Simulated Annealing for Rounded Capacity Inequality

Sangil Han

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Abstract

Capacitated Vehicle Routing Problem is a well-known routing problem. By using it, we can solve many logistics problems. However, this problem is known as NP-hard so we use the the Branch-and-Cut algorithm. In Branch-and-Cut algorithm, we must get a violated Rounded Capacity Inequality(RCI) to cut. The exact RCI separation algorithm is also known as NP-hard. Therefore, we use a heuristic algorithm for RCI separation to get violated RCI in a reasonable time. The most well-known heuristic algorithm is CVRPSep, but it just finds violated RCI not considering the performance. To get the best violated RCI, we would like to use the Simulated Annealing(SA) algorithm. To improve the performance of SA, we also design a specific neighborhood solution searching method for RCI separation.

Contents

1 Introduction

1.1 Branch-and-Cut Algorithms for CVRP

Capacitated Vehicle Routing Problem(CVRP) is an optimization problem distributing goods from a central depot to multiple customers using a fixed number of vehicles while considering each vehicle's limited capacity. Naddef and Rinaldi(2002) defined CVRP as follows[1]. V is a node-set containing $n+1$ nodes numbered $0, 1, ..., n$. The node 0 corresponds to the depot, and the other nodes correspond to the n customers. V_0 is the set of customers, i.e., $V_0 = V \backslash 0$. Each customer i wants as much as d_i and each edge e has a positive cost c_e. Let K be the fixed number of vehicles. Each vehicle has the same capacity C. x_e is the number of times e appears in the K-route R which is the union of K routes $R_1,\ R_2,\ ...,\ R_K$. For a subset F of edge set E, and for a vector $x \in \mathbb{R}^E$, $x(F)$ means $\sum_{e \in F} x_e$. For $S \subset V$, $\delta(S)$ means an edge-set of edges that one endpoint is in $\delta(S)$ and the other in $V \backslash S$. The ILP of CVRP is given by:

$$min \sum_{e \in E} c_e x_e \tag{1}$$

subject to

$$x(\delta(i)) = 2 \qquad \forall i \in V_0 \tag{2}$$

$$x(\delta(0)) = 2K \tag{3}$$

$$x(\delta(S)) \geq 2 \left\lceil \frac{d(S)}{C} \right\rceil \qquad \forall \emptyset \neq S \subseteq V_0 \tag{4}$$

$$0 \leq x_e \leq 1 \qquad \forall e \in E \backslash \delta(0) \tag{5}$$

$$0 \leq x_e \leq 2 \qquad \forall e \in \delta(0) \tag{6}$$

$$x_e \ integer \qquad \forall e \in E \tag{7}$$

We must focus on (4), Rounded Capacity Inequality. The left-hand-side means the number of times the vehicles in the K-route cross between the set S and $V \backslash S$. The right-hand-side roughly means two times the number of vehicles needed to serve all customers in S.

Because it's hard to solve ILP, we first do linear relaxation by dropping (7). The LP is still difficult because the size of (4) is exponential. We can use the cutting plane technique.

The cutting plane algorithm is as follows. We remove (4) and get optimal solution x^*. If x^* is feasible for ILP, it is an optimal solution for ILP. Otherwise, we must use a separation algorithm. It gives us one constraint in (4) violated by x^* if exists. We can get an updated optimal solution after adding the constraint. Repeat it until getting the optimal solution for ILP or relaxation for ILP. If we get an optimal solution of relaxation of ILP, we must use the branch-and-bound algorithm to get the optimal solution of ILP.

1.2 Exact Separation for Rounded Capacity Inequality

The exact separation algorithm for RCI can be represented by MILP. Fukasawa et al.,(2004) defined the exact RCI separation algorithm as follows[2]. $\bar{x_{ij}}$ is fractional value. Define y_i as a binary variable equal to 1 if $i \in S \subset V$ and w_{ij} as a variable equal to 1 if an edge (i, j) belongs to $\delta(S)$. For each value of M in $\{0, ..., \left\lceil \frac{d(V_0)}{C} \right\rceil - 1\}$, we solve:

