

Slides for BST Q&A

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Plan

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Today we see how it implements the following

1. SEARCH
2. INSERT
3. SUCC (also PRED)
4. DELETE

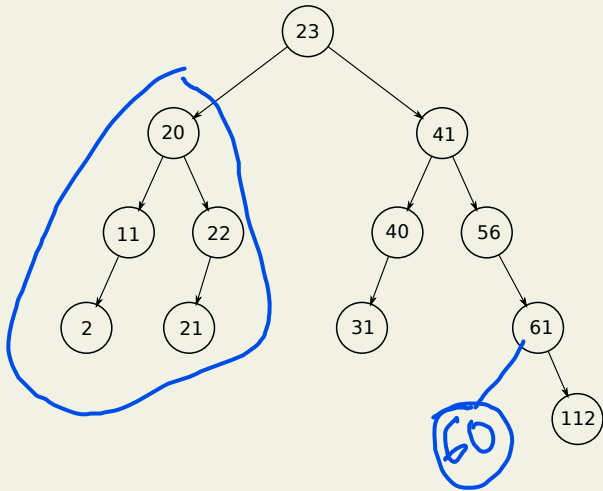
Binary Search Trees

Recall that a Binary Search Tree (BST) has the following crucial property:

For every node X in the BST, we have:

- ▶ Every node in the left subtree of X contains a value smaller than that of X .
- ▶ Every node in the right subtree of X contains a value larger than that of X .

Example BST



```

graph TD
    0((0)) --> 2((2))
    2 --> 3((3))
    3 --> 0_4((0))
    0_4 --> 0_5((0))
    0_5 --> 0_6((0))
    0_6 --> 7((7))
    7 --> 7
  
```

INSERT procedure

The INSERT(*node*, *x*) procedure:

- ▶ If *node* = NULL, create new node with *x* and attach to parent.
- ▶ Else If $x < \text{value}(\textit{node})$,
 - ▶ INSERT(*node* → *left*, *x*)
- ▶ Else If $x > \text{value}(\textit{node})$ Then,
 - ▶ INSERT(*node* → *right*, *x*)

SEARCH procedure

- ▶ $\text{SEARCH}(node, x)$:
- ▶ If $node = \text{NULL}$, then return NULL
- ▶ Else If $x = \text{value}(node)$, then return $node$
- ▶ Else If $x < \text{value}(node)$, then
 - ▶ Return $\text{SEARCH}(node \rightarrow \text{left}, x)$
- ▶ Else
 - ▶ Return $\text{SEARCH}(node \rightarrow \text{right}, x)$

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- ▶ Find the node which stores *val*. Refer to this node as “*node*”.
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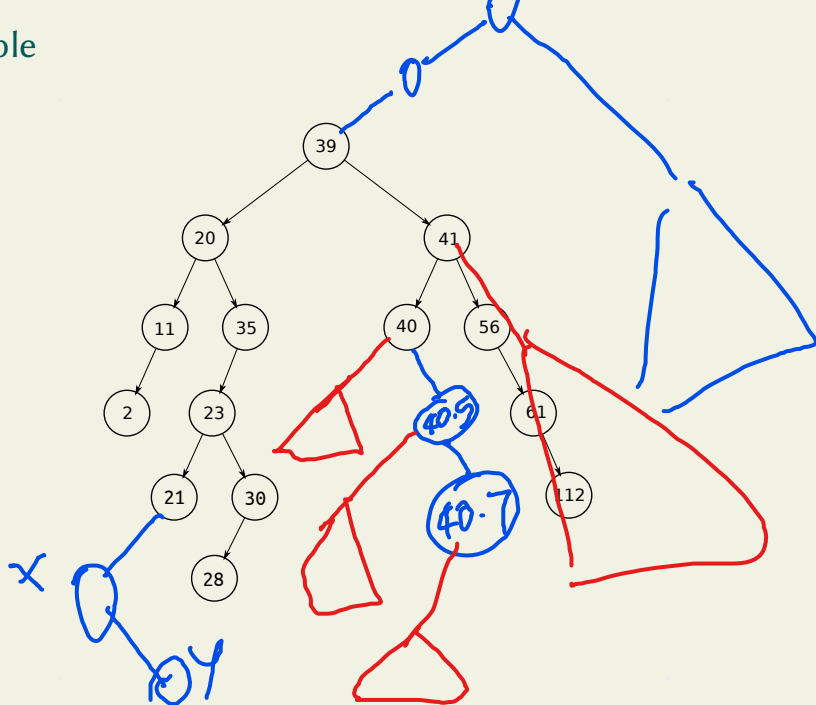
Case 1: *node* has a right child.

