UNIVERSITY NAME

DOCTORAL THESIS

Thesis Title

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"Thanks to my solid academic training, today I can write hundreds of words on virtually any topic without possessing a shred of information, which is how I got a good job in journalism."

Dave Barry

UNIVERSITY NAME

Abstract

Faculty Name
Department or School Name

Doctor of Philosophy

Thesis Title

by John SMITH

The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too...

Acknowledgements

The acknowledgments and the people to thank go here, don't forget to include your project advisor…

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For/Dedicated to/To my…

Chapter 1

Proving semantic preservation in HILECOP

- Change σ_{injr} and σ_{injf} into σ_i.
- Define the $\texttt{Inject}_{\downarrow}$ and $\texttt{Inject}_{\uparrow}$ relations.
- Keep the *sitpn* argument in the SITPN full execution relation, but remove it from the SITPN execution, cycle and state transition relations.

1.1 Preliminary Definitions

1.2 Behavior Preservation Theorem

1.3 Initial States

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1.5 Rising Edge

1.6 Falling Edge

Definition 1 (Falling Edge Hypotheses). *Given an* $sitpn \in SITPN$, $d \in design$, $\gamma \in WM(sitpn, d)$, $E_c \in \mathbb{N} \to \mathcal{C} \to \mathbb{B}$, $\Delta \in ElDesign(d, \mathcal{D}_{\mathcal{H}})$, $E_p \in (\mathbb{N} \times \{\uparrow, \downarrow\}) \to Ins(\Delta) \to value$, $\tau \in \mathbb{N}$, $s, s' \in S(sitpn)$, $\sigma_e, \sigma, \sigma_i, \sigma_{\downarrow}, \sigma' \in \Sigma(\Delta)$, *assume that:*

- $\lfloor sitpn \rfloor_{\mathcal{H}} = (d, \gamma)$ *and* $\gamma \vdash E_p \stackrel{env}{=} E_c$ *and* $\mathcal{D}_{\mathcal{H}}, \emptyset \vdash \mathrm{d} \xrightarrow{elab} \Delta, \sigma_e$
- $\gamma, E_c, \tau \vdash s \stackrel{\uparrow}{\sim} \sigma$
- $E_c, \tau \vdash sitpn, s \xrightarrow{\downarrow} s'$
- $\texttt{Inject}_{\downarrow}(\sigma, E_p, \tau, \sigma_i)$ *and* $\Delta, \sigma_i \vdash \mathrm{d.cs} \xrightarrow{\downarrow} \sigma_{\downarrow}$ *and* $\Delta, \sigma_{\downarrow} \vdash \mathrm{d.cs} \xrightarrow{\leadsto} \sigma'$
- *State* σ *is a stable design state:* $\mathcal{D}_{\mathcal{H}}, \Delta, \sigma \vdash \mathrm{d.cs} \xrightarrow{comb} \sigma$

Lemma 1 (Falling Edge). *For all $sitpn$, d, γ, Δ, σ_e, E_c, E_p, τ, s, s', σ, σ_i, $\sigma_\downarrow$, σ' that verify the hypotheses of Def. 1, then $\gamma \vdash s' \overset{\downarrow}{\sim} \sigma'$.*

Proof. By definition of **??**, there are 12 points to prove.

