$a_0, a_1, \ldots\ldots a_n$ 3/abril/2023

$f: \mathbb{Z} \to \mathbb{R}$

$a(n) = a_n = f(n), \quad n \xrightarrow{f} a_n$

$$A(x) = \sum_{k=0}^{\infty} a_k X^k, \quad \lim_{x \to 1} A(x) = \sum_{k=0}^{\infty} a_k$$

$$A'(x) = \sum_{k=0}^{\infty} a_k k x^{k-1}, \quad \lim_{x \to 1} A(x) = E[X]$$

$$f(x) = (a+x)^n, \quad \frac{d^k}{dx^k}(a+x)^n \Big|_{x=0} = n(n-1)\cdots(n-(k-1))$$

$$\frac{d^3}{dx^3}(a+x)^5 = \frac{d}{dx}\left(\frac{d}{dx}\left(\frac{d}{dx}(a+x)^5\right)\right) =$$

$$5(a+x)^4$$

$$20(a+x)^3$$

$$5 \cdot 4 \cdot 3 (a+x)^2$$

$$\frac{d^n}{dx^n}(a+x)^n = n(n-1)\cdots(n-(n-1))a^{n-n} = n!$$

$$\frac{d^{n+1}}{dx^{n+1}} = \frac{d}{dx}\left(\frac{d^n}{dx^n}(a+x)^n\right) = 0$$

$$n(n-1)(n-2)(a+x)^{n-k}$$

$$F(x) = \sum_{k=0}^{\infty} (a+x)^{(k)}\Big|_{x=0} x^k$$

$$n(n-1)(n-(k-1)(a+x)^{n-k}$$

$$= \sum_{k=0}^{n} \left(\frac{n(n-1)\cdots(n(k-n)}{k!}\right)!$$

$$= n(n-1)\ldots(n-(k-1))a^{n-k}$$

$$\frac{d^n}{dx^n}(a+x)^n\Big|_{x=0} \; n(n-1)\ldots(n-(k-1))\, a^{n-n} = n!$$

$n = \alpha$

$$(a+x)^{\alpha} = \sum_{k=0}^{\infty} \left[\frac{\alpha(k-1)\cdots(\alpha-(k-1))}{k!} \right] a^{\alpha-k} x^{n}$$

$$(a+b)^{\alpha} = \sum_{k=0}^{\infty} \binom{\alpha}{k} a^{\alpha-k} b^{k}$$

$$\frac{\alpha!}{(\alpha-k)k!}$$

$$\binom{2n}{n} = \frac{(2n)!}{(2n-n)!\,n!} = \frac{(2n)!}{n!\,n!} = \frac{(2n)(2n-1)\overline{(2n-2)}\cdot\cdots\cdot \bar{6}\cdot 5\cdot \bar{4}\cdot 3\cdot \bar{2}\cdot 1}{n!\,n!}$$

$$= \frac{\prod_{k=0}^{n-1}(2n-2k)\prod_{k=1}^{n}(2n-(2k-1))}{n!\,n!} = \frac{\left[\cancel{2^{n}\left(\prod_{k=0}^{n-1}(n-k)\right)}\right]\left[\prod_{k=1}^{n}(2n-(2k-1))\right]}{\cancel{n!}\,n!}$$

$$\binom{2n}{n} = \frac{\left(\frac{2^{n}}{2^{n}}\right) 2^{n} \prod_{k=1}^{n}(2n-(2k-1))}{n!} = \frac{2^{n}2^{n}\prod_{k=1}^{n}\frac{(2n-(2k-1))}{2}}{n!}$$

$$= \frac{4^{n}}{n!}(-1)^{n}(-1)^{n}\prod_{k=1}^{n}\left(\tfrac{1}{2}+n-k\right)$$