Due Date: 10-11-2016 (Tue.), BEFORE the class starts

In problem 1 to 2, solve the following recurrence with different methods that we learned in class.

$$T(n) = \begin{cases} \Theta(1) & n \le k \\ 6T\left(\frac{n}{2}\right) + 2T\left(\frac{n}{2}\right) + 9^{(\log_3 n)} & n > k \end{cases}$$

where $n = 2^j$ for a positive integer j, and k is a small positive integer. That is, find a function g(n) such that $T(n) \in \Theta(g(n))$. The $\Theta(1)$ terminating condition is intended to represent some small constant.

1. (10 points) Use the **Master Theorem** to solve this recurrence.
2. (20 points) Use the **recursion tree** to solve the recurrence.

3. Resolve Recurrence

(1) (10 points) Find a tight upper and lower bound solution for the following recurrence:

$$T(n) = 3T(\frac{n}{3}) + T(\frac{n}{3}) + 2^{\lg n}$$

That is, find a function g(n) such that $T(n) \in \Theta(g(n))$. You may assume that $n=3^k$ for some positive integer k. Justify your answer.

(2) (20 points) prove your answer in (1) using the **substitution method**.

4. (10 points) For each of the following recurrences, give an expression for the runtime T(n) if the recurrence can be solved with the Master Theorem. Otherwise, explain why the Master Theorem does not apply. Justify your answer.

(1) $T(n) = 3^n T(\frac{n}{3}) + n^3$

(2) $T(n) = 5T(\frac{n}{2}) + \sqrt{10}n^3$

(3) $T(n) = \frac{1}{4} T(\frac{n}{4}) + n \lg n$

(4) $T(n) = T(n-1) + 2n$

(5) $T(n) = 16T(\frac{n}{4}) + n^2$

5. **Design an Algorithm** (30 points)

You are given a sorted array *A,* which stores *n* integers, and a value *key*. Design an efficient **divide-and-conquer** algorithm that returns the index of the value *key* if it can be found in array *A. Otherwise, the algorithm returns 0*.

(1) (10 points) Pseudocode *(please use the textbook conventions)*

(2) (10 points) Analysis: Provide a tight lower and upper bound on the worst-case asymptotic running time of your pseudocode (provide the recurrence and resolve it). Justify your answer (i.e., list the cost for executing each line of code and how you calculate the total running time).

(3) (10 points) Prove your answer using the **substitution method**

Algorithms -- COMP.4040 Honor Statement
(Courtesy of Prof. Tom Costello and Karen Daniels with modifications)

Must be attached to each submission

Academic achievement is ordinarily evaluated on the basis of work that a student produces independently. Infringement of this Code of Honor entails penalties ranging from reprimand to suspension, dismissal or expulsion from the University.

Your name on any exercise is regarded as assurance and certification that what you are submitting for that exercise is the result of your own thoughts and study. Where collaboration is authorized, you should state very clearly which parts of any assignment were performed with collaboration and name your collaborators.

In writing examinations and quizzes, you are expected and required to respond entirely on the basis of your own memory and capacity, without any assistance whatsoever except such as what is specifically authorized by the instructor.

I certify that the work submitted with this assignment is mine and was generated in a manner consistent with this document, the course academic policy on the course website on Blackboard, and the UMass Lowell academic code.

Date: _____

Name (please print): _____

Signature: _____