$$z = min \sum_{e \in E} \bar{x_{ij}} w_{ij} \tag{8}$$

subject to

$$w_{ij} \geq y_i - y_j \qquad \forall \{i, j\} \in E \tag{9}$$

$$w_{ij} \geq y_j - y_i \qquad \forall \{i, j\} \in E \tag{10}$$

$$\sum_{i \in V_0} d_i y_i \geq (M \cdot C) + 1 \tag{11}$$

$$y_0 = 0 \tag{12}$$

$$y_i \in \{0, 1\} \qquad \forall i \in V_0 \tag{13}$$

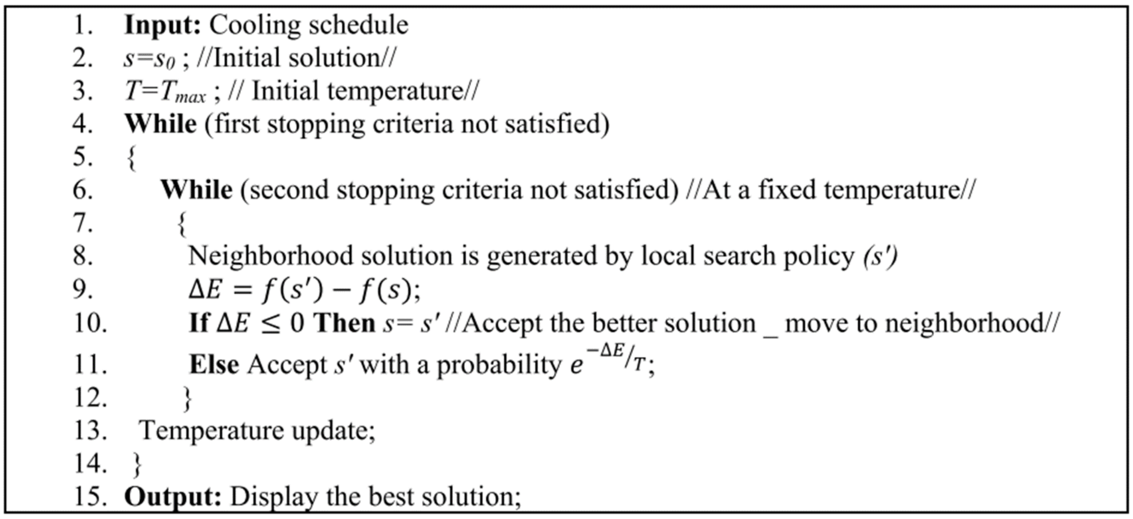
$$w_{ij} \geq 0 \qquad \forall \{i, j\} \in E \tag{14}$$

If $z < 2(M + 1)$, then y is the violated set. The MILP is known as NP-hard. Therefore, it's hard to use exact separation.

2 Simulated Annealing

2.1 Overview of SA

To get the violated RCI in a reasonable time, we can use the Simulated Annealing algorithm. It is a well-known meta-heuristic algorithm. Thanks to an efficient Metropolis acceptance criterion, SA can escape from local minima. The pseudo-code of SA is as follows.



```
**Input:** Cooling schedule
$s=s_0$ ; //Initial solution//
$T=T_{max}$ ; // Initial temperature//
**While** (first stopping criteria not satisfied)
{
    **While** (second stopping criteria not satisfied) //At a fixed temperature//
    {
        Neighborhood solution is generated by local search policy $(s')$
        $\Delta E = f(s') - f(s)$;
        **If** $\Delta E \leq 0$ **Then** $s = s'$ //Accept the better solution _ move to neighborhood//
        **Else** Accept $s'$ with a probability $e^{-\Delta E / T}$;
    }
  Temperature update;
}
**Output:** Display the best solution;
```

Figure 1: Pseudo code of SA

To perform SA, we need to set a cooling schedule, an initial solution, and how to get a neighborhood solution. The cooling schedule consists of initial temperature, temperature update function, stopping criterion, and length of Markov chain.

2.2 Initial Temperature

When setting the initial temperature, we must consider the acceptance rate. It means the acceptance probability of the neighborhood solution for the initial solution. The higher the acceptance rate is, the better the performance of SA is. However, the number of iterations increases as the acceptance rate increases and it makes the longer runtime. The recommended value for acceptance rate is 0.8.

Delahye, Chaimatanan, and Mongeat(2019) find the acceptance rate and initial temperature as follows[3]. Let m_1 be the number of transitions that improves the objective function value, and m_2 be the number of the other transitions. Moreover, let $\bar{\Delta}_f^{(+)}$ be the average of the objective function value differences over all the not improved transitions.

Then, the acceptance rate can be approximated by:

$$\chi(c) \simeq \frac{m_1 + m_2 \cdot e^{-(\frac{\bar{\Delta_f}^{(+)}}{c})}}{m_1 + m_2} \tag{15}$$

which yields

$$c \simeq \frac{\bar{\Delta_f}^{(+)}}{ln(\frac{m_2}{m_2 \cdot \chi(c) - m_1 \cdot (1-\chi(c)})} \tag{16}$$

If we set m_1, m_2 arbitrary, and $\chi(c)$ to 0.8, we can get the initial temperature.

