1. If $f_x(a,b) = f_y(a,b) = 0$, does it follow that f has a local maximum or local minimum at (a,b)? Explain.

Choose the correct answer below.

- **A.** Yes. The point (a,b) is a critical point and must be a local maximum or local minimum.
- **B.** No. One (or both) of f_x and f_y must also not exist at (a,b) to be sure that f has a local maximum or local minimum at (a,b).
- **C.** Yes. The tangent plane to f at (a,b) is horizontal. This indicates the presence of a local maximum or a local minimum at (a,b).
- **D.** No. It follows that (a,b) is a critical point of f, and (a,b) is a candidate for a local maximum or local minimum.

2. What is the discriminant and how do you compute it?

Choose the correct answer below.

- **A.** The discriminant is a determinant; it is computed using $D(x,y) = f_{xx}f_{yy} - f_{xy}^2$.
- **B.** The discriminant is a point where f(x,y) does not exist; it is computed using $D(x,y) = f_x^2 - f_y^2$.
- **C.** The discriminant is a critical point; it is computed using $D(x,y) = f_{xy}f_{yx} - f_{xx}^2 - f_{yy}^2$.
- **D.** The discriminant is the absolute value of f(x,y); it is computed using $D(x,y) = \left(f_x - f_y\right)^2 - f_{xy}^2$.

3. What is the procedure for locating absolute maximum and minimum values on a closed bounded domain R?

Select the correct choice below.

- **A.** Determine the values of the function at all critical points in R. Then find the maximum and minimum values on the boundary of R. The greatest of these values is the absolute maximum on R, and the least of these is the absolute minimum.
- **B.** Determine the values of the function at all critical points of R. The greatest of these values is the absolute maximum on R, and the least of these is the absolute minimum.
- **C.** Because the absolute extreme values always occur on the boundary, find the maximum and minimum values of the function on the boundary of R.
- **D.** A continuous function on a closed bounded domain may not have absolute extreme values.

4. Find all critical points of the following function.

$$f(x,y) = -8xy + 2x^4 + 2y^4$$

What are the critical points? Select the correct choice below and, if necessary, fill in the answer box within your choice.

- **A.** The critical point(s) is/are (0,0), (−1,−1), (1,1).
(Type an ordered pair. Use a comma to separate answers as needed.)
- **B.** There are no critical points.

5. Find all critical points of the following function.

$$f(x,y) = x^2 + 12x + y^2 - 19$$

What are the critical points? Select the correct choice below and fill in any answer boxes within your choice.

A. The critical point(s) is/are $(-6,0)$.
(Type an ordered pair. Use a comma to separate answers as needed.)

B. There are no critical points.

6. Find the critical points of the following function. Use the Second Derivative Test to determine (if possible) whether each critical point corresponds to a local maximum, local minimum, or saddle point. Confirm your results using a graphing utility.

$$f(x,y) = \frac{2x}{49 + x^2 + y^2}$$

What are the critical points? Select the correct choice below and fill in any answer boxes within your choice.

A. The critical point(s) is/are $(7,0),(-7,0)$.
(Type an ordered pair. Use a comma to separate answers as needed.)

B. There are no critical points.

Identify any local maxima. Select the correct choice below and fill in any answer boxes within your choice.

A. There are local maxima at __________.
(Type an ordered pair. Use a comma to separate answers as needed.)

B. There are no local maxima.

Identify any local minima. Select the correct choice below and fill in any answer boxes within your choice.

A. There are local minima at __________.
(Type an ordered pair. Use a comma to separate answers as needed.)

B. There are no local minima.

Identify any saddle points. Select the correct choice below and fill in any answer boxes within your choice.

A. There are saddle points at __________.
(Type an ordered pair. Use a comma to separate answers as needed.)

B. There are no saddle points.

7. Find the critical points of the following function. Use the Second Derivative Test to determine (if possible) whether each critical point corresponds to a local maximum, local minimum, or saddle point. Confirm your results with a graphing utility.

$$f(x,y) = x^4 + 4x^2(y-2) + 7(y-1)^2$$

The critical points of f(x,y) are $(0,1),\left(\sqrt{\frac{14}{3}},-\frac{1}{3}\right),\left(-\sqrt{\frac{14}{3}},-\frac{1}{3}\right)$

(Type an ordered pair. Use a comma to separate answers as needed.)

Use the Second Derivative Test to determine the behavior of f at each critical point. At what points does f have a local maximum? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** f has (a) local maximum (maxima) at ________.
  (Type an ordered pair. Use a comma to separate answers as needed.)
- **B.** f does not have a local maximum.

