Finite and Boundary Element Methods in Manufacturing
Assignment-1 (Aug 8, 2025)

Q1. Consider the governing differential equation:

$$-\frac{d^2u}{dx^2} = cos(\pi x) \text{ for } 0 < x < 1$$

subject to the following three sets of boundary conditions:

(a) u(0)=0, u(1)=0

(b) u(0)=0, $\frac{du}{dx}(1) = 0$

(c) $\frac{du}{dx}(0) = 0$, $\frac{du}{dx}(1) = 0$

Determine a N-parameter approximate solution with trigonometric functions using (a) Ritz, (b) Galerkin, (c) Least square, and (d) Collocation methods for $N = 1, 2, 4, 8$ and compare results with exact solutions given as:

(a) $u = \pi^{-2}\left[\cos(\pi x) + 2x - 1\right]$

(b) $u = \pi^{-2}\left[\cos(\pi x) - 1\right]$

(c) $u = \pi^{-2}\cos(\pi x)$

Q2. Obtain a one-parameter Galerkin solution of the boundary value problem given by

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -1 \text{ in } \Omega = \text{unitsquare}$$

with $u = 0$ on square boundary. Use (a) algebraic and (b) trignometric approximation functions.

Q3. Obtain the stiffness matrix and force vector for N-parameter Rayleigh-Ritz approximation of the following problem:

$$\frac{d}{dx^2}\left(EI\frac{d^2w}{dx2}\right) = q_0 \text{ for } 0 < x < L$$

with

$$w = EI\frac{d^2w}{dx^2} = 0 \text{ at } x = 0, L$$

using (a) algebraic polynomials and (b) trignometric functions.