Matching algorithms (Cont...)

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Blocking set of augmenting paths

- If G is a graph (bipartite or not) and M is a maximum matching, a ***blocking set of augmenting paths*** with respect to M is a set $\{P_1, P_2, \ldots, P_k\}$ of augmenting paths such that
  1. the paths $P_1, P_2, \ldots, P_k$ are vertex disjoint paths
  2. all the paths have the same length, say l
  3. l is the minimum length of an M-augmenting path
  4. every augmenting path of length l has at least one vertex in common with $P_1 \cup P_2 \cup \cdots \cup P_k$
- In other words, a blocking set of augmenting paths is a (set wise) ***maximal*** **collection of vertex-disjoint minimum-length augmenting paths**

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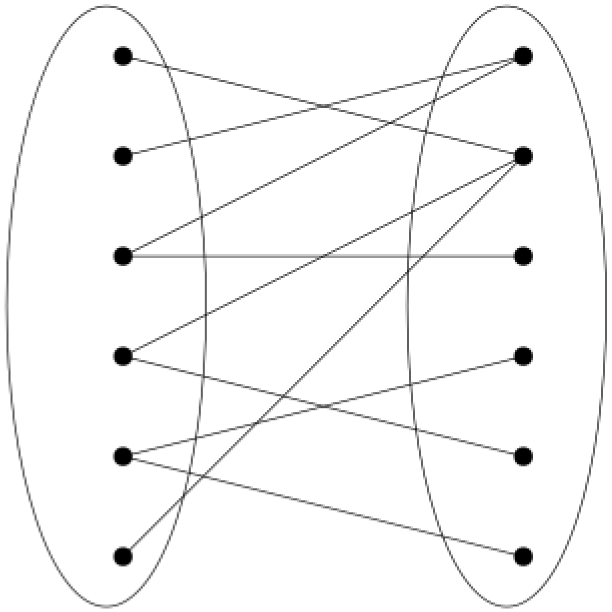

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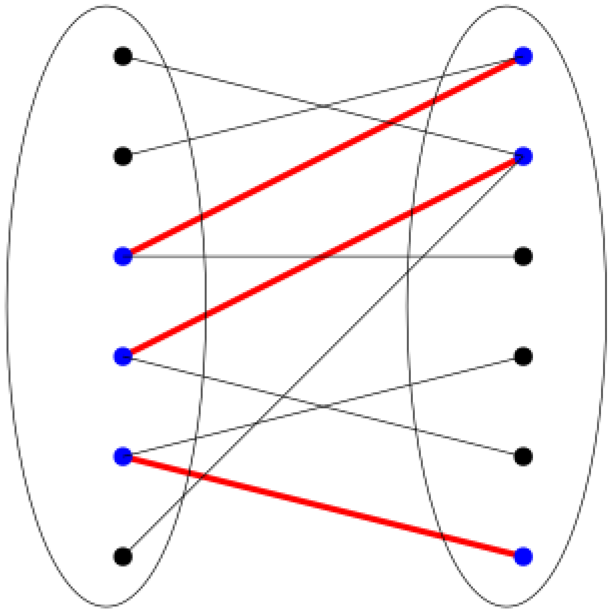

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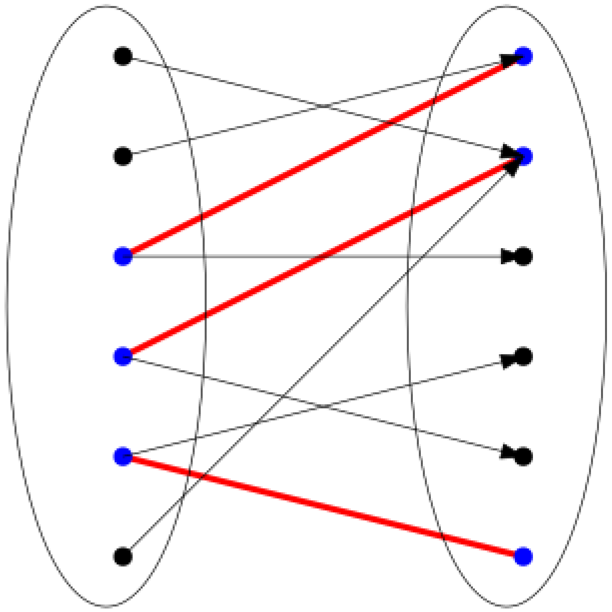