Find the smallest element of the right subtree.

Case 2: *node* does not have a right child.

Keep going to the parent till we reach an ancestor *x* that is a left child of its parent. The parent of *x* is the successor.

Example



DELETE

The DELETE(*VAL*) procedure is as follows:
Find the node that has value *val*.

DELETE

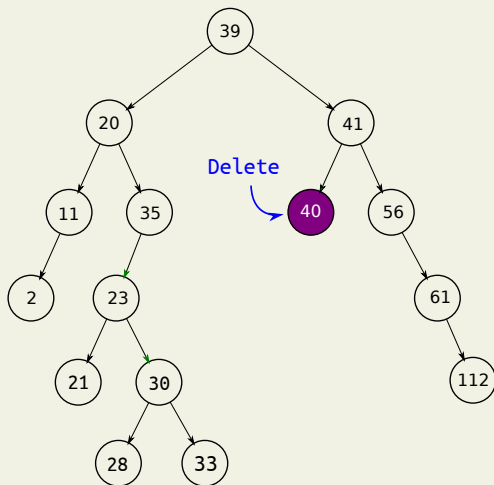
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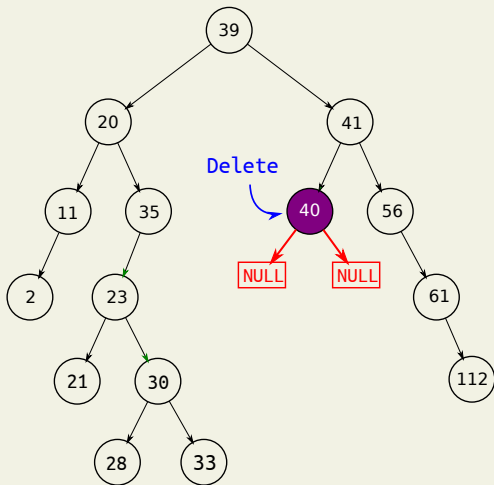
Three cases:

1. *node* has 0 children. (trivial)
2. *node* has 1 child. (splice)
3. *node* has 2 children:
 - ▶ Find successor node *X* with value *x*.
 - ▶ Splice *X* out of the tree.
 - ▶ Replace *val* with *x*.
 - ▶ Delete node *X*.

