MACHINE INTELLIGENCE

UNIT - 4

Particle Swarm Optimisation

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Swarm

- Loosely structured collection of interacting agents
- Agents contribute to and benefit from their group
- Eg: swarm of bees, ant colony, flock of birds, human crowds, cells & molecules

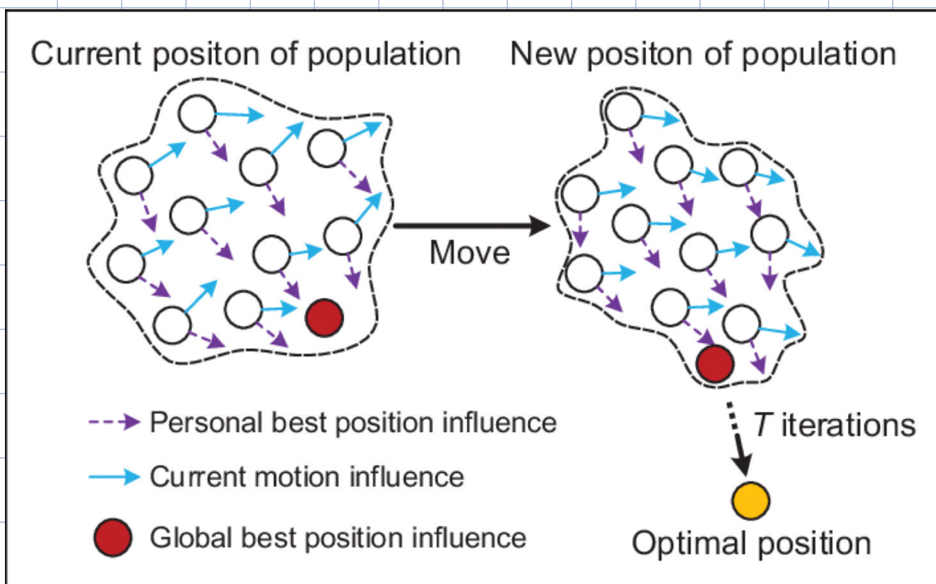
Swarm Intelligence

- No centralised control structures

PARTICLE SWARM OPTIMISATION

- Population-based stochastic optimisation technique
- Potential solutions - particles in the problem space
- Particles fly through the problem space by following current optimum particles
- Each particle searches for optimal solution

- Each particle has velocity
- Each particle remembers its personal best position (most optimal solution it has visited so far)
- Swarm keeps track of overall best solution



Components of PSO

- Each particle: internal state + neighbours
- Let the position of particle i at time t be $x_i(t)$
- The position of particle i at time t+1 is

$$x_i(t+1) = x_i(t) + v_i(t+1)$$

$$x_i(0) = U(x_{min}, x_{max})$$

uniform random

- Velocity at timestep t+1 influenced by

  1. inertia: previous velocity
  2. pbest: personal best position
  3. lbest: local best (best position found by itself and its social neighbours)