2.3 Temperature Update Function

The temperature update function makes the temperature decrease. The most well-known and simple temperature update function is given by:

$$T_{k+1} = \alpha \cdot T_k \qquad (0.8 \leq \alpha \leq 0.99) \tag{17}$$

2.4 Stopping Criterion

Stop criterion can be set using current temperature, run time, iteration number, etc. The general method is using the current temperature. If T_k is closer to 0, we can stop SA. The most well-known and simple stop criterion is given by:

$$T_k < 0.01 \tag{18}$$

2.5 Length of Markov Chains

The length of Markov chains(L_k) is a transition number at a fixed temperature. Because the neighborhood solution and Metropolis acceptance criterion are influenced only by the current solution, we can regard the transitions as a Markov chain. Delahye et al.,(2019) find L_k as follows[3]. L_k is related to the probability that a neighborhood solution can be visited. The average number of elements of set S visited during N iteration is given by:

$$|S| \cdot [1 - e^{-\frac{N}{|S|}}] \tag{19}$$

for large N and large S. Let S_i be a neighborhood solution set for a given solution i. If no transition is accepted and if $N = |S_i|$, the probability that a neighborhood solution is visited is $1 - e^{-1} \simeq \frac{2}{3}$. A good choice for L_k is $|S_i|$.

2.6 Initial Solution

To start SA, we need an initial feasible solution. In RCI, the feasible solution is a set that doesn't include a depot node. In the study, 4 different initial solutions, selecting customers randomly, including all customers, selecting one customer with the most degree, and a solution of CVRPSep are used. CVRPSep is a package that finds violated sets using other heuristic algorithms.

2.7 Neighborhood Solution

The most well-known method to get a neighborhood solution is n-flip. It is a method that chooses n nodes randomly and changes the value of selected nodes. n-flip is easy to use when the solution consists of binary variables.

Another method to get a neighborhood solution is a hamming ball operator. Let k be the number of balls, and m be the number of flips. The current solution is divided by k. At each ball, m nodes are selected randomly and the value of nodes are changed.

These two methods are general method. To get a better solution, a more specific method focused on RCI separation can be used. When S is a node-set, the objective function to get good RCI is given by:

$$min\ f = x(\delta(S)) - 2\left\lceil \frac{d(S)}{C} \right\rceil \tag{20}$$

To decrease the objective function value, we must decrease $x(\delta(S))$ or increase $\left\lceil \frac{d(S)}{C} \right\rceil$. Assume that we use n-flip. The probability that $\left\lceil \frac{d(S)}{C} \right\rceil$ is changed is low because n is a small value due to the neighborhood solution set size. If it is too large, L_k is also a big value and the runtime can be too long. Therefore, it's hard to decrease $\left\lceil \frac{d(S)}{C} \right\rceil$. Let's see $x(\delta(S))$. If the node that isn't an endpoint of edge in $\delta(S)$ is selected, then $|\delta(S)|$ must be increased and $x(\delta(S))$ also be increased after a flip. However, if the node that is an endpoint of edge in $\delta(S)$ is selected, then $|\delta(S)|$ can be decreased and $x(\delta(S))$ also can be decreased after a flip. We can assume that it's better to select nodes that are an endpoint of edge in $\delta(S)$. Let's say this method to ***n*-flip in $\boldsymbol{\delta(S)}$**. [Figure 2] is an example of n-flip in $\delta(S)$. The numbers that are written in the nodes are the demand of nodes. $x_e = 1$ for the line, $x_e = 0.5$ for the dotted line. The nodes that can be selected are red.

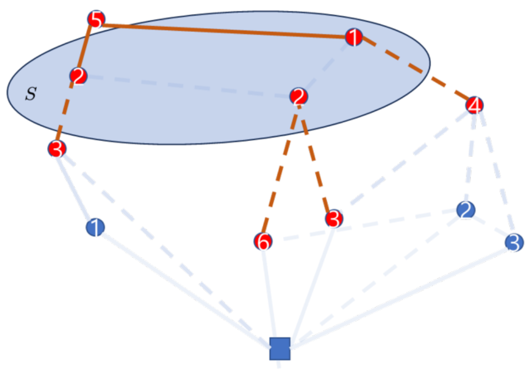


Figure 2: Example of n-flip in $\delta(S)$

Moreover, let k_i be the number of edges that are in $\delta(S)$ and whose one endpoint is node i. If k_i is the same or bigger than $degree(i) - k_i$, $|\delta(S)|$ must not be increased after flipping for node i. It makes the probability that $x(\delta(S))$ is decreased higher. Let's say this method to **tight n-flip in $\delta(S)$**. [Figure 3] is an example of tight n-flip in $\delta(S)$.