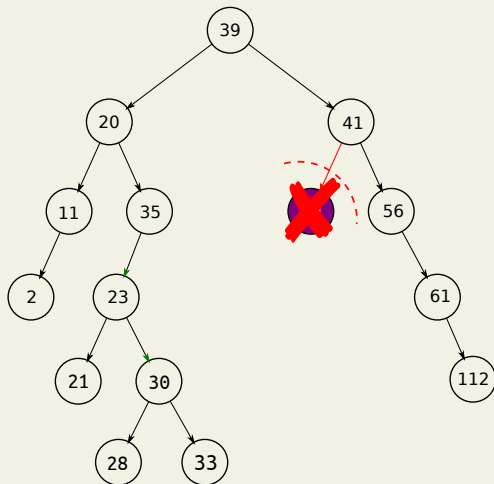
Example



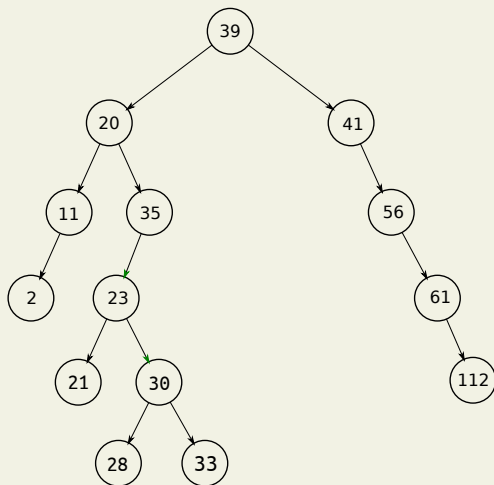
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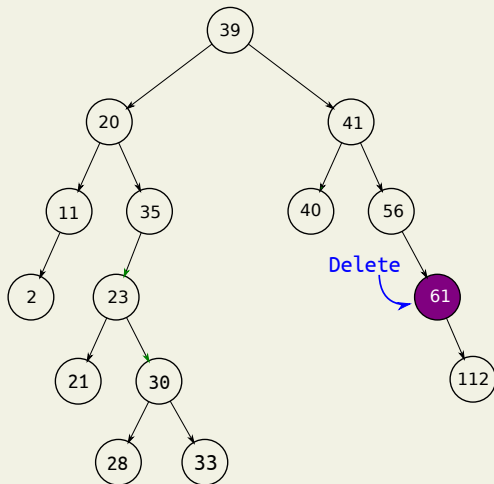
Example



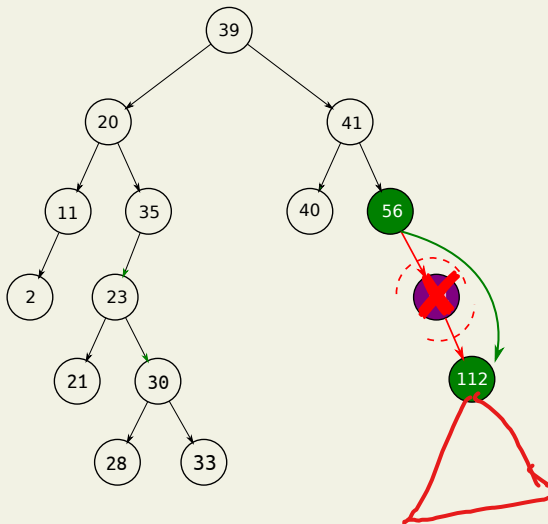
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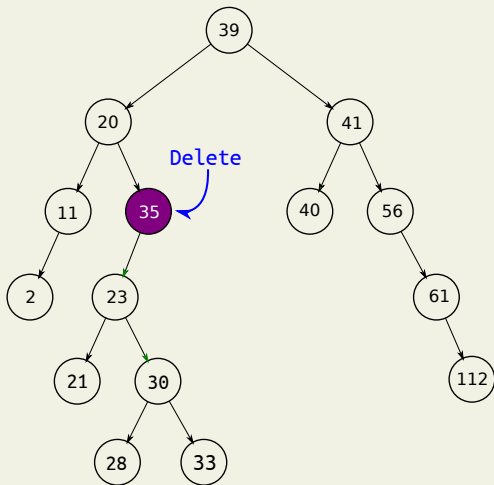
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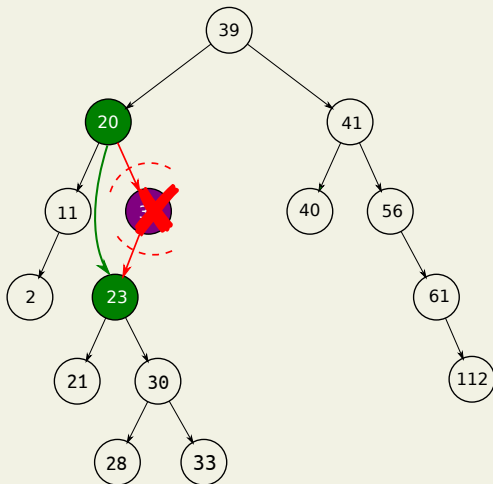
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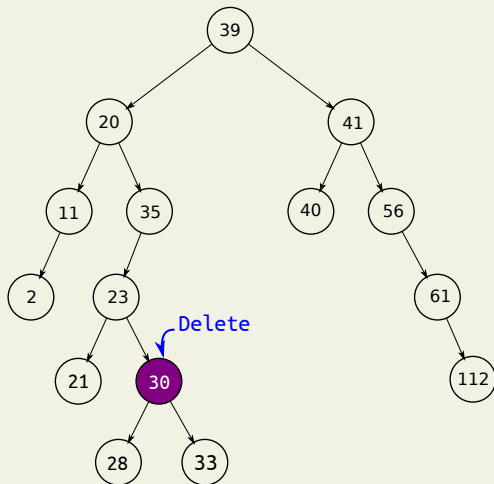
Example