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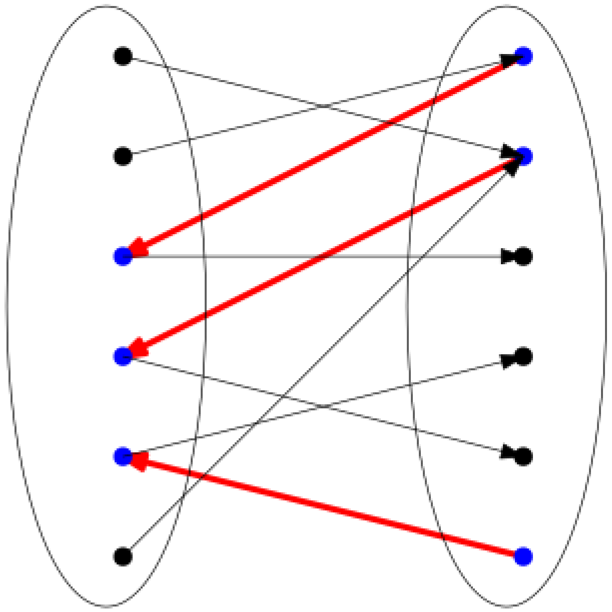

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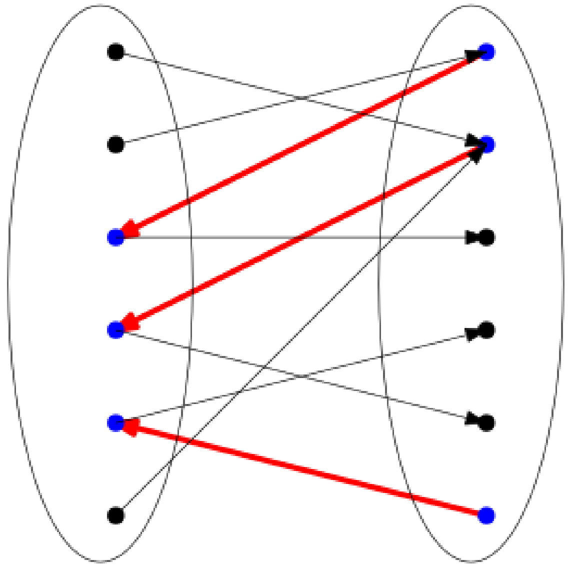

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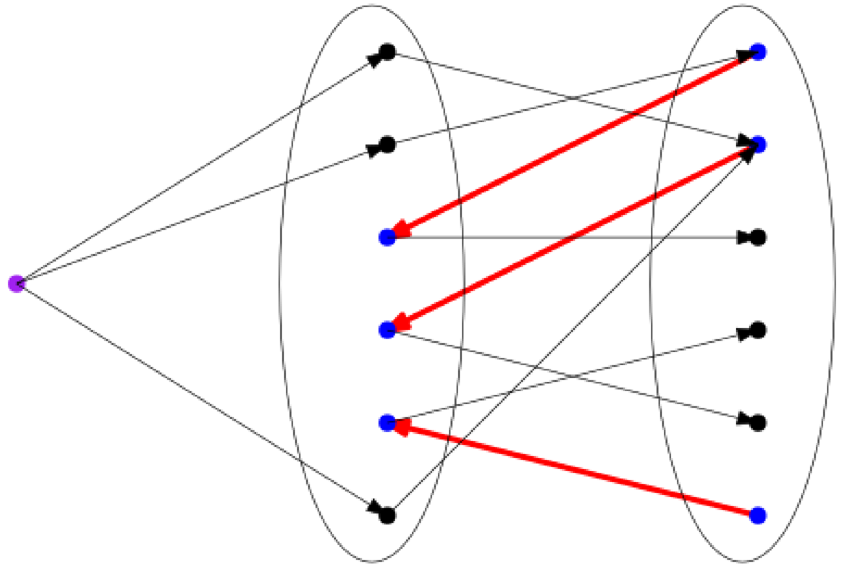