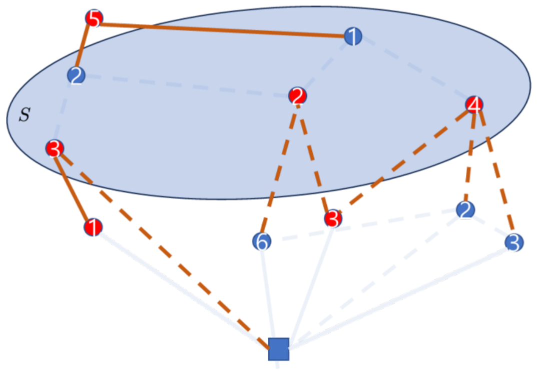


Figure 3: Example of tight n-flip in $\delta(S)$

Nodes that are an endpoint of edge in $\delta(S)$ can be divided into nodes in set S and not in S. Let these two sets be S_{in} and S_{out}. When we select nodes in S_{out} to flip, we can select nodes that increase $\left\lceil \frac{d(S)}{C} \right\rceil$. We also can select nodes in S_{in} that don't decrease $\left\lceil \frac{d(S)}{C} \right\rceil$. If we select nodes to flip in these nodes, the probability that the objective function value improves is high. Let's say this method to ***n*-flip in $\delta(S)$ considering demand**. [Figure 4] is an example of n-flip in $\delta(S)$ considering demand. Assume that the capacity is 10, select two nodes in S_{out} and one node in S_{in}.

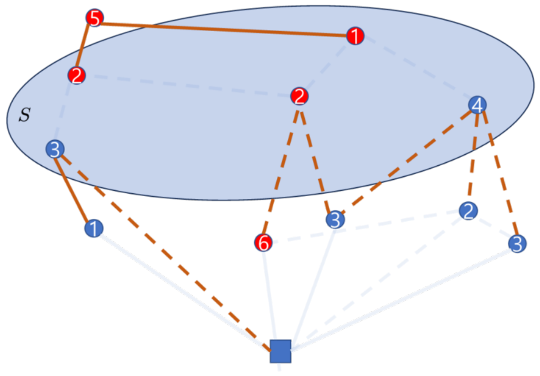


Figure 4: Example of n-flip in $\delta(S)$ considering demand

After a transition, we can improve the objective function value by adding the nodes that increase $\left\lceil \frac{d(S)}{C} \right\rceil$ to S. If $\left\lceil \frac{d(S)}{C} \right\rceil$ can be increased by just adding a node i, the objective function value must be the same or decrease because $x(\delta(i)) \leq 2$. If we need more than one node to increase $\left\lceil \frac{d(S)}{C} \right\rceil$, the objective function value can be increased after adding these nodes. However, if we allow to addition of only one node, the probability that the neighbor solution set is an empty set is high. Also, If we add more than one node, the solution that can be visited becomes diversified. When we set the number of added nodes as k, the node i that can be selected must satisfy the condition, $\left\lceil \frac{d(S)+k \cdot d(i)}{C} \right\rceil > \left\lceil \frac{d(S)}{C} \right\rceil$. Let's say this method to **improvement method**. [Figure 5] is an example of the improvement method. Assume that the capacity is 10, and the number of added nodes is up to 2.

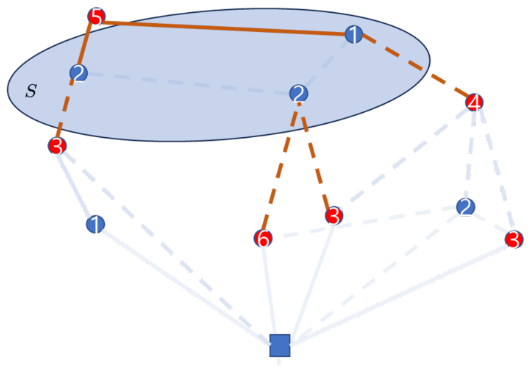


Figure 5: Example of improvement method

These methods can improve the objective function value after one transition. However, it doesn't guarantee that a better solution will be obtained after performing SA.