- when entire system taken as neighbourhood, gbest or global best taken

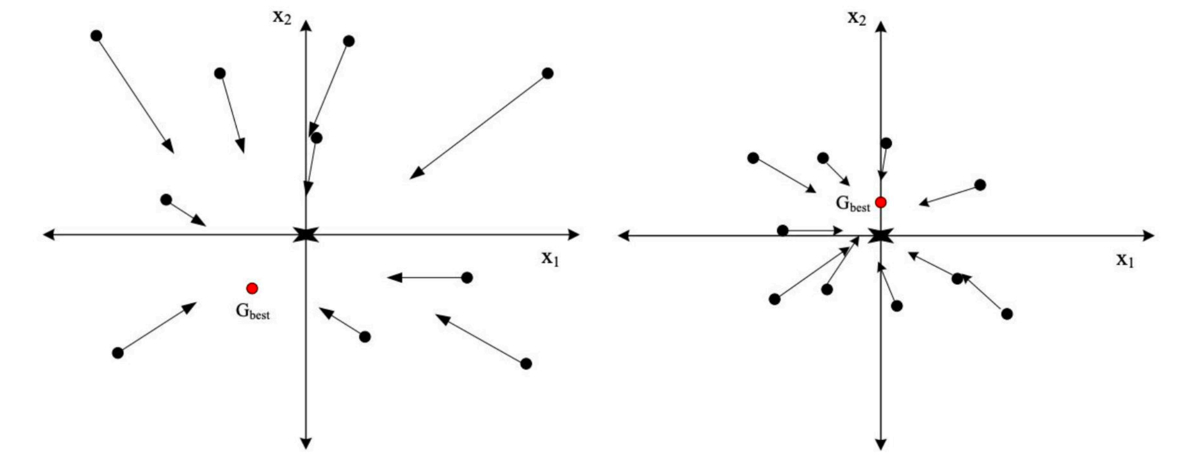
TOPOLOGIES

- Topology defines social neighbourhood of each particle

(1) Global Best PSO

- star topology
- all particles communicate with all others

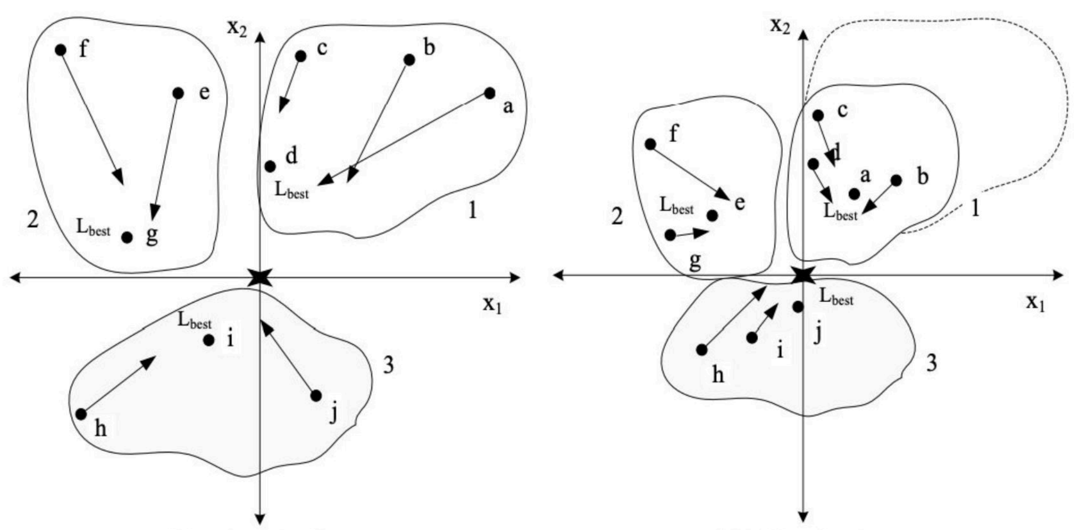
- flaw: local minimum, less diversity
- advantage: converges fast



(a) at time t = 0 (b) at time t = 1

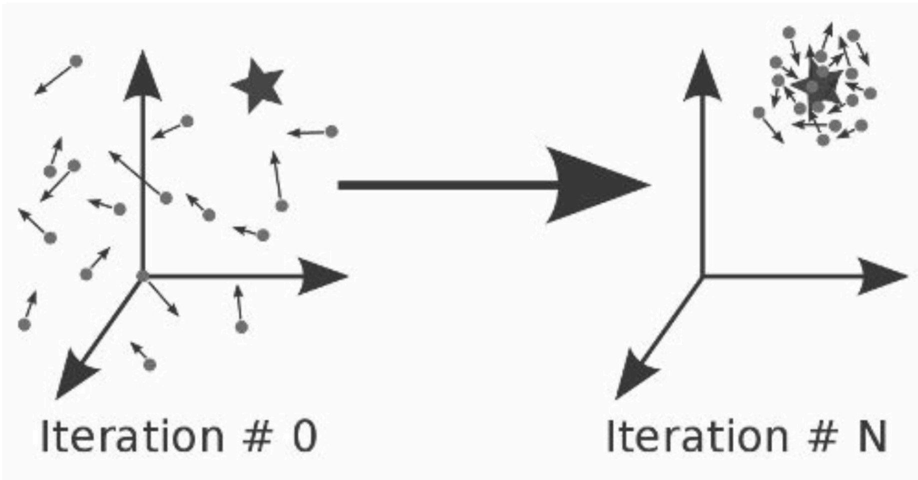
(2) Local Best PSO

- eg: ring topology – every particle connected to 2 neighbours
- advantage: not easily trapped in local minima, more diversity