Non-bipartite matching

- The algorithms discussed for bipartite graphs can be extended to non-bipartite graphs
- Unfortunately, the lack of bipartite structure makes the task of finding augmenting paths difficult
- In bipartite case, every augmenting path starts at a vertex from the left partition and ends at a vertex in the right partition
- This is not the case with non-bipartite graphs
- Non-bipartite graphs contain odd cycles

Existing algorithms

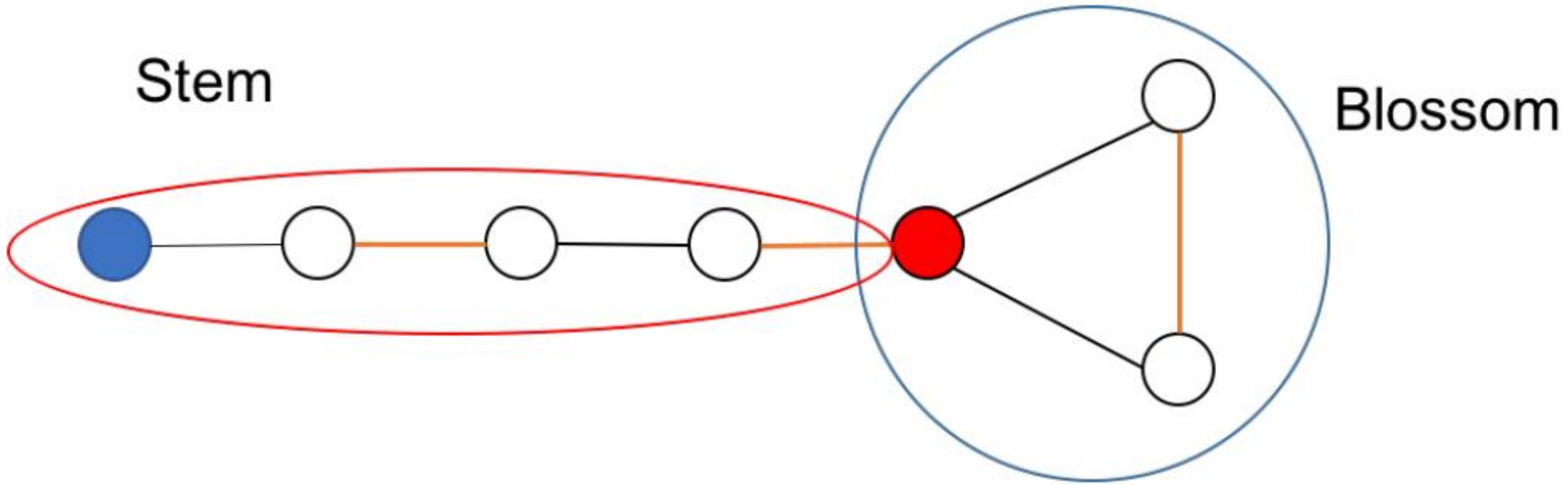
- The first algorithm was proposed by Edmonds in 1965 which runs in $O(n^4)$
- After that a lot of improvements have been added to his algorithm with the following running times:
  - $O(n^3)$ (Gabow, 1976)
  - $O(nm)$ (Kameda and Munro, 1974)
  - $O(n^{2.5})$ (Even and Kariv, 1975)
  - $O(\sqrt{n}m)$ (Micali and Vazirani, 1980)

