MACHINE INTELLIGENCE

UNIT - 5

RNNs

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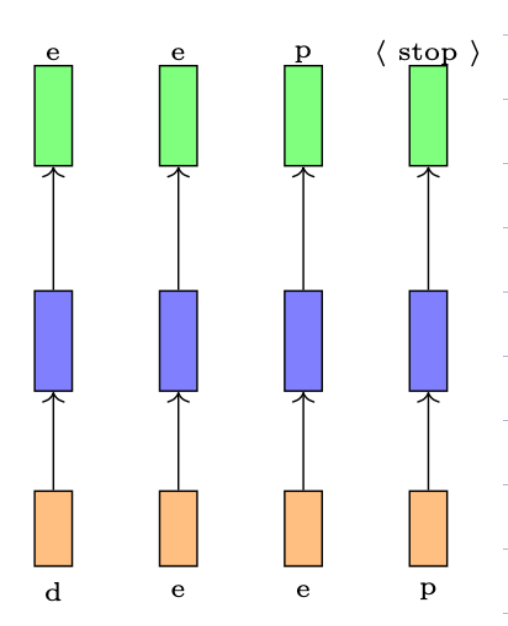
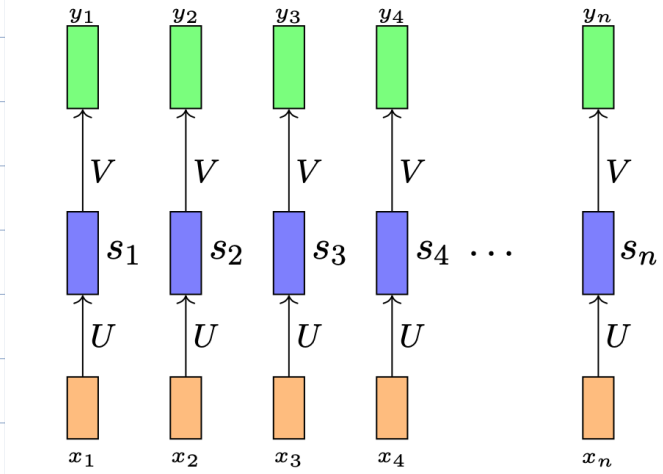
Why RNNs?

- Feedforward NNs : fixed input size, each input independent of prev/future inputs
- Image classification: each image independent of prev
- Auto-completion: successive inputs not independent
  - predicting next letter depends on prev & cur inputs
  - length of input, no. of predictions not fixed
  - each orange-blue-green network performs same task



