FFBS for Coupled HMM

Notation:

$Z_t^{[c]}$ denote the hidden state variable of chain $c \in \{1, \ldots, C\}$ at time $t \in \{1, \ldots, T\}$ with a finite set of possible states, G – assuming the same number of states for each chain

$Z_t^{[c]} \in \Omega = \{1, \ldots, G\}$

Nonhomogeneous Markov chain in which the transition probabilities depend on time given by:

$Pr(Z_t^{[c]} = j | Z_{t-1}^{[c]} = i, \boldsymbol{Z}_{t-1}^{[c]}, \boldsymbol{\theta})$

$\forall i, j \in \Omega$, for $Z_{t-1}^{[-c]}$ denotes $\left(Z_{t-1}^{[1]}, Z_{t-1}^{[2]}, \ldots, Z_{t-1}^{[C]}\right)$ with Z_{t-1}^{c} removed

$Y_t^{[c]}$ denotes the observation variables, which we assume to be generated according to the underlying state process $Z_t^{[c]}$

Let $f(Y_t^{[c]} = y_t^{[c]} | Z_t^{[c]} = g, \boldsymbol{\theta}) = f_g(y_t^{[c]} | \boldsymbol{\theta}) \quad g \in G$ denote the probability density/mass function for $y_t^{[c]}$. If missing, we set $f_g(y_t^{[c]} | \boldsymbol{\theta}) = 1$.

where $\boldsymbol{\theta}$ represents any and all other parameters requiring estimation

Assumptions:

- Every chain has the same number of observations T – (in the future, what would happen with time series of varying length? In what cases would this make sense?)
- Every chain has the same number of states (for now – Sherlock extended this)
- Assumption that the observation is missing 'at random'

Likelihood and Posterior

Complete-data likelihood

Let $\pi(\boldsymbol{\theta})$ denote the joint prior distribution of all parameters.