1. $\forall p \in P, id_p \in Comps(\Delta)$ s.t. $\gamma(p) = id_p,\ s'.M(p) = \sigma'(id_p)(''s\_marking'')$.
2. $\forall t \in T_i, id_t \in Comps(\Delta)$ s.t. $\gamma(t) = id_t$,
$(upper(I_s(t)) = \infty \wedge s'.I(t) \leq lower(I_s(t)) \Rightarrow s'.I(t) = \sigma'(id_t)(''s\_time\_counter''))$
$\wedge(upper(I_s(t)) = \infty \wedge s'.I(t) > lower(I_s(t)) \Rightarrow \sigma'(id_t)(''s\_time\_counter'') = lower(I_s(t)))$
$\wedge(upper(I_s(t)) \neq \infty \wedge s'.I(t) > upper(I_s(t)) \Rightarrow \sigma'(id_t)(''s\_time\_counter'') = upper(I_s(t)))$
$\wedge(upper(I_s(t)) \neq \infty \wedge s'.I(t) \leq upper(I_s(t)) \Rightarrow s'.I(t) = \sigma'(id_t)(''s\_time\_counter''))$.
3. $\forall t \in T_i, id_t \in Comps(\Delta)$ s.t. $\gamma(t) = id_t, s'.reset_t(t) = \sigma'(id_t)(''s\_reinit\_time\_counter'')$.
4. $\forall c \in \mathcal{C}, id_c \in Ins(\Delta)$ s.t. $\gamma(c) = id_c,\ s'.cond(c) = \sigma'(id_c)$.
5. $\forall a \in \mathcal{A}, id_a \in Outs(\Delta)$ s.t. $\gamma(a) = id_a,\ s'.ex(a) = \sigma'(id_a)$.
6. $\forall f \in \mathcal{F}, id_f \in Outs(\Delta)$ s.t. $\gamma(f) = id_f,\ s'.ex(f) = \sigma'(id_f)$.
7. $\forall t \in T, id_t \in Comps(\Delta)$ s.t. $\gamma(t) = id_t, t \in Firable(s') \Leftrightarrow \sigma'(id_t)(''s\_firable'') = \texttt{true}$.
8. $\forall t \in T, id_t \in Comps(\Delta)$ s.t. $\gamma(t) = id_t, t \notin Firable(s') \Leftrightarrow \sigma'(id_t)(''s\_firable'') = \texttt{false}$.
9. $\forall t \in T, id_t \in Comps(\Delta)$ s.t. $\gamma(t) = id_t, t \in Fired(s') \Leftrightarrow \sigma'(id_t)(''fired'') = \texttt{true}$.
10. $\forall t \in T, id_t \in Comps(\Delta)$ s.t. $\gamma(t) = id_t, t \notin Fired(s') \Leftrightarrow \sigma'(id_t)(''fired'') = \texttt{false}$.
11. $\forall p \in P, id_p \in Comps(\Delta)$ s.t. $\gamma(p) = id_p,\ \sum\limits_{t \in Fired(s')} pre(p,t) = \sigma'(id_p)(''s\_output\_token\_sum'')$.
12. $\forall p \in P, id_p \in Comps(\Delta)$ s.t. $\gamma(p) = id_p,\ \sum\limits_{t \in Fired(s')} post(t,p) = \sigma'(id_p)(''s\_input\_token\_sum'')$.

Each point is proved by a separate lemma:

- Apply Lemma Falling Edge Equal Marking to solve 1.
- Apply Lemma Falling Edge Equal Time Counters to solve 2.
- Apply Lemma Falling Edge Equal Output Token Sum to solve 11.
- Apply Lemma Falling Edge Equal Input Token Sum to solve 12.

□

1.6.1 Falling Edge and marking

Lemma 2 (Falling Edge Equal Marking). *For all $sitpn$, d, γ, Δ, σ_e, E_c, E_p, τ, s, s', σ, σ_i, $\sigma_\downarrow$, σ' that verify the hypotheses of Def. 1, then $\forall p \in P, id_p \in Comps(\Delta)$ s.t. $\gamma(p) = id_p,\ s'.M(p) = \sigma'(id_p)(''s\_marking'')$.*

Proof. Given a $p \in P$ and an $id \in Comps(\Delta)$ s.t. $\gamma(p) = id_p$, let us show $\boxed{s'.M(p) = \sigma'(id_p)(\text{"}s\_marking\text{"}).}$

By definition of $E_c, \tau \vdash sitpn, s \xrightarrow{\downarrow} s'$:

$$s.M(p) = s'.M(p) \tag{1.1}$$

By property of the $\mathtt{Inject}_{\downarrow}$ relation, the $\mathcal{H}$-VHDL falling edge relation, the stabilize relation and $\mathtt{comp}(id_p, \text{"}place\text{"}, gm_p, ipm_p, opm_p) \in d.cs$:

$$\sigma'(id_p)(\text{"}s\_marking\text{"}) = \sigma(id_p)(\text{"}s\_marking\text{"}) \tag{1.2}$$

Rewriting the goal with (1.1) and (1.2): $\boxed{s.M(p) = \sigma(id_p)(\text{"}s\_marking\text{"}).}$

By definition of $\gamma, E_c, \tau \vdash s \overset{\uparrow}{\sim} \sigma$: $s.M(p) = \sigma(id_p)(\text{"}s\_marking\text{"}).$

