Phys 331 - Numerical Techniques for the Sciences I.
Homework 08: Ordinary Differential Equations Part 1

Posted November 4, 2024
Due November 7, 2024

Problem 1: Setting up the Three-body Problem [10 pts]

Using intermediate derivatives, any higher-order ordinary differential equation can be written as a series of first-order ordinary differential equation. E.g. a second-order ODE like Newton's second law:

$$\frac{d^2x(t)}{dt^2} = a(t) = F(t)/m \tag{1}$$

can be rewritten as two first-order equations

$$\frac{d}{dt}\begin{bmatrix} v(t) \\ x(t) \end{bmatrix} = \begin{bmatrix} F/m \\ v(t) \end{bmatrix} \tag{2}$$

where we have used the velocity, $\frac{dx(t)}{dt} = v(t)$, as an intermediate equation. This is important, because almost all numerical ODE solvers are geared to solve first-order ODEs. For studio this Friday, we are going to solve the 3-body problem, but first we need to rewrite it in terms of a series of 1st-order ODEs.

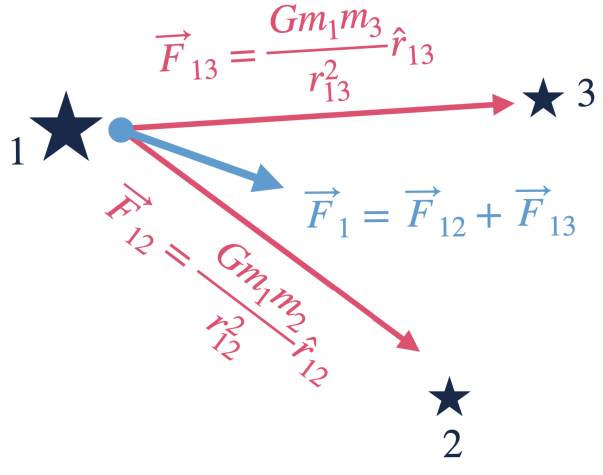

Figure 1: Force diagram for the 3-body problem.

To write the 3-body problem as a first-order ODE, you will need three positions (x_i, y_i, z_i) and three velocities $(v_{x,i}, v_{y,i}, v_{z,i})$ for each of $i = 1, 2, 3$. This means your 1st-order ODE

(equation 2) will have 18 independent equations: 6 for each particle corresponding to its 3D position and velocity.

(a) *Force Equations:* Write the 3 force equations for particle 1 ($F_{x,1}$, $F_{y,1}$, $F_{z,1}$) in Cartesian coordinates. Think carefully about how to write $r_{12} = |\vec{x}_1 - \vec{x}_2|$ and the displacement unit vector $\hat{r}_{12}$.

(b) *Vector Equations:* Write explicitly the 18-length vector form of the ODE for the 3-body problem. You may write individual forces as $\vec{F}_{12}$. **Hint:** you will only need three individual forces if you use Newton's 3rd law.