Quantum teleportation using three-particle entanglement

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(Received 13 February 1998)

We investigate the ''teleportation'' of a quantum state using three-particle entanglement to either one of two receivers in such a way that, generally, either one of the two, but only one, can fully reconstruct the quantum state conditioned on the measurement outcome of the other. We furthermore delineate the similarities between this process and a quantum nondemolition measurement. [S1050-2947(98)08812-X]

PACS number(s): 03.67.Hk, 03.65.Bz, 42.50.Dv

I. INTRODUCTION

By quantum teleportation a process is denoted by which the complete information about a quantum state can be sent using a classical transmission of information with the aid of long-range Einstein-Podolsky-Rosen (EPR) correlations [1] in an entangled quantum state [2]. The truly interesting aspect of quantum teleportation is the light it sheds on the nature of classical and quantum information. Experimentally, quantum teleportation was recently demonstrated [3,4] using parametric down-conversion [5], in [3] interferometric Bell-state analyzers [6], and in [4] k-vector entanglement.

Given that teleportation has been demonstrated using two-particle entanglement, and the general interest and quest to demonstrate a three-particle entangled Greenberger-Horne-Zeilinger (GHZ) state [8,9], we may ask the following: What new scheme can be developed using a three-particle entangled state? It is clear that it does not permit by any means the faithful transmission of an unknown quantum state to two locations. This would be forbidden by the no-cloning/broadcast theorems [12,13]. In view of this, we instead find that one may teleport to either of the two locations considered, but not to both. However, there is an interesting midway case where both parties have some information about the original state. This, of course, is very similar to a quantum copier (cloning device) [14–17]. Recently, it was also brought to our attention that a scheme similar to ours had been studied in a more general context by Bruß *et al.* [18]. We will comment on the similarity and difference between their proposal and ours.

The paper is outlined as follows. In Sec. II, we briefly review quantum teleportation using two-particle entanglement. In Sec. III, we present the three-particle entanglement teleportation scheme. In both Secs. II and III, we only consider the case of polarization entanglement. In Sec. IV we discuss the similarities to a quantum nondemolition measurement, and in Sec. V we analyze how much information both receivers have on the state. Finally, in Sec. VI we discuss the results and present some conclusions.

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II. A BRIEF REVIEW OF QUANTUM TELEPORTATION USING TWO-PARTICLE ENTANGLEMENT

Let us begin with a brief review of quantum teleportation using a two-particle polarization entanglement. Quantum teleportation can be accomplished using a two-particle entangled state, such as from a type II parametric down-conversion [5]. The state generated from a type II down-conversion crystal can be written as [5]

$$|\psi_{i,j}\rangle=\frac{1}{\sqrt{2}}(|\updownarrow\rangle_i|\leftrightarrow\rangle_j+e^{i\alpha}|\leftrightarrow\rangle_i|\updownarrow\rangle_j), \tag{2.1}$$

where α is a birefringent phase shift of the crystal, and the subscripts denote particles i and j, respectively. Using appropriate birefringent phase shifts and polarization conversion, one may easily convert the above state into any of the four Bell states [5]:

$$|\psi^{\pm}_{i,j}\rangle=\frac{1}{\sqrt{2}}(|\updownarrow\rangle_i|\leftrightarrow\rangle_j\pm|\leftrightarrow\rangle_i|\updownarrow\rangle_j), \tag{2.2}$$

and

$$|\phi^{\pm}_{i,j}\rangle=\frac{1}{\sqrt{2}}(|\updownarrow\rangle_i|\updownarrow\rangle_j\pm|\leftrightarrow\rangle_i|\leftrightarrow\rangle_j). \tag{2.3}$$

Experimentally, shifting between these states (actually between all four Bell states) has been demonstrated in Bell state analysis [6] and quantum dense coding experiments [7]. By making a shift of basis from a 0° and 90° base $\{|\leftrightarrow\rangle,|\updownarrow\rangle\}$ to a 45° and a 135° polarization base $\{|\nearrow\rangle,|\nwarrow\rangle\}$, the states become $|\phi^+\rangle=(|\nwarrow\rangle|\nwarrow\rangle+|\nearrow\rangle|\nearrow\rangle)/\sqrt{2}$ or $|\phi^-\rangle=(|\nwarrow\rangle|\nearrow\rangle+|\nearrow\rangle|\nwarrow\rangle)/\sqrt{2}$. For the reader who is more versed in spin measurement, we may rewrite the state in terms of spin 1/2 particles putting $|\leftrightarrow\rangle=|z+\rangle,|\updownarrow\rangle=|z-\rangle$. In the same terminology, a 45° and a 135° polarized photon become $|\nearrow\rangle=|x+\rangle$ and $|\nwarrow\rangle=|x-\rangle$, where $|z+\rangle$ denotes a spin eigenstate in the positive z direction, etc. See any quantum mechanics textbook for details.

