CHAPTER 2

NUMBERS AND CODES

Numbers

11 ? Eleven?
Three?
Five?
Seventeen?
Nine?

Radix, Base

Table 2.1 Example of five number systems.

System	Radix (base)	Digits
Binary	2	0, 1
Ternary	3	0, 1, 2
Octal	8	0, 1, 2, 3, 4, 5, 6, 7
Decimal	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Hexadecimal	16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Number representation

$$751.36 = 7\times10^2 + 5\times10^1 + 1\times10^0 + 3\times10^{-1} + 6\times10^{-2}$$

Positional representation

Polynomial representation

Radix point

$$N_{10} = (a_{n-1}a_{n-2} \;.......... \; a_2a_1a_0 \,.\, a_{-1}a_{-2} \;.....\; a_{-p+1}a_{-p})_{10}$$

$$= (a_{n-1} \times 10^{n-1} + a_{n-2} \times 10^{n-2} + ... + a_2 \times 10^2 + a_1 \times 10^1 + a_0 \times 10^0 + a_{-1} \times 10^{-1} + a_{-2} \times 10^{-2} + ... + a_{-p} \times 10^{-p})_{10}$$

$$= \sum_{i=-p}^{n-1} a_i \times 10^i \qquad (2.1)$$

For N in base R and $R \neq 10$,

$$N_R = (b_{m-1}b_{m-2} \;.......... \; b_2b_1b_0 \,.\, b_{-1}b_{-2} \;.....\; b_{-q+1}b_{-q})_R$$

$$= (b_{m-1} \times 10^{m-1} + b_{m-2} \times 10^{m-2} + ... + b_2 \times 10^2 + b_1 \times 10^1 + b_0 \times 10^0 + b_{-1} \times 10^{-1} + b_{-2} \times 10^{-2} + ... + b_{-q} \times 10^{-q})_R$$

$$= (\sum_{i=-q}^{m-1} a_i \times 10^i)_R \qquad (2.2)$$

Base	10	2	4	8	16
	0	0	0	0	0
	1	1	1	1	1
	2	10	2	2	2
	3	11	3	3	3
	4	100	10	4	4
	5	101	11	5	5
	6	110	12	6	6
	7	111	13	7	7
	8		20	10	8
	9		21	11	9
	10		22	12	A
	11		23	13	B
	12		30	14	C
	13		31	15	D
	14		32	16	E
	15		33	17	F
	16			20	10

$$0 \leq b_i \leq (R-1)$$

$$R_{10} = 10_R$$

Number Conversions

Conversion from Non-Decimal to Decimal

Polynomial substitution

- Example 2.1

$(356.1)_8 = (\ ? \ )_{10}$

$(356.1)_8 = (3 \times 10^2 + 5 \times 10^1 + 6 \times 10^0 + 1 \times 10^{-1})_8$

$(3 \times 8^2 + 5 \times 8^1 + 6 \times 8^0 + 1 \times 8^{-1})_{10} = (238.125)_{10}$

$$N_R = (b_{m-1}b_{m-2} \text{ .... } b_2b_1b_0 \, . \, b_{-1}b_{-2} \text{ ..... } b_{-q+1}b_{-q})_R$$

$$= (b_{m-1} \times R^{m-1} + b_{m-2} \times R^{m-2} + \text{ .... } + b_2 \times R^2 + b_1 \times R^1 + b_0 \times R^0 + b_{-1} \times R^{-1} + b_{-2} \times R^{-2} + \text{ ..... } + b_{-q} \times R^{-q})_{10} \quad (2.3)$$

- Example 2.2

$$(130.2)_4 = (1 \times 4^2 + 3 \times 4^1 + 0 \times 4^0 + 2 \times 4^{-1})_{10} = (28.5)_{10}$$

- Example 2.3

$$(1101.01)_2 = (1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2})_{10} = (13.25)_{10}$$

- Example 2.4

This example shows the conversion for R > 10.

$$(15C.F)_{16} = (1 \times 10^2 + 5 \times 10^1 + C \times 10^0 + F \times 10^{-1})_{16}$$

$$= (1 \times 16^2 + 5 \times 16^1 + 12 \times 16^0 + 15 \times 16^{-1})_{10} = (348.9375)_{10}$$