Example

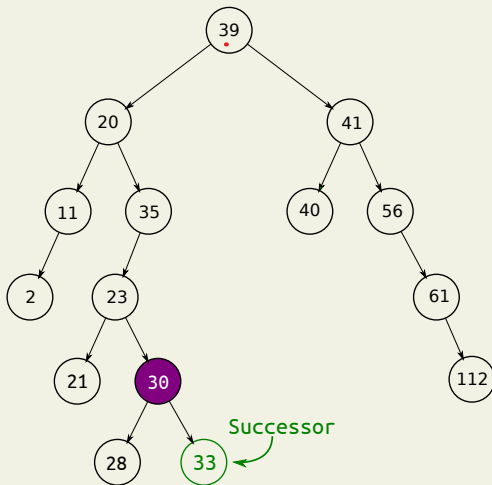


Example

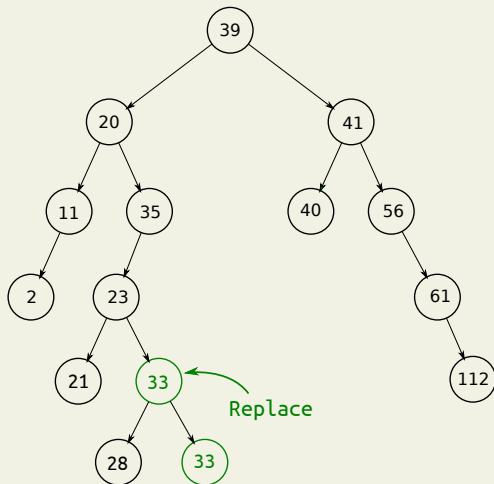


Example

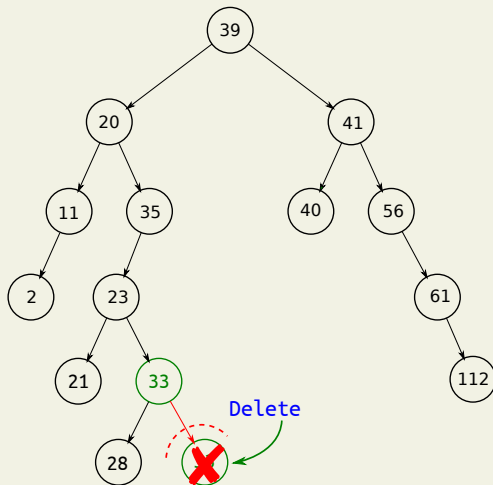
1, 2, 3, 4, 5, 18, 20, 26, ..