Suppose now that a person ''Alice'' wants to send a quantum state

$$|\Psi_A\rangle_1=a|\updownarrow\rangle_1+b|\leftrightarrow\rangle_1 \tag{2.4}$$

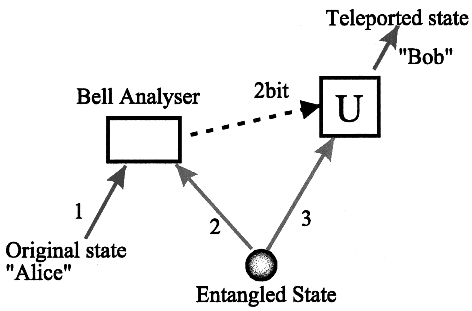

FIG. 1. Schematical picture of a two-particle entanglement teleportation scheme. The box ''U'' denotes the unitary transformation Bob (generally) must perform in order to retrieve the original state.

to a person ''Bob.'' However, for some reason she does not want to send the state itself, which we can assume is fragile, but instead only sends sufficient information for Bob to regenerate the state. To do this, she makes use of a teleporting ''machine'' (as in Fig. 1) for a schematic of the two-particle entanglement teleportation scheme. To initiate the teleportation, the teleporting machine has a source of pure entangled EPR pairs,

$$|\Phi_{23}^{-}\rangle=\frac{1}{\sqrt{2}}(|\updownarrow\rangle_2|\leftrightarrow\rangle_3-|\leftrightarrow\rangle_2|\updownarrow\rangle_3), \tag{2.5}$$

from which one particle (particle 2) is kept, and another particle (particle 3) is sent to the recipient Bob. The joint product state of Alice state and the apparatus state $|\Psi_A\rangle\otimes|\Phi_{23}^{-}\rangle$ can be rewritten in terms of the Bell states for particles 1 and 2 as

$$\begin{aligned}|\Psi_A\rangle\otimes|\Phi_{23}^{-}\rangle=\frac{1}{2}[&|\phi_{12}^{+}\rangle\otimes(a|\leftrightarrow\rangle_3-b|\updownarrow\rangle_3)\\&+|\phi_{12}^{-}\rangle\otimes(a|\leftrightarrow\rangle_3+b|\updownarrow\rangle_3)\\&+|\psi_{12}^{+}\rangle\otimes(-a|\updownarrow\rangle_3+b|\leftrightarrow\rangle_3)\\&+|\psi_{12}^{-}\rangle\otimes(-a|\updownarrow\rangle_3-b|\leftrightarrow\rangle_3)].\end{aligned} \tag{2.6}$$

To swap the information, that is, the (a,b) coefficients of the state from particle 1 to particle 3, Alice uses a Bell state analyzer to measure particle 1 and particle 2. Suppose she obtains the result $|\psi_{12}^{-}\rangle$ and then we immediately see that the conditioned state of particle 3 at the recipient Bob becomes

$$|\Psi_{Bob}\rangle=a|\updownarrow\rangle_3+b|\leftrightarrow\rangle_3, \tag{2.7}$$

that is, the state $|\Psi_A\rangle$ has been ''teleported'' from Alice to Bob. Generally, as seen from Eq. (2.6), Alice may obtain one of four outcomes, and Bob will not obtain the desired state directly, but must perform a (simple) unitary operation to retrieve the state [2]. To know which operation to perform, Alice must send a classical (two-bit) message to Bob. This, of course, is why quantum teleportation does not violate causality. Experimentally, quantum teleportation has recently been demonstrated using an interferometric Bell state analyzer, which may distinguish two out of four Bell states, with the other two states giving the same result [3,6].

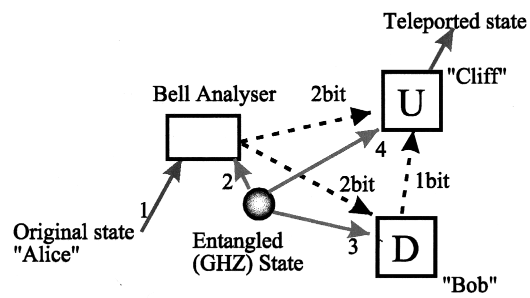

FIG. 2. Schematical picture of a three-particle entanglement teleportation scheme. In this case, Cliff must perform a unitary transform, i.e., the box U, and Bob does a von Neumann measurement, i.e., the box ''D.''