- Sequence learning problems
- Functions at each layer



$$s_i = \sigma(Ux_i + b)$$

or

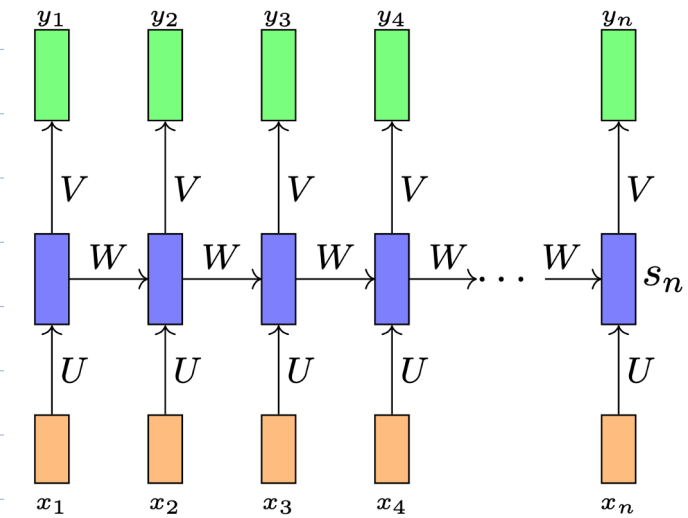
$$s_i = \tanh(Ux_i + b)$$

$$y_i = O(Vs_i + c)$$

or

$$y_i = \text{softmax}(Vs_i + c)$$

RNNs



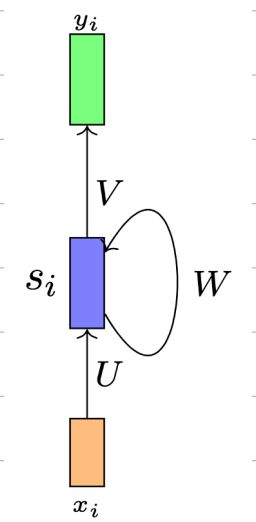
parameters shared across timestamps

$$s_i = \sigma(Ux_i + Ws_{i-1} + b)$$

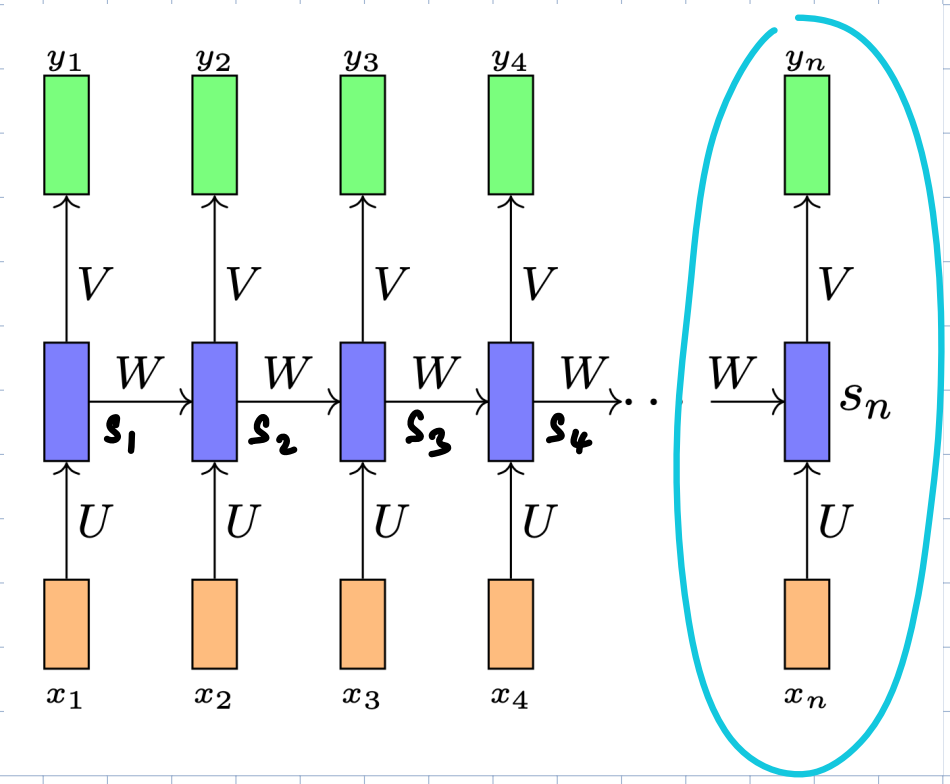
$$y_i = O(Vs_i + c)$$

or

$$y_i = f(x_i, s_{i-1}, U, V, W, b, c)$$



Dimensions



each is → a fully connected ANN

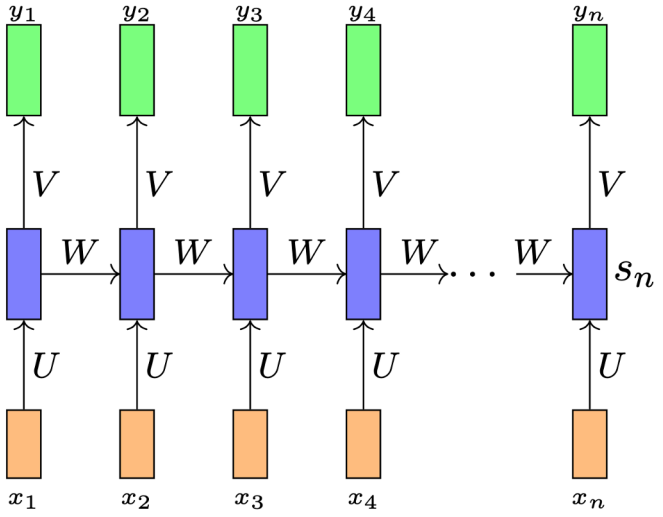
$x_i \in R^n$ (n-d input)
$s_i \in R^d$ (d-d state)
$y_i \in R^K$ (K classes)

