f) show $z^n = r^n(\cos(n\phi) + i\sin(n\phi))$

Euler formula. $z = re^{i\phi}$

$z^n = (re^{i\phi})^n = r^n e^{in\phi} = r^n(\cos(n\phi) + i\sin(n\phi))$ $\blacksquare$

g) Show $\ln(z) = \ln r + i(\phi + 2\pi n)$

$z = re^{i\phi}$

$\ln(z) = \ln(re^{i\phi}) = \ln r + \ln(e^{i\phi}) = \ln r + i\phi$

$\ln(z) = \ln r + i(\phi + 2\pi r)$ $\blacksquare$

Problem 1

a) Show $R[z^2] = x^2 - y^2$, $Im[z^2] = 2xy$

$z = x + iy$

$z^2 = (x+iy)^2 = x^2 + 2ixy - y^2 = (x^2 - y^2) + i(2xy)$

$R(z^2) = x^2 - y^2 \quad Im[z^2] = 2xy$

b) $z^* z$, show $z^* z = x^2 + y^2$

$z = x + iy \quad z^* = x - iy$

$z^* z = (x+iy)(x-iy) = x^2 - ixy + ixy + y^2 = x^2 + y^2$

c) Show $R(1/z) = \frac{x}{x^2+y^2}$, $Im\left[\frac{1}{z}\right] = \frac{-y}{x^2+y^2}$

$$\frac{1}{z} = \frac{1}{x+iy} = \frac{x-iy}{(x+iy)(x-iy)} = \frac{x-iy}{x^2+y^2}$$

$$\frac{1}{z} = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$$

$$R\left[\frac{1}{z}\right] = \frac{x}{x^2+y^2} \quad Im[1/z] = \frac{-y}{x^2+y^2}$$

d) Show $e^z = e^x(\cos y + i\sin y)$

$e^{iy} = \cos y + i\sin y \rightarrow$ Eulers formula

$e^z = e^{x+iy} = e^x \cdot e^{iy} = e^x(\cos y + i\sin y)$

e) Show $\sinh(z) = \sinh(x)\cos(y) + i\cosh(x)\sin(y)$

$$\sinh(z) = \frac{e^z - e^{-z}}{2}$$

$z = x + iy$

$$\sinh(z) = \frac{e^{x+y} - e^{-(x+iy)}}{2}$$

$e^{x+iy} = e^x(\cos y + i\sin y)$, $e^{-(x+iy)} = e^{-x}(\cos y - i\sin y)$

$$\sinh(z) = \frac{e^x(\cos y + i\sin y) - e^{-x}(\cos y - i\sin y)}{2}$$

$\sinh(z) = \sinh(x)\cos(y) + i\cosh(x)\sin(y)$

Problem 3

$f(x) = x^4 - 20x^2 + x$

a) How many roots do you expect?

Since its a degree of 4, we excpect 4 roots (real or complex)

$f(x) = x(x^3 - 20x + 1) = 0$

$\boxed{x_1 = 0}$ we expect 3 more roots from cubic equation

Problem 3

$f(x) = x^4 - 20x^2 + x$

$f(x) = x(x^3 - 20x + 1) = 0$

$g_1(x) = \frac{x^3 + 1}{20} \longrightarrow g_1'(x) = \frac{3x^2}{20}$ $\quad \left|\frac{3x^2}{20}\right| < 1 \quad 3x^2 < 20$

$$\boxed{-\sqrt{\frac{20}{3}} < x_1 < \sqrt{\frac{20}{3}}}$$

$g_2(x) = \sqrt[3]{20x - 1}$

$$\left|\frac{20}{3(20x-1)^{2/3}}\right| < 1 \qquad \frac{20}{3(20x-1)^{2/3}} < 1$$

