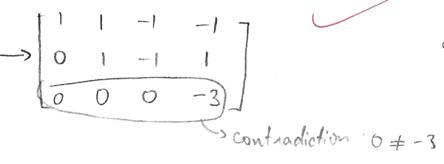Name: PHONG VO

Linear Algebra: Quiz 2
Show ALL work, as unjustified answers may receive no credit. Calculators are not allowed on any quiz or test paper. Make sure to exhibit skills discussed in class. Box all answers and clean up answers as much as possible.

$\frac{10}{10}$

5 1. [5pts] Is $\vec{b} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$ a linear combination of the vectors $\vec{a_1} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $\vec{a_2} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $\vec{a_3} = \begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix}$?

$$\begin{bmatrix} 0 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & 2 & -2 & 3 \end{bmatrix} \xrightarrow{} \begin{bmatrix} 1 & 1 & -1 & -1 \\ 0 & 1 & -1 & 1 \\ 1 & 2 & -2 & 3 \end{bmatrix} \xrightarrow[=nR_3]{R_3-R_2} \begin{bmatrix} 1 & 1 & -1 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & 4 \end{bmatrix} \xrightarrow[uR_3]{\substack{R_2 \\ -R_3}}$$

$$\xrightarrow{} \begin{bmatrix} 1 & 1 & -1 & -1 \\ 0 & 1 & -1 & 1 \\ \boxed{0 & 0 & 0 & -3} \end{bmatrix}$$

$\rightarrow$ contradiction $0 \neq -3$

So, $\vec{b}$ is $\boxed{\text{NOT}}$ a linear combination of $\{ \vec{a_1}, \vec{a_2}, \vec{a_3} \}$

2. [5pts] For the following vectors, what value(s) of h is $\vec{y} \in span\{ \vec{v_1}, \vec{v_2} \}$:

5

$$\vec{v_1} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \quad \vec{v_2} = \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}, \quad \vec{y} = \begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ -2 & 8 & -3 \end{bmatrix} \xrightarrow[=nR_3]{2R_1 + R_3} \begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ 0 & 2 & 2h-3 \end{bmatrix} \xrightarrow[=nR_3]{\frac{R_3}{2}}$$

$$\xrightarrow{} \begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ 0 & 1 & \frac{2h-3}{2} \end{bmatrix} \xrightarrow{} \frac{2h-3}{2} = -5$$

To let $\vec{y} \in span\{ \vec{v_1}, \vec{v_2} \}$

$\rightarrow 2h - 3 = -10$

$2h = -7 \rightarrow \boxed{h = \frac{-7}{2}}$