2.3.3

$$\begin{bmatrix} 2 & 0 & 0 \\ -3 & -5 & 0 \\ 9 & 6 & -2 \end{bmatrix} \xrightarrow{\frac{R_1}{2}} \begin{bmatrix} 1 & 0 & 0 \\ -3 & -5 & 0 \\ 9 & 6 & -2 \end{bmatrix} \xrightarrow[-9R_1+R_3 = nR_3]{3R_1+R_2 = nR_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 6 & -2 \end{bmatrix} \xrightarrow[R_3/2]{R_2/-5} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & -1 \end{bmatrix} \xrightarrow{3R_2-R_3 = nR_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

There are 3 pivot positions in 3 rows

→ **invertible**

2.3.4

$$\begin{bmatrix} 4 & 0 & -4 \\ 3 & 0 & 5 \\ -4 & 0 & 9 \end{bmatrix}$$

Column #2 is 0's ⇒ system has at most 2 pivot positions

⇒ free variable, nontrivial solution ∃

⇒ **NOT invertible**

2.3.6

$$\begin{bmatrix} 1 & -3 & -6 \\ 0 & 4 & 3 \\ -2 & 3 & 0 \end{bmatrix} \xrightarrow{2R_1+R_3 = nR_3} \begin{bmatrix} 1 & -3 & -6 \\ 0 & 4 & 3 \\ 0 & -3 & -12 \end{bmatrix} \xrightarrow{\frac{R_3}{-3}} \begin{bmatrix} 1 & -3 & -6 \\ 0 & 4 & 3 \\ 0 & 1 & 4 \end{bmatrix}$$

$$\xrightarrow{-R_2+4R_3 = nR_3} \begin{bmatrix} 1 & -3 & -6 \\ 0 & 4 & 3 \\ 0 & 0 & 13 \end{bmatrix}$$

3 pivot positions ⇒ **invertible**

2.3.7

$$\begin{bmatrix} 1 & 4 & 0 & -1 \\ 5 & 7 & 26 & -5 \\ 1 & 4 & -1 & -1 \\ 0 & -1 & 2 & 1 \end{bmatrix} \xrightarrow[R_3/-3]{-R_1} \xrightarrow{R_1-R_3 = nR_3} \begin{bmatrix} 1 & 4 & 0 & -1 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 5 & 7 & 26 & -5 \end{bmatrix} \xrightarrow{5R_1-R_4 = nR_4}$$

$$\xrightarrow{} \begin{bmatrix} 1 & 4 & 0 & -1 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 13 & -26 & 0 \end{bmatrix} \xrightarrow{\frac{R_4}{13}} \begin{bmatrix} 1 & 4 & 0 & -1 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \end{bmatrix} \xrightarrow{-R_2+R_4 = nR_4}$$

$$\xrightarrow{} \begin{bmatrix} 1 & 4 & 0 & -1 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4 pivots positions in 4 rows

$\Rightarrow$ **invertible**

2.3.34 $T(x_1, x_2) = (5x_1 - 9x_2, -5x_1 + 7x_2)$

$$A = \begin{bmatrix} 5 & -9 \\ -5 & 7 \end{bmatrix}$$

$\det(A) = 35 - 45 = \boxed{-10} \neq 0$ ✓

$\Rightarrow$ invertible

$$T^{-1} = \frac{1}{\det(A) = -10} \begin{bmatrix} 7 & 9 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} \frac{-7}{10} & \frac{-9}{10} \\ \frac{-1}{2} & \frac{-1}{2} \end{bmatrix}$$

$$\Rightarrow T^{-1}(x_1, x_2) = \boxed{\left(\frac{-7}{10}x_1 - \frac{9}{10}x_2, \frac{-1}{2}x_1 - \frac{1}{2}x_2\right)}$$