$20 < 3(30x-1)^{2/3}$ $\qquad \left(\frac{20}{3}\right)^{3/2} < 20x - 1$

$\left(\frac{20}{3}\right)^{3/2} \approx 10.65$ $\qquad 11.65 < 20x$

$\boxed{x_2 > 0.5825}$

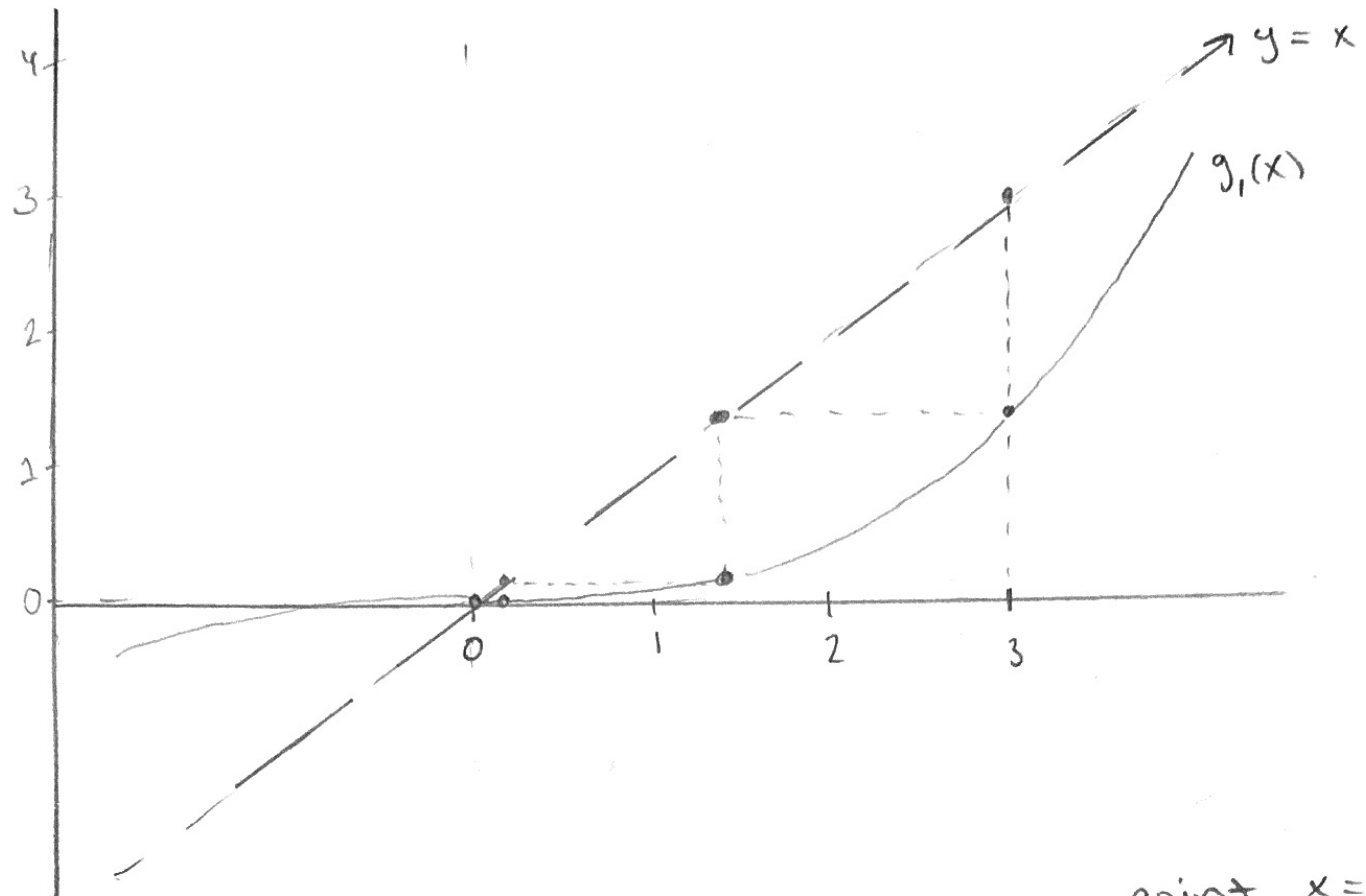
$g_3(x) = \frac{1}{20} + \frac{x^3}{20}$

