contas

Bia

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Latex

MQ $\rightarrow$ mínimos quadrados

$y_i = \alpha + \beta x_i + \varepsilon_i \rightarrow \varepsilon_i = y_i - \alpha - \beta x_i$

Como x tem valores 0 ou 1:

$x = 0 \rightarrow \varepsilon_i = y_i - \alpha$

$x = 1 \rightarrow \varepsilon_i = y_i - \alpha - \beta$

$\varepsilon_i = y_i - \alpha - \beta x_i \rightarrow \sum_{i=i}^{n}(\varepsilon_i)^2 = \sum_{i=1}^{n}(y_i - \alpha - \beta x_i)^2 = \sum_{i=1}^{n}(y_i^2 + \alpha^2 + \beta^2 x_i^2 - 2\alpha y_i - 2\beta x_i y_i + 2\alpha\beta x_i)$

Fazendo os mínimos quadrados:

$\frac{d}{d\alpha}\sum_{i=1}^{n}(y_i^2 + \alpha^2 + \beta^2 x_i^2 - 2\alpha y_i - 2\beta x_i y_i + 2\alpha\beta x_i) = \sum_{i=1}^{n} 2\alpha - 2y_i + 2\beta x_i = 2[n\alpha - \sum_{i=1}^{n} y_i + \beta\sum_{i=1}^{n} x_i]$

Para provar que é o mínimo:

$\frac{d}{d\alpha}\sum_{i=1}^{n} 2\alpha - 2y_i + 2\beta x_i = 2 > 0$

$\frac{d}{d\beta}\sum_{i=1}^{n}(y_i^2 + \alpha^2 + \beta^2 x_i^2 - 2\alpha y_i - 2\beta x_i y_i + 2\alpha\beta x_i) = \sum_{i=1}^{n} 2\beta x_i^2 - 2x_i y_i + 2\alpha x_i$

$= 2[\beta\sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} x_i y_i + \alpha\sum_{i=1}^{n} x_i]$

Para provar que é o mínimo:

$\frac{d}{d\beta}\sum_{i=1}^{n} 2\beta x_i^2 - 2x_i y_i + 2\alpha = 2\sum_{i=1}^{n} x_i^2 > 0$ (é sempre positiva dado que x_i está ao quadrado)

Se:

$n\alpha - \sum_{i=1}^{n} y_i + \beta\sum_{i=1}^{n} x_i = 0$

$\beta\sum_{i=1}^{n} x_i^2 - \sum_{i=1}^{n} x_i y_i + \alpha\sum_{i=1}^{n} = 0$

então

$\alpha = \frac{1}{n}\sum_{i=1}^{n} y_i - \frac{\beta}{n}\sum_{i=1}^{n} x_i$

$\beta = \frac{\sum_{i=1}^{n} x_i y_i - \alpha\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} x_i^2}$

Substituindo α em β temos:

$\beta = \frac{\sum_{i=1}^{n} x_i y_i - \frac{1}{n}\sum_{i=1}^{n} y_i \sum_{i=1}^{n} x_i + \frac{\beta}{n}\sum_{i=1} n x_i \sum_{i=1}^{n}}{\sum_{i=1}^{n} x_i^2}$

$\rightarrow \beta = \frac{\sum_{i=1}^{n} x_i Y_i - \frac{1}{n}\sum_{i=1}^{n} y_i \sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i}$

$\sum_{i=1}^{n} y_i = \sum_{j=1}^{n} y_{1j} + \sum_{j=1}^{n} y_{2j}$

sendo y_1 os valores de y quando $x = 0$ e y_2 os valores de y correspondentes a $x = 1$

Sabemos que $\sum_{i=1}^{n} x_i y_i = \sum_{j=1}^{n} y_{2j}$ e que $\sum_{i=1}^{n} x_i = n_2 = \sum_{i=1}^{n} x_i^2$

$\beta = \frac{\sum_{i=1}^{n} y_{2j} - \frac{1}{n}\sum_{i=1}^{n} y_i (n_2)}{n_2 - \frac{(n_2)^2}{n}} \rightarrow \beta = \frac{n\sum_{i=1} ny_{2j} - \sum_{i=1}^{n} y_i (n_2)(\frac{1}{n_2})}{n(n_2)-(n_2)^2} \rightarrow \beta = \frac{n\bar{y}_2 - \sum_{i=1}^{n} y_i}{n-n_2} \rightarrow \beta = \frac{n\bar{y}_2 - \sum_{i=1}^{n} y_i}{n_1}$

$\rightarrow \beta = \frac{n\bar{y}_2}{n_1} - \frac{\sum_{i=1}^{n} y_{1j}}{n_1} - \frac{\sum_{i=1}^{n} y_{2j}}{n_1} \rightarrow \beta = \frac{n}{n_1}\bar{y}_2 - \bar{y}_1 - \frac{n_2\bar{y}_2}{n_1} \rightarrow \beta = \frac{(n_1+n_2)\bar{y}_2 - n_1\bar{y}_1 - n_2\bar{y}_2}{n_1} \rightarrow$

$\beta = \bar{y}_2 - \bar{y}_1$

Substituindo $\beta = \bar{y}_2 - \bar{y}_1$ em α

$\alpha = \frac{1}{n}\sum_{i=1}^{n} y_1 - \frac{\bar{y}_2 - \bar{y}_1}{n}\sum_{i=1}^{n} x_i = \frac{\sum_{i=1}^{n} y_1 - n_2\bar{y}_2 + n_2\bar{y}_1}{n} = \frac{\sum_{j=1}^{n} y_{1j} + \sum_{j=1}^{n} y_{2j} - n_2\bar{y}_2 + n_2\bar{y}_1}{n}$

$= \frac{n_1\frac{\sum_{j=1}^{n} y_{1j}}{n_1} + n_2\frac{\sum_{j=1}^{n} y_{2j}}{n_2} - n_2\bar{y}_2 + n_2\bar{y}_1}{n} = \frac{n_1\bar{y}_1 + (n_2\bar{y}_2 - n_2\bar{y}_2) + n_2\bar{y}_1}{n} = \frac{(n_1+n_2)\bar{y}_1}{n}$

$\alpha = \bar{y}_1$