(a) at time t = 0 (b) at time t = 1

GLOBAL BEST PSO



- Population of particles with random positions and velocities
- Evaluate fitness function for each particle
- compare with pbest and gbest and update accordingly
- Assume D dimensional space
- Position x

$$X_i = (x_{i1}, x_{i2}, \ldots, x_{iD})^T$$

- Velocity V

$$V_i = (V_{i1}, V_{i2}, \ldots, V_{iD})^T$$

- Personal best

$$P_i = (P_{i1}, P_{i2}, \ldots, P_{iD})^T$$

- Global best

$$P_g = (P_{g1}, P_{g2}, \ldots, P_{gD})^T$$

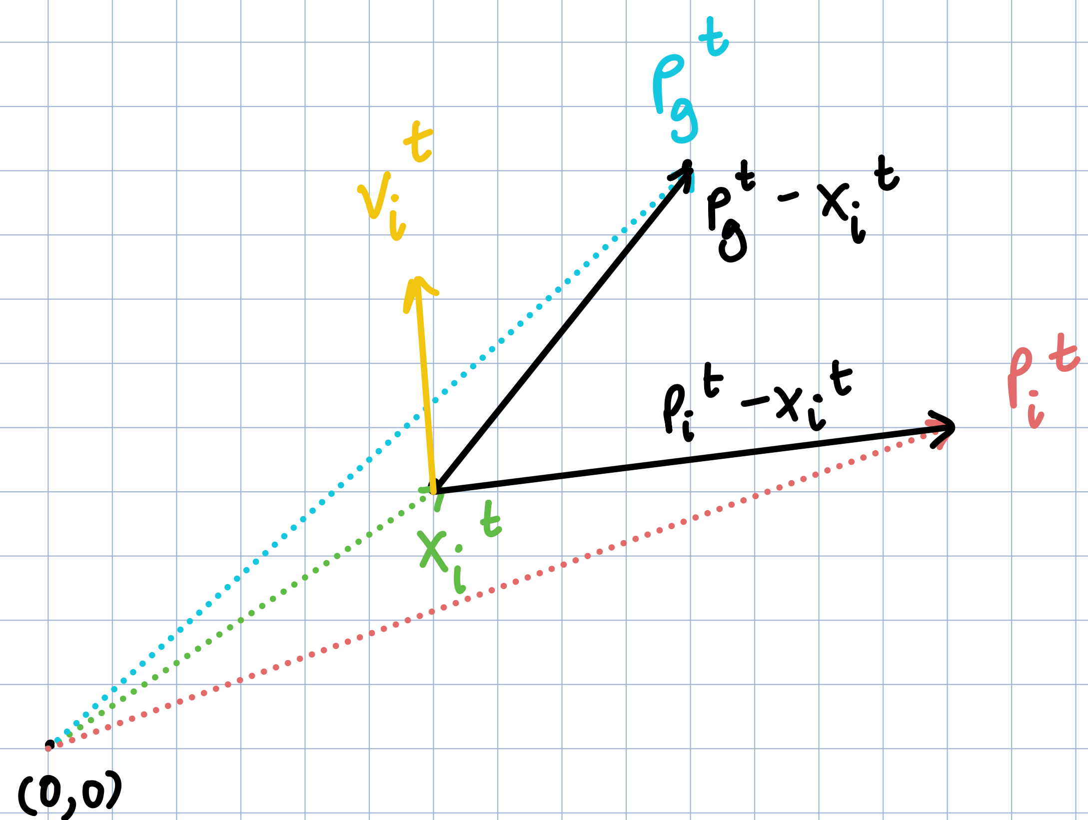
Updating Position

cognitive component c_1

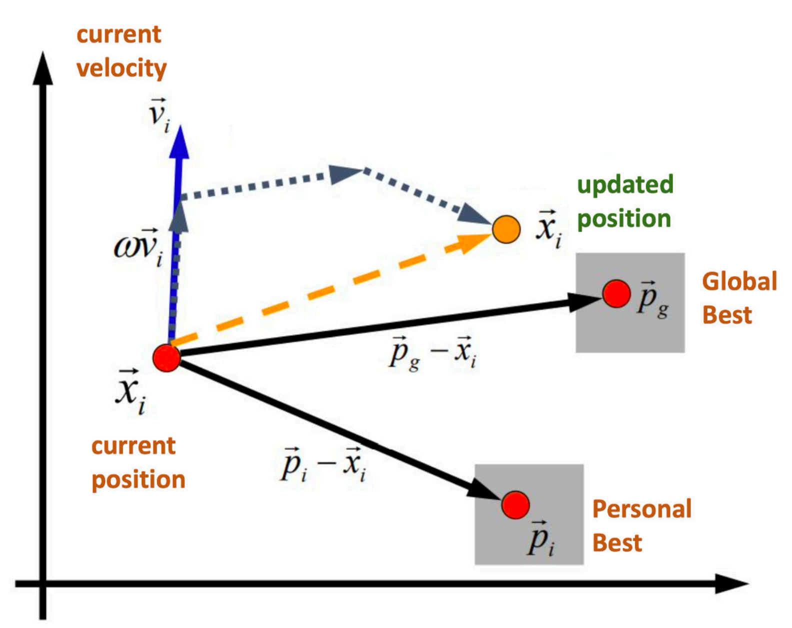
social component c_2

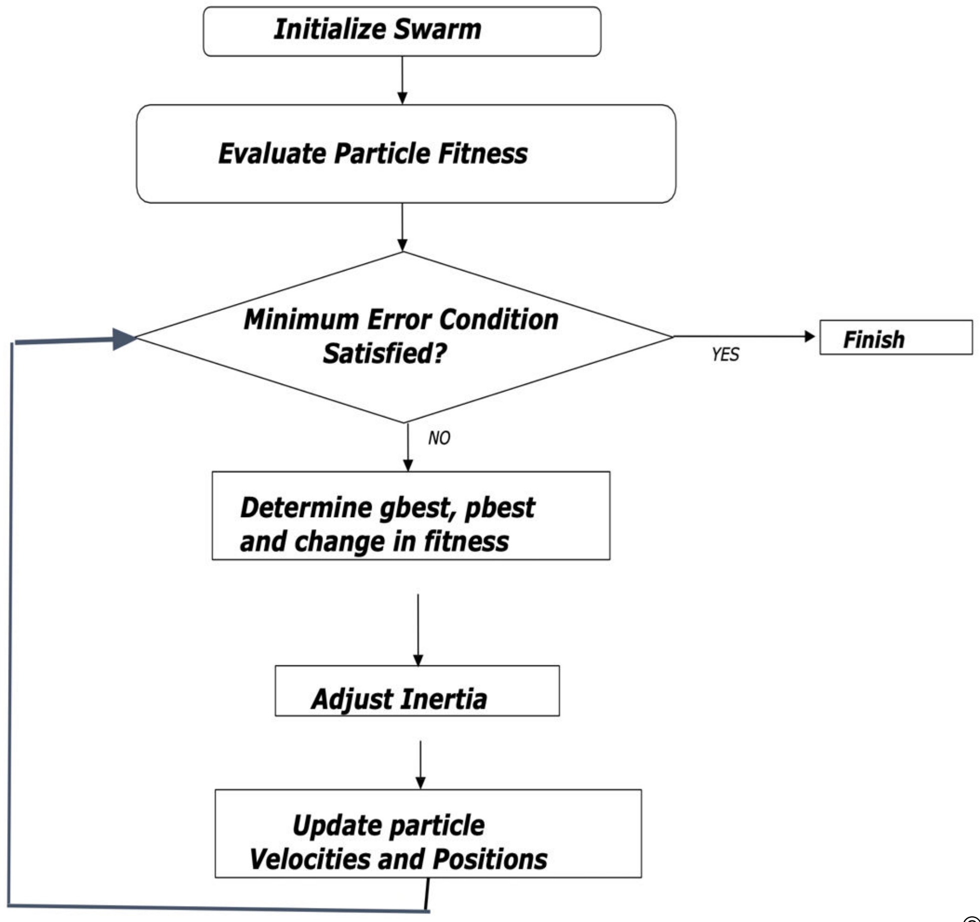
$$V_i^{t+1} = V_i^t + c_1\, rand_1 \times (P_i^t - X_i^t) + c_2\, rand_2 \times (P_g^t - X_i^t)$$

$$X_i^{t+1} = X_i^t + V_i^{t+1}$$



- c_1 & c_2 : acceleration coefficients
- $rand_1$ & $rand_2$: uniformly ([0,1]) dist. random numbers