$\downarrow$

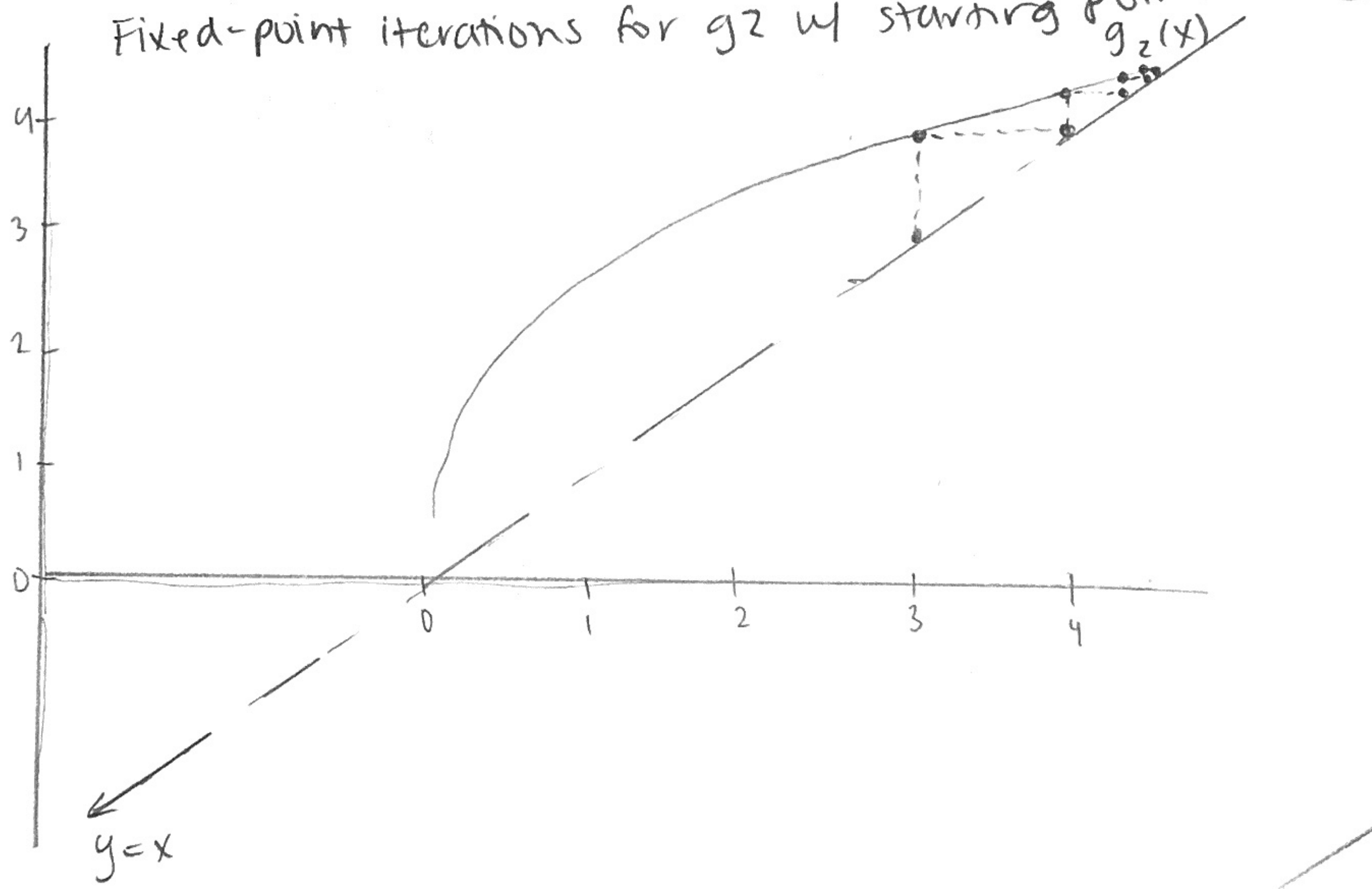
$g_3'(x) = \frac{3x^2}{20}$ same

$$\boxed{-\sqrt{\frac{20}{3}} < x_3 < \sqrt{\frac{20}{3}}}$$

Fixed-point iterations for g1 w/ starting point x=3



Fixed-point iterations for g2 w/ starting point x = 3



Fixed point iterations for g3 w/ starting point x = 3