Student: Phong Vo **Date:** 02/19/20	**Instructor:** Erica Yankowskas **Course:** Linear Algebra I (Spring 2020)	**Assignment:** Section 1.4 Homework

1. Compute the product using (a) the definition where A**x** is the linear combination of the columns of A using the corresponding entries in **x** as weights, and (b) the row-vector rule for computing A**x**. If a product is undefined, explain why.

$$\begin{bmatrix} -9 & 10 \\ 5 & 1 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 8 \\ -2 \\ 9 \end{bmatrix}$$

(a) Compute the product using the definition where A**x** is the linear combination of the columns of A using the corresponding entries in **x** as weights. If the product is undefined, explain why. Select the correct choice below and, if necessary, fill in any answer boxes to complete your choice.

- **A.** A**x** =
- **B.** The matrix-vector A**x** is not defined because the number of rows in matrix A does not match the number of entries in the vector **x**.
- **C.** The matrix-vector A**x** is not defined because the number of columns in matrix A does not match the number of entries in the vector **x**.

(b) Compute the product using the row-vector rule for computing A**x**. If the product is undefined, explain why. Select the correct choice below and, if necessary, fill in any answer boxes to complete your choice.

- **A.** A**x** =
- **B.** The matrix-vector A**x** is not defined because the row-vector rule states that the number of columns in matrix A must match the number of entries in the vector **x**.
- **C.** The matrix-vector A**x** is not defined because the row-vector rule states that the number of rows in matrix A must match the number of entries in the vector **x**.

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2. Use the definition of A**x** to write the vector equation as a matrix equation.

$$x_1\begin{bmatrix} 8 \\ 3 \\ 1 \\ 2 \end{bmatrix} + x_2\begin{bmatrix} 1 \\ 6 \\ -3 \\ -5 \end{bmatrix} + x_3\begin{bmatrix} 2 \\ 1 \\ -9 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 6 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 1 & 2 \\ 3 & 6 & 1 \\ 1 & -3 & -9 \\ 2 & -5 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 6 \\ 7 \end{bmatrix}$$

(Type an integer or simplified fraction for each matrix element.)

3. Write the system first as a vector equation and then as a matrix equation.

$$\begin{aligned} 9x_1 + x_2 - 3x_3 &= 9 \\ 9x_2 + 4x_3 &= 0 \end{aligned}$$

Write the system as a vector equation. Select the correct choice below and, if necessary, fill in any answer boxes to complete your choice.

A. $x_1 \begin{bmatrix} 9 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 9 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$

B. $\underline{\qquad} \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} = \underline{\qquad}$

C. $\underline{\qquad} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \underline{\qquad}$

Write the system as a matrix equation. Select the correct choice below and, if necessary, fill in any answer boxes to complete your choice.

A. $\begin{bmatrix} 9 & 1 & -3 \\ 0 & 9 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$

B. $\underline{\qquad} \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} = \underline{\qquad}$

C. $x_1 \underline{\qquad} + x_2 \underline{\qquad} + x_3 \underline{\qquad} = \underline{\qquad}$

4. Write the system first as a vector equation and then as a matrix equation.

$$\begin{aligned} 2x_1 - x_2 &= 5 \\ 8x_1 + 3x_2 &= 4 \\ 5x_1 - x_2 &= 1 \end{aligned}$$

Write the system as a vector equation. Select the correct choice below and, if necessary, fill in any answer boxes to complete your choice.

A. $x_1 \begin{bmatrix} 2 \\ 8 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix}$

B. $\quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \quad$

C. $\quad \begin{bmatrix} x_1 & x_2 \end{bmatrix} = \quad$

Write the system as a matrix equation. Select the correct choice below and, if necessary, fill in any answer boxes to complete your choice.

A. $\begin{bmatrix} 2 & -1 \\ 8 & 3 \\ 5 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix}$

B. $x_1 \quad + x_2 \quad =$

C. $\quad \begin{bmatrix} x_1 & x_2 \end{bmatrix} = \quad$

5. Given A and **b** to the right, write the augmented matrix for the linear system that corresponds to the matrix equation A**x** = **b**. Then solve the system and write the solution as a vector.

