Pumping Lemma Examples

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We use Pumping lemma to show that some languages are not regular. The steps that are followed are

* Assume the language is regular.
* If regular, there is a pumping length p.
* For a specific string s, where $|s| \geq p$, and for some i, we show that $xy^iz \notin$ language for any split xyz of s.
* Contradiction!

(1) Let $B = \{0^n 1^n \mid n \geq 0\}$.

Assume B is regular $\Rightarrow \exists p$, pumping length by pumping lemma.

Consider $s = 0^p 1^p$. The string $s \in B$, $|s| \geq p$. This implies that s can be written as $s = xyz$, satisfying the rules of pumping lemma. (s can be pumped). We will show that this is not possible.

$s = 00\ldots0111\ldots1$

(3)

(2)

$\leftarrow xy \rightarrow$

show that this is not

$\leftarrow xy \rightarrow$ (2)

There are 3 possibilities.

(1) y has only 0's. $\Rightarrow$ $s' = xyyz$ has more 0's than 1's.

(2) y has only 1's $\Rightarrow$ $s' = xyyz$ has more 1's than 0's

(3) y has 0's & 1's $\Rightarrow$ $y = 00..011...1$

$$s' = xyyz = \underbrace{0\ldots0}_{x}\,\underbrace{0\ldots01\ldots1}_{y}\,\underbrace{0\ldots01\ldots1}_{y}\,\underbrace{11\ldots1}_{z}$$

This is not of the form $0^* 1^*$ and hence $s' \notin B$. Hence $s = 0^p 1^p$ cannot be pumped.

Hence B is not regular.

Note: We did not use condition (3) of the lemma. (3) $\Rightarrow$ $|xy| \le p$. Using this, we see that x and y consist only of 0's. Hence $xyyz$ must have more 0's than 1's.

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(2) $C = \{ w \mid w$ has an equal number of 0's and 1's $\}$

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If C is regular, there is a pumping length p. Consider $s = 0^p 1^p \in C$. Also $|s| \geq p$.

$$\underbrace{0000}_{x}\underbrace{011111}_{y=0011}\ldots \qquad xyyz$$

By pumping lemma, s can be pumped.

Suppose $s = xyz$. By (3), $|xy| \leq p$.

Hence $xy = 00\ldots0$. Hence $y = 0^l$ for some $l \geq 1$. So

$xyyz$ contains more 0's than 1's.

So $xyyz \notin C$. Contradiction.

<u>Note</u>: The choice of s is critical here. There may be strings that can be pumped. Say, if we choose $s = (01)^p$. This can be pumped by $x = 01$, $y = 01$, $z = (01)^{p-2}$.

<u>Note</u>: Another way to show that C is not regular is by appealing to closure properties.

If C was regular, then $C \cap 0^* 1^*$ is also regular. This is because regular languages are closed under intersection.

regular.

are closed under intersection.

But $C \cap 0^* 1^* = B$, which we saw is not regular. So C is also not regular.

Read Example 1.75: $F = \{ww \mid w \in \{0,1\}^*\}$.

(3) Unary language: $D = \{1^{n^2} \mid n \geq 0\}$

The length of all the strings are a perfect square.

D is regular $\Rightarrow$ pumping length p.

Consider $s = 1^{p^2}$. If $s = xyz$,

we have $|xy| \leq p \Rightarrow |y| \leq p$.

$$|xyz| < |xy^2z| = p^2 + |y| \leq p^2 + p < (p+1)^2$$

So $$p^2 < |xyyz| < (p+1)^2.$$

So $xyyz \notin D$. So D is not regular.

Read Example 1.77 where "pumping down" is necessary. $E = \{0^i 1^j \mid i > j\}$.

$xz \in$ lang.

Summary: If A is regular, $\forall s$ such

<u>Summary</u>: If A is regular, $\forall s$ such that $|s| \geq p$, $\exists$ a split $s = xyz$, such that $|xy| \leq p$, $|y| > 0$, and $\forall i \geq 0$, $xy^iz \in A$.