$U \in R^{n \times d}$

$W \in R^{d \times d}$

$V \in R^{d \times K}$

BACKPROPAGATION (for RD)



$$\mathcal{L}(\theta) = \sum_{t=1}^{T} \mathcal{L}_t(\theta)$$

$$\mathcal{L}_t(\theta) = -\log(y_{tc})$$

y_{tc} : predicted prob of true character at timestep t

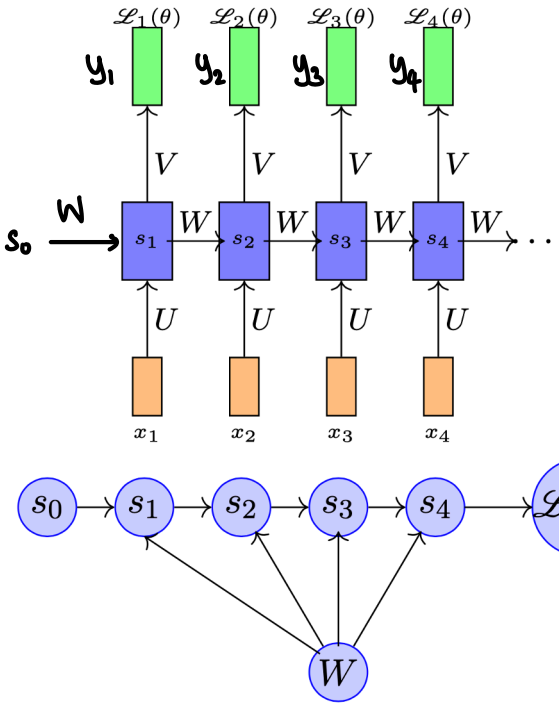
(1) Derivative of loss wrt outer layer weights V (matrix)

$$\frac{\partial \mathcal{L}(\theta)}{\partial V} = \sum_{t=1}^{T} \frac{\partial \mathcal{L}_t(\theta)}{\partial V}$$

(2) Derivative of loss wrt recurrent weights W (matrix)

$$\frac{\partial \mathcal{L}(\theta)}{\partial W} = \sum_{t=1}^{T} \frac{\partial \mathcal{L}_t(\theta)}{\partial W}$$

Consider $\mathcal{L}_4(\theta)$



$\mathcal{L}_4(\theta)$ depends on s_4

s_4 depends on s_3 & W

s_3 depends on s_2 & W

s_2 depends on s_1 & W

s_1 depends on s_0

Timestamp 1

$$\frac{\partial \mathcal{L}_1(\theta)}{\partial W} = \frac{\partial \mathcal{L}_1(\theta)}{\partial y_1} \frac{\partial y_1}{\partial s_1} \frac{\partial s_1}{\partial W}$$

Timestamp 2

$$\frac{\partial \mathcal{L}_2(\theta)}{\partial W} = \frac{\partial \mathcal{L}_2(\theta)}{\partial y_2} \frac{\partial y_2}{\partial s_2} \frac{\partial s_2}{\partial W} + \frac{\partial \mathcal{L}_2(\theta)}{\partial y_2} \frac{\partial y_2}{\partial s_2} \frac{\partial s_2}{\partial s_1} \frac{\partial s_1}{\partial W}$$

treat s_2 as independent

s_2 depends on s_1

Timestamp t

$$\frac{\partial \mathcal{L}_t(\theta)}{\partial W} = \sum_{k=1}^{t} \frac{\partial \mathcal{L}_t(\theta)}{\partial y_t} \frac{\partial y_t}{\partial s_t} \left(\prod_{i=k+1}^{t} \frac{\partial s_i}{\partial s_{i-1}} \right) \frac{\partial s_k}{\partial W}$$