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 1 & 5 & 2 \\ 4 & 2 & 4 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -8 \\ 10 \\ -26 \end{bmatrix}$$

Write the augmented matrix for the linear system that corresponds to the matrix equation A**x** = **b**. Select the correct choice below and fill in any answer boxes within your choice.

A. $\begin{bmatrix} 1 & 3 & -2 \\ 1 & 5 & 2 \\ 4 & 2 & 4 \end{bmatrix}$ **B.** $\begin{bmatrix} -8 & & \\ 10 & & \\ -26 & & \end{bmatrix}$

Solve the system and write the solution as a vector. Select the correct choice below and fill in any answer boxes within your choice.

A. $\mathbf{x} = \begin{bmatrix} \\ \end{bmatrix}$

B. $\mathbf{x} = \begin{bmatrix} -11 \\ 3 \\ 3 \end{bmatrix}$

6.

Given A and **b** to the right, write the augmented matrix for the linear system that corresponds to the matrix equation A**x** = **b**. Then solve the system and write the solution as a vector.

$$A = \begin{bmatrix} 1 & 6 & -7 \\ -4 & -2 & 6 \\ 2 & 3 & 3 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -14 \\ 56 \\ -4 \end{bmatrix}$$

Write the augmented matrix for the linear system that corresponds to the matrix equation A**x** = **b**. Select the correct choice below and fill in any answer boxes within your choice.

A. $\begin{bmatrix} 1 & \square & \square \\ \square & -2 & \square \\ \square & \square & 3 \end{bmatrix}$

B. $\left[\begin{array}{ccc|c} 1 & 6 & -7 & -1 \\ -4 & -2 & 6 & 56 \\ 2 & 3 & 3 & -4 \end{array}\right]$ ✔

Solve the system and write the solution as a vector. Select the correct choice below and fill in any answer boxes within your choice.

A. $\mathbf{x} = \begin{bmatrix} \square \\ \square \end{bmatrix}$

B. $\mathbf{x} = \begin{bmatrix} -11 \\ 3 \\ 3 \end{bmatrix}$ ✔

7.

Let $A = \begin{bmatrix} 1 & -2 & -1 \\ -3 & 3 & 0 \\ 2 & -1 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Show that the equation A**x** = **b** does not have a solution for all possible **b**, and describe the set of all **b** for which A**x** = **b** does have a solution.

How can it be shown that the equation A**x** = **b** does not have a solution for all possible **b**? Choose the correct answer below.

- **A.** Row reduce the augmented matrix $\begin{bmatrix} A & \mathbf{b} \end{bmatrix}$ to demonstrate that $\begin{bmatrix} A & \mathbf{b} \end{bmatrix}$ has a pivot position in every row.
- **B.** Row reduce the matrix A to demonstrate that A does not have a pivot position in every row. ✔
- **C.** Row reduce the matrix A to demonstrate that A has a pivot position in every row.
- **D.** Find a vector **x** for which A**x** = **b** is the zero vector.
- **E.** Find a vector **b** for which the solution to A**x** = **b** is the zero vector.

Describe the set of all **b** for which A**x** = **b** does have a solution.

$0 = b_1 + b_2 + b_3$

(Type an expression using b_1, b_2, and b_3 as the variables and 1 as the coefficient of b_3.)

8.

Let $\mathbf{v}_1 = \begin{bmatrix} 0 \\ 0 \\ -5 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ -3 \\ 10 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 5 \\ -2 \\ -15 \end{bmatrix}$. Does $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ span $\mathbb{R}^3$? Why or why not?

Choose the correct answer below.

- **A.** Yes. Any vector in $\mathbb{R}^3$ except the zero vector can be written as a linear combination of these three vectors.
- **B.** No. When the given vectors are written as the columns of a matrix A, A has a pivot position in only two rows.
- **C.** No. The set of given vectors spans a plane in $\mathbb{R}^3$. Any of the three vectors can be written as a linear combination of the other two.
- **D.** Yes. When the given vectors are written as the columns of a matrix A, A has a pivot position in every row.