□

Lemma 3 (Falling Edge Equal Output Token Sum)**.** *For all $sitpn$, d, γ, Δ, σ_e, E_c, E_p, τ, s, s', σ, σ_i, $\sigma_{\downarrow}$, σ' that verify the hypotheses of Def. 1, then $\forall p, id_p$ s.t. $\gamma(p) = id_p$, $\sum\limits_{t \in Fired(s')} pre(p,t) = \sigma'(id_p)(\text{"}s\_output\_token\_sum\text{"})$.*

Proof. Given a $p \in P$ and an $id_p \in Comps(\Delta)$, let us show $\boxed{\sum\limits_{t \in Fired(s')} pre(p,t) = \sigma'(id_p)(\text{"}s\_output\_token\_sum\text{"}).}$

By definition of id_p, there exist gm_p, ipm_p, opm_p s.t. $\mathtt{comp}(id_p, \text{"}place\text{"}, gm_p, ipm_p, opm_p) \in d.cs$. By property of the stabilize relation and $\mathtt{comp}(id_p, \text{"}place\text{"}, gm_p, ipm_p, opm_p) \in d.cs$:

$$\sigma'(id_p)(\text{"}sots\text{"}) = \sum_{i=0}^{\Delta(id_p)(\text{"}oan\text{"})-1} \begin{cases} \sigma'(id_p)(\text{"}oaw\text{"})[i] \ \mathtt{if}\ (\sigma'(id_p)(\text{"}otf\text{"})[i] \\ \qquad\qquad . \ \sigma'(id_p)(\text{"}oat\text{"})[i] = \mathtt{BASIC}) \\ 0 \ otherwise \end{cases} \tag{1.3}$$

Rewriting the goal with (1.3):

$$\boxed{\sum_{t \in Fired(s')} pre(p,t) = \sum_{i=0}^{\Delta(id_p)(\text{"}oan\text{"})-1} \begin{cases} \sigma'(id_p)(\text{"}oaw\text{"})[i] \ \mathtt{if}\ (\sigma'(id_p)(\text{"}otf\text{"})[i] \\ \qquad\qquad . \ \sigma'(id_p)(\text{"}oat\text{"})[i] = \mathtt{BASIC}) \\ 0 \ otherwise \end{cases}}$$

Let us unfold the definition of the left sum term:

$$\boxed{\begin{gathered} \sum_{t \in Fired(s')} \begin{cases} \omega \ \mathtt{if}\ pre(p,t) = (\omega, \mathtt{basic}) \\ 0 \ otherwise \end{cases} \\ = \\ \sum_{i=0}^{\Delta(id_p)(\text{"}oan\text{"})-1} \begin{cases} \sigma'(id_p)(\text{"}oaw\text{"})[i] \ \mathtt{if}\ (\sigma'(id_p)(\text{"}otf\text{"})[i] \\ \qquad\qquad . \ \sigma'(id_p)(\text{"}oat\text{"})[i] = \mathtt{BASIC}) \\ 0 \ otherwise \end{cases} \end{gathered}}$$

To ease the reading, let us define functions $f \in Fired(s') \rightarrow \mathbb{N}$ and $g \in [0, |output(p)| - 1] \rightarrow \mathbb{N}$ s.t.

$f(t) = \begin{cases} \omega \texttt{ if } pre(p,t) = (\omega, \texttt{basic}) \\ 0 \textit{ otherwise} \end{cases}$ and $g(i) = \begin{cases} \sigma'(id_p)(''oaw'')[i] \texttt{ if } (\sigma'(id_p)(''otf'')[i] \\ \qquad . \sigma'(id_p)(''oat'')[i] = \texttt{BASIC}) \\ 0 \textit{ otherwise} \end{cases}$

Then, the goal is: $\boxed{\sum_{t \in Fired(s')} f(t) = \sum_{i=0}^{\Delta(id_p)(''oan'')-1} g(i)}$

Let us perform case analysis on $output(p)$; there are two cases:

1. $output(p) = \emptyset$:

   By construction, $<\texttt{output\_arcs\_number} \Rightarrow \texttt{1}> \in gm_p$, $<\texttt{output\_arcs\_types}(0) \Rightarrow \texttt{BASIC}> \in ipm_p$, $<\texttt{output\_transitions\_fired}(0) \Rightarrow \texttt{true}> \in ipm_p$, and $<\texttt{output\_arcs\_weights}(0) \Rightarrow \texttt{0}> \in ipm_p$.