- Example 2.5

$$(120.2)_3 = (1 \times 3^2 + 2 \times 3^1 + 0 \times 3^0 + 2 \times 3^{-1})_{10} = (15.666\ldots)_{10}$$

Conversion from Decimal to Non-Decimal

❖ Example 2.6

$(39)_{10} = (\ ?\ )_4$

$$(3\,9)_{10} = (3 \times 10^1 + 9 \times 10^0)_{10} \qquad (2.4)$$

0	1	2	3	4	5	6	7	8	9	10	(for R = 10)
0	1	2	3	10	11	12	13	20	21	22	(for R = 4)

$$(3\,9)_{10} = (3 \times 10^1 + 9 \times 10^0)_{10} = (3 \times 22^1 + 21 \times 22^0)_4 = (3 \times 22 + 21)_4$$

$$\begin{array}{r} 2\ 2 \\ \times\quad 3 \\ \hline 1\ 2 \\ +\ 1\ 2\ \\ \hline 1\ 3\ 2 \end{array} \qquad \begin{array}{r} 1\ 3\ 2 \\ +\quad 2\ 1 \\ \hline 2\ 1\ 3 \end{array}$$

Verification: $(2\,1\,3)_4 = (2 \times 4^2 + 1 \times 4^1 + 3 \times 4^0)_{10} = (3\,9)_{10}$

Division method for converting an integer

$$(a_{n-1}a_{n-2} \;\ldots\ldots\ldots\; a_2a_1a_0\,.\,/10)_{10} = (a_{n-1}a_{n-2} \;\ldots\ldots\ldots\; a_2a_1\,.\,a_0)_{10}$$

Quotient = $(a_{n-1}a_{n-2} \;\ldots\ldots\ldots\; a_2a_1)_R$ Remainder = $(a_0)_R$

In general

$$(b_{m-1}b_{m-2} \;\ldots\ldots\ldots\; b_2b_1b_0\,/10\,)_R = (b_{m-1}b_{m-2} \;\ldots\ldots\ldots\; b_2b_1\,.\,b_0\,)_R$$

$$(\,213\,/\,10\,)_4 = (\,21.3\,)_4$$

Given Unknown

$$(\,a_{n-1}a_{n-2} \;\ldots\ldots\ldots\; a_2a_1a_0\,)_{10} = (\,b_{m-1}b_{m-2} \;\ldots\ldots\ldots\; b_2b_1b_0\,)_R$$

$$(\,a_{n-1}a_{n-2} \;\ldots\ldots\ldots\; a_2a_1a_0\,/R\,)_{10} = (\,b_{m-1}b_{m-2} \;\ldots\ldots\ldots\; b_2b_1b_0\,/\,10)_R$$

$$= (\,b_{m-1}b_{m-2} \;\ldots\ldots\ldots\; b_2b_1\,.\,b_0\,)_R$$

Division method
for conversion of decimal integer to non-decimal integer

- Example 2.6 (revisit)

$$(39)_{10} = (\ ?\ )_4$$

	Quotient	Remainder
$39 \div 4 =$	9	3
$9 \div 4 =$	2	1
$2 \div 4 =$	0	2

$$(39)_{10} = (2\ 1\ 3)_4$$

❖ Example 2.7

Find the binary equivalent of $(153)_{10}$.

	Quotient	Remainder
$153 \div 2 =$	76	1
$76 \div 2 =$	38	0
$38 \div 2 =$	19	0
$19 \div 2 =$	9	1
$9 \div 2 =$	4	1
$4 \div 2 =$	2	0
$2 \div 2 =$	1	0
$1 \div 2 =$	0	1

$$(1\,5\,3)_{10} = (1\;0\;0\;1\;1\;0\;0\;1)_2$$

- Example 2.8

$(1905)_{10} = (\ ?\ )_8$

		Quotient	Remainder
$1905 \div 8$	=	238	1
$238 \div 8$	=	29	6
$29 \div 8$	=	3	5
$3 \div 8$	=	0	3