3 Experiments

Using diverse initial solutions, neighborhood solution searching methods, and cooling schedule settings, we can perform SA with several settings. In the study, we used data about 50 nodes, 75 nodes, 100 nodes, and 200 nodes. Each data has different 100 graph data. The result can be evaluated by the gap and success rate. The gap is given by:

$$Avg\left(\frac{(-f_{\text{exact}}) - (-f_{\text{SA}})}{(-f_{\text{exact}})}\right) \tag{21}$$

The success rate is given by:

$$\frac{\text{The number of success to find violated RCI}}{100} \tag{22}$$

The gaps for several settings are as follows.

	Initial Solution	Neighborhood Solution	L_k	Gap_50	Gap_75	Gap_100	Gap_200
1	all	1 flip	n	55.7964	67.7561	72.1	90.9774
2	all	2 flip	$4 \cdot \binom{n}{2}$	11.4562	31.3429	39.9459	85.57
3	all	2 flip in $\delta(S)$	$4 \cdot \binom{n_{in\ \delta(S)}}{2}$	32.5252	45.0197	50.0597	73.4208
4	one	2 flip	$4 \cdot \binom{n}{2}$	10.0987	24.7909	39.4349	67.5102
5	random	2 flip	$4 \cdot \binom{n}{2}$	10.8368	30.9756	37.6189	69.7466
6	CVRPSep	2 flip	$4 \cdot \binom{n}{2}$	6.6654	15.4478	20.9001	38.1849
7	CVRPSep	1 flip & improvement	$4 \cdot \binom{n}{2}$	9.1247	15.5408	15.178	17.5413
8	CVRPSep	2 flip & improvement	$4 \cdot \binom{n}{2}$	3.4905	7.8612	10.8351	16.8602
9	CVRPSep	2 flip in $\delta(S)$ & improvement	$4 \cdot \binom{n}{2}$	0.9912	7.29	7.357	8.5917
10	CVRPSep	hamming ball operator_3 & improvement	$(\frac{n}{3})^3$	3.0259	5.4181	11.6079	12.8734
11	CVRPSep	hamming ball operator_3 & improvement	$4 \cdot \binom{n}{2}$	2.6748	7.6122	10.8091	21.2108
12	CVRPSep	hamming ball operator_4 & improvement	$4 \cdot \binom{n}{2}$	2.6925	6.9041	7.6679	22.2573
13	CVRPSep	hamming ball operator_5 & improvement	$4 \cdot \binom{n}{2}$	3.9527	9.0045	10.8749	22.4901
14	CVRPSep	hamming ball operator_2 & improvement	$4 \cdot \binom{n}{2}$	3.4663	9.5997	11.6219	16.2258
15	CVRPSep	3 flip & improvement	$4 \cdot \binom{n}{2}$	3.1141	8.2442	8.6585	18.2919
16	CVRPSep	3 flip in $\delta(S)$ & improvement	$4 \cdot \binom{n}{2}$	0.2748	4.3907	6.2751	15.9961
17	CVRPSep	3 flip in $\delta(S)$ considering demand & improvement	$4 \cdot \binom{n}{2}$	7.1066	-	-	-
18	CVRPSep	2 flip in $\delta(S)$ considering demand & improvement	$4 \cdot \binom{n}{2}$	8.5847	-	19.8768	-
19	CVRPSep	2 flip in $\delta(S)$ & 2 flip in $\delta(S)$ considering demand & improvement	$4 \cdot \binom{n}{2}$	-	10.2669	-	-
20	CVRPSep	2 flip in $\delta(S)$ considering demand & improvement(accept only improved)	$4 \cdot \binom{n}{2}$	0.4815	4.0245	8.2993	11.605
21	CVRPSep	2 flip in $\delta(S)$ & improvement(upto 1 node)	$4 \cdot \binom{n}{2}$	0.2533	-	10.4175	29.0582
22	CVRPSep	2 flip in $\delta(S)$ & improvement(upto 3 nodes)	$4 \cdot \binom{n}{2}$	-	-	11.6856	-
23	CVRPSep	2 flip in $\delta(S)$ & 2 flip in $\delta(S)$ & improvement	$4 \cdot \binom{n}{2}$	0.1266	4.3646	6.6659	35.2763
24	CVRPSep	tight 2 flip in $\delta(S)$ & improvement	$4 \cdot \binom{n}{2}$	-	-	26.8237	-

Table 1: Gap between the result of SA and the exact RCI separation algorithm

The cooling schedule except for L_k is the same. $\alpha = 0.95$. L_k is originally the cardinality of the neighborhood solution set. However, to compare the performance of the searching methods, we also set L_k to $4 \cdot \binom{n}{2}$. If two neighborhood solutions except for the improvement method that is case 19 and 23 are used, the second neighborhood solution is accepted only when it is an improved solution. Because some settings(case17, 18, 20, 21, 22, and 24) didn't show good value for some data, we don't perform SA with the setting for the other data. The default number of added nodes in the improvement method is 2. [Table 1] shows that the best initial solution is CVRPSEP, and the best searching method is n-flip in $\delta(S)$

except for $n = 1$. When the improvement method with the default setting or accept-only-improved setting is added, the performance is much better.