   By property of the elaboration relation and $\texttt{comp}(id_p, ''place'', gm_p, ipm_p, opm_p) \in d.cs$:

$$\Delta(id_p)(''oan'') = 1 \tag{1.4}$$

   By property of the stabilize relation and $\texttt{comp}(id_p, ''place'', gm_p, ipm_p, opm_p) \in d.cs$:

$$\begin{aligned} \sigma'(id_p)(''oat'')[0] &= \texttt{BASIC} \qquad (1.5) \\ \sigma'(id_p)(''otf'')[0] &= \texttt{true} \qquad (1.6) \\ \sigma'(id_p)(''oaw'')[0] &= 0 \qquad (1.7) \end{aligned}$$

   By property of $output(p) = \emptyset$:

$$\sum_{t \in Fired(s')} \begin{cases} \omega \texttt{ if } pre(p,t) = (\omega, \texttt{basic}) \\ 0 \textit{ otherwise} \end{cases} = 0 \tag{1.8}$$

   Rewriting the goal with (1.4), (1.5), (1.6), (1.7) and (1.8), tautology.

2. $output(p) \neq \emptyset$:

   By construction, $<\texttt{output\_arcs\_number} \Rightarrow |output(p)|> \in gm_p$, and by property of the elaboration relation:

$$\Delta(id_p)(''oan'') = |output(p)| \tag{1.9}$$

   Rewriting the goal with (1.9): $\boxed{\sum_{t \in Fired(s')} f(t) = \sum_{i=0}^{|output(p)|-1} g(i).}$

   Let us reason by induction on the right sum term of the goal.

   - **BASE CASE**:

In that case, $0 > |output| - 1$ and $\sum_{i=0}^{|output(p)|-1} g(i) = 0$.

As $0 > |output| - 1$, then $|output(p)| = 0$, thus contradicting $output(p) \neq \emptyset$.

- **INDUCTION CASE**:

In that case, $0 \leq |output(p)| - 1$.

$$\forall F \subseteq Fired(s'), g(0) + \sum_{t \in F} f(t) = g(0) + \sum_{i=1}^{|output(p)|-1} g(i)$$

$$\boxed{\sum_{t \in Fired(s')} f(t) = g(0) + \sum_{i=1}^{|output(p)|-1} g(i)}$$

By definition of g:

$$g(0) = \begin{cases} \sigma'(id_p)(''oaw'')[0] \texttt{ if } (\sigma'(id_p)(''otf'')[0] \\ \qquad\qquad\qquad\qquad . \sigma'(id_p)(''oat'')[0] = \texttt{BASIC}) \\ 0 \ otherwise \end{cases} \tag{1.10}$$

Let us perform case analysis on the value of $\sigma'(id_p)(''otf'')[0] \ . \ \sigma'(id_p)(''oat'')[0] = \texttt{BASIC}$; there are two cases:

(a) $(\sigma'(id_p)(''otf'')[0] \ . \ \sigma'(id_p)(''oat'')[0] = \texttt{BASIC}) = \texttt{false}$:
In that case, $g(0) = 0$, and then we can apply the induction hypothesis with $F = Fired(s')$ to solve the goal: $\sum_{t \in Fired(s')} f(t) = \sum_{i=1}^{|output(p)|-1} g(i)$.

(b) $(\sigma'(id_p)(''otf'')[0] \ . \ \sigma'(id_p)(''oat'')[0] = \texttt{BASIC}) = \texttt{true}$:
In that case, $g(0) = \sigma'(id_p)(''oaw'')[0]$, $\sigma'(id_p)(''otf'')[0] = \texttt{true}$ and $\sigma'(id_p)(''oat'')[0] = \texttt{BASIC}$.
By construction, there exist a $t \in output(t)$, $id_t \in Comps(\Delta)$ s.t. $\gamma(t) = id_t$. Let us take such a $t \in output(p)$.
By definition of id_t, there exist gm_t, ipm_t, opm_t s.t. $\texttt{comp}(id_t, ''transition'', gm_t, ipm_t, opm_t) \in d.cs$.
As $t \in output(p)$, there exist $\omega \in \mathbb{N}^*$ and $a \in \{\texttt{BASIC}, \texttt{TEST}, \texttt{INHIB}\}$ s.t. $pre(p,t) = (\omega, a)$. Let us take an ω and a s.t. $pre(p,t) = (\omega, a)$.
By construction, $<\texttt{output\_arcs\_types}(0) \Rightarrow a> \in ipm_p$, $<\texttt{output\_arcs\_weights}(0) \Rightarrow \omega> \in ipm_p$, and there exists $id_{ft} \in Sigs(\Delta)$ s.t. $<\texttt{fired} \Rightarrow id_{ft}> \in opm_t$ and $<\texttt{output\_transitions\_fired}(0) \Rightarrow \texttt{id}_{\texttt{ft}}> \in ipm_p$
By property of the stabilize relation, $\sigma'(id_p)(''oat'')[0] = \texttt{BASIC}$ and $<\texttt{output\_arcs\_types}(0) \Rightarrow \texttt{a}> \in ipm_p$:

$$pre(p,t) = (\omega, \texttt{basic}) \tag{1.11}$$

By property of the stabilize relation, $<\texttt{fired} \Rightarrow \texttt{id}_{\texttt{ft}}> \in opm_t$, $<\texttt{output\_transitions\_fired}(0) \Rightarrow \texttt{id}_{\texttt{ft}}> \in ipm_p$ and $\sigma'(id_p)(''otf'')[0] = \texttt{true}$:

$$\sigma'(id_t)(''fired'') = \texttt{true} \tag{1.12}$$

Appealing to Lemma **??**, we know $t \in Fired(s')$.
As $t \in Fired(s')$, we can rewrite the left sum term of the goal as follows:

$$\boxed{f(t) + \sum_{t' \in Fired(s') \setminus \{t\}} f(t') = g(0) + \sum_{i=1}^{|output(p)|-1} g(i)}$$

We know that $g(0) = \sigma'(id_p)(''oaw'')[0]$, and by property of the stabilize relation and $<$`output_arcs_weights`$(0) \Rightarrow \omega> \in ipm_p$:

$$\sigma'(id_p)(''oaw'')[0] = \omega \tag{1.13}$$

Rewriting the goal with (1.13):

$$\boxed{f(t) + \sum_{t' \in Fired(s') \setminus \{t\}} f(t') = \omega + \sum_{i=1}^{|output(p)|-1} g(i)}$$

By definition of f, and as $pre(p,t) = (\omega, \texttt{basic})$, then $f(t) = \omega$; thus, rewriting the goal:

$$\boxed{\omega + \sum_{t' \in Fired(s') \setminus \{t\}} f(t') = \omega + \sum_{i=1}^{|output(p)|-1} g(i)}$$

Then, knowing that $g(0) = \omega$, we can apply the induction hypothesis with $F = Fired(s') \setminus \{t\}$: $g(0) + \sum_{t' \in Fired(s') \setminus \{t\}} f(t') = g(0) + \sum_{i=1}^{|output(p)|-1} g(i)$.

$\square$

Lemma 4 (Falling Edge Equal Input Token Sum)**.** *For all sitpn, d, γ, Δ, σ_e, E_c, E_p, τ, s, s', σ, σ_i, $\sigma_\downarrow$, σ' that verify the hypotheses of Def. 1, then $\forall p, id_p$ s.t. $\gamma(p) = id_p$, $\sum_{t \in Fired(s')} post(t,p) = \sigma'_p(''s\_input\_token\_sum'')$.*

Proof. Given a $p \in P$ and an $id_p \in Comps(\Delta)$, let us show

$$\boxed{\sum_{t \in Fired(s')} post(t,p) = \sigma'(id_p)(''s\_input\_token\_sum'').}$$

By definition of id_p, there exist gm_p, ipm_p, opm_p s.t. $\texttt{comp}(id_p, ''place'', gm_p, ipm_p, opm_p) \in d.cs$.
By property of the stabilize relation and $\texttt{comp}(id_p, ''place'', gm_p, ipm_p, opm_p) \in d.cs$:

$$\sigma'(id_p)(''sits'') = \sum_{i=0}^{\Delta(id_p)(''ian'')-1} \begin{cases} \sigma'(id_p)(''iaw'')[i] \texttt{ if } \sigma'(id_p)(''itf'')[i] \\ 0 \ otherwise \end{cases} \tag{1.14}$$

Rewriting the goal with (1.14):

$$\boxed{\sum_{t \in Fired(s')} post(t,p) = \sum_{i=0}^{\Delta(id_p)(''ian'')-1} \begin{cases} \sigma'(id_p)(''iaw'')[i] \texttt{ if } \sigma'(id_p)(''otf'')[i] \\ 0 \ otherwise \end{cases}}$$