$(1905)_{10} = (3561)_8$

❖ Example 2.9

$$(47)_{10} = (\ ?\ )_3$$

	Quotient	Remainder
$47 \div 3 =$	15	2
$15 \div 3 =$	5	0
$5 \div 3 =$	1	2
$1 \div 3 =$	0	1

$$(47)_{10} = (1\ 2\ 0\ 2)_3$$

Example 2.10

$$(5583)_{10} = (\ ?\ )_{16}$$

	Quotient	Remainder: Decimal	Remainder: Hexadecimal
$5583 \div 16 =$	348	15	F
$348 \div 16 =$	21	12	C
$21 \div 16 =$	1	5	5
$1 \div 16 =$	0	1	1

$$(5\,5\,8\,3)_{10} = (1\ 5\ C\ F)_{16}$$

Multiplication method for converting a fraction

$$(.a_{p-1}a_{p-2} \;\ldots\ldots\ldots\; a_{p-m+1}a_{p-m} \times 10)_{10} = (a_{p-1}\,.\,a_{p-2} \;\ldots\ldots\ldots\; a_{p-m+1}a_{p-m})_{10}$$

In general,

$$(\,.b_{q-1}b_{q-2} \;\ldots\ldots\ldots\; b_{q-m+1}b_{q-m} \times 10\,)_R = (\,b_{q-1}\,.\,b_{q-2} \;\ldots\ldots\ldots\; b_{q-m+1}b_{q-m}\,)_R$$

Given Unknown

$$(\,.a_{p-1}a_{p-2} \;\ldots\ldots\ldots\; a_{p-m+1}a_{p-m}\,)_{10} = (\,.b_{q-1}b_{q-2} \;\ldots\ldots\ldots\; b_{q-m+1}b_{q-m}\,)_R$$

$$\begin{aligned}(\,.a_{p-1}a_{p-2} \;\ldots\ldots\ldots\; a_{p-m+1}a_{p-m} \times R\,)_{10} &= (\,.b_{q-1}b_{q-2} \;\ldots\ldots\ldots\; b_{q-m+1}b_{q-m} \times 10)_R \\ &= (\,b_{q-1}\,.\,b_{q-2} \;\ldots\ldots\ldots\; b_{q-m+1}b_{q-m}\,)_R\end{aligned}$$

❖ Example 2.11

$$(0.4375)_{10} = (\ ?\ )_8$$

$$(0.4375)_{10} = (0.34)_8$$

$$0.4375 \times 8 = 3.5 = .5 \quad 3$$

$$0.5 \times 8 = 4.0 = .0 \quad 4$$

Fraction Integer

❖ Example 2.12

$$(0.625)_{10} = (\ ?\ )_2$$

$$(0.625)_{10} = (0.101)_2$$

		Fraction	Integer
$0.625 \times 2 = 1.25$	=	.25	1
$0.25 \times 2 = 0.5$	=	.5	0
$0.5 \times 2 = 1.0$	=	.0	1

❖ Example 2.13

$$(0.32)_{10} = (\ ?\ )_5$$

$$(0.32)_{10} = (0.13)_5$$

$0.32 \times 5 = 1.6 =$ .6 1

$0.6 \times 5 = 3.0 =$ .0 3

Fraction Integer

- Example 2.14

$$(0.75)_{10} = (\ ?\ )_5$$

$$(0.75)_{10} = (0.33333\ldots\ldots)_5$$

$0.75 \times 5 = 3.75 =$ .75 3

$0.75 \times 5 = 3.75 =$ .75 3

Fraction Integer

- Example 2.15

$$(0.90625)_{10} = (\ ?\ )_{16}$$

$$(0.90625)_{10} = (0.E8)_{16}$$

$$0.90625 \times 16 = 14.5 = .5 \qquad 14 \rightarrow E$$

$$0.5 \times 16 = 8.0 = .0 \qquad 8 \rightarrow 8$$

Decimal Hexadecimal

Fraction Integer

Addition method for conversion of binary integer to decimal integer

$$(a_{n-1}a_{n-2} \ \ldots\ldots\ldots \ a_2a_1a_0)_2$$

$$= \ (a_{n-1} \times 2^{n-1} + a_{n-2} \times 2^{n-2} + \ldots\ldots\ldots + a_2 \times 2^2 + a_1 \times 2^1 + a_0 \times 2^0)_{10}$$

