Image

$z = x_1 y_1 + x_2 y_2 + x_3 y_3$

Consider (α, β) as Y is unit norm vector.

$\Rightarrow y_1 = \sin\alpha\cos\beta$

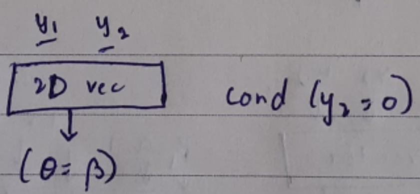
$y_2 = \sin\alpha\sin\beta$

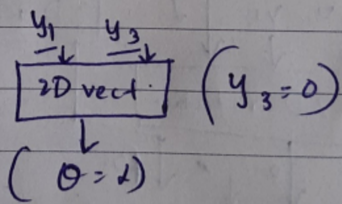
$y_3 = \cos\alpha$

$$z = x_1 \sin\alpha\cos\beta + x_2 \sin\alpha\sin\beta + x_3\cos\alpha$$
$$= (x_1\cos\beta + x_2\sin\beta)\sin\alpha + x_3\cos\alpha$$
$$= [Rot_x(x_1, x_2, \beta)]\sin\alpha + x_3\cos\alpha$$
$$= Rot_x(x_3, Rot_x(x_1, x_2, \beta), \alpha)$$

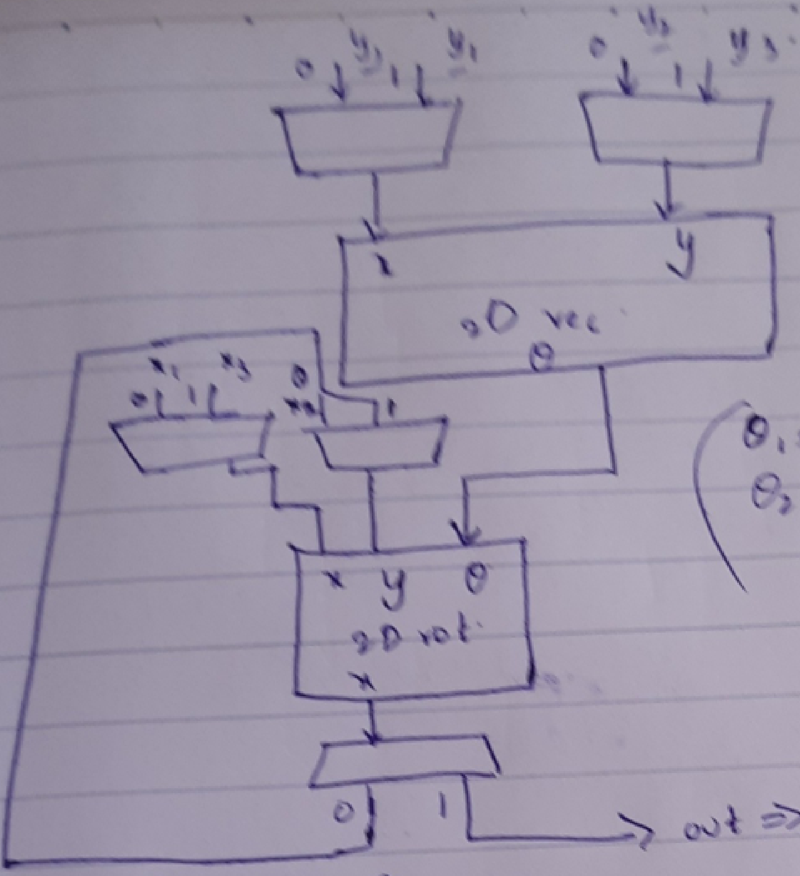
$\bar{Y} = [y_1, y_2, y_3]$.

For β,





$$z = Rot_x[x_3 Rot_x(x_1, x_2, \beta), \alpha]$$



$Rot_x(x_1, x_2, \beta)$

cond 1 $\Rightarrow y_2 = 0$

cos $\theta_2 \Rightarrow y_3 = 0$.

$\left(\begin{matrix} \theta_1 = \beta \\ \theta_2 = \alpha \end{matrix} \right)$

$\rightarrow$ out $\Rightarrow z = Rot_x(x_3, Rot_x(x_1, x_2, \beta), \alpha)$

$= x_1 y_1 + x_2 y_2 + x_3 y_3$