At what points does f have a local minimum? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** f has (a) local minimum (minima) at ________.
  (Type an ordered pair. Use a comma to separate answers as needed.)
- **B.** f does not have a local minimum.

What are the saddle points of f? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** f has (a) saddle point(s) at ________.
  (Type an ordered pair. Use a comma to separate answers as needed.)
- **B.** f does not have a saddle point.

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8. A delivery company requires that any box delivered must have a length plus girth (distance around) totaling no more than 126 inches. Find the dimensions of the box with maximum volume that can be sent.

The dimensions of the box with maximum volume that can be sent are 21,21,42 inches.
(Simplify your answer. Use a comma to separate answers as needed.)

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9. A rectangular metal tank with an open top is to hold 500 cubic feet of liquid. What are the dimensions of the tank that require the least material to build?

The tank that requires the least amount of material to build has length 10 ft, width 10 ft, and height 5 ft.
(Simplify your answers.)

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10. Show that the Second Derivative Test is inconclusive when applied to the following function at (0,0). Describe the behavior of the function at the critical point.

$$f(x,y) = 8x^2y - 5$$

Evaluate the discriminant $D = f_{xx}f_{yy} - \left(f_{xy}\right)^2$ at (0,0).

$D(0,0) =$ 0

The Second Derivative Test is inconclusive because $D(0,0) = 0$. What is the behavior of f at (0,0)? Choose the correct answer below.

- **A.** f has a local maximum at (0,0).
- **B.** f has a local minimum at (0,0).
- **C.** f has a saddle point at (0,0).

11. Show that the Second Derivative Test is inconclusive when applied to the following function at (0,0). Describe the behavior of the function at the critical point.

$f(x,y) = \sin\left(x^2y^2\right)$

First, determine whether the given function meets the conditions of the Second Derivative Test. Find the first partial derivatives. Start with $f_x(x,y)$.

$f_x(x,y) = 2xy^2 \cos\left(x^2y^2\right)$

Find $f_y(x,y)$.

$f_y(x,y) = 2x^2y \cos\left(x^2y^2\right)$

Are the first partial derivatives both equal to 0 at the point (0,0)?

- (x) Yes
- ( ) No

Now, find the second partial derivatives. Start with $f_{xx}(x,y)$.

$f_{xx}(x,y) = 2y^2\left(\cos\left(x^2y^2\right) - 2x^2y^2 \sin\left(x^2y^2\right)\right)$

Find $f_{yy}(x,y)$.

$f_{yy}(x,y) = 2x^2\left(\cos\left(x^2y^2\right) - 2x^2y^2 \sin\left(x^2y^2\right)\right)$

Find $f_{xy}(x,y)$.

$f_{xy}(x,y) = 4xy \cos\left(x^2y^2\right) - 4x^3y^3 \sin\left(x^2y^2\right)$

Are all of these second partial derivatives continuous throughout an open disk centered at the point (0,0)?

- (x) Yes
- ( ) No

Evaluate the discriminant of f at (0,0).

$D(0,0) =$ 0 (Simplify your answer.)

The value of D(0,0) indicates that the Second Derivative Test is inconclusive for the given function because $D(a,b) = 0$.

The critical point (0,0) is a(n) local, but not absolute, minimum. There are local, but not absolute, minima all along the x- and y-axes.

12. If $f(x,y,z) = 2x^2 + y^2 + 5z^2$ and $g(x,y,z) = 4x + 3y - 8z + 6 = 0$, write the Lagrange multiple conditions that must be satisfied by a point that maximizes or minimizes f subject to $g(x,y,z) = 0$.

Choose the correct answer below.

- (x) **A.** $4x = 4\lambda$, $2y = 3\lambda$, $10z = -8\lambda$, $4x + 3y - 8z + 6 = 0$
- ( ) **B.** $2x = 8\lambda$, $4y = -3\lambda$, $10z = 10\lambda$, $2x^2 + y^2 + 5z^2 = 0$
- ( ) **C.** $x = 4\lambda$, $2y = 5\lambda$, $3z = 8\lambda$, $6x^2 + 5y^2 + z^2 = 0$
- ( ) **D.** $3x = -3\lambda$, $8y = 4\lambda$, $10z = -4\lambda$, $3x + 8y - 4z + 6 = 0$

13. Use Lagrange multipliers to find the maximum and minimum values of f (if they exist) subject to the given constraint.

$f(x,y) = 3x + 3y$ subject to $9x^2 - 9xy + 9y^2 = 9$

If there is a maximum value, what is it? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The maximum value is 6 . (Type an exact answer.)
- **B.** There is no maximum value.

If there is a minimum value, what is it? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The minimum value is -6 . (Type an exact answer.)
- **B.** There is no minimum value.