

contas

Bia

24 de maio de 2017

Latex

MQ \rightarrow mínimos quadrados

$$y_i = \alpha + \beta x_i + \varepsilon_i \rightarrow \varepsilon_i = y_i - \alpha - \beta x_i$$

Como x tem valores 0 ou 1:

$$x = 0 \rightarrow \varepsilon_i = y_i - \alpha$$

$$x = 1 \rightarrow \varepsilon_i = y_i - \alpha - \beta$$

$$\varepsilon_i = y_i - \alpha - \beta x_i \rightarrow \sum_{i=1}^n (\varepsilon_i)^2 = \sum_{i=1}^n (y_i - \alpha - \beta x_i)^2 = \sum_{i=1}^n (y_i^2 + \alpha^2 + \beta^2 x_i^2 - 2\alpha y_i - 2\beta x_i y_i + 2\alpha\beta x_i)$$

Fazendo os mínimos quadrados:

$$\frac{d}{d\alpha} \sum_{i=1}^n (y_i^2 + \alpha^2 + \beta^2 x_i^2 - 2\alpha y_i - 2\beta x_i y_i + 2\alpha\beta x_i) = \sum_{i=1}^n 2\alpha - 2y_i + 2\beta x_i = 2[n\alpha - \sum_{i=1}^n y_i + \beta \sum_{i=1}^n x_i]$$

Para provar que é o mínimo:

$$\frac{d}{d\alpha} \sum_{i=1}^n 2\alpha - 2y_i + 2\beta x_i = 2 > 0$$

$$\begin{aligned} \frac{d}{d\beta} \sum_{i=1}^n (y_i^2 + \alpha^2 + \beta^2 x_i^2 - 2\alpha y_i - 2\beta x_i y_i + 2\alpha\beta x_i) &= \sum_{i=1}^n 2\beta x_i^2 - 2x_i y_i + 2\alpha x_i \\ &= 2[\beta \sum_{i=1}^n x_i^2 - \sum_{i=1}^n x_i y_i + \alpha \sum_{i=1}^n x_i] \end{aligned}$$

Para provar que é o mínimo:

$$\frac{d}{d\beta} \sum_{i=1}^n 2\beta x_i^2 - 2x_i y_i + 2\alpha = 2 \sum_{i=1}^n x_i^2 > 0 \text{ (é sempre positiva dado que } x_i \text{ está ao quadrado)}$$

Se:

$$n\alpha - \sum_{i=1}^n y_i + \beta \sum_{i=1}^n x_i = 0$$

$$\beta \sum_{i=1}^n x_i^2 - \sum_{i=1}^n x_i y_i + \alpha \sum_{i=1}^n x_i = 0$$

então

$$\alpha = \frac{1}{n} \sum_{i=1}^n y_i - \frac{\beta}{n} \sum_{i=1}^n x_i$$

$$\beta = \frac{\sum_{i=1}^n x_i y_i - \alpha \sum_{i=1}^n x_i}{\sum_{i=1}^n x_i^2}$$

Substituindo α em β temos:

$$\begin{aligned} \beta &= \frac{\sum_{i=1}^n x_i y_i - \frac{1}{n} \sum_{i=1}^n y_i \sum_{i=1}^n x_i + \frac{\beta}{n} \sum_{i=1}^n n x_i \sum_{i=1}^n x_i}{\sum_{i=1}^n x_i^2} \\ \rightarrow \beta &= \frac{\sum_{i=1}^n x_i y_i - \frac{1}{n} \sum_{i=1}^n y_i \sum_{i=1}^n x_i}{\sum_{i=1}^n x_i^2 - \frac{1}{n} \sum_{i=1}^n x_i \sum_{i=1}^n x_i} \end{aligned}$$

$$\sum_{i=1}^n y_i = \sum_{j=1}^n y_{1j} + \sum_{j=1}^n y_{2j}$$

sendo y_1 os valores de y quando $x = 0$ e y_2 os valores de y correspondentes a $x = 1$

Sabemos que $\sum_{i=1}^n x_i y_i = \sum_{j=1}^n y_{2j}$ e que $\sum_{i=1}^n x_i = n_2 = \sum_{i=1}^n x_i^2$

$$\beta = \frac{\sum_{i=1}^n y_{2j} - \frac{1}{n} \sum_{i=1}^n y_i (n_2)}{n_2 - \frac{(n_2)^2}{n}}$$

$$\beta = \frac{n \sum_{i=1}^n y_{2j} - \sum_{i=1}^n y_i (n_2) (\frac{1}{n_2})}{n(n_2) - (n_2)^2}$$

$$\beta = \frac{n \bar{y}_2 - \sum_{i=1}^n y_i}{n - n_2}$$

$$\beta = \frac{n \bar{y}_2 - \sum_{i=1}^n y_i}{n_1}$$

$$\beta = \frac{n \bar{y}_2}{n_1} - \frac{\sum_{i=1}^n y_{1j}}{n_1} - \frac{\sum_{i=1}^n y_{2j}}{n_1}$$

$$\beta = \frac{n}{n_1} \bar{y}_2 - \bar{y}_1 - \frac{n_2 \bar{y}_2}{n_1}$$

$$\beta = \frac{(n_1 + n_2) \bar{y}_2 - n_1 \bar{y}_1 - n_2 \bar{y}_2}{n_1}$$

$$\rightarrow \beta = \bar{y}_2 - \bar{y}_1$$

Substituindo $\beta = \bar{y}_2 - \bar{y}_1$ em α

$$\alpha = \frac{1}{n} \sum_{i=1}^n y_1 - \frac{\bar{y}_2 - \bar{y}_1}{n} \sum_{i=1}^n x_i$$

$$\alpha = \frac{\sum_{i=1}^n y_1 - n_2 \bar{y}_2 + n_2 \bar{y}_1}{n}$$

$$\alpha = \frac{\sum_{j=1}^n y_{1j} + \sum_{j=1}^n y_{2j} - n_2 \bar{y}_2 + n_2 \bar{y}_1}{n}$$

$$\alpha = \frac{n_1 \frac{\sum_{j=1}^n y_{1j}}{n_1} + n_2 \frac{\sum_{j=1}^n y_{2j}}{n_2} - n_2 \bar{y}_2 + n_2 \bar{y}_1}{n}$$

$$\alpha = \frac{n_1 \bar{y}_1 + (n_2 \bar{y}_2 - n_2 \bar{y}_2) + n_2 \bar{y}_1}{n}$$

$$\alpha = \frac{(n_1 + n_2) \bar{y}_1}{n}$$

$$\rightarrow \alpha = \bar{y}_1$$