III. THREE-PARTICLE ENTANGLEMENT TELEPORTATION

Let us now see what three-particle entanglement (in polarization here) can add to the problem. Suppose again that the unknown quantum state we seek to teleport is

$$|\Psi_A\rangle=a|\updownarrow\rangle_1+b|\leftrightarrow\rangle_1. \tag{3.1}$$

To perform the teleportation, Alice now uses a three-particle entangled state (a GHZ state) [8,9] as

$$|\psi_{GHZ}\rangle=\frac{1}{\sqrt{2}}(|\updownarrow\rangle_2|\updownarrow\rangle_3|\updownarrow\rangle_4+|\leftrightarrow\rangle_2|\leftrightarrow\rangle_3|\leftrightarrow\rangle_4). \tag{3.2}$$

The above state can be generated in the laboratory using, for instance, entanglement swapping starting from three down-converters [10] or as very recently demonstrated experimentally using two pairs of entangled photons [11]. In [18] a more general three-particle entangled state of the form

$$\begin{aligned}|\psi_B\rangle=&\sqrt{\frac{2}{3}}|\updownarrow\rangle_2|\leftrightarrow\rangle_3|\leftrightarrow\rangle_4-\sqrt{\frac{1}{6}}|\leftrightarrow\rangle_2|\updownarrow\rangle_3|\leftrightarrow\rangle_4\\&-\sqrt{\frac{1}{6}}|\leftrightarrow\rangle_2|\leftrightarrow\rangle_3|\updownarrow\rangle_4\end{aligned} \tag{3.3}$$

is used in order to reach the optimal condition of the ''universal quantum cloning machine'' [14–17]. However, for reasons to be specified below, here we choose to keep a simple, equally weighted state. To set up the teleportation, particle 2 is kept by Alice, while particles 3 and 4 have been sent to some remote locations, which we denote by ''Bob'' and ''Cliff.'' See Fig. 2 for a schematic of the three-particle entanglement teleportation scheme.

Using the decomposition into Bell states as before, we can rewrite the initial product state $|\Psi_A\rangle\otimes|\psi_{GHZ}\rangle$ as

$$\begin{aligned}|\Psi_A\rangle\otimes|\psi_{GHZ}\rangle=\frac{1}{2}[&|\phi_{12}^{+}\rangle\otimes(a|\updownarrow\rangle_3|\updownarrow\rangle_4+b|\leftrightarrow\rangle_3|\leftrightarrow\rangle_4)\\&+|\phi_{12}^{-}\rangle\otimes(a|\updownarrow\rangle_3|\updownarrow\rangle_4-b|\leftrightarrow\rangle_3|\leftrightarrow\rangle_4)\\&+|\psi_{12}^{+}\rangle\otimes(a|\updownarrow\rangle_3|\leftrightarrow\rangle_4+b|\leftrightarrow\rangle_3|\updownarrow\rangle_4)\\&+|\psi_{12}^{-}\rangle\otimes(a|\updownarrow\rangle_3|\leftrightarrow\rangle_4-b|\leftrightarrow\rangle_3|\updownarrow\rangle_4)].\end{aligned} \tag{3.4}$$

A measurement using Bell state analyzers on particles 1 and 2 will project the state of particles 3 and 4 onto the joint, generally entangled states seen in the equation above. Let us analyze here the case in which the Bell state analyzers give the readout $|\phi_{12}^{+}\rangle$, which occurs with a probability 1/4. All of the other cases can be treated in a similar fashion as was the case of the two-particle teleportation scheme. The state of particles 3 and 4 becomes

$$|\psi_{34}\rangle = a|\updownarrow\rangle_3|\updownarrow\rangle_4 + b|\leftrightarrow\rangle_3|\leftrightarrow\rangle_4. \tag{3.5}$$

Let us now see how the original state can be reconstructed. Suppose we seek to reconstruct the state at location 4 ''Cliff'' with the help of ''Bob'' at location 3. If Bob uses a von Neumann measurement, for instance, a linear polarization (or generally spin-state) analyzer with two outcomes x_1 or x_2, we can decompose the incoming states in the new basis $\{|x_1\rangle,|x_2\rangle\}$ as

$$|\updownarrow\rangle_3 = \sin\,\theta|x_1\rangle_3 + \cos\,\theta|x_2\rangle_3,$$

$$|\leftrightarrow\rangle_3 = \cos\,\theta|x_1\rangle_3 - \sin\,\theta|x_2\rangle_3, \tag{3.6}$$

where θ describes the analyzer angle. Rewriting the output state in the new basis $\{|x_1\rangle,|x_2\rangle\}$ gives

$$|\psi_{34}\rangle = (a\ \sin\ \theta|\updownarrow\rangle_4 + b\ \cos\ \theta|\leftrightarrow\rangle_4)|x_1\rangle_3 + (a\ \cos\ \theta|\updownarrow\rangle_4 - b\ \sin\ \theta|\leftrightarrow\rangle_4)|x_2\rangle_3. \tag{3.7}$$

We note from this expression that θ can be used as a parameter to nonlocally affect the state of particle 4. This is not really surprising, since the same ''action at a distance'' is inherent in any entangled state. The difference here is that the coefficients a and b are unknown to Bob at location 3.