Exploratory vs Exploitative Behaviour

- Exploration: explore diff regions of search space to find optimum
- Exploitation: concentrate search around promising area
- PSO parameters and algorithm should be chosen to balance exploration and exploitation
- Avoid premature convergence
  1. Velocity clamping
  2. Inertia weight
  3. Constriction factor

1. Velocity clamping

if $v_i^t > V_{max}$, $v_i^t = V_{max}$

if $v_i^t < -V_{max}$, $v_i^t = -V_{max}$

- Small V_{max} : encourages exploitation (local)
- Large V_{max} : encourages exploration (global)

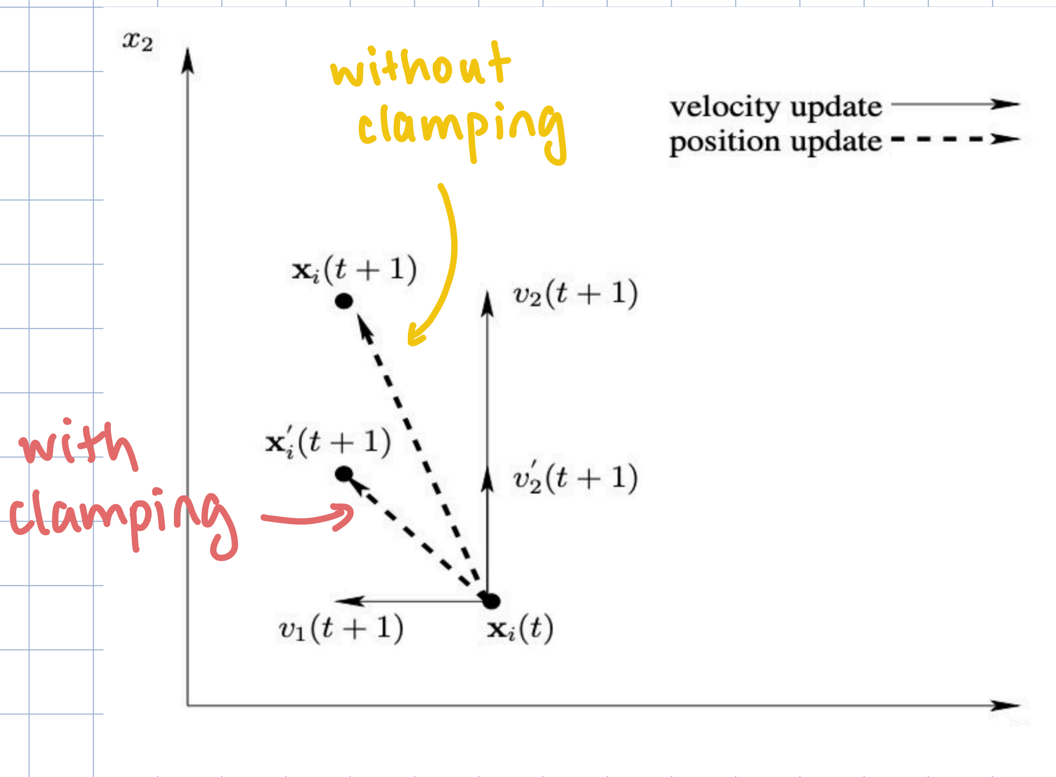
- V_{max} must balance between exploration and exploitation

- Usually, V_{max} chosen to be fraction of domain of each dimension of the search space

$$V_{max} = \delta(X_{max} - X_{min})$$

$$\delta \in [0,1]$$

- Velocity clamping changes direction of velocity



- Problem: all velocities may eventually equal V_{max} and particles search only boundries of hypercube

- Solutions

1. change V_{max} when gbest does not change for k iterations
2. Exponentially decay V_{max}
3. Introduce inertia weight
4. Constriction factor

2. Inertia Weight w

- Controls momentum of particle

$$V_i^{t+1} = wV_i^t + c_1\, rand_1 \times (P_i^t - X_i^t) + c_2\, rand_2 \times (P_g^t - X_i^t)$$