9. Determine whether each statement below is true or false. Justify each answer.

a. The equation A**x** = **b** is referred to as a vector equation. Choose the correct answer below.

- **A.** False. The equation A**x** = **b** is referred to as a matrix equation because A is a matrix.
- **B.** True. The equation A**x** = **b** is referred to as a vector equation because it consists of scalars multiplied by vectors.
- **C.** True. The equation A**x** = **b** is referred to as a vector equation because A is constructed from column vectors.
- **D.** False. The equation A**x** = **b** is referred to as a linear equation because **b** is a linear combination of vectors.

b. A vector **b** is a linear combination of the columns of a matrix A if and only if the equation A**x** = **b** has at least one solution. Choose the correct answer below.

- **A.** False. If the matrix A is the identity matrix, then the equation A**x** = **b** has at least one solution, but **b** is not a linear combination of the columns of A.
- **B.** True. The equation A**x** = **b** is unrelated to whether the vector **b** is a linear combination of the columns of a matrix A.
- **C.** True. The equation A**x** = **b** has the same solution set as the equation $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \cdots + x_n\mathbf{a}_n = \mathbf{b}$.
- **D.** False. If the equation A**x** = **b** has infinitely many solutions, then the vector **b** cannot be a linear combination of the columns of A.

c. The equation A**x** = **b** is consistent if the augmented matrix $\begin{bmatrix} A & \mathbf{b} \end{bmatrix}$ has a pivot position in every row. Choose the correct answer below.

- **A.** True. If the augmented matrix $\begin{bmatrix} A & \mathbf{b} \end{bmatrix}$ has a pivot position in every row, then the equation A**x** = **b** has a solution for each **b** in $\mathbb{R}^m$.
- **B.** False. The augmented matrix $\begin{bmatrix} A & \mathbf{b} \end{bmatrix}$ cannot have a pivot position in every row because it has more columns than rows.
- **C.** False. If the augmented matrix $\begin{bmatrix} A & \mathbf{b} \end{bmatrix}$ has a pivot position in every row, the equation equation A**x** = **b** may or may not be consistent. One pivot position may be in the column representing **b**.
- **D.** True. The pivot positions in the augmented matrix $\begin{bmatrix} A & \mathbf{b} \end{bmatrix}$ always occur in the columns that represent A.

d. The first entry in the product A**x** is a sum of products. Choose the correct answer below.

- **A.** False. The first entry in A**x** is the product of x_1 and the column $\mathbf{a}_1$.
- **B.** False. The first entry in A**x** is the sum of the corresponding entries in **x** and the first entry in each column of A.
- **C.** True. The first entry in A**x** is the sum of the products of corresponding entries in **x** and the first entry in each column of A.
- **D.** True. The first entry in A**x** is the sum of the products of corresponding entries in **x** and the first column of A.

e. If the columns of an $m \times n$ matrix A span $\mathbb{R}^m$, then the equation A**x** = **b** is consistent for each **b** in $\mathbb{R}^m$. Choose the correct answer below.

- **A.** True. Since the columns of A span $\mathbb{R}^m$, the augmented matrix $\begin{bmatrix} A & \mathbf{b} \end{bmatrix}$ has a pivot position in every row.

- B. False. Since the columns of A span $\mathbb{R}^m$, the matrix A has a pivot position in exactly m − 1 rows.
- C. True. If the columns of A span $\mathbb{R}^m$, then the equation A**x** = **b** has a solution for each **b** in $\mathbb{R}^m$.
- D. False. If the columns of A span $\mathbb{R}^m$, then the equation A**x** = **b** is inconsistent for each **b** in $\mathbb{R}^m$.

f. If A is an $m \times n$ matrix and if the equation A**x** = **b** is inconsistent for some **b** in $\mathbb{R}^m$, then A cannot have a pivot position in every row. Choose the correct answer below.