We may achieve a similar weighting also in quantum teleportation using two-particle entanglement. Suppose we define a more general set of Bell states as

$$|\psi_{\theta}^{+}\rangle = \cos\ \theta|\updownarrow\rangle_i|\leftrightarrow\rangle_j + \sin\ \theta|\leftrightarrow\rangle_i|\updownarrow\rangle_j,$$

$$|\psi_{\theta}^{-}\rangle = -\sin\ \theta|\updownarrow\rangle_i|\leftrightarrow\rangle_j + \cos\ \theta|\leftrightarrow\rangle_i|\updownarrow\rangle_j, \tag{3.8}$$

and

$$|\phi_{\theta}^{+}\rangle = \cos\ \theta\ |\updownarrow\rangle_i|\updownarrow\rangle_j + \sin\ \theta|\leftrightarrow\rangle_i|\leftrightarrow\rangle_j$$

$$|\phi_{\theta}^{-}\rangle = -\sin\ \theta|\updownarrow\rangle_i|\updownarrow\rangle_j + \cos\ \theta|\leftrightarrow\rangle_i|\leftrightarrow\rangle_j. \tag{3.9}$$

From this it is easy to show that for the two-particle teleportation scheme one may project, or remotely prepare, the state

$$|\psi_{3,out}\rangle = \frac{a\ \cos\ \theta|\updownarrow\rangle_3 + b\ \sin\ \theta|\leftrightarrow\rangle_3}{\sqrt{|a|^2\cos^2\theta + |b|^2\sin^2\theta}}. \tag{3.10}$$

Returning now to the three-particle teleportation scheme, if the outcome is x_1 and we choose $\theta = +\pi/4$, the state of particle 4 becomes

$$|\psi_4\rangle = a|\updownarrow\rangle_4 + b|\leftrightarrow\rangle_4. \tag{3.11}$$

That is, we have again successfully ''teleported'' the state from particle 1 to particle 4. If the outcome is x_2, Cliff can simply flip the $|\leftrightarrow\rangle_4$ state by a factor of π to retrieve the desired state. Note that Cliff must receive a one-bit message from Bob, telling him which outcome occurred. In a similar way, Cliff must phase shift the states, when the outcomes of the Bell state analyzer at Alice's location give another readout other than $|\phi_{12}^{+}\rangle$. Of course, regardless of the outcome, the teleportation is feasible. We like to stress that this ability for Bob and Cliff to decide to exactly recreate the state at one of the two locations has, to our knowledge, not been pointed out before. This may also be possible to do using the state $|\psi_B\rangle$ of Eq. (3.3) as well. However, we think the choice of the $|\psi_{GHZ}\rangle$ makes the projection particularly simple. It should also be noted that teleporting to two locations could be done by first ''copying'' the state and then two ordinary quantum teleportations to the two location. Again, however, unless one has control of the ancilla state of the quantum cloner [14–17], it is not possible to select if one of the two receivers should receive the state perfectly or not.

IV. COMPARISON TO QUANTUM-NONDEMOLITION MEASUREMENTS

Let us elaborate a bit on the role of the angle θ in the measurement, and on the similarities between the conditioned teleportation, quantum gates [19], and quantum nondemolition (QND) measurements [20]. Suppose we start with a product state

$$|\psi_{34,in}\rangle = (a|\updownarrow\rangle_3 + b|\leftrightarrow\rangle_3)\otimes|\leftrightarrow\rangle_4. \tag{4.1}$$

If we now apply a quantum controlled-not gate with the rule that qubit 4 is flipped $|\leftrightarrow\rangle_4$ to $|\updownarrow\rangle_4$ if bit 3 is $|\updownarrow\rangle_3$, otherwise it is left untouched [18]. Applying this to the state $|\psi_{34,in}\rangle$ of Eq. (4.1) gives the output

$$|\psi_{34,out}\rangle = a|\updownarrow\rangle_3|\updownarrow\rangle_4 + b|\leftrightarrow\rangle_3|\leftrightarrow\rangle_4, \tag{4.2}$$

which is exactly the entangled state of the teleportation. To see how this relates to a QND measurement, suppose that the measurement of particle 3 is made with the angle setting $\theta = 0$. Then with a probability $|a|^2$ the state is projected onto $|\updownarrow\rangle_4$, and with a probability $|b|^2$ the state is projected onto $|\leftrightarrow\rangle_4$. Thus, for $(a,b)=(0,1)$ or $(a,b)=(1,0)$, the meter 3 perfectly measures the state without destroying it, as required for a QND measurement. We can also view θ as selecting a preferred base for the measurement, or the probabilities by which $|\updownarrow\rangle_3$ is projected onto $|x_1\rangle_3$ or $|x_2\rangle_3$ (and similarly for $|\updownarrow\rangle_4$). For $\theta = n\pi + \pi/4$, the probabilities of the projection onto $|x_1\rangle_3$ or $|x_2\rangle_3$ are equal $(=1/2)$, and therefore the original superposition of states is kept.

Let us finish this section by noting that by having the GHZ state, and an interferometric Bell state analyzer, which may distinguish two out of four Bell states and gives identical results for the other two, one may mimic a QND measurement on the single-photon level using only linear photon manipulations. A QND measurement would require nonlinear (Kerr-type) optical phase shifts of π on the single-photon level, nearly, but not yet in reach at present [21].

