 1 0 0 0
	$0\ 0\ 0\ 0$	0111	$0\ 0\ 0\ 0\ 0$
Unused	0001	0101	$0\ 0\ 0\ 0\ 1$
(invalid)	0010	0100	•••••
codes	1 1 0 1	1100	•••••
	1110	1101	11110
	1111	1111	11111

(a)
$$(25701)_{10}$$

= $(0010\ 0101\ 0111\ 0000\ 0001)_{BCD}$
= $(0101\ 1000\ 1010\ 0011\ 0100)_{Excess-3}$
(b) $(1000000000)_2$
= $(256)_{10}$
= $(0010\ 0101\ 0110)_{BCD}$
= $(0101\ 1000\ 1001)_{Excess-3}$
(c) $(0011\ 0100\ 0110\ 1001\ 0001)_{BCD}$
= $(34691)_{10}$
(d) $(0011\ 0100\ 1110\ 1001\ 0001)_{BCD}$
= $(?)_{10}$

Error Detection

Table 2.5 Single-error detection codes.

$a_3 a_2 a_1 a_0$	Even Parity $a_3 \ a_2 \ a_1 \ a_0 \ P$	Odd Parity a ₃ a ₂ a ₁ a ₀ P
0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1 1 0 0 0 0 1 1 0 0 0 0 1	0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 0 0 1 0 0 1 0 0 1	0 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 0 1 1 1 0 1 0 0 0 0 1 0 1 1 0 1 1 0 1 0 1 1 1 0 1 0 0 0 0 1 0 0 1 1 1 0 1 0 1 1 0 1 1 0 1 1 0 0 1 1 1 0 0 1 1 1 0 0 0
1 1 1 1	1 1 1 1 0	1 1 1 1 1