Let us unfold the definition of the left sum term:

$$\boxed{\begin{array}{c}\displaystyle\sum_{t \in Fired(s')} \begin{cases} \omega \text{ if } post(t,p) = \omega \\ 0 \; otherwise \end{cases} \\ = \\ \displaystyle\sum_{i=0}^{\Delta(id_p)(''ian'')-1} \begin{cases} \sigma'(id_p)(''iaw'')[i] \text{ if } \sigma'(id_p)(''itf'')[i] \\ 0 \; otherwise \end{cases}\end{array}}$$

Let us perform case analysis on $input(p)$; there are two cases:

1. $input(p) = \emptyset$:

   By construction, $<\texttt{input\_arcs\_number} \Rightarrow 1> \in gm_p$, $<\texttt{input\_transitions\_fired}(0) \Rightarrow \texttt{true}> \in ipm_p$, and $<\texttt{input\_arcs\_weights}(0) \Rightarrow 0> \in ipm_p$.
   By property of the elaboration relation and $\texttt{comp}(id_p, ''place'', gm_p, ipm_p, opm_p) \in d.cs$:

   $$\Delta(id_p)(''ian'') = 1 \tag{1.15}$$

   By property of the stabilize relation and $\texttt{comp}(id_p, ''place'', gm_p, ipm_p, opm_p) \in d.cs$:

   $$\sigma'(id_p)(''itf'')[0] = \texttt{true} \tag{1.16}$$
   $$\sigma'(id_p)(''iaw'')[0] = 0 \tag{1.17}$$

   By property of $input(p) = \emptyset$:

   $$\sum_{t \in Fired(s')} \begin{cases} \omega \text{ if } post(t,p) = \omega \\ 0 \; otherwise \end{cases} = 0 \tag{1.18}$$

   Rewriting the goal with (1.15), (1.16), (1.17), and (1.18), and simplifying the goal, tautology.

2. $input(p) \neq \emptyset$:

   By construction, $<\texttt{input\_arcs\_number} \Rightarrow |input(p)|> \in gm_p$, and by property of the elaboration relation:

   $$\Delta(id_p)(''ian'') = |input(p)| \tag{1.19}$$

   To ease the reading, let us define functions $f \in Fired(s') \to \mathbb{N}$ and $g \in [0, |input(p)| - 1] \to \mathbb{N}$ s.t. $f(t) = \begin{cases} \omega \text{ if } post(t,p) = \omega \\ 0 \; otherwise \end{cases}$ and
   $g(i) = \begin{cases} \sigma'(id_p)(''iaw'')[i] \text{ if } \sigma'(id_p)(''itf'')[i] \\ 0 \; otherwise \end{cases}$

   Then, the goal is: $\boxed{\displaystyle\sum_{t \in Fired(s')} f(t) = \sum_{i=0}^{\Delta(id_p)(''ian'')-1} g(i)}$

Rewriting the goal with (1.19): $\boxed{\sum_{t \in Fired(s')} f(t) = \sum_{i=0}^{|input(p)|-1} g(i).}$

Let us reason by induction on the right sum term of the goal.

- **BASE CASE**:
  In that case, $0 > |input(p)| - 1$ and $\sum_{i=0}^{|input(p)|-1} g(i) = 0$.
  As $0 > |input(p)| - 1$, then $|input(p)| = 0$, thus contradicting $input(p) \neq \emptyset$.
- **INDUCTION CASE**:
  In that case, $0 \leq |input(p)| - 1$.

  $$\forall F \subseteq Fired(s'),\ g(0) + \sum_{t \in F} f(t) = g(0) + \sum_{i=1}^{|input(p)|-1} g(i)$$

  $$\boxed{\sum_{t \in Fired(s')} f(t) = g(0) + \sum_{i=1}^{|input(p)|-1} g(i)}$$

  By definition of g:

  $$g(0) = \begin{cases} \sigma'(id_p)(\text{"}iaw\text{"})[0] \texttt{ if } \sigma'(id_p)(\text{"}itf\text{"})[0] \\ 0\ otherwise \end{cases} \tag{1.20}$$

  Let us perform case analysis on the value of $\sigma'(id_p)(\text{"}itf\text{"})[0]$; there are two cases:

  (a) $\sigma'(id_p)(\text{"}itf\text{"})[0] = \texttt{false}$:
  In that case, $g(0) = 0$, and then we can apply the induction hypothesis with $F = Fired(s')$ to solve the goal: $\sum_{t \in Fired(s')} f(t) = \sum_{i=1}^{|input(p)|-1} g(i)$.

