

$$\textcircled{1} \quad \frac{dy}{dx} = -2x - y$$

$$y(0) = -1$$

$$g(x) = ?$$

$$p(x) = ?$$

$$p(x) = ?$$

Method 1

$$\frac{dy}{dx} + p(x)y = g(x)$$

$$\frac{dy}{dx} + y = -2x \Rightarrow \begin{cases} p(x) = 1 \\ g(x) = -2x \end{cases}$$

$$\mu(x) = e^{\int p(x) dx}$$

$$\mu(x) = e^{\int 1 dx} = e^x \Rightarrow \boxed{\mu(x) = e^x}$$

$$\Rightarrow \mu(x) \left( \frac{dy}{dx} + y \right) = \mu(x) g(x)$$

$$e^x \left( \frac{dy}{dx} + y \right) = e^x \cdot (-2x)$$

$$e^x \frac{dy}{dx} + e^x y = -2xe^x$$

$$\frac{d}{dx} (e^x y) = -2xe^x$$

$$\frac{d}{dx} (e^x y) = -2xe^x$$

$$\int \frac{d}{dx} (e^x y) dx = \int -2xe^x dx$$

$$e^x y = \int -2xe^x dx = -2xe^x + 2e^x + C \Rightarrow e^x y = -2xe^x + 2e^x + C$$

$$u = -2x \quad dv = e^x dx$$

$$\frac{du}{dx} = -2 \quad v = e^x$$

$$y = -2x + 2 + \frac{C}{e^x}$$

$$y(0) = -2(0) + 2 + \frac{C}{e^0} = -1$$

$$C + 2 = -1$$

$$C = -3$$

$$y = -2x + 2 - \frac{3}{e^x}$$

$$\Rightarrow \boxed{y = -3e^{-x} - 2x + 2}$$



## Method 2

$$\frac{dy}{dx} + p(x) = q(x)$$

$$\rightarrow \frac{dy}{dx} + y = -2x$$

$$p(x) = 1$$

$$q(x) = -2x$$

homogenize and char eqn

$$\frac{dy}{dx} + y = 0$$

$$r^2 + 1 = 0$$

$$r = -1$$

$$y_H = C_0 e^{rx} = C_0 e^{-x} \rightarrow \boxed{y_H = C_0 e^{-x}}$$

let  $y_p = Ax + B$ ,  $A, B \in \mathbb{R}$ ,  $A, B = \text{const}$

particular  
soln

$$\frac{dy_p}{dx} = A$$

$$\frac{dy}{dx} + y = -2x$$

$$\frac{dy_p}{dx} + y_p = -2x$$

$$A + Ax + B = -2x$$

$$\text{coeff of } x: A = -2$$

$$\text{coeff of } 1: A + B = 0 \quad B = 2$$

(constant)

$$y_p = -2x + 2$$

$$y = y_H + y_p = C_0 e^{-x} - 2x + 2$$

$$y(0) = C_0 \cdot 1 - 2(0) + 2 = -1 \Rightarrow C_0 = -3$$

$$\boxed{y = -3e^{-x} - 2x + 2}$$