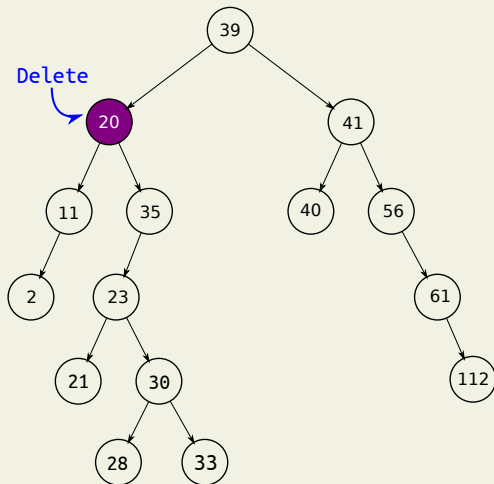
Example



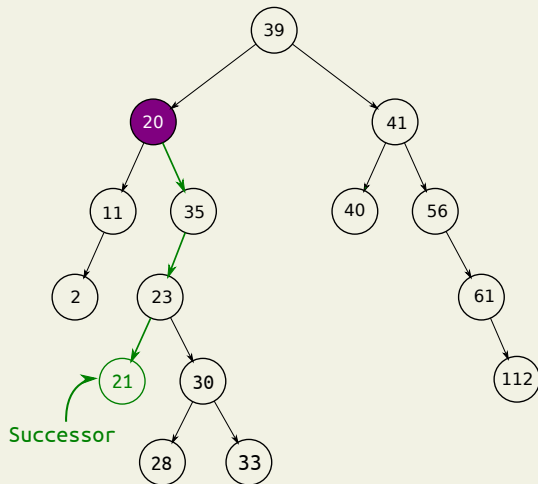
Example



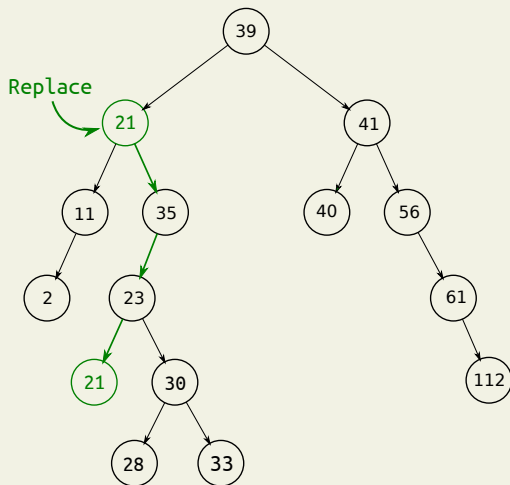
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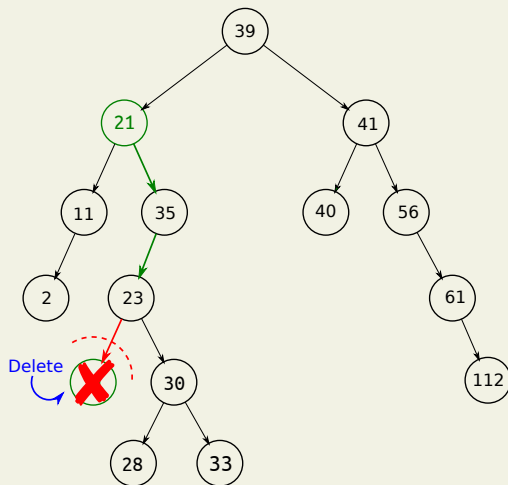
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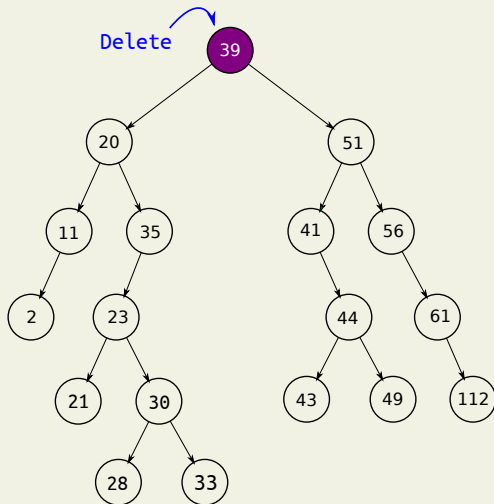
Example