  (b) $\sigma'(id_p)(\text{"}itf\text{"})[0] = \texttt{true}$:
  In that case, $g(0) = \sigma'(id_p)(\text{"}iaw\text{"})[0]$ and $\sigma'(id_p)(\text{"}itf\text{"})[0] = \texttt{true}$.
  By construction, there exist a $t \in input(t)$, $id_t \in Comps(\Delta)$ s.t. $\gamma(t) = id_t$. Let us take such a $t \in input(p)$.
  By definition of id_t, there exist gm_t, ipm_t, opm_t s.t. $\texttt{comp}(id_t, \text{"}transition\text{"}, gm_t, ipm_t, opm_t) \in d.cs$.
  As $t \in input(p)$, there exist $\omega \in \mathbb{N}^*$ s.t. $post(t,p) = \omega$. Let us take an ω s.t. $post(t,p) = \omega$.
  By construction, $<\texttt{input\_arcs\_weights}(0) \Rightarrow \omega> \in ipm_p$, and there exists $id_{ft} \in Sigs(\Delta)$ s.t. $<\texttt{fired} \Rightarrow id_{ft}> \in opm_t$ and $<\texttt{input\_transitions\_fired}(0) \Rightarrow \texttt{id}_{\texttt{ft}}> \in ipm_p$
  By property of the stabilize relation and $<\texttt{input\_arcs\_types}(0) \Rightarrow \texttt{a}> \in ipm_p$:

  $$post(t,p) = \omega \tag{1.21}$$

By property of the stabilize relation, $<\texttt{fired} \Rightarrow \texttt{id}_{\texttt{ft}}> \in opm_t$, $<\texttt{input\_transitions\_fired}(0) \Rightarrow \texttt{id}_{\texttt{ft}}> \in ipm_p$ and $\sigma'(id_p)(''itf'')[0] = \texttt{true}$:

$$\sigma'(id_t)(''fired'') = \texttt{true} \tag{1.22}$$

Appealing to Lemma **??** and (1.22), we know $t \in Fired(s')$.
As $t \in Fired(s')$, we can rewrite the left sum term of the goal as follows:

$$\boxed{f(t) + \sum_{t' \in Fired(s') \setminus \{t\}} f(t') = g(0) + \sum_{i=1}^{|input(p)|-1} g(i)}$$

We know that $g(0) = \sigma'(id_p)(''iaw'')[0]$, and by property of the stabilize relation and $<\texttt{input\_arcs\_weights}(0) \Rightarrow \omega> \in ipm_p$:

$$\sigma'(id_p)(''iaw'')[0] = \omega \tag{1.23}$$

Rewriting the goal with (1.23):

$$\boxed{f(t) + \sum_{t' \in Fired(s') \setminus \{t\}} f(t') = \omega + \sum_{i=1}^{|input(p)|-1} g(i)}$$

By definition of f, and as $post(t, p) = \omega$, then $f(t) = \omega$; thus, rewriting the goal:

$$\boxed{\omega + \sum_{t' \in Fired(s') \setminus \{t\}} f(t') = \omega + \sum_{i=1}^{|input(p)|-1} g(i)}$$

Then, knowing that $g(0) = \omega$, we can apply the induction hypothesis with $F = Fired(s') \setminus \{t\}$: $g(0) + \sum_{t' \in Fired(s') \setminus \{t\}} f(t') = g(0) + \sum_{i=1}^{|input(p)|-1} g(i)$.

□

1.6.2 Falling edge and time counters

Lemma 5 (Falling Edge Equal Time Counters). *For all $sitpn$, d, γ, Δ, σ_e, E_c, E_p, τ, s, s', σ, σ_i, $\sigma_\downarrow$, σ' that verify the hypotheses of Def. 1, then $\forall t \in T_i, id_t \in Comps(\Delta)$ s.t. $\gamma(t) = id_t$,*
$(upper(I_s(t)) = \infty \wedge s'.I(t) \leq lower(I_s(t)) \Rightarrow s'.I(t) = \sigma'(id_t)(''s\_time\_counter''))$
$\wedge (upper(I_s(t)) = \infty \wedge s'.I(t) > lower(I_s(t)) \Rightarrow \sigma'(id_t)(''s\_time\_counter'') = lower(I_s(t)))$
$\wedge (upper(I_s(t)) \neq \infty \wedge s'.I(t) > upper(I_s(t)) \Rightarrow \sigma'(id_t)(''s\_time\_counter'') = upper(I_s(t)))$
$\wedge (upper(I_s(t)) \neq \infty \wedge s'.I(t) \leq upper(I_s(t)) \Rightarrow s'.I(t) = \sigma'(id_t)(''s\_time\_counter'')).$

