**Indian Institute of Information Technology Raichur**
**Assignment – 1**

**Course Title**: Combinatorial Optimization
**Instructor**: Dr. Ramesh K. Jallu
**Course Code**: CS331
**Total Marks**: 20

**Class**: CSB19
**Semester**: 6
**Date**: February 14, 2022
**Due date**: February 21, 2022, 5:00 PM

1. Solve all the questions.
2. Feel free to discuss with me if you do not understand any question.
3. Do not discuss with others and do not dig internet for solutions. Any means of cheating is not acceptable and if found you will be awarded 0 marks.
4. Assignments submitted after the due date will not be considered.
5. Please let me know if there are any typographical errors.

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1. Consider the algorithm discussed in class for finding a matching in a given graph $G = (V, E)$:

   a. Start with any arbitrary edge as the initial matching
   b. Find another edge that doesn't have a vertex common with the current matching. If one exists, add it to the current matching
   c. Repeat step b till no more edges can be added
   d. Return the matching

   Answer the following:

   (i) Does the algorithm always find a maximum matching? If not, give a counter example. **2 Marks**

   (ii) Is it true that the matching returned by the algorithm always has at least half as many edges as a maximum matching? That is, if $M$ is a matching returned by the algorithm, and $M'$ is a *maximum* matching, then is it true that $|M| \geq \frac{|M'|}{2}$? Argue your claim with a proper justification. **3 Marks**

2. Consider the game of placing dominoes on a checkerboard[1]. The goal is to tile the given checkerboard with dominoes. You can assume that you are given abundant number of dominoes. A tiling is said to be a *perfect tiling* if no square on the checker board left uncovered with dominoes (see Figure 1).

[1]A checker board is a collection of unit squares, and a domino is two adjacent squares sharing a common side. Hence, a domino can cover two adjacent cells

(i) Show that the game can be formulated as a matching problem in bipartite graphs. **2 Marks**

(ii) Argue that the given checkerboard has a perfect tiling if and only if the bipartite graph constructed in (i) has a perfect matching. **3 Marks**