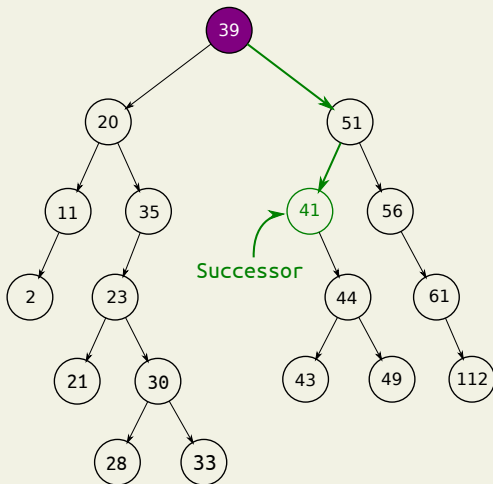
Example



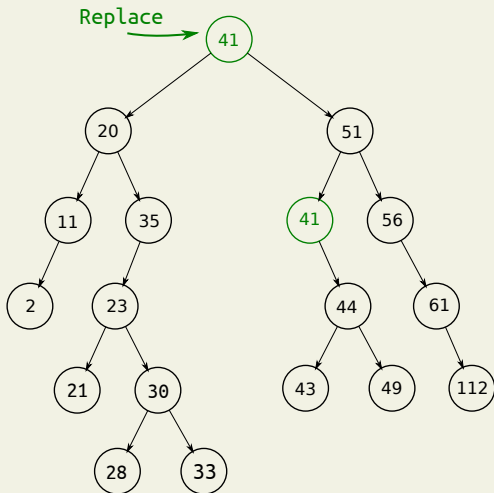
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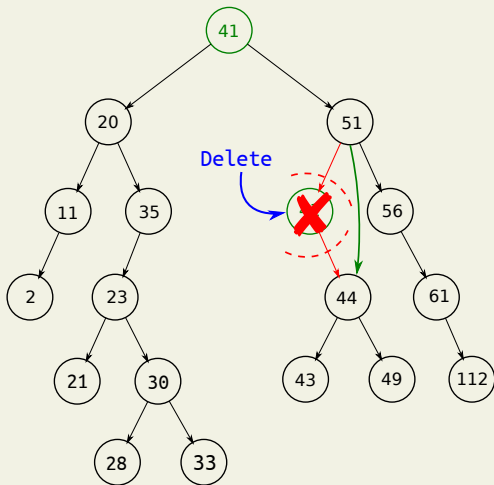
Example



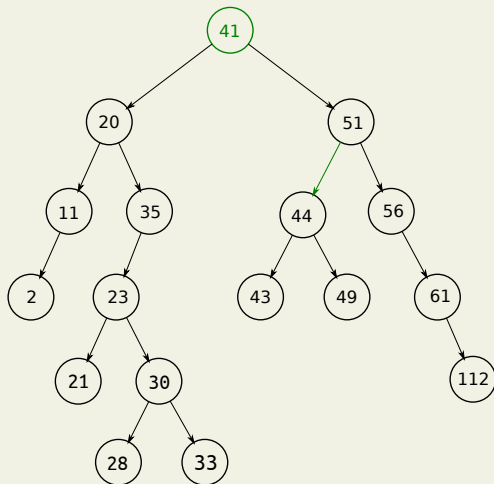
Example



Example



Example



Running Time

Worst case running times for a BST of height h :

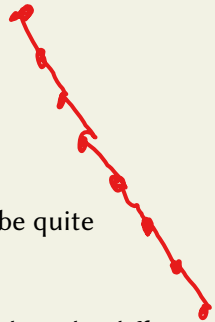
- ▶ INSERT – $O(h)$.
- ▶ SUCC – $O(h)$.
- ▶ SEARCH – $O(h)$.
- ▶ DELETE – $O(h)$.

The height of a BST depends on the input sequence and can be n after inserting n elements in the worst case.

Balancing a BST

The biggest drawback of BSTs are that they can be quite “unbalanced”.

One way to measure if a tree is balanced is to look at the difference between the longest path from root to leaf and the shortest path from root to leaf.



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We want to make sure this difference does not get too large.

Thank You!