OPTIMIZATION FUNCTION:

- SUPPOSE WE ESTABLISH A 2D CARTESIAN COORDINATE SYSTEM TO PLACE EACH RECEIVER NODE ON.

- EACH RECEIVER NODE i IS ASSIGNED A COORDINATE MATCHING ITS PHYSICAL LOCATION (x_i, y_i)

- GIVEN THE RECEIVED SIGNAL STRENGTH IN dB WE CAN ESTIMATE THE RANGE TO TARGET FROM EACH RECEIVER

FRIIS TRANSMISSION EQN:

$$P_r = P_t + G_R + G_T + 20 \log_{10}\left(\frac{\lambda}{4\pi R}\right)$$

A-PRIORI, WE KNOW:

P_t TRANSMIT POWER

G_R RECEIVER GAIN

G_T TRANSMITTER GAIN

λ WAVELENGTH

WE MEASURE:

P_r RECEIVED POWER

- SOLVING THE EQUATION FOR EACH RECEIVER, WE OBTAIN RADII OF CIRCLES, R_i

THE GOAL NOW IS TO FIND THE POINT ON THE PLANE THAT MINIMIZES THE DISTANCE TO ALL CIRCLES

FOR ANY POINT (x', y'), THE DISTANCE TO CIRCLE WITH CENTER (x_i, y_i) OF RADIUS R_i IS

$$D_i = \left| R_i - \sqrt{(x' - x_i)^2 - (y' - y_i)^2} \right|$$

THUS THE FUNCTION TO OPTIMIZE BECOMES

$$\phi(x,y) = \sum_{i=0}^{2} D_i = \left| R_0 - \sqrt{(x-x_0)^2+(y-y_0)^2} \right| + \left| R_1 - \sqrt{(x-x_1)^2+(y-y_1)^2} \right| + \left| R_2 - \sqrt{(x-x_2)^2+(y-y_2)^2} \right|$$

THIS FUNCTION IS CONCAVE, THEREFORE WE CAN USE A DECREASING GRADIENT APPROACH WHERE OUR PREDICTION POINT JUST FOLLOWS:

$$\nabla \phi(x,y)$$

THAT IS, ADDING SOME TIME UNIT:

$$(x', y', t+1) = (x,y) + \nabla\phi(x,y) \cdot \text{stepsize}$$

$$\text{WHERE } \nabla\phi = \frac{\partial}{\partial x}\phi(x,y)\,\hat{x} + \frac{\partial}{\partial y}\phi(x,y)\,\hat{y}$$

GIVEN $\phi(x,y)$ FIND $\nabla\phi(x,y)$

FOR THE SAKE OF DOING CALCULUS APPROXIMATE ABS( ) WITH SQUARING:

$$\phi(x,y) \approx \left(R_0 - \sqrt{(x-x_0)^2+(y-y_0)^2}\right)^2 + \dots$$

$$\frac{\partial\phi}{\partial x} = 2\left(R_0 - \sqrt{(x-x_0)^2+(y-y_0)^2}\right)\left(-\frac{1}{2}\left((x-x_0)^2+(y-y_0)^2\right)^{-\frac{1}{2}}\right)\left(-2x+2x_0\right) + \dots$$

SIMPLIFIED, LET $K_i = \sqrt{(x-x_i)^2+(y-y_i)^2}$

$$\nabla\phi(x,y) = \hat{x}\,2\left[(R_0-K_0)\left(\frac{-1}{2K_0}\right)(-2x+2x_0) + (R_1+K_1)\left(\frac{-1}{2K_1}\right)(-2x+2x_1) + (R_2+K_2)\left(\frac{-1}{2K_2}\right)(-2x+2x_2)\right]$$

$$+\hat{y}\,2\left[(R_0-K_0)\left(\frac{-1}{2K_0}\right)(-2y+2y_0) + (R_1+K_1)\left(\frac{-1}{2K_1}\right)(-2y+2y_1) + (R_2+K_2)\left(\frac{-1}{2K_2}\right)(-2y+2y_2)\right]$$