V. FIDELITY MEASURES

Let us now assess how well one will generally succeed in teleporting the state. We may view the teleporter as trying to implement a quantum cloning machine [14–18], albeit not an optimal one. Suppose that the ideal target state is Alice's state

$$|\Psi_T\rangle_i = a|\updownarrow\rangle_i + b|\leftrightarrow\rangle_i, \tag{5.1}$$

where $i\in\{3,4\}$. The reduced density operator of either of the outputs by itself, not using the information in the other measurements, is

$$\hat{\rho}_{3,nc} = \mathrm{Tr}_4\{|\psi_{34,out}\rangle\langle\psi_{34,out}|\} = |a|^2|\updownarrow\rangle_{33}\langle\updownarrow| + |b|^2|\leftrightarrow\rangle_{33}\langle\leftrightarrow|, \tag{5.2}$$

and

$$\hat{\rho}_{4,nc} = \mathrm{Tr}_3\{|\psi_{34,out}\rangle\langle\psi_{34,out}|\} = |a|^2|\updownarrow\rangle_{44}\langle\updownarrow| + |b|^2|\leftrightarrow\rangle_{44}\langle\leftrightarrow|. \tag{5.3}$$

As a criterion for successful copying, we compute the fidelity [14,15], which is the probability of the received state to pass as the desired state $|\Psi_T\rangle_x$. In the present case, the nonconditioned fidelity may be written as

$$\mathcal{F}_{3,nc} = \mathcal{F}_{4,nc} = {}_3\langle\psi_T|\hat{\rho}_{3,nc}|\psi_T\rangle_3 = |a|^4 + |b|^4. \tag{5.4}$$

The fidelity varies between $\mathcal{F}_{3,nc}=\mathcal{F}_{4,nc}=1/2$ for $|a|^2=|b|^2=1/2$, up to $\mathcal{F}_{3,nc}=\mathcal{F}_{4,nc}=1$ for $(|a|,|b|)=(0,1)$ or $(1,0)$. It should be noted that a fidelity $\mathcal{F}=1/2$ corresponds to a random result. To see this, suppose Cliff or Bob regenerates the state $\hat{\rho}_{nc}=1/2|\updownarrow\rangle\langle\updownarrow|+1/2|\leftrightarrow\rangle\langle\leftrightarrow|$, the fidelity of this state is $\mathcal{F}=1/2$, but no use was made of a and b.

In order to compute the fidelity averaged over all input states (assuming all states appear equally often), we may parametrize (a,b) in polar coordinates as

$$|\Psi_T\rangle_x = a|\updownarrow\rangle_x + b|\leftrightarrow\rangle_x = \cos(\vartheta/2)|\updownarrow\rangle_x + e^{i\phi}\sin(\vartheta/2)|\leftrightarrow\rangle_x, \tag{5.5}$$

and compute an averaged fidelity

$$\overline{\mathcal{F}} \equiv \int_0^{2\pi} d\phi \int_0^{\pi} \mathcal{F}\sin\vartheta\, d\vartheta/4\pi. \tag{5.6}$$

Computing the value of the averaged nonconditioned fidelity gives $\overline{\mathcal{F}}_{3,nc}=\overline{\mathcal{F}}_{4,nc}=2/3\approx 0.67$. Actually, as shown in [22], $\overline{\mathcal{F}}=2/3$ is the limit for a ''classical'' teleportation device, i.e., where a single input state is measured at Alice site, and the result of the measurement is sent over a classical channel to Bob and Cliff who, in turn, try to reconstruct the state. We may compare our result to the result for the $1\leftrightarrow M$ cloning device investigated by Gisin and Massar [15] and Bruß *et al.* [18], where it was shown that the optimum average fidelity for a cloning device is $\mathcal{F}_{opt}=(2M+1)/3M=5/6\approx 0.83$ for $M=2$. Note that the fidelity approaches the classical limit $\mathcal{F}_{opt}\to 2/3$, when $M\to\infty$. At first sight, it would appear that nothing is gained from ''our'' three-particle entanglement using the $|\psi_{GHZ}\rangle$ state. However, this is not so, as will be shown below.

What about the fidelity of the conditioned state? Using the conditioned outcomes we expect that Cliff at 4 is able to succeed much better than without this information. Suppose that we look at the fidelity conditioned on detecting the eigenvalue x_1 of detector 3. The conditioned state for the output becomes

$$|\psi_c^1\rangle_4 = \frac{a\sin\theta|\updownarrow\rangle_4 + b\cos\theta|\leftrightarrow\rangle_4}{\sqrt{|a|^2\sin^2\theta + |b|^2\cos^2\theta}}. \tag{5.7}$$

The fidelity becomes

$$\mathcal{F}_{c,4}^1 = |{}_4\langle\psi_T|\psi_c^1\rangle_4|^2 = \frac{(|a|^2\sin\theta + |b|^2\cos\theta)^2}{|a|^2\sin^2\theta + |b|^2\cos^2\theta}. \tag{5.8}$$

