CS4321 Midterm Exam

Oct. 24, 2006 (70 points plus 5 points optional)

NAME: ________________.

1	2	3	4	5	6	7	8	9	10	Total

You may need the following equations.

$\log(ab) = \log a + \log b$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

$$\sum_{i=0}^{n} r^i = \frac{r^{n+1}-1}{r-1}, when \quad r \neq 1.$$

1. (5 points) Prove that $log^2 n$ is smooth.

2. (17 points) We are going to analyze the efficiency of the following algorithm consisting of three routines where mainRoutine() calls subRoutine1() and subRoutine2() as follows.

```
void mainRoutine(int n, boolean b)
{
        if  (b) {
                subRoutine1(n);
        } else {
                subRoutine2(n);
        }
}

void subRoutine1(int n)
{
      int i;
      int counter1 = 0;

       for (i = 1; i <= n; i ++) {
             for (j=1; j<=i; j++)
                counter1++;
        }
}

void subRoutine2(int n)
{
      int i = 1;
      int counter2 = 0;

       while (i < n) {
            counter2++;
           i *= 2;
        }
}
```

a. (4 points) Develop a big-Θ bound for subRoutine1() and subRoutine2() respectively;

b. (6 points) Assume that there’s 40% chance that the Boolean variable *b* is true and 60% chance that *b* is false. What’s the worst-case, best-case, and average cost of the mainRoutine() respectively?

c. (7 points) Determine if each of the following statements about mainRoutine() is true or false. You do not need to prove your answer.

i. The algorithm takes time in $O(n^2)$. (   )

ii. The algorithm takes time in $\Theta(n^2)$. (   )

iii. The algorithm takes time in $\Omega(n\log n)$. (   )

iv. The algorithm takes time in $O(n^2\log n)$. (   )

v. The algorithm takes best case time in $\Omega(\log n)$. (   )

vi. The algorithm takes worst case time in $O(n\log n)$. (   )

vii. The algorithm takes worst case time in $\Theta(n^2)$. (   )

3. (8 points) Assuming that n is a power of 5, we are going to analyze the following algorithm.
    a. Which instruction do you want to use as the barometer?
    b. Develop an exact function for the number of times your barometer executes.
    c. Express the cost of this algorithm using the big-Θ notation.

```
int counter = 0;
void Foo (int n) { // n is an positive integer and exact power of 5.
{
    int i;

    while (n > 0) {
          for (i = 0; i < 2*n; i++) {
               counter++;
          }
           n = n /5;
    }
 }
```

4. (6 points) Using the relations "$\subset$" and "=" to put the order of the following functions into a sequence. Prove your answers.

$$n\log^2 n \qquad n^2 \qquad (100n+5)^2 \qquad n^{1.01}\log n$$

5. (7 points) Solving the following two recurrences. Express your answer as simply as possible using the Θ notation.

a. $t_n = \begin{cases} 0, & n = 0 \\ 2t_{n-1} + 3^{n-1}, & n > 0 \end{cases}$

b. n is a power of 3, and $t(n) = \begin{cases} 1, & n = 1 \\ 30t(n/3) + n^3, & n > 1 \end{cases}$.

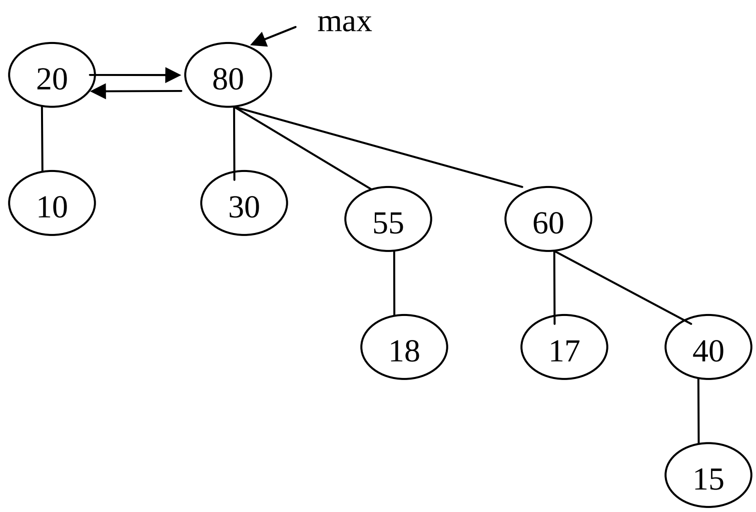
6. (4 points) Given a heap T[0..n-1] where the index starts from 0, i.e., the root is T[0], list the equations that find the indexes of the parent and two children of node T[i]. (Proof is not needed)

7. (8 points) The following table lists the values of *set*[1..8] after a series of calls to merge3() (merge shorter tree to taller tree) and find3() (find with path compression).

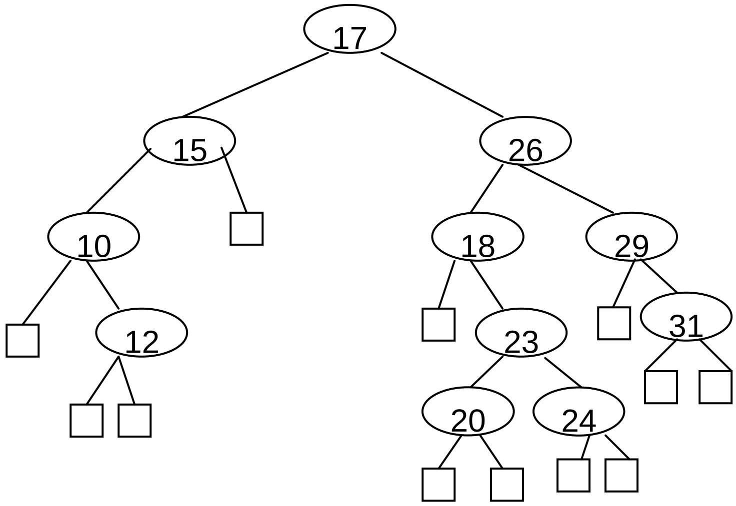
i	1	2	3	4	5	6	7	8	9
set[i]	1	1	3	6	1	3	6	8	3

   a. (2 points) Show the sets in the form of trees.
   b. (2 points) Is merge3(5, 8) legal now? Why?
   c. (2 points) Draw the tree after merge3(1,3) is called.
   d. (2 points) Draw the tree after find3(4) is called on the original sets.

8. (10 points) We discussed in class an algorithm that deletes the node with the maximum value from a binomial heap. Now design an O(log n) algorithm that deletes any node from a binomial heap.
    a. (4 points) Write pseudo code of your algorithm.
    b. (3 points) Show that your algorithm takes time in O(log n).
    c. (3 points) Demonstrate how your algorithm works when you delete the node with value 18 in the following binomial heap.



9. (5 points) Draw the splay tree after search(23) is called on the splay tree below.



10. (5 points) This question is **optional**. Prove the following routine convert a digit string to the corresponding integer value. For example, the input string “123” will yield a return value of 123.

```
int atoi(char s[])
{
    int n = strlen(s);  // n is the length of the string
    int v = 0;

    for (i = 0; i < len; i++)
        v = v*10 + (s[i] – ‘0’);

    return v;
}
```