Exam One

CS202: Design & Analysis of Algorithms

Total Marks: 28 Time: 09:00-13:00

Instructions:

- Write clearly. For every algorithm that you describe, state its time complexity with brief justification.
- You can use any algorithm/analysis that we've seen in class/assignments.
- Best of luck!

Section A

1. Give asymptotic upper bounds for the functions f, T. **[4 marks]**

   (i) $T(n) \leq 2T(5n/7) + n \log n$; (ii) $f(n) \leq 4f(n/4) + n$.

2. The input is an array $A[1...n]$ with a unique index k such that $A[i] = i + 1$ for $i < k$, $A[k] = 1$ and $A[i] = i$ for $i > k$. An example is the array $[2, 3, 4, 5, 1, 6, 7]$, where $k = 5$. The goal is to find k. Describe an algorithm to do this in $O(\log n)$ time. **[6 marks]**

3. Two strings are said to be anagrams (of each other) if one can be obtained by permuting the letters of the other. For example, ASTRONOMER is an anagram of MOONSTARER.

   (a) Give an efficient algorithm that accepts two strings of English letters, each of length n, and checks if they are anagrams. **[2 marks]**

   (b) Suggest an efficient algorithm to re-arrange a given list of m strings of length n so that any set of anagrams is listed consecutively. For

example, if the list is *abc*, *baa*, *acb*, *abd*, *cab*, *dab*, then one feasible rearrangement is *abc*, *acb*, *cab*, *baa*, *abd*, *dab*. **[4 marks]**

Section B: Answer ANY TWO questions only.

4. We have an array $A[1...n]$ of distinct integers such that $n = mk$ where m, k are integers. We wish to find the following elements of A: the kth smallest element, the $2k$th smallest element, and so on, up to the mkth smallest element. Describe an $O(n \log k)$ algorithm for this. **[6 marks]**

5. You are given two sets of n points, one set $P_1, P_2, \ldots, P_n$ on the line $y = 0$ and the other set $Q_1, Q_2, \ldots, Q_n$ on the line $y = 1$. The points can be in any order with respect to the x co-ordinates. Consider the n line segments obtained by joining P_i and Q_i. Describe and analyze an algorithm to determine how many pairs of these line segments intersect, in $O(n \log n)$ time. **[6 marks]**

6. (a) Find $DFT_4(1, 2, 3, 4)$. **[2 marks]**

   (b) Consider two sets A and B, each having n integers in the range $\{0, 1, \ldots, 10n\}$. We wish to find the set $C = A + B$ which is defined as $C = A + B = \{x + y | x \in A, y \in B\}$. Note that the integers in C are in the range from 0 to $20n$. We want to find the elements of C and the number of times that each element of C is realized as a sum of elements in A and B. Show how to solve this problem in $O(n \log n)$ time. [Hint: Represent A, B as polynomials of degree at most $10n$.] **[4 marks]**