Likewise, if the eigenvalue x_2 is obtained, the conditioned state, *after a phase flip of* π for the $|\leftrightarrow\rangle_4$ state, becomes

$$|\psi_c^2\rangle_4 = \frac{a\cos\theta|\updownarrow\rangle_4 + b\sin\theta|\leftrightarrow\rangle_4}{\sqrt{|a|^2\cos^2\theta + |b|^2\sin^2\theta}}, \tag{5.9}$$

giving the fidelity

$$\mathcal{F}_{c,4}^2 = |{}_4\langle\psi_T|\psi_c^2\rangle_4|^2 = \frac{(|a|^2\cos\theta + |b|^2\sin\theta)^2}{|a|^2\cos^2\theta + |b|^2\sin^2\theta}. \tag{5.10}$$

The fidelity averaged over both outcomes (eigenvalues) x_1 and x_2 can be written

$$\mathcal{F}_{ave,4} = P(x_1)\mathcal{F}_{c,4}^1 + P(x_2)\mathcal{F}_{c,4}^2 = \cdots = |a|^4 + |b|^4 + 4|a|^2|b|^2\cos\theta\sin\theta, \tag{5.11}$$

where we have used the probabilities for the x_1 and x_2 outcomes $P(x_1)=|a|^2\sin^2\theta+|b|^2\cos^2\theta$, and $P(x_2)=|a|^2\cos^2\theta+|b|^2\sin^2\theta$. Plugging in $\theta=\pi/4$, we find an averaged fidelity of $\mathcal{F}_{ave,4}=1$. Computing the fidelity averaged over all input states gives

$$\overline{\mathcal{F}}_{ave,4} = \frac{2}{3} + \frac{1}{3}\sin(2\theta), \tag{5.12}$$

which varies between $\overline{\mathcal{F}}_{ave,4}=2/3$ for $\theta=0$ and $\theta=\pi/2$, and $\overline{\mathcal{F}}_{ave,4}=1$ for $\theta=\pi/4$.

What about the fidelity of Bob for particle 3? A reasonable assumption for Bob is to assume that the state $|x_1\rangle_3$ was the sent state as well. As shown in [22], this is actually the optimal guess given only a single input particle. This would correspond to a fidelity

$$\mathcal{F}_{c,3}^1 = |{}_3\langle\psi_T|x_1\rangle_3|^2 = |(a\sin\theta + b\cos\theta)|^2. \tag{5.13}$$

For a click in the $|x_2\rangle_3$ detector, we may either assume that the state was $|x_2\rangle_3$ or, parallel with the discussion above, that the state was $|x_2'\rangle_3=\cos\theta|\updownarrow\rangle_3+\sin\theta|\leftrightarrow\rangle_3$.

Therefore, suppose now that the $|x_2'\rangle_3$ is chosen. The fidelity becomes

$$\mathcal{F}_{c,3}^2 = |{}_3\langle\psi_T|x_2'\rangle_3|^2 = |(a\cos\theta + b\sin\theta)|^2. \tag{5.14}$$

The average fidelity for both outcomes x_1 and x_2 can be writen

$$\mathcal{F}_{ave,3} = P(x_1)\mathcal{F}_{c,3}^1 + P(x_2)\mathcal{F}_{c,3}^2. \tag{5.15}$$