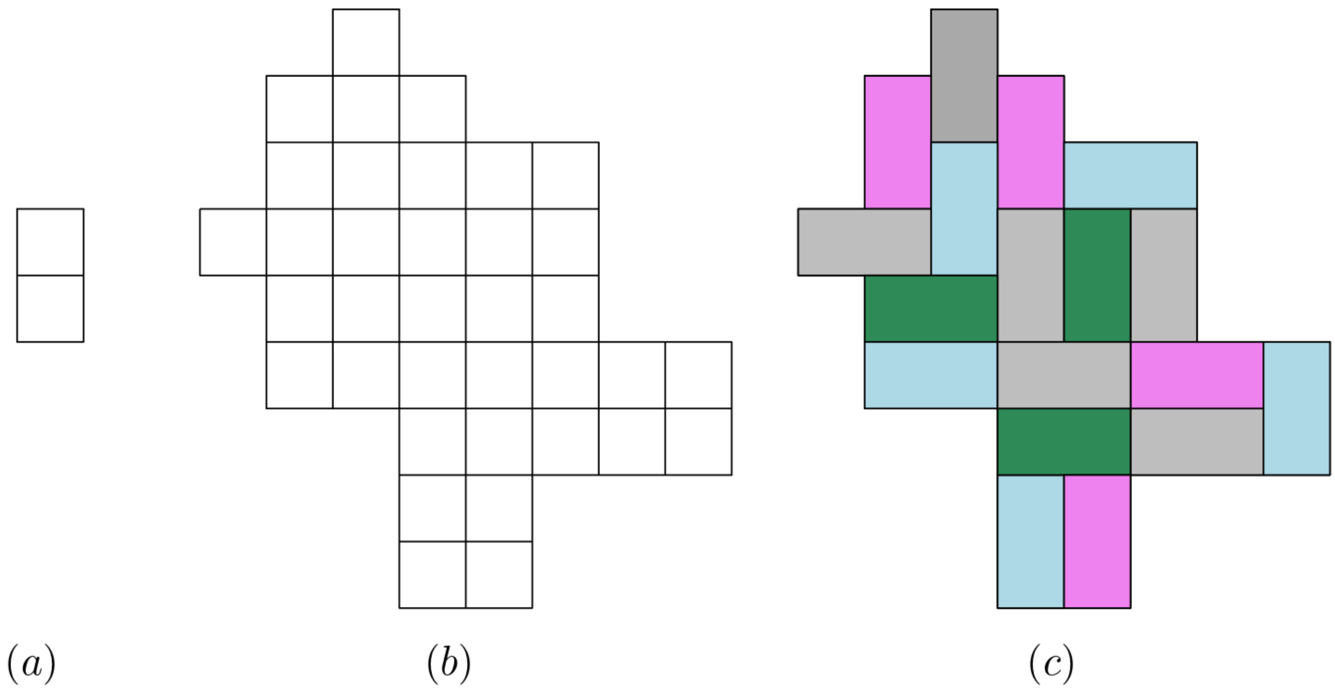

Figure 1: (a) A domino, (b) A checkerboard, and (c) a perfect tiling (I have used colors to show the tiling with dominoes)

3. Let $G = (V, E)$ be a graph with no isolated vertices. Let $C$ be a vertex cover[2] of minimum size. Also, let $M$ be a maximum matching in $G$. What is the relation between the cardinalities (sizes) of $C$ and $M$? **3 Marks**

4. Let $G = (V, E)$ be a flow network. Let $f_1$ and $f_2$ be two distinct flows in the network. Consider $f = \alpha f_1 + (1 - \alpha) f_2$ for some $\alpha \in [0, 1]$. Test whether $f$ is indeed a flow in $G$. **2 Marks**

5. Consider the following flow network in Figure 2. Suppose that the Ford-Fulkerson algorithm chooses the augmenting paths in-order as follows.

   **1st iteration** $s \to v \to w \to t$

   **2nd iteration** $s \to x \to w \to v \to u \to t$

[2] Vertex cover in a graph is a subset of vertices that includes at least one end point of every edge in the graph. A vertex cover of smallest size/cardinality is known as minimum vertex cover

**3rd iteration** $s \to v \to w \to x \to t$

**4th iteration** $s \to x \to w \to v \to u \to t$

**5th iteration** $s \to u \to v \to w \to t$

**6th iteration** $s \to x \to w \to v \to u \to t$

**7th iteration** $s \to v \to w \to x \to t$

**8th iteration** $s \to x \to w \to v \to u \to t$

**9th iteration** $s \to u \to v \to w \to t$

For each iteration, clearly draw the residual graph with its residual capacities. Also, mention the flow value after every iteration. **5 Marks**

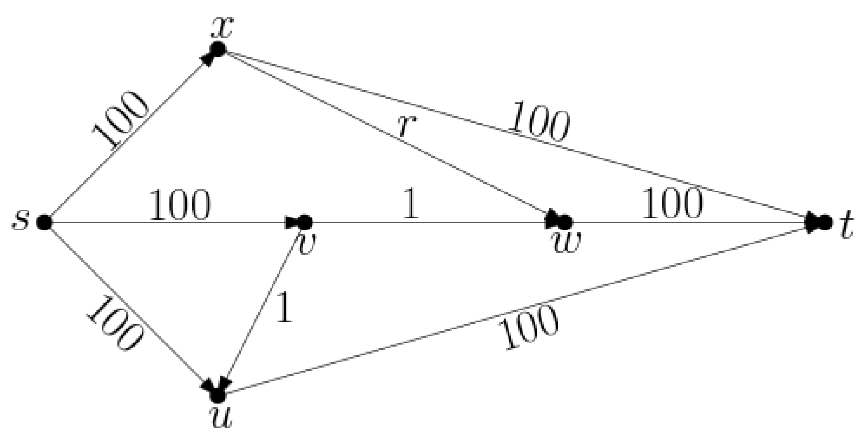


Figure 2: A flow network, where $r = \frac{\sqrt{5}-1}{2} \implies r^2 = 1 - r$

Good luck!