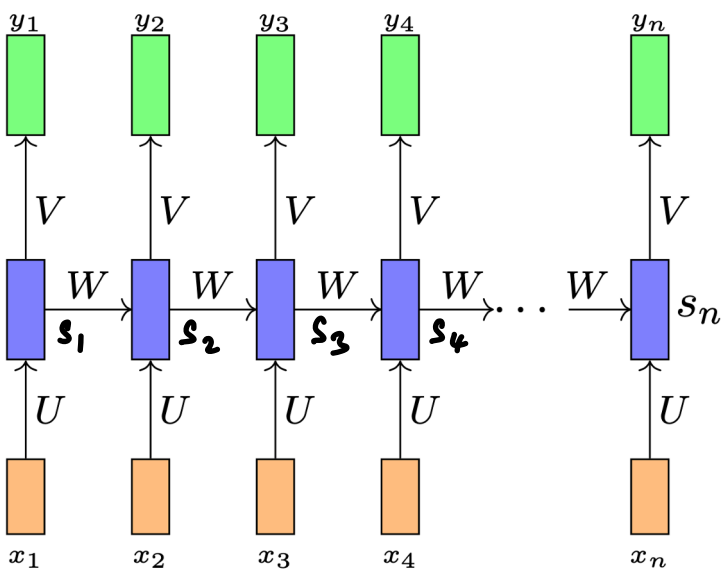
$$= \frac{\partial \mathcal{L}_t(\theta)}{\partial y_t} \sum_{k=1}^{t} \frac{\partial y_t}{\partial s_t} \left(\prod_{i=k+1}^{t} \frac{\partial s_i}{\partial s_{i-1}} \right) \frac{\partial s_k}{\partial W}$$

Exploding / Vanishing Gradient

$$\frac{\partial s_i}{\partial s_{i-1}}$$

$$a_i = W s_{i-1} + b + U x_i$$

$$s_i = \sigma(a_i)$$



$$\frac{\partial s_i}{\partial s_{i-1}} = \underbrace{\frac{\partial s_i}{\partial a_i}}_{\text{(a)}} \underbrace{\frac{\partial a_i}{\partial s_{i-1}}}_{\text{(b)}}$$

$$a_i = [a_{i1}, a_{i2}, \ldots, a_{id}]$$

$$s_i = [\sigma(a_{i1}), \sigma(a_{i2}), \ldots, \sigma(a_{id})]$$

(a) $\frac{\partial s_i}{\partial a_i}$ is a $d \times d$ matrix

$$\frac{\partial s_i}{\partial a_i} = \begin{bmatrix} \frac{\partial s_{i1}}{\partial a_{i1}} & \frac{\partial s_{i2}}{\partial a_{i1}} & \cdots & \frac{\partial s_{id}}{\partial a_{i1}} \\ \frac{\partial s_{i1}}{\partial a_{i2}} & \frac{\partial s_{i2}}{\partial a_{i2}} & & \vdots \\ \vdots & & \ddots & \\ \frac{\partial s_{i1}}{\partial a_{id}} & \cdots & & \frac{\partial s_{id}}{\partial a_{id}} \end{bmatrix}$$

$$\frac{\partial s_i}{\partial a_i} = \begin{bmatrix} \sigma'(a_{i1}) & 0 & \cdots & 0 \\ 0 & \sigma'(a_{i2}) & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma'(a_{id}) \end{bmatrix}$$

$$= \text{diag}(\sigma'(a_i))$$

(b) $\frac{\partial a_i}{\partial s_{i-1}}$ $\qquad a_i = W s_{i-1} + b$

$$\frac{\partial a_i}{\partial s_{i-1}} = W$$

Magnitude of $\frac{\partial s_i}{\partial s_{i-1}}$

$$\sigma'(a_i) \leq \frac{1}{4} = \gamma \quad \text{(sigmoid)}$$

$$\sigma'(a_i) \leq 1 = \gamma \quad \text{(tanh)}$$

$$\left\| \frac{\partial s_i}{\partial s_{i-1}} \right\| = \left\| \text{diag}(\sigma'(a_i))\, W \right\|$$