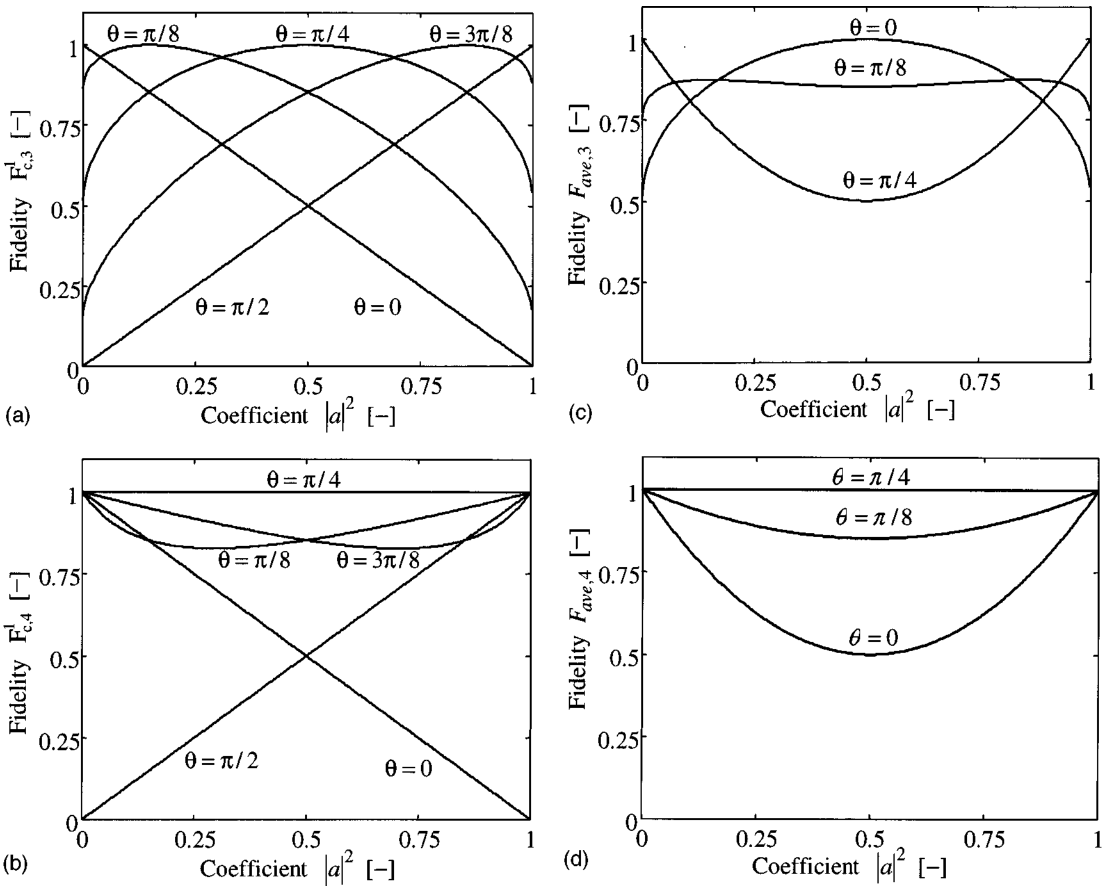

FIG. 3. Fidelity is plotted as a function of $|a|^2$ (a real), in (a) for $\mathcal{F}^1_{c,3}$, (b) for $\mathcal{F}^1_{c,4}$, (c) for $\mathcal{F}_{ave,3}$, and (d) for $\mathcal{F}_{ave,4}$. In all figures the setting θ is used as a parameter.

Computing the average for over all (a,b) coefficients, however, Bob faces some problems. Suppose he only looks at the outcomes in x_2. Then, since there is no information on (a,b) in $|x_2\rangle_3$ itself, it is easily shown that $\overline{\mathcal{F}}^1_{c,3}=\overline{\mathcal{F}}^2_{c,3}=1/2$, which appears to be a completely ''no win'' situation for Bob. If, however, one takes into account that the probabilities $P(x_1)$ and $P(x_2)$ depend on (a,b), it is realized that the average fidelity $\overline{\mathcal{F}}_{ave,3}$ will be larger than 1/2. Performing the average gives

$$\overline{\mathcal{F}}_{ave,3}=\frac{2}{3}-\frac{1}{6}\sin^2(2\theta), \tag{5.16}$$

which varies between $\overline{\mathcal{F}}_{ave,3}=2/3$ for $\theta=0$ and $\theta=\pi/2$, and $\overline{\mathcal{F}}_{ave,3}=1/2$ for $\theta=\pi/4$, which was expected since $\overline{\mathcal{F}}_{ave,4}=1$ for $\theta=\pi/4$. As was mentioned, the fidelity $\overline{\mathcal{F}}_{ave,3}=2/3$ is the optimum value for estimating the polarization state, given only one particle [21].

In Fig. 3, the fidelity is plotted as a function of $|a|^2$ (a real) in (a) for $\mathcal{F}^1_{c,3}$, (b) for $\mathcal{F}^1_{c,4}$, (c) for $\mathcal{F}_{ave,3}$, and (d) for $\mathcal{F}_{ave,4}$. As can be seen, when the fidelity for Cliff is ideal $\mathcal{F}^1_{c,4}=1$, the fidelity for Bob is $\mathcal{F}^1_{c,3}=1/2$. Note also that $\mathcal{F}^2_{c,3}$ follows from $\mathcal{F}^1_{c,3}$ by replacing $\theta\leftrightarrow\pi/2-\theta$, and similarly for $\mathcal{F}^2_{c,4}$. However, as seen for some angles, both Bob and Cliff can get a substantial amount of information on the state. This is a (trivial) example of state-dependent cloning [18]. Due to the choice of measurement basis, the teleportation here works very well for states with real and positive (a,b) coefficients.

The most interesting and generally most useful quantity, however, is the fidelity averaged over all (a,b) coefficients. This is shown in Fig. 4, where the average fidelity is plotted as a function of θ, in (a) for $\overline{\mathcal{F}}^1_{c,3}$ and $\overline{\mathcal{F}}^2_{c,2}$, and (b) for $\overline{\mathcal{F}}_{ave,3}$ and $\overline{\mathcal{F}}_{ave,4}$. The maximum of the average of both fidelities $(\overline{\mathcal{F}}_{ave,3}+\overline{\mathcal{F}}_{ave,4})/2=3/4\approx0.75$ (which by itself is perhaps not so meaningful), is pretty close to the optimum $\overline{\mathcal{F}}\approx0.83$ for the optimal cloners [14,15,18]. The fact that we may get $\overline{\mathcal{F}}_{ave,4}$ well above 2/3, while keeping $\overline{\mathcal{F}}_{ave,3}$ reasonably high, shows the improvement using the three-particle entanglement for the ''teleportation'' instead of a ''classical'' scheme. It should be emphasized, however, that the joint mutual information from Bob and Cliff, about the original state $|\Psi_A\rangle$ should not, in our scheme, nor for the cloning machine [14,15,18], exceed that which is available by an optimal measurement at Alice's site [22].

