Name:

Linear Algebra I: Exam 1 (Spring 2020)

<u>Show ALL work, as unjustified answers may receive no credit</u>. Calculators are not allowed on any quiz or test paper. *<u>Make sure to exhibit skills discussed in class</u>*. Box all answers and simplify answers as much as possible.

Good Luck! ☺

1. <u>Systems of Linear Equations</u>

[6pts] Determine the value(s) of h for which the following linear system is consistent:

$$\begin{cases} 9x_1 + hx_2 = 9 \\ hx_1 + x_2 = -3 \end{cases}$$

2. ***The Matrix Equation, $A\vec{x} = \vec{b}$***

Consider the following matrix equation:

$$\begin{bmatrix} 1 & 2 & 13 \\ 1 & -1 & -2 \\ 2 & 4 & 26 \\ 2 & 1 & 11 \\ 3 & 3 & 24 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -13 \\ 2 \\ -26 \\ -11 \\ -24 \end{bmatrix}$$

(a) [3pts] Write the given Matrix Equation as a System of Linear Equations.

(b) [9pts] Solve the system and write the general solution in a parametric vector form.

3. ***Solution Sets of Linear Systems***

Consider the following:

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} -4 \\ -2 \\ 2 \end{bmatrix}$$

(a) [9pts] Solve the Nonhomogeneous System $A\vec{x} = \vec{b}$ and write the solution in parametric-vector form.

(b) [3pts] Using the parametric vector form of the solution in part (a), determine a particular solution.

(c) [3pts] Write the general solution for the Homogeneous System, $A\vec{x} = \vec{0}$, in parametric vector form.

4. *Linear Independence*

Consider the following vectors:

$$\vec{v_1} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \vec{v_3} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \vec{v_4} = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$$

(a) [3pts] Show that the following set of vectors is Linearly Dependent: $\cancel{\{ \vec{v_1}, \vec{v_2} \}}$ **(-3) on my test; (+3) on your test ☺*

(b) [7pts] Show that the following set of vectors is Linearly Independent: $\{ \vec{v_1}, \vec{v_2}, \vec{v_3} \}$

(c) [7pts] Write $\vec{v_4}$ as a Linear Combination of $\{ \vec{v_1}, \vec{v_2}, \vec{v_3} \}$, if possible.

Bonus Question [5pts]:

Let $\overrightarrow{e_1}, \overrightarrow{e_2}, \overrightarrow{e_3} \in \mathbb{R}^3$ be the elementary vectors in $\mathbb{R}^3$, and let $\overrightarrow{y_1} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\overrightarrow{y_2} = \begin{bmatrix} -4 \\ 5 \\ 6 \end{bmatrix}$, & $\overrightarrow{y_3} = \begin{bmatrix} 7 \\ 8 \\ -9 \end{bmatrix}$.

Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a Linear Transformation that maps $\overrightarrow{e_1}$ to $\overrightarrow{y_1}$, maps $\overrightarrow{e_2}$ to $\overrightarrow{y_2}$, and maps $\overrightarrow{e_3}$ to $\overrightarrow{y_3}$.

Find the image under T of $\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$.

Scratch Work (Not Graded)