$$\leq \left\| \text{diag}(\sigma'(a_i)) \right\| \|W\|$$

$$\leq \gamma \|W\|$$

$$\leq \gamma \lambda$$

$$\left\| \frac{\partial s_t}{\partial s_{t-1}} \frac{\partial s_{t-1}}{\partial s_{t-2}} \cdots \frac{\partial s_{k+1}}{\partial s_k} \right\| = \left\| \prod_{j=k+1}^{t} \frac{\partial s_j}{\partial s_{j-1}} \right\|$$

$$\leq \left(\prod_{j=k+1}^{t} \gamma \lambda \right)$$

$$\leq (\gamma\lambda)^{t-k}$$

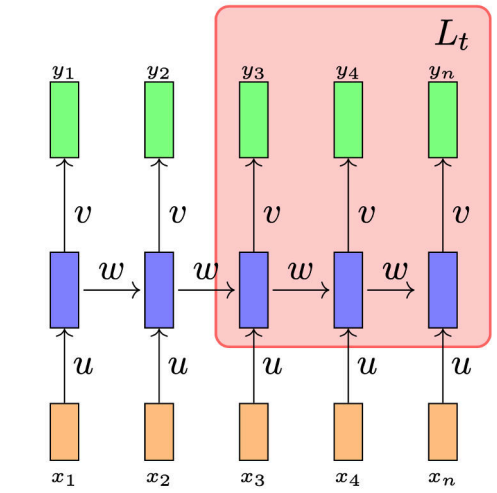
if $\gamma\lambda < 1$: vanishing gradient

if $\gamma\lambda > 1$: exploding gradient

How to avoid?

1. Truncated backpropagation

- only t-k timestep to t timestep



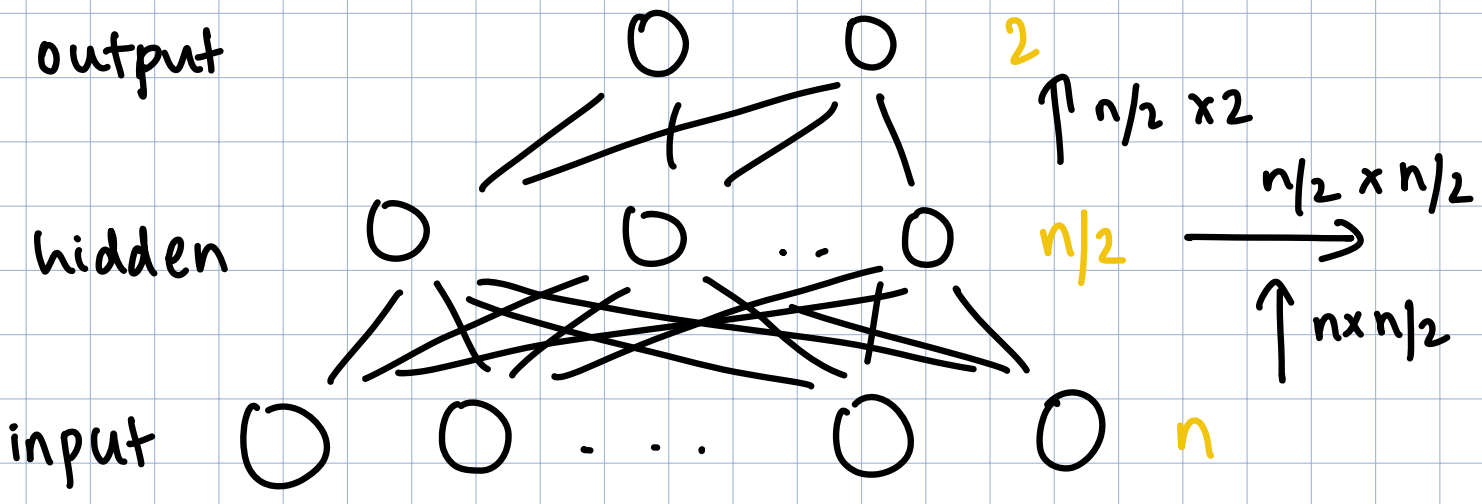
2. Gradient clipping

- Normalise gradients according to a vector norm (eg: L2)
- Let $\hat{g} = \frac{\partial \mathcal{L}(\theta)}{\partial W}$

$$\hat{g} = \frac{\text{threshold}}{\|\hat{g}\|} \cdot \hat{g}$$

3. LSTM

Q: An RNN takes input of words each as a vector of length n, one hidden layer with n/2 neurons and one output layer with 2 neurons. If total no. of weights = 161, find n.