VI. DISCUSSION

We have shown how ''quantum teleportation'' to one of two locations can be realized using three-particle entanglement. The teleportation to more than one location is also related to quantum copying [14,15,17]. However, unlike in-the ''universal quantum copying machine,'' where the fidel-

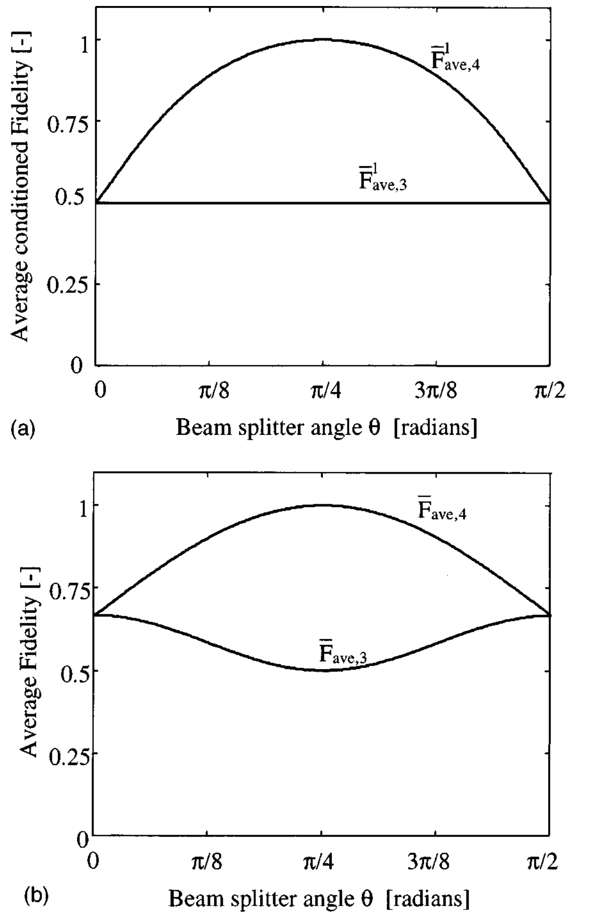

FIG. 4. Fidelity averaged over all (a,b) coefficients is plotted as a function of θ, in (a) for $\overline{\mathcal{F}}^{\,1}_{c,1}$ and $\overline{\mathcal{F}}^{\,1}_{c,4}$ and (b) for $\overline{\mathcal{F}}_{ave,3}$ and $\overline{\mathcal{F}}_{ave,4}$.

ity of the output is state independent [14,17,18], in the present case, the fidelity is state dependent, as well as requiring an additional classical transmission between the copying sites. Nevertheless, in the teleportation scheme, a previously shared entanglement is used to convey the quantum state, and only classical information is transmitted at the moment of teleportation. As mentioned, a similar, but more elaborate and perhaps more clever scheme by Bruß *et al.* [18] was brought to our attention at the end of this work. In their work, a weighted three-particle entangled state was used to construct the universal quantum cloning machine of Bužek and Hillery [16]. Here, a less ambitious goal was sought using an equally weighted entangled state and a simpler measurement strategy. Still, the present scheme permits an average fidelity that is rather close to the optimum one. Also, what should be stressed is that our strategy allows for a simple way to get a perfect reception of the state at either Bob or Cliff after the Bell measurement has been done, but of course before the state is ''collapsed'' at Bob or Cliff. We believe it would be interesting to try to improve our scheme toward higher fidelities, while also retaining the possibility of exactly reproducing the original state at one of the two sites by simple means.

It should furthermore be stressed that the quantum cloners in [14–18] operate using only unitary transforms. Here, as well as in parts of the teleportation scheme considered in [18], a combination of simple unitary transformation, nonunitary evolution (detection), and conditioning is used. This in turn implies that if a nonperfect success rate is acceptable, only linear photon manipulation is needed. We believe that this tradeoff between the success of conditioning and the complexity of the nonunitary interactions needed would be of interest to study further. In terms of experimental implementations, besides the ability to generate a GHZ state, the scheme requires the detection of correlations between four detectors. With a nonunity detection quantum efficiency, this gives a low overall efficiency of the scheme. This problem, however, is inherent in any quantum information scheme requiring many-particle manipulation.

ACKNOWLEDGMENTS

We would like to acknowledge Gunnar Björk and Jonas Söderholm of the Royal Institute of Technology (KTH), and Henrik Carlsen of Quantum Chemistry, Uppsala University, for useful comments. This work was supported by the Swedish Natural Science Research Council (NFR) and the Swedish Technical Science Research Council (TFR).

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