- A. False. Since the equation A**x** = **b** has a solution for each **b** in $\mathbb{R}^m$, the equation A**x** = **b** is consistent for each **b** in $\mathbb{R}^m$.
- B. True. If A is an $m \times n$ matrix and if the equation A**x** = **b** is inconsistent for some **b** in $\mathbb{R}^m$, then the equation A**x** = **b** has no solution for some **b** in $\mathbb{R}^m$.
- C. False. If A is an $m \times n$ matrix and if the equation A**x** = **b** is inconsistent for some **b** in $\mathbb{R}^m$, then the equation A**x** = **b** has a solution for each **b** in $\mathbb{R}^m$.
- D. True. If A is an $m \times n$ matrix and if the equation A**x** = **b** is inconsistent for some **b** in $\mathbb{R}^m$, then the columns of A span $\mathbb{R}^m$.

10. Determine whether each of statements a through f below are true or false. Justify each answer.

a. Every matrix equation A**x** = **b** corresponds to a vector equation with the same solution set. Choose the correct answer below.

- **A.** False. The matrix equation A**x** = **b** does not correspond to a vector equation with the same solution set.
- **B.** False. The matrix equation A**x** = **b** only corresponds to an inconsistent system of vector equations.
- **C.** True. The matrix equation A**x** = **b** is simply another notation for the vector equation $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \cdots + x_n\mathbf{a}_n = \mathbf{b}$, where $\mathbf{a}_1, \ldots, \mathbf{a}_n$ are the columns of A.
- **D.** True. The matrix equation A**x** = **b** is simply another notation for the vector equation $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \cdots + x_n\mathbf{a}_n = \mathbf{b}$, where $\mathbf{a}_1, \ldots, \mathbf{a}_n$ are the rows of A.

b. If the equation A**x** = **b** is consistent, then **b** is in the set spanned by the columns of A. Choose the correct answer below.

- **A.** False. **b** is only included in the set spanned by the columns of A if A**x** = **b** is inconsistent.
- **B.** False. A**x** = **b** is only consistent if the values of **b** are nonzero.
- **C.** True. The equation A**x** = **b** has a nonempty solution set if and only if **b** is a linear combination of the columns of A.
- **D.** True. The equation A**x** = **b** has a solution set if and only if A has a pivot position in every row.

c. Any linear combination of vectors can always be written in the form A**x** for a suitable matrix A and vector **x**. Choose the correct answer below.

- **A.** True. The matrix A is the matrix of coefficients of the system of vectors.
- **B.** True. A**x** can be written as a linear combination of vectors because any two vectors can be combined by addition.
- **C.** False. A and **x** cannot be written as a linear combination because the matrices do not have the same dimensions.
- **D.** False. A and **x** can only be written as a linear combination of vectors if and only if in A**x** = **b**, **b** is nonzero.

d. If the coefficient matrix A has a pivot position in every row, then the equation A**x** = **b** is inconsistent. Choose the correct answer below.

- **A.** False. If a coefficient matrix A has a pivot position in every row, then the equation A**x** = **b** may or may not be consistent.
- **B.** True. A pivot position in every row of a matrix indicates an inconsistent system of equations because the augmented column will always be zeros.
- **C.** False. If A has a pivot position in every row, the echelon form of the augmented matrix could not have a row such as [0 0 0 1], and A**x** = **b** must be consistent.
- **D.** True. If A has a pivot position in every row, then the augmented matrix must have a row of all zeros, indicating an inconsistent system of equations.

e. The solution set of a linear system whose augmented matrix is $\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{b} \end{bmatrix}$ is the same as the solution set of A**x** = **b**, if A= $\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{bmatrix}$. Choose the correct answer below.

- **A.** True. If A is an $m \times n$ matrix with columns $\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_n \end{bmatrix}$, and **b** is a vector in $\mathbb{R}^m$, the matrix equation A**x** = **b** has the same solution set as the system of linear equations whose augmented matrix is $\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_n & \mathbf{b} \end{bmatrix}$.

- **B.** True. The linear system whose augmented matrix is $\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{b} \end{bmatrix}$ will have the same solution set as $A\mathbf{x} = \mathbf{b}$ if and only if $\mathbf{b}$ is nonzero.
- **C.** False. If A is an $m \times n$ matrix with columns $\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_n \end{bmatrix}$, then $\mathbf{b}$ cannot be found in $\mathbb{R}^m$, and the system is inconsistent.
- **D.** False. The solution set of a linear system whose augmented matrix is $\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{b} \end{bmatrix}$ is the same as the solution set of $A\mathbf{x} = \mathbf{b}$ if and only if $\mathbf{x}$ has the same number of rows as A.

f. If A is an $m \times n$ matrix whose columns do not span $\mathbb{R}^m$, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbb{R}^m$.
Choose the correct answer below.