The success rates for several settings are as follows.

	Initial Solution	Neighborhood Solution	L_k	Success rate_50	Success rate_75	Success rate_100	Success rate_200
1	all	1 flip	n	0.83	0.66	0.75	0.59
2	all	2 flip	$4 \cdot \binom{n}{2}$	0.99	0.91	0.81	0.52
3	all	2 flip in $\delta(S)$	$4 \cdot \binom{n_{in\ \delta(S)}}{2}$	0.85	0.75	0.75	0.45
4	one	2 flip	$4 \cdot \binom{n}{2}$	0.99	0.87	0.79	0.52
5	random	2 flip	$4 \cdot \binom{n}{2}$	0.97	0.83	0.82	0.49
6	CVRPSep	2 flip	$4 \cdot \binom{n}{2}$	0.98	0.9	0.89	0.85
7	CVRPSep	1 flip & improvement	$4 \cdot \binom{n}{2}$	0.99	0.94	0.89	0.93
8	CVRPSep	2 flip & improvement	$4 \cdot \binom{n}{2}$	0.99	0.95	0.93	0.93
9	CVRPSep	2 flip in $\delta(S)$ & improvement	$4 \cdot \binom{n}{2}$	1	0.95	0.95	0.98
10	CVRPSep	hamming ball operator_3 & improvement	$(\frac{n}{3})^3$	0.98	0.97	0.91	0.93
11	CVRPSep	hamming ball operator_3 & improvement	$4 \cdot \binom{n}{2}$	0.99	0.96	0.92	0.89
12	CVRPSep	hamming ball operator_4 & improvement	$4 \cdot \binom{n}{2}$	1	0.95	0.96	0.88
13	CVRPSep	hamming ball operator_5 & improvement	$4 \cdot \binom{n}{2}$	0.99	0.96	0.92	0.9
14	CVRPSep	hamming ball operator_2 & improvement	$4 \cdot \binom{n}{2}$	0.99	0.94	0.93	0.92
15	CVRPSep	3 flip & improvement	$4 \cdot \binom{n}{2}$	0.99	0.95	0.94	0.9
16	CVRPSep	3 flip in $\delta(S)$ & improvement	$4 \cdot \binom{n}{2}$	1	0.96	0.95	0.9
17	CVRPSep	3 flip in $\delta(S)$ considering demand & improvement	$4 \cdot \binom{n}{2}$	0.99	-	-	-
18	CVRPSep	2 flip in $\delta(S)$ considering demand & improvement	$4 \cdot \binom{n}{2}$	0.98	-	0.88	-
19	CVRPSep	2 flip in $\delta(S)$ & 2 flip in $\delta(S)$ considering demand & improvement	$4 \cdot \binom{n}{2}$	-	0.94	-	-
20	CVRPSep	2 flip in $\delta(S)$ considering demand & improvement(accept only improved)	$4 \cdot \binom{n}{2}$	1	0.97	0.94	0.95
21	CVRPSep	2 flip in $\delta(S)$ & improvement(upto 1 node)	$4 \cdot \binom{n}{2}$	1	-	0.94	0.85
22	CVRPSep	2 flip in $\delta(S)$ & improvement(upto 3 nodes)	$4 \cdot \binom{n}{2}$	-	-	0.92	-
23	CVRPSep	2 flip in $\delta(S)$ & 2 flip in $\delta(S)$ & improvement	$4 \cdot \binom{n}{2}$	1	0.96	0.95	0.76
24	CVRPSep	tight 2 flip in $\delta(S)$ & improvement	$4 \cdot \binom{n}{2}$	-	-	0.85	-

Table 2: Success rate for the result of SA

[Table 2] also shows that the best initial solution is CVRPSep and n-flip, n-flip in $\delta(S)$, and hamming ball operator with default or accept-only-improved improvement method are similar good searching methods. However, The success rate for 200 nodes data is especially high when the searching method is 2-flip in $\delta(S)$.

