

## Algorithms: Quiz Two

1. A palindrome is a string that reads the same from right-to-left and left-to-right, i.e. a string of the form  $S[1]S[2] \dots S[n-1]S[n]$ , which is equal to  $S[n]S[n-1] \dots S[2]S[1]$ . Some examples are LOL and RADAR. Describe and analyze an algorithm to find a longest subsequence of a given input string  $T[1, 2, \dots, n]$  that is a palindrome. For example, if the input string is ABRACADABRA, then ABACABA is one longest palindrome subsequence. For each  $i \leq j$ , let  $f(i, j)$  denote the length of the longest palindrome subsequence in  $T[i, i+1, \dots, j]$ . Write a recurrence relation for  $f(i, j)$ .
2. There are  $n$  successive days of an event with  $a_i$  activities on the  $i$ th day, where the  $a_i$ s are positive integers. Participating on the  $i$ th day will leave you tired and not able to participate for the next  $b_i$  days. Your goal is to choose a subset  $S$  of  $\{1, 2, \dots, n\}$  and participate in all the activities of the days in  $S$  so that the total number of activities is maximized. Let  $M_i$  denote the maximum number of activities that you can participate in among the first  $i$  days. Write a recurrence for  $M_i$  or describe how to find it assuming that the values of  $M_1, M_2, \dots, M_{i-1}$  are known.
3. In the previous problem, suppose that you are given the values  $M_1, M_2, \dots, M_n$ . Describe how to find a set  $S$  of days that satisfy the constraints and whose total number of activities is equal to the value of  $M_n$ .