Funções de teste

1 Função 1:

$$f : \mathbb{R}^8 \to R, \qquad f(\mathbf{x}) = ||\mathbf{x}||^2 = \sum_{i=1}^{8} x_i^2. \tag{1}$$

$$\nabla f(\mathbf{x}) = 2\mathbf{x} = \begin{bmatrix} 2x_1 \\ 2x_2 \\ 2x_3 \\ 2x_4 \\ 2x_5 \\ 2x_6 \\ 2x_7 \\ 2x_8 \end{bmatrix}, \quad \nabla^2 f(\mathbf{x}) = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}. \tag{2}$$

1.1 Restrições $(A \times \mathbf{x} = b)$

$$A = \begin{bmatrix} 1 & 0 & -1 & 2 & 0 & 1 & 3 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 2 & 4 \\ 2 & 3 & -5 & 0 & 0 & 1 & 3 & 4 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \tag{3}$$

2 Função 2[Hock & Schittkowski Model 48]:

$$f : \mathbb{R}^5 \to R, \qquad f(\mathbf{x}) = (x_1 - 1)^2 + (x_2 - x_3)^2 + (x_4 - x_5)^2. \tag{4}$$

$$\nabla f(\mathbf{x}) = \begin{bmatrix} 2(x_1 - 1) \\ 2(x_2 - x_3) \\ -2(x_2 - x_3) \\ 2(x_4 - x_5) \\ -2(x_4 - x_5) \end{bmatrix}, \quad \nabla^2 f(\mathbf{x}) = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & -2 & 0 & 0 \\ 0 & -2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & -2 & 2 \end{bmatrix}. \tag{5}$$

2.1 Restrições $(A \times \mathbf{x} = b)$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -2 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ -3 \end{bmatrix}. \tag{6}$$

3 Função 3[Exemplo de Linear and Nonlinear Optimization Por Igor Griva,Stephen G. Nash,Ariela Sofer]:

$$f : \mathbb{R}^5 \to R, \qquad f(\mathbf{x}) = x_1^2 - 2x_1 + x_2^2 - x_3^2 + 4x_3. \tag{7}$$

$$\nabla f(\mathbf{x}) = \begin{bmatrix} 2(x_1 - 1) \\ 2x_2 \\ 4 - 2x_3) \end{bmatrix}, \quad \nabla^2 f(\mathbf{x}) = \begin{bmatrix} 2 & -0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}. \tag{8}$$

3.1 Restrições $(A \times \mathbf{x} = b)$

$$A = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \end{bmatrix}. \tag{9}$$

4 Função 4:

$$f : \mathbb{R}^5 \to R, \qquad f(\mathbf{x}) = (x_1 - x_2)^2 + (x_3 - 1)^2 + (x_4 - 1)^4 + (x_5 - 1)^6. \tag{10}$$

$$\nabla f(\mathbf{x}) = \begin{bmatrix} 2(x_1 - x_2) \\ -2(x_1 - x_2) \\ 2(x_3 - 1) \\ 4(x_4 - 1)^3 \\ 6(x_5 - 1)^5 \end{bmatrix}, \quad \nabla^2 f(\mathbf{x}) = \begin{bmatrix} 2 & -2 & 0 & 0 & 0 \\ -2 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 12(x_4 - 1)^2 & 0 \\ 0 & 0 & 0 & 0 & 30(x_5 - 1)^4 \end{bmatrix}. \tag{11}$$

4.1 Restrições $(A \times \mathbf{x} = b)$

$$A = \begin{bmatrix} 1 & 1 & 1 & 4 & 0 \\ 0 & 0 & 1 & 0 & 5 \end{bmatrix}, \quad b = \begin{bmatrix} 7 \\ 6 \end{bmatrix}. \tag{12}$$

5 Função 5:

$$f : \mathbb{R}^6 \to R, \qquad f(\mathbf{x}) = x_1 + 2x_2 + 4x_5 + \exp(x_1 x_4). \tag{13}$$

$$\nabla f(\mathbf{x}) = \begin{bmatrix} 1 + x_4 \exp(x_1 x_4) \\ 2 \\ 0 \\ x_1 \exp(x_1 x_4) \\ 4 \\ 0 \end{bmatrix}, \tag{14}$$

$$\nabla^2 f(\mathbf{x}) = \begin{bmatrix} 2 & -2 & 0 & 0 & 0 \\ -2 & 4 & -2 & 0 & 0 \\ 0 & -2 & 12(x_3 - x_4)^2 + 2 & -12(x_3 - x_4)^2 & 0 \\ 0 & 0 & -12(x_3 - x_4)^2 & 12(x_3 - x_4)^2 + 2 & -2 \\ 0 & 0 & 0 & -2 & 2 \end{bmatrix}. \tag{15}$$

5.1 Restrições $(A \times \mathbf{x} = b)$

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 & 5 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 6 \\ 3 \\ 2 \\ 1 \\ 2 \\ 2 \end{bmatrix}. \tag{16}$$

Observar que a soma das duas primeiras linhas de A é igual à soma das 3 últimas linhas de A.

6 Função 6[Extended Rosenbrock function]:

$$f : \mathbb{R}^8 \to R, \qquad f(\mathbf{x}) = \sum_{i=1}^{4} (1 - x_{2i-1})^2 + \sum_{i=1}^{4} 100(x_{2i} - x_{2i-1}^2)^2. \tag{17}$$

$$\begin{cases} \nabla f(\mathbf{x})_{2i-1} & = & -400(x_{2i} - x_{2i-1}^2)x_{2i-1} - 2(1 - x_{2i-1}) \\ \nabla f(\mathbf{x})_{2i} & = & 200(x_{2i} - x_{2i-1}^2) \end{cases} \qquad i = 1, 2, 3, 4 \tag{18}$$

$$\nabla^2 f(\mathbf{x}) = \begin{bmatrix} B_1 & 0 & 0 & 0 \\ 0 & B_2 & 0 & 0 \\ 0 & 0 & B_3 & 0 \\ 0 & 0 & 0 & B_4 \end{bmatrix}, \quad B_i = \begin{bmatrix} (1200x_{2i-1}^2 - 400x_{2i} + 2) & -400x_{2i-1} \\ -400x_{2i-1} & 200 \end{bmatrix}, \quad i = 1, 2, 3, 4. \tag{19}$$

6.1 Restrições $(A \times \mathbf{x} = b)$

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -1 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{20}$$