- If $w \geq 1$, velocities increase over time and swarm diverges
  - particles fail to change direction
- If $w < 1$, veocities decrease and reach 0
- Dynamic w

negative value

$$w = w_{max} + \frac{(w_{min} - w_{max})(iter - 1)}{itermax - 1}$$

- w and acceleration coefficients

$$w > \frac{1}{2}(c_1 + c_2) - 1$$

- Typically, $c_1 = c_2 = 2.0$

3. Constriction coefficient φ

- Velocities constricted by φ

$$V_i^{t+1} = \varphi\left(wV_i^t + c_1\, rand_1 \times \left(P_i^t - X_i^t\right) + c_2\, rand_2 \times \left(P_g^t - X_i^t\right)\right)$$

- where φ is given by

$$\varphi = \frac{2}{\left|2 - \psi - \sqrt{\psi^2 - 4\psi}\right|}$$

- and ψ is $(\psi \geq 4)$

$$\psi = c_1 + c_2$$

COGNITION ONLY MODEL

- $c_2 = 0$
- Excludes social component

$$v_i^{t+1} = wv_i^t + c_1 \, rand_1 * (p_i^t - x_i^t)$$

SOCIAL ONLY MODEL

- $c_1 = 0$
- Excludes cognitive component

$$v_i^{t+1} = wv_i^t + c_2 \, rand_2 * (p_g^t - x_i^t)$$

- Faster than full & cognitive models for dynamic environments

Q: Minimise $f(x,y) = x^2 + y^2$

ST constraints $-1 \le x \le 1$
$-1 \le y \le 1$

Assume 5 particles

Let ω (inertia) = 0.3

Let $c_1 = c_2 = 2$

Assume $P_i^0 = P_g^0 = 10000$ (large no.)

Initialisation

Choose $P_i \sim U(1,1)$ for 5 particles in 2 dimensions

$P_1 = (1,1)$ $v_1 = (0,0)$

$P_2 = (-1,1)$ $v_2 = (0,0)$

$P_3 = (0.5,-0.5)$ $V_3 = (0,0)$

$P_4 = (1,-1)$ $V_4 = (0,0)$

$P_5 = (0.25, 0.25)$ $V_5 = (0,0)$

Fitness value = $f(x,y) = x^2+y^2$

Run 1

TABLE 1: Initial positions, velocity, and best positions of all particles.

Particle No.	Initial Positions		Velocity		Best Solution	Best Position		Fitness Value
	x	y	x	y		x	y	
P_1	1	1	0	0	1000	-	-	2
P_2	-1	1	0	0	1000	-	-	2
P_3	0.5	-0.5	0	0	1000	-	-	0.5
P_4	1	-1	0	0	1000	-	-	2
P_5	0.25	0.25	0	0	1000	-	-	0.125

Global best value = 0.125

For P1:

$P_g^1 = (0.25, 0.25)$

Let $r_1 = r_2 = 0.5$

Local best of $P_1 = (1,1)$

$$v_1^1 = w v_1^0 + c_1 r_1 (P_1^0 - x_1^0) + c_2 r_2 (P_g^0 - x_1^0)$$

$$= 0.3 \times (0,0) + 2 \times 0.5 (0,0) + 2 \times 0.5 (1-0.25, 1-0.25)$$

$$v_1^1 = (-0.75, -0.75)$$

$$P_1^1 = (1,1) + (-0.75, -0.75)$$

$P_1' = (0.25, 0.25) \rightarrow$ new PB!

$f(P_1') = 0.125$

and so on

Global best value = 0.125 and Global best position = 0.25, 0.25

	Current Position			Updated Velocity		Personal Best Position			Updated Position	
Particle (i)	xi(t)	yi(t)	Fitness value f(xi(t), yi(t))	vi(t+1)	vi(t+1)	P_b(i)	P_b(i)	global best value	xi(t+1)	yi(t+1)
1	1	1	2	-0.75	-0.75	1	1	0.125	0.25	0.25
2	-1	1	2	1.25	-0.75	-1	1	0.125	0.25	0.25
3	0.5	-0.5	0.5	-0.25	0.75	0.5	-0.5	0.125	0.25	0.25
4	1	-1	2	-0.75	1.25	1	-1	0.125	0.25	0.25
5	0.25	0.25	0.125	0	0	0.25	0.25	0.125	0.25	0.25

Applications

1. Transportation planning
2. Neural networks
3. Clustering