The average runtimes for several settings are as follow. The unit of runtime is sec.

	Initial Solution	Neighborhood Solution	L_k	Runtime_50	Runtime_75	Runtime_100	Runtime_200
1	all	1 flip	n	0.009	0.011	0.014	0.023
2	all	2 flip	$4 \cdot \binom{n}{2}$	0.5	1.2	2.25	9.42
3	all	2 flip in $\delta(S)$	$4 \cdot \binom{n_{in\ \delta(S)}}{2}$	0.17	0.8	1.8	8.7
4	one	2 flip	$4 \cdot \binom{n}{2}$	0.47	1.1	2	8.7
5	random	2 flip	$4 \cdot \binom{n}{2}$	0.36	0.9	1.5	6.5
6	CVRPSep	2 flip	$4 \cdot \binom{n}{2}$	0.5	1.1	2.1	8.6
7	CVRPSep	1 flip & improvement	$4 \cdot \binom{n}{2}$	0.8	1.9	3.4	14.3
8	CVRPSep	2 flip & improvement	$4 \cdot \binom{n}{2}$	0.8	2	3.7	15.4
9	CVRPSep	2 flip in $\delta(S)$ & improvement	$4 \cdot \binom{n}{2}$	1.2	3.3	6.3	30.6
10	CVRPSep	hamming ball operator_3 & improvement	$(\frac{n}{3})^3$	1.1	4	9	73.2
11	CVRPSep	hamming ball operator_3 & improvement	$4 \cdot \binom{n}{2}$	1	2.4	4.6	18.7
12	CVRPSep	hamming ball operator_4 & improvement	$4 \cdot \binom{n}{2}$	1.2	3	5.5	23.1
13	CVRPSep	hamming ball operator_5 & improvement	$4 \cdot \binom{n}{2}$	1.4	3.4	6.4	25.7
14	CVRPSep	hamming ball operator_2 & improvement	$4 \cdot \binom{n}{2}$	0.8	2	3.8	15.5
15	CVRPSep	3 flip & improvement	$4 \cdot \binom{n}{2}$	1.1	2.8	5	22
16	CVRPSep	3 flip in $\delta(S)$ & improvement	$4 \cdot \binom{n}{2}$	1.5	4.2	8.3	43.2
17	CVRPSep	3 flip in $\delta(S)$ considering demand & improvement	$4 \cdot \binom{n}{2}$	1.8	-	-	-
18	CVRPSep	2 flip in $\delta(S)$ considering demand & improvement	$4 \cdot \binom{n}{2}$	1.6	-	7.6	-
19	CVRPSep	2 flip in $\delta(S)$ & 2 flip in $\delta(S)$ considering demand & improvement	$4 \cdot \binom{n}{2}$	-	6.8	-	-
20	CVRPSep	2 flip in $\delta(S)$ considering demand & improvement(accept only improved)	$4 \cdot \binom{n}{2}$	4.8	15.4	30	171.6
21	CVRPSep	2 flip in $\delta(S)$ & improvement(upto 1 node)	$4 \cdot \binom{n}{2}$	1.2	-	6.7	35.6
22	CVRPSep	2 flip in $\delta(S)$ & improvement(upto 3 nodes)	$4 \cdot \binom{n}{2}$	-	-	6.7	-
23	CVRPSep	2 flip in $\delta(S)$ & 2 flip in $\delta(S)$ & improvement	$4 \cdot \binom{n}{2}$	6	18	36	198
24	CVRPSep	tight 2 flip in $\delta(S)$ & improvement	$4 \cdot \binom{n}{2}$	-	-	5.5	-

Table 3: Rumtime for the result of SA

[Table 3] shows that case 10, 20 and 23, the runtime is too long.

Using CVRPSep, n-flip in $\delta(S)$, and default improvement, the performance is best. If SA for that setting iterates more, a better solution can be obtained. To make more iterations, we set the acceptance rate to 0.9, $\alpha = 0.98$, and the stopping criterion to

$T_k \leq 0.001$. The result is as follows. To know how good the performance of SA is, we compared the result to the performance of CVRPSep. The results are shown in [Table 4], [Table 5], and [Table 6].