$$\text{total} = n \times \frac{n}{2} + \frac{n}{2} \times \frac{n}{2} + \frac{n}{2} \times 2 = 161$$

$$\frac{3n^2}{4} + n = 161$$

$$n = 14$$

(3) Derivative of loss wrt input weights U (matrix)

$$\frac{\partial \mathcal{L}(\theta)}{\partial U} = \sum_{t=1}^{T} \frac{\partial \mathcal{L}_t(\theta)}{\partial U}$$

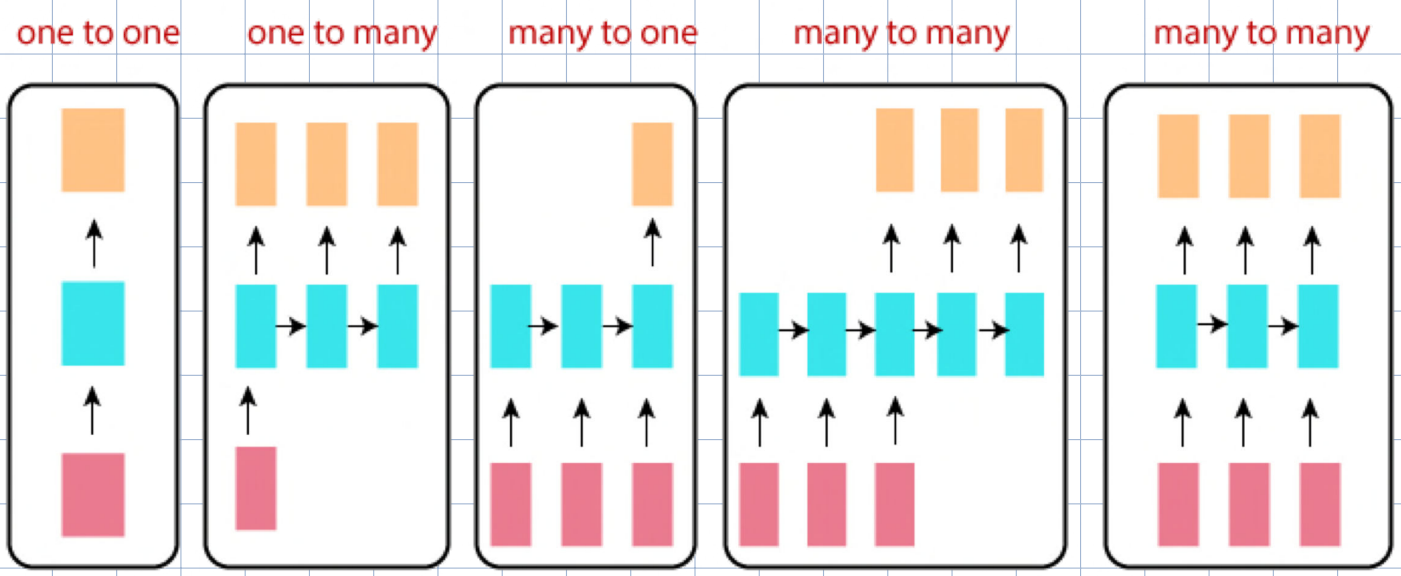
RNN Architectures

(1) One-to-one

- Single input to single output
- Output from t fed as input to t+1
- Eg: predictive text

(2) One-to- Many

- Single input to multiple hidden states and multiple output values
- Share hidden states across timesteps
- Eg: image captioning, music generation



(3) Many-to-One

- Sequence of inputs mapped to single output
- Eg: sentiment analysis

(4) Many-to-Many

- Input of arbitrary length, output of arbitrary length
- Eg: language translation