	a_{n-1}	a_{n-2}	a_{n-3}	………	a_3	a_2	a_1	a_0
Weight	2^{n-1}	2^{n-2}	2^{n-3}	………	2^3	2^2	2^1	2^0
	2^{n-1}	2^{n-2}	2^{n-3}	………	8	4	2	1

❖ Example 2.16

$$(10011010)_2 = (\ ? \ )_{10}$$

Binary number	1	0	0	1	1	0	1	0	
Weight	128	64	32	16	8	4	2	1	
Decimal number	128 +			16 +	8 +		2		= 154

- Example 2.17

Convert $(111111111)_2$ to decimal.

$111111111 = 1000000000 - 1$

$(1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)_2 \quad 2^9 = 512.$

$(111111111)_2 = (511)_{10}$

Range of an n-bit binary number N is

$$0 \le N \le (2^n - 1)$$

❖ Example 2.18 $(1658)_{10} = (\ ?\ )_2$

Weight		
	1658	decimal number N
$2^{10} = 1024$	$-\ 1024$	
	634	difference
$2^9 = 512$	$-\ 512$	
	122	difference
$2^6 = 64$	$-\ 64$	
	58	difference
$2^5 = 32$	$-\ 32$	
	26	difference
$2^4 = 16$	$-\ 16$	
	10	difference
$2^3 = 8$	$-\ 8$	
	2	difference
$2^1 = 2$	$-\ 2$	
	0	stop

$a_{10} = a_9 = a_6 = a_5 = a_4 = a_3 = a_1 = 1$

$a_8 = a_7 = a_2 = a_0 = 0$

$$(1658)_{10} = (a_{10}\, a_9\, a_8\, a_7\, a_6\, a_5\, a_4\, a_3\, a_2\, a_1 a_0)_2 = (11001111010)_2$$

Extension to Fraction

	a_{-1}	a_{-2}	a_{-3}	....	a_{-q+3}	a_{-q+2}	a_{-q+1}	a_{-q}
Weight	2^{-1}	2^{-2}	2^{-3}	....	2^{-q+3}	2^{-q+2}	2^{-q+1}	2^{-q}
	.5	.25	.125	....	2^{-q+3}	2^{-q+2}	2^{-q+1}	2^{-q}

Conversion without Computations

$$R_1 = (R_2)^n \qquad n \geq 2$$

One digit in base R_1 $\Longleftrightarrow$ n digits in R_2

Table 2.2 (a) Conversion between binary and octal. (b) Conversion between binary and hexadecimal.

(a)

R =2	R = 8
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

(b)

R =2	R = 16
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

❖ Example 2.19

(a) $(F8.A7)_{16}$

$= (1111\ 1000.1010\ 0111)_2$

$= (011\ 111\ 000.101\ 001\ 110)_2$

$= (370.516)_8$

(b) $(20000)_{16}$

$= (0010\ 0000\ 0000\ 0000\ 0000)_2$

$= (100\ 000\ 000\ 000\ 000\ 000)_2$

$= (400000)_8$

(c) $(320.71)_8$

$= (011\ 010\ 000.111\ 001)_2$

$= (1101\ 0000.1110\ 0100)_2$

$= (D0.E4)_{16}$

Binary Codes

Table 2.3 Weighted codes for decimal digits.