	Initial Solution	Neighborhood Solution	L_k	Gap_50	Gap_75	Gap_100	Gap_200
1	CVRPSep	2 flip in $\delta(S)$ & improvement	$4 \cdot \binom{n}{2}$	0.2983	5.1523	7.3447	5.4615
2	CVRPSep	3 flip in $\delta(S)$ & improvement	$4 \cdot \binom{n}{2}$	0.2533	3.096	5.0226	8.1804
3	CVRPSep	-	-	16.7526	30.4392	32.5385	43.4472

Table 4: Gap for the best result of SA and CVRPSep

	Initial Solution	Neighborhood Solution	L_k	Success rate_50	Success rate_75	Success rate_100	Success rate_200
1	CVRPSep	2 flip in $\delta(S)$ & improvement	$4 \cdot \binom{n}{2}$	1	0.96	0.94	0.99
2	CVRPSep	3 flip in $\delta(S)$ & improvement	$4 \cdot \binom{n}{2}$	1	0.97	0.95	0.97
3	CVRPSep	-	-	0.97	0.87	0.85	0.73

Table 5: Success rate for the best result of SA and CVRPSep

	Initial Solution	Neighborhood Solution	L_k	Runtime_50	Runtime_75	Runtime_100	Runtime_200
1	CVRPSep	2 flip in $\delta(S)$ & improvement	$4 \cdot \binom{n}{2}$	5.2	14.1	26.6	141
2	CVRPSep	3 flip in $\delta(S)$ & improvement	$4 \cdot \binom{n}{2}$	6.3	16.5	32.5	174

Table 6: Runtime for the best result of SA

Both the gap and success rate are improved a little. Compared to CVRPSep, both the gap and success rate are improved significantly. The runtimes are about 4 times longer.

4 Discussion

n-flip, n-flip in $\delta(S)$, tight n-flip in $\delta(S)$, and n-flip in $\delta(S)$ considering demand are more tight searching methods in order. That is, the probability that the solution after a transition is better than the current solution is higher in order. However, the performance is not better in order. Using n-flip in $\delta(S)$, the performance is the best. If tight n-flip in $\delta(S)$ or n-flip in $\delta(S)$ considering demand is used, the cardinality of the neighborhood solution set is too small and it means it's hard to search diverse solutions. n-flip in $\delta(S)$ also makes a small cardinality of neighborhood solution set compared to n-flip, but it means only unnecessary solutions are excluded from searching using n-flip in $\delta(S)$. These results show it's important to design the appropriate solution-excluding method. Hamming ball operator also excludes some solutions, but it excludes some good solutions and shows worse performance.

When the improvement method is used additionally, the performance is much better. The method makes SA search for more diverse solutions, particularly necessary solutions. n-flip in $\delta(S)$ only considers $x(\delta(S))$, but $2 \cdot \left\lceil \frac{d(S)}{C} \right\rceil$ more influences to objective function value because n is a small value. To consider $2 \cdot \left\lceil \frac{d(S)}{C} \right\rceil$, we designed other searching methods like tight n-flip in $\delta(S)$, and n-flip in $\delta(S)$. However, using these methods, the cardinality of the neighborhood solution set is too small and the performance is worse. On the contrary, the improvement method only searches for other solutions considering $2 \cdot \left\lceil \frac{d(S)}{C} \right\rceil$ and it makes the performance much better.

The performance of 1-flip in $\delta(S)$ is much worse than the other n-flip in $\delta(S)$ even with the same L_k. Using 1-flip in $\delta(S)$, the cardinality of the neighborhood solution set is n and it's too small compared to L_k. Therefore, SA with 1-flip in $\delta(S)$ and $L_k = 4 \cdot \binom{n}{2}$ may search the same neighborhood solution more compared to the other n-flip in $\delta(S)$. It makes it hard to search for a better solution.

5 Conclusion

The exact RCI separation algorithm is known as an NP-hard problem. To get the violated RCI in a reasonable time, we must use a heuristic algorithm. The most well-known method is using CVRPSep, but it doesn't consider the value of violation, i.e., $x(\delta(S)) - 2 \cdot \left\lceil \frac{d(S)}{C} \right\rceil$. The value of violation determines the performance of the violated RCI. The higher the value is, the better the violated RCI. To get the best violated RCI, we use the Simulated Annealing algorithm. When the general method for neighborhood solution and arbitrary initial solution are used, the performance isn't good. However, If the output of CVRPSep is used for the initial solution and more specific neighborhood solution searching methods for RCI are used, the performance is improved much better. SA for RCI is better in performance than CVRPSep, but not in runtime. It's hard to use SA in the Branch-and-Cut algorithm because it needs many violated RCIs. SA can be used to make a comparative group for other heuristic algorithms about RCI separation, instead. The specific neighborhood solution searching methods also can be used to design other heuristic algorithms like the genetic algorithm.

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