$\boxed{4.3.3}$

$$\det(A) = \begin{vmatrix} 1 & -3 & -5 \\ 0 & 1 & 2 \\ -3 & 7 & 11 \end{vmatrix} \begin{matrix} 1 & -3 \\ 0 & 1 \\ -3 & 7 \end{matrix}$$

15 14 0

11 18 0

$= 11 + 18 - 15 - 14$

$= \emptyset$

$\Rightarrow \det(A) = 0 \Rightarrow$ the system is dependent.

$\boxed{4.3.4}$

$$\det(A) = \begin{bmatrix} 3 & 2 & -12 \\ -1 & -2 & 4 \\ 1 & 2 & 4 \end{bmatrix} \begin{matrix} 3 & 2 \\ -1 & -2 \\ 1 & 2 \end{matrix}$$

$24 \quad 12 \quad -8 = 28$

$-24 \quad 8 \quad 24 = 8$

$= 8 - 28 = -20$

$\det(A) = -20 \neq 0 \Rightarrow$ the system is linearly independent

$\boxed{4.3.10}$ $[A \,\vdots\, \vec{0}] =$

$$= \left[\begin{array}{ccccc|c} 1 & 1 & -4 & -1 & 5 & 0 \\ 0 & 1 & 0 & -3 & -1 & 0 \\ 0 & 0 & -8 & 0 & 8 & 0 \end{array}\right] \xrightarrow{R_3/-8} \left[\begin{array}{ccccc|c} 1 & 1 & -4 & -1 & 5 & 0 \\ 0 & 1 & 0 & -3 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{array}\right]$$

$$\xrightarrow[= nR_1]{R_1 - R_2} \left[\begin{array}{ccccc|c} 1 & 0 & -4 & 2 & 6 & 0 \\ 0 & 1 & 0 & -3 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{array}\right]$$

$$\xrightarrow[= nR_1]{R_1 + 4R_3} \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 2 & 2 & 0 \\ 0 & 1 & 0 & -3 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{array}\right]$$

$$\begin{cases} x_3 = x_5 \\ x_2 = 3x_4 + x_5 \\ x_1 = -2x_4 - 2x_5 \end{cases}$$

x_4, x_5 : free variables

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_4 \begin{bmatrix} -2 \\ 3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$\boxed{4.3.11}$ $x - 5y + 2z = 0 \Rightarrow \begin{cases} x = 5y - 2z \\ y, z \text{ are free} \end{cases}$

$$\vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = y \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

$x - 5y + 4z = 0 \Rightarrow \begin{cases} x = 5y - 4z \\ y, z \text{ are free} \end{cases}$

$$\Rightarrow \vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = y \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}$$

$\boxed{4.3.14}$

$$A = \begin{bmatrix} \boxed{1} & 2 & \boxed{4} & -2 & \boxed{5} \\ 1 & 2 & 0 & 2 & 5 \\ 2 & 4 & -5 & 9 & 8 \\ 4 & 8 & 0 & 8 & 8 \end{bmatrix} \qquad B = \begin{bmatrix} \underline{1} & 2 & 0 & 2 & 3 \\ 0 & 0 & \underline{4} & -4 & 4 \\ 0 & 0 & 0 & 0 & \underline{-7} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Basic col A

Nul A:

$$A = \begin{bmatrix} 1 & 2 & 4 & -2 & 5 \\ 1 & 2 & 0 & 2 & 5 \\ 2 & 4 & -5 & 9 & 8 \\ 4 & 8 & 0 & 8 & 8 \end{bmatrix} \xrightarrow{R_4/4} \left[\begin{array}{ccccc|c} 1 & 2 & 4 & -2 & 5 & 0 \\ 1 & 2 & 0 & 2 & 5 & 0 \\ 2 & 4 & -5 & 9 & 8 & 0 \\ 1 & 2 & 0 & 2 & 2 & 0 \end{array}\right]$$

$$\xrightarrow[R_1 - R_4 = nR_4]{R_1 - R_2 = nR_2,\ 2R_1 - R_3 = nR_3} \left[\begin{array}{ccccc|c} 1 & 2 & 4 & -2 & 5 & 0 \\ 0 & 0 & 4 & -4 & 0 & 0 \\ 0 & 0 & 13 & -13 & 2 & 0 \\ 0 & 0 & 4 & -4 & 3 & 0 \end{array}\right] \times \frac{13}{4}$$

$$\frac{39}{4} - 2 = \frac{31}{4}$$

$$\xrightarrow[R_3 - \frac{13}{4}R_4 = nR_3]{R_4 - R_2 = nR_4} \left[\begin{array}{ccccc|c} 1 & 2 & 4 & -2 & 5 & 0 \\ 0 & 0 & 4 & -4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{31}{4} & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \end{array}\right]$$

$$\xrightarrow[=nR_3]{\frac{13}{4}R_4 - R_3} \left[\begin{array}{ccccc|c} 1 & 2 & 4 & -2 & 5 & 0 \\ 0 & 0 & 4 & -4 & 3 & 0 \\ 0 & 0 & 0 & 0 & 31/4 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \end{array}\right]$$

$$\frac{39}{4} - 2 = \frac{39-8}{4} = \frac{31}{4}$$

$$\xrightarrow[=nR_1]{R_1 - R_2} \left[\begin{array}{ccccc|c} 1 & 2 & 0 & 2 & 2 & 0 \\ 0 & 0 & 4 & -4 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\Rightarrow \begin{cases} x_5 = 0 \\ x_3 = x_4 - \frac{3}{4}x_5 \\ x_1 = -2x_2 - 2x_4 - 2x_5 \\ x_2, x_4, x_5 : \text{free} \end{cases}$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -2 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -2 \\ 0 \\ -3/4 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{4.3.16}$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 14 & 7 & 0 \\ 0 & 0 & -2 & -8 & -4 & 0 \\ 0 & 0 & 2 & 8 & 4 & 0 \\ 1 & -2 & -2 & -13 & -5 & 0 \end{array}\right]$$

$$\xrightarrow[\substack{R_1 - R_4 \\ = nR_4}]{\substack{R_2 + R_3 \\ = nR_3}} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 14 & 7 & 0 \\ 0 & 0 & 1 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 5 & 27 & 12 & 0 \end{array}\right]$$

$$\xrightarrow{\substack{2R_1 - R_4 \\ = nR_4}} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 14 & 7 & 0 \\ 0 & 0 & 1 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & & & & & 0 \end{array}\right]$$