Decimal digit	BCD code (8, 4, 2, 1) weighted code	(6, 3, 1, 1) weighted code	(6, 3, 1, 1) weighted code
0	0 0 0 0	0 0 0 0	0 0 0 0
1	0 0 0 1	0 0 0 1	0 0 1 0
2	0 0 1 0	0 0 1 1	0 0 1 1
3	0 0 1 1	0 1 0 0	0 1 0 0
4	0 1 0 0	0 1 0 1	0 1 1 0
5	0 1 0 1	0 1 1 1	0 1 1 1
6	0 1 1 0	1 0 0 0	1 0 0 0
7	0 1 1 1	1 0 0 1	1 0 1 0
8	1 0 0 0	1 0 1 1	1 0 1 1
9	1 0 0 1	1 1 0 0	1 1 0 0
Unused (invalid) codes	1 0 1 0	0 0 1 0	0 0 0 1
	1 0 1 1	0 1 1 0	0 1 0 1
	1 1 0 0	1 0 1 0	1 0 0 1
	1 1 0 1	1 1 0 1	1 1 0 1
	1 1 1 0	1 1 1 0	1 1 1 0
	1 1 1 1	1 1 1 1	1 1 1 1

Table 2.4 Other binary codes.

Decimal digit	Excess-3 code	Reflected (Gray) code	2-out-of-5 code
0	0 0 1 1	0 0 0 0	0 0 0 1 1
1	0 1 0 0	0 0 0 1	0 0 1 0 1
2	0 1 0 1	0 0 1 1	0 0 1 1 0
3	0 1 1 0	0 0 1 0	0 1 0 0 1
4	0 1 1 1	0 1 1 0	0 1 0 1 0
5	1 0 0 0	1 1 1 0	0 1 1 0 0
6	1 0 0 1	1 0 1 0	1 0 0 0 1
7	1 0 1 0	1 0 1 1	1 0 0 1 0
8	1 0 1 1	1 0 0 1	1 0 1 0 0
9	1 1 0 0	1 0 0 0	1 1 0 0 0
Unused (invalid) codes	0 0 0 0	0 1 1 1	0 0 0 0 0
	0 0 0 1	0 1 0 1	0 0 0 0 1
	0 0 1 0	0 1 0 0	……….
	1 1 0 1	1 1 0 0	……….
	1 1 1 0	1 1 0 1	1 1 1 1 0
	1 1 1 1	1 1 1 1	1 1 1 1 1

❖ Example 2.20

(a) $(2\ 5\ 7\ 0\ 1)_{10}$

$= (0010\ \ 0101\ \ 0111\ \ 0000\ \ 0001)_{BCD}$

$= (0101\ \ 1000\ \ 1010\ \ 0011\ \ 0100)_{Excess\text{-}3}$

(b) $(1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)_2$

$= (2\ 5\ 6)_{10}$

$= (0010\ \ 0101\ \ 0110)_{BCD}$

$= (0101\ \ 1000\ \ 1001)_{Excess\text{-}3}$

(c) $(0011\ \ 0100\ \ 0110\ \ 1001\ \ 0001)_{BCD}$

$= (3\ 4\ 6\ 9\ 1)_{10}$

(d) $(0011\ \ 0100\ \ 1110\ \ 1001\ \ 0001)_{BCD}$

$= (\ ?\ )_{10}$

Error Detection

Table 2.5 Single-error detection codes.

				Even Parity					Odd Parity				
a_3	a_2	a_1	a_0	a_3	a_2	a_1	a_0	P	a_3	a_2	a_1	a_0	P
0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	1	0	0	0	1	1	0	0	0	1	0
0	0	1	0	0	0	1	0	1	0	0	1	0	0
0	0	1	1	0	0	1	1	0	0	0	1	1	1
0	1	0	0	0	1	0	0	1	0	1	0	0	0
0	1	0	1	0	1	0	1	0	0	1	0	1	1
0	1	1	0	0	1	1	0	0	0	1	1	0	1
0	1	1	1	0	1	1	1	1	0	1	1	1	0
1	0	0	0	1	0	0	0	1	1	0	0	0	0
1	0	0	1	1	0	0	1	0	1	0	0	1	1
1	0	1	0	1	0	1	0	0	1	0	1	0	1
1	0	1	1	1	0	1	1	1	1	0	1	1	0
1	1	0	0	1	1	0	0	0	1	1	0	0	1
1	1	0	1	1	1	0	1	1	1	1	0	1	0
1	1	1	0	1	1	1	0	1	1	1	1	0	0
1	1	1	1	1	1	1	1	0	1	1	1	1	1