Proof. Given a $t \in T_i$ and an $id_t \in Comps(\Delta)$ s.t. $\gamma(t) = id_t$, let us show

$$\boxed{\begin{array}{l}(upper(I_s(t)) = \infty \wedge s'.I(t) \leq lower(I_s(t)) \Rightarrow s'.I(t) = \sigma'(id_t)(''s\_time\_counter''))\\ \wedge (upper(I_s(t)) = \infty \wedge s'.I(t) > lower(I_s(t)) \Rightarrow \sigma'(id_t)(''s\_time\_counter'') = lower(I_s(t)))\\ \wedge (upper(I_s(t)) \neq \infty \wedge s'.I(t) > upper(I_s(t)) \Rightarrow \sigma'(id_t)(''s\_time\_counter'') = upper(I_s(t)))\\ \wedge (upper(I_s(t)) \neq \infty \wedge s'.I(t) \leq upper(I_s(t)) \Rightarrow s'.I(t) = \sigma'(id_t)(''s\_time\_counter''))\end{array}}$$

By definition of id_t, there exist gm_t, ipm_t, opm_t s.t. $\texttt{comp}(id_t, ''transition'', gm_t, ipm_t, opm_t) \in d.cs$.

Then, there are 4 points to show:

1. $\boxed{upper(I_s(t)) = \infty \wedge s'.I(t) \leq lower(I_s(t)) \Rightarrow s'.I(t) = \sigma'(id_t)(\text{"}s\_time\_counter\text{"})}$

   Assuming $upper(I_s(t)) = \infty$ and $s'.I(t) \leq lower(I_s(t))$, let us show $\boxed{s'.I(t) = \sigma'(id_t)(\text{"}s\_time\_counter\text{"}).}$
   By property of the $\texttt{Inject}_{\uparrow}$, $\mathcal{H}$-VHDL rising edge and stabilize relations, and $\texttt{comp}(id_t, \text{"}transition\text{"}, gm_t, ipm_t, opm_t) \in d.cs$:

$$\sigma'(id_t)(\text{"}s\_time\_counter\text{"}) = \sigma(id_t)(\text{"}s\_time\_counter\text{"}) \tag{1.24}$$

2. $\boxed{upper(I_s(t)) = \infty \wedge s'.I(t) > lower(I_s(t)) \Rightarrow \sigma'(id_t)(\text{"}s\_time\_counter\text{"}) = lower(I_s(t).}$

3. $\boxed{upper(I_s(t)) \neq \infty \wedge s'.I(t) > upper(I_s(t)) \Rightarrow \sigma'(id_t)(\text{"}s\_time\_counter\text{"}) = upper(I_s(t)).}$

4. $\boxed{upper(I_s(t)) \neq \infty \wedge s'.I(t) \leq upper(I_s(t)) \Rightarrow s'.I(t) = \sigma'(id_t)(\text{"}s\_time\_counter\text{"}).}$

□

1.6.3 Falling edge and firable transitions

Lemma 6 (Falling Edge Equal Firable). *For all $sitpn$, d, γ, Δ, σ_e, E_c, E_p, τ, s, s', σ, σ_i, $\sigma_{\downarrow}$, σ' that verify the hypotheses of Def. 1, and $\forall t, id_t$ s.t. $\gamma(t) = id_t$ and $\sigma'(id_t) = \sigma'_t$, then $t \in Firable(s') \Leftrightarrow \sigma'_t(\text{"}s\_firable\text{"}) = \texttt{true}$.*

Proof. □

1.7 A detailled proof: equivalence of fired transitions

Appendix A

Reminder on natural semantics

Appendix B

Reminder on induction principles

- Present all the material that will be used in the proof, and that needs clarifying for people who do not come from the field (e.g, automaticians and electronicians)
  - structural induction
  - induction on relations
  - ...