- **A.** True. If $A\mathbf{x} = \mathbf{b}$ is consistent, then the rows of A must span $\mathbb{R}^m$.
- **B.** True. If the columns of A do not span $\mathbb{R}^m$, $\mathbf{b}$ may or may not span $\mathbb{R}^m$.
- **C.** False. If the columns of A do not span $\mathbb{R}^m$, $A\mathbf{x} = \mathbf{b}$ cannot be consistent.
- **D.** False. If the columns of A do not span $\mathbb{R}^m$, then A does not have a pivot position in every row, and row reducing $\begin{bmatrix} A & \mathbf{b} \end{bmatrix}$ could result in a row of the form $\begin{bmatrix} 0 & 0 & \cdots & 0 & c \end{bmatrix}$, where c is a nonzero real number.

11.

Let $\mathbf{u} = \begin{bmatrix} -2 \\ -6 \\ -4 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -4 \\ 6 \\ -2 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 30 \\ -18 \\ 24 \end{bmatrix}$. It can be shown that $-3\mathbf{u} - 6\mathbf{v} - \mathbf{w} = 0$. Use this fact (and no row operations) to find x_1 and x_2 that satisfy the equation $\begin{bmatrix} -2 & -4 \\ -6 & 6 \\ -4 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 30 \\ -18 \\ 24 \end{bmatrix}$.

$x_1 =$ -3

$x_2 =$ -6

(Simplify your answers.)

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12. Could a set of three vectors in $\mathbb{R}^4$ span all of $\mathbb{R}^4$? Explain. What about n vectors in $\mathbb{R}^m$ when n is less than m?

Could a set of three vectors in $\mathbb{R}^4$ span all of $\mathbb{R}^4$? Explain. Choose the correct answer below.

- **A.** Yes. Any number of vectors in $\mathbb{R}^4$ will span all of $\mathbb{R}^4$.
- **B.** No. There is no way for any number of vectors in $\mathbb{R}^4$ to span all of $\mathbb{R}^4$.
- **C.** No. The matrix A whose columns are the three vectors has four rows. To have a pivot in each row, A would have to have at least four columns (one for each pivot.)
- **D.** Yes. A set of n vectors in $\mathbb{R}^m$ can span $\mathbb{R}^m$ when $n < m$. There is a sufficient number of rows in the matrix A formed by the vectors to have enough pivot points to show that the vectors span $\mathbb{R}^m$.

Could a set of n vectors in $\mathbb{R}^m$ span all of $\mathbb{R}^m$ when n is less than m? Explain. Choose the correct answer below.

- **A.** No. Without knowing values of n and m, there is no way to determine if n vectors in $\mathbb{R}^m$ will span all of $\mathbb{R}^m$.
- **B.** Yes. A set of n vectors in $\mathbb{R}^m$ can span $\mathbb{R}^m$ if $n < m$. There is a sufficient number of rows in the matrix A formed by the vectors to have enough pivot points to show that the vectors span $\mathbb{R}^m$.
- **C.** Yes. Any number of vectors in $\mathbb{R}^m$ will span all of $\mathbb{R}^m$.
- **D.** No. The matrix A whose columns are the n vectors has m rows. To have a pivot in each row, A would have to have at least m columns (one for each pivot.)

13. Determine if the columns of the matrix to the right span $\mathbb{R}^4$.

$$\begin{bmatrix} 11 & -5 & 4 & 2 & 6 \\ -7 & 5 & -5 & 3 & -4 \\ -7 & 9 & -6 & 8 & -7 \\ 2 & -2 & 9 & -9 & 11 \end{bmatrix}$$

Choose the correct answer below.

- **A.** The columns of the matrix span $\mathbb{R}^4$.
- **B.** The columns of the matrix do not span $\mathbb{R}^4$.