Unit - IV (Complex Differentiation).

Important Qustions

Short Answer Tyle :

1) State the Necessary & sufficient condition for $f(z)$ to be analytic. (2017-18)

2) Define harmonic function. Find constant 'b' s.t. $u = e^{bx} \cos 5y$ is harmonic.

3) Define analytic function with example. (2017-18)

4) Show that $f(z) = z^3$ is analytic. (2018-19)

5) Define Conformal mapping. (2018-19)

6) Write cauchy's Reimann equation in polar coordinates system. (2016)

7) Show that $u = x^2 - y^2$ is harmonic. (2016)

8) If $f(z) = u + iv$ is an analytic function (2016) and $v = y^2 - x^2$. Find its conjugate harmonic function.

9) The function $f(z) = e^x(\cos y + i \sin y)$ is holomorphic or not.

10) Find the constants a, b, c such that $f(z)$ where $f(z) = -x^2 + xy + y^2 + i(ax^2 + bxy + cy^2)$ is analytic.

Long Answer Type

1) Given an example of a function in which Cauchy-Riemann equation are satisfied at origin yet function is not differentiable at origin. Justify your answer. (2017-18)

2) Show that $u = x^4 - 6x^2y^2 + y^4$ is harmonic. (2018-19) Find complex function $f(z)$ whose real part is u.

3) Examin the nature of the function at the origin.

$$f(z) = \begin{cases} \dfrac{x^3y(y - ix)}{x^6 + y^2}, & z \neq 0 \\ f(0) = 0. \end{cases}$$

4) If $f(z) = \begin{cases} \dfrac{x^2y^5(x+iy)}{x^4+y^{10}} & ; \quad z \neq 0 \\ 0 & ; \quad z = 0 \end{cases}$

Prove that C-R equations are satisfied at origin but $f(z)$ is not diff at $z=0$.

5) Construct the analytic function $f(z)$ if

$$u - v = \frac{\cos x + \sin x - e^{y}}{2\cos x - 2\cosh y} \; ; \; f\left(\frac{\pi}{2}\right) = 0$$

6) Find Möbius transformation that maps (2018-19) the points $z = 0, -i, 2i$ in $w = 5i, \infty, -\frac{i}{3}$ resp.

7) Prove that the function $f(z)$ defined by

$$f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2} \; , \quad z \neq 0$$

$$f(0) = 0$$

is not diff. at origin though it satisfies C-R equations at origin.

8) Prove that $\sinh z$ is analytic and find its derivative. (2017)

9) Show that $f(z) = z|z|$ is not analytic anywhere.

10) Find the regular function $f(z)$ in term of z

(i) whose imaginary part is $e^{x}(x\sin y + y\cos y)$

(ii) whose real part is $\frac{x-y}{x^2+y^2}$.

Q: 11 Determin analytic function $f(z) = u + iv$ in terms of z, if

$$u - v = (x-y)(x^2 + 4xy + y^2)$$

12) Show that $v(x,y) = e^x \sin y$ is harmonic. Find its conjugate harmonic $u(x,y)$ and the corresponding analytic function $f(z)$.

13) Show that for $f(z) = \begin{cases} \dfrac{2xy(x+iy)}{x^2+y^2} & , z \neq 0 \\ 0 & , z = 0 \end{cases}$

the C-R equations are satisfied at origin but derivative of $f(z)$ does not exist at origin.