Basic definitions

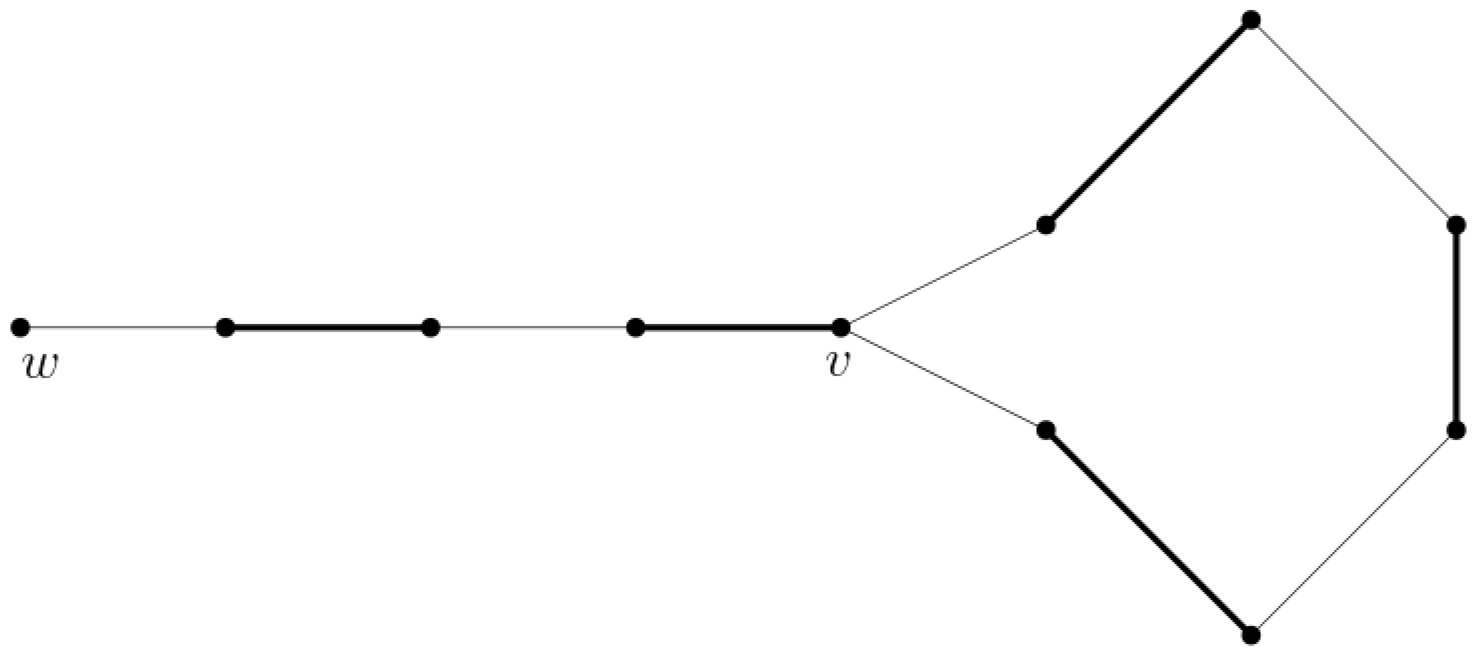
- Given a graph $G = (V, E)$ and a matching M of G

  1. A ***stem*** with respect to M is an alternating path of even length from an unmatched vertex v (called the ***root*** of the stem)
     - Observation: The last edge on the stem belongs to M
  2. A ***blossom*** B with respect to M is a
     I. a cycle in G consisting of $2k + 1$ edges
     II. exactly k edges of the cycle belong to M, and
     III. one of the vertices v of the cycle (we call it the ***base***) is such that there exists a stem from an unmatched vertex w to v
        - Observation: The two edges of the blossom touching the base are not in M. Other than that, every second edge on the blossom belongs to M

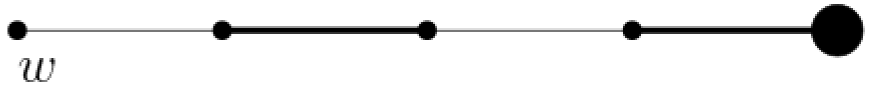
Example 1